



Article

Positive Mathematical Programming to model regional or basin-wide implications of producer adoption of practices emerging from plot-based research.

Nicolas Quintana-Ashwell ^{1*}, Gurpreet Kaur ¹, Gurbir Singh ¹, Drew Gholson ¹, Chris Delhom ², L. Jason Krutz ³, Shraddha Hegde⁴

¹ National Center for Alluvial Aquifer Research, Mississippi State University, 4006 Old Leland Rd, Leland, MS 38756, USA; n.quintana@msstate.edu (N.Q.), gk340@msstate.edu (GK), gs1064@msstate.edu (GS), drew.gholson@msstate.edu (DG)

² United States Department of Agriculture, Agricultural Research Service; chris.delhom@usda.gov (C.D.)

³ Mississippi Water Resources Research Institute, Mississippi State University; j.krutz@msstate.edu (L.K.)

⁴ Delta Research and Extension Center, Mississippi State University; sgh234@msstate.edu (S.H.)

* Correspondence: n.quintana@msstate.edu; Tel.: 1-662-390-8508

Version September 15, 2021 submitted to Journal Not Specified

1 **Abstract:** A method for calibrating models of agricultural production and resource use presented
2 by Howitt [1] for policy analysis is proposed to leverage multidisciplinary agricultural research at
3 the National Center for Alluvial Aquifer Research (NCAAR). An illustrative example for Sunflower
4 County, MS is presented to show how plot-level research can be extended to draw systemic region
5 or basin wide implications. A hypothetical improvement in yields for dryland soybean varieties is
6 incorporated to the model and shown to have a positive impact on aquifer outcomes and producer
7 profits. The example illustrates that a change in one practice-crop combination can have system-wide
8 impacts as evidenced by the change in acreages for all crops and practices.

9 **Keywords:** positive mathematical programming; integrated multidisciplinary research; aquifer
10 depletion; land use allocations; groundwater use; irrigation; conservation; profitability; water
11 economics; groundwater; alluvial aquifer; row crops; Mississippi Delta; Lower Mississippi River
12 Valley

13 1. Introduction

14 The National Center for Alluvial Aquifer Research (NCAAR) was created to conduct research
15 aimed at developing novel irrigation and agricultural water management technologies to improve
16 water productivity, decrease irrigation water withdrawal from, and increase the groundwater
17 recharge to the Mississippi River Valley Alluvial Aquifer (MRVAA) with the overall objective of
18 ensuring sustainable agricultural water supplies in the Lower Mississippi River Basin (LMRB). The
19 complexity of natural resource management in general, and groundwater resources in particular,
20 require multidisciplinary research efforts that are reflected in the diverse background of the NCAAR
21 researchers, from natural to social scientists. The complexity of the problem and the composition
22 of NCAAR is represented in the conceptual diagram for the proposed USDA Agricultural Research
23 Service (ARS) project under National Program 211: Water Availability and Watershed Management
24 which funds NCAAR (see figure 1). The complexity of the the NCAAR mission is magnified by
25 the challenge that the region receives significant rainfall annually, but the timing does not coincide
26 with crop production. The rainfall timing is paired with evolving land use, long-term irrigation



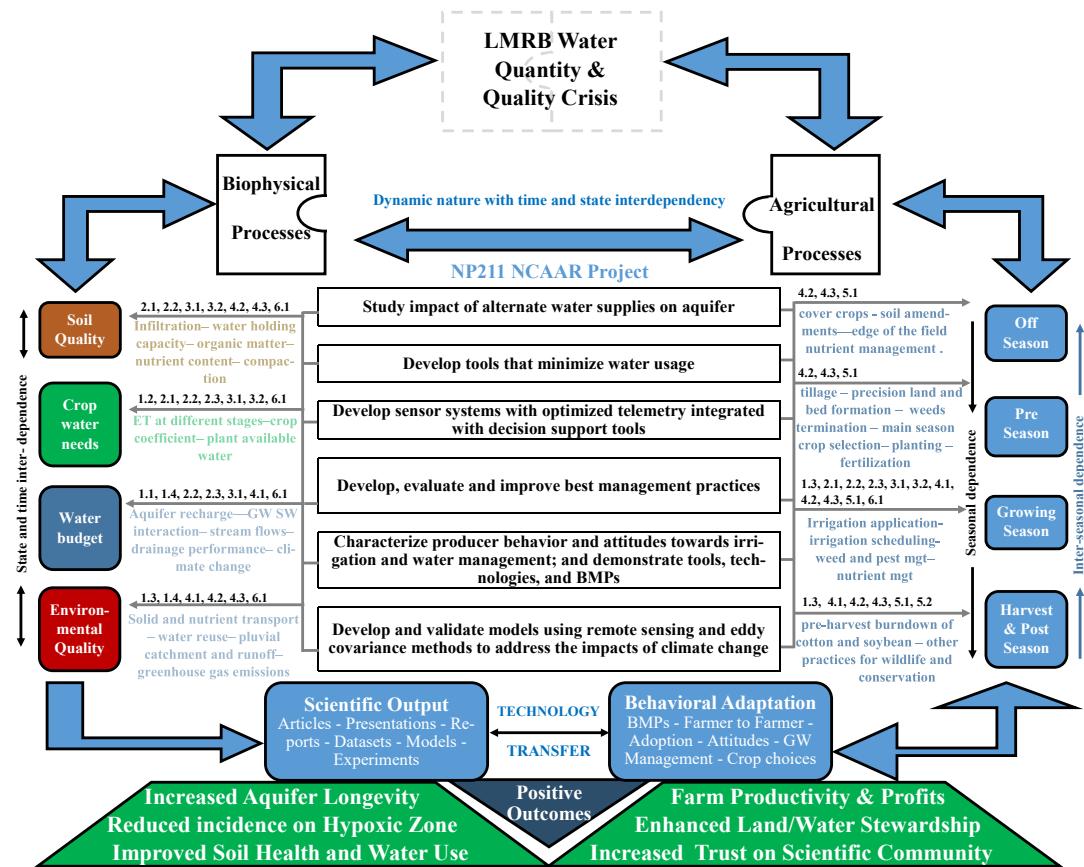


Figure 1. Conceptual diagram of USDA ARS NP211 that funds the National Center for Alluvial Aquifer Research (NCAAR).

27 practices which must change, and a wide range of socio-economic classes of producers who must
 28 all adopt new practices. This paper presents a methodology that can bridge the inter-disciplinary
 29 obstacles to translate plot and field level research results to regional or basin-wide potential outcomes
 30 that incorporate implicit producer behavior with minimal data requirements: Positive Mathematical
 31 Programming.

32 The Mississippi River Valley Alluvial Aquifer (MRVAA, see figure 2) is the primary source of water
 33 for irrigation for the Lower Mississippi River Basin (LMRB) and is depleting at an unsustainable rate [2,
 34 3]. The increase in global population, the resulting growing demand for food, and the receding irrigated
 35 acreage in areas where aquifers are depleting require ever increasing levels of productivity from
 36 agricultural areas that are relatively rich in water resources such as the LMRB [4,5]. NCAAR's mission
 37 leverages multidisciplinary agricultural research to alleviate and ultimately contribute to solving the
 38 problem of a depleting MRVAA. Aligned with this mission is research at the experimental plot or
 39 field level that reduces crop water use without a significant impact on baseline yields, increases crop
 40 productivity for a baseline level of water use, or increases the capture of available water by allowing
 41 earlier planting to capture natural precipitation or developing infrastructure to capture irrigation
 42 or pluvial runoff for reuse. Plot and field level research in this area show growing evidence that
 43 important water savings are achievable with relatively minor modifications to existing irrigation and
 44 agronomic practices in the Mid-South USA [3,4,6–11]. However, regional or basin-wide implications
 45 of the potential results of wide producer adoption of these practices have not been explored.

46 Positive Mathematical Programming (PMP) is a methodology widely used for agricultural
 47 economic policy analysis because it requires minimal data; it is capable of characterizing resource,
 48 environmental, or policy constraints; and models that employ it are consistent with economic
 49 production theory [1]. Basically, PMP uses the shadow prices of calibration constraints from a profit

50 maximization linear program (LP) to specify (calibrate) a non-linear objective function such that
 51 observed activity levels are reproduced by the optimal solution of the new unconstrained programming
 52 problem [12,13]. The form of the unconstrained programming model can be subsequently modified
 53 to incorporate farming, environmental, resource, or policy conditions not explicitly modelled [13].
 54 The calibration step avoids the problem of over-specialization or corner solutions in which all the
 55 acres are assigned to the most profitable crops [14]. The analysis proceeds by evaluating changes
 56 in optimal allocations induced by changes introduced in the variables or parameters of interest.
 57 Furthermore, in the case of groundwater, dynamic simulations that update the state of the aquifer and
 58 other constraining resources over time allows to project the impact of those changes into the future.

59 The PMP methodology is particularly
 60 useful when data on individual decision units
 61 is unavailable, insufficient or inadequate
 62 for econometric analysis. The absence of
 63 observations over a wide range of prices
 64 required the use of programming approaches
 65 to estimate the elasticities of the derived
 66 demand for water[15,16]. A growing literature
 67 has employed PMP to study water use or
 68 aquifer depletion implications in a variety of
 69 settings. For example, Pulido-Velazquez *et al.*
 70 [17] calibrate a set of functions of marginal
 71 economic benefit of surface-groundwater use in
 72 a hydroeconomic model of a river basin in Spain.
 73 Clark [18] explores the impact of high commodity
 74 price scenarios on irrigated crop production,
 75 groundwater application to irrigation and
 76 aquifer outcomes in Western Kansas. Esteban
 77 and Albiac [19] use PMP to calibrate a model of
 78 groundwater management under three aquifer
 79 management scenarios that incorporate
 80 ecosystem damages from groundwater
 81 over-pumping. Employing a formulation
 82 similar to Clark [18], Garay-Armoa [14]
 83 assesses the impact of two water conservation
 84 practices (water use restrictions and permanent
 85 conversions to dryland crops) on the Ogallala
 86 Aquifer and on producer welfare for a set of
 87 counties in Kansas.

88 A major criticism of the programming approach is that the pre-specified functions may not
 89 precisely represent the biological and physical processes of, for example, plant growth [15,16]. However,
 90 several studies have been able to address this issue by applying PMP iteratively in combination
 91 with separate crop growth and hydrological models. Aistrup *et al.* [20] apply the formulation to
 92 Groundwater Management District 3 (GMD3) in southwestern Kansas in which PMP is used with a
 93 plant growth model integrating water and land use patterns, changing climate, economic trends, and
 94 population dynamics. In California, MacEwan *et al.* [21] develop a modular hydroeconomic modeling
 95 approach integrating California's C2VSim groundwater-surface water simulation model with the
 96 Statewide Agricultural Production (SWAP) economic model. Similarly, PMP is the core of the Central
 97 Valley Production Model (CVPM), a "multi-regional model of irrigated agricultural production that can
 98 forecast changes in crop acres as a function of changes in the availability of water supplies," presented
 99 by Dale *et al.* [22]. Finally, Qureshi *et al.* [13] developed a biophysical-economic mathematical model

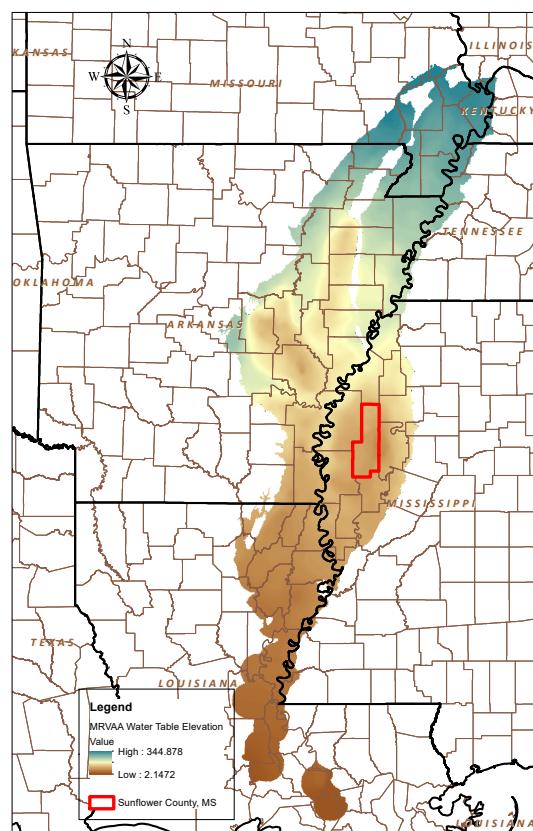


Figure 2. Potentiometric map of the Mississippi River Valley Alluvial Aquifer based on U.S. Geological Survey data from 2016.

100 with PMP that calibrated against the observed multi-period land use data to evaluate the impacts of
101 droughts and a set of policy options on agricultural production in the Murray-Darling Basin, Australia.

102 In the following sections we describe the PMP methodology and how it can help integrate
103 multidisciplinary plot or field level research to project likely aquifer and producer welfare outcomes.

104 Then we present a case study to illustrate the methodology and conclude with a discussion of the
105 implications.

106 **2. Integrating multidisciplinary research with Positive Mathematical Programming (PMP)**

107 Disciplinary research offers important insights into processes within a specific domain and
108 rarely incorporate interactions with other natural or social processes [23]. The way career researchers
109 are evaluated by their academic department tends to incentivize disjoint disciplinary research that
110 result in shorter publication timelines and favor “preferred field-journals.” This effect is particularly
111 evident with Early Career Researchers (ECRs) who are underutilised in multidisciplinary research [24].
112 However, the scientific community is increasingly pushing and demanding research that integrates
113 the insights of multiple disciplines to address global environmental challenges [23,25,26]. Far from
114 being an integration of multidisciplinary models, Positive Mathematical Programming is an economic
115 analysis tool that allows the incorporation of otherwise disjoint disciplinary research into economic
116 analyses and simulation of biophysical and socio-economic impacts that may result if certain practices
117 or policies are adopted (see figure 3).

118 Next, we describe the type of disciplinary research that can be fed into a PMP model to draw
119 aquifer and policy implication insights.

120 *2.1. From plot and field level research to economic behavior*

121 Farmers operate in an increasingly risky environment and are more likely to adopt practices that
122 improve productivity (including water productivity), increase profits or reduce risks [3]. Producers
123 who want to be good stewards of their environment and are attracted to natural resource conservation
124 still need assurances that the practices they adopt do not adversely affect their net income [27]. Plot
125 and field level research develops practices or prescriptions that hold the potential to deliver increased
126 crop productivity but often times it is hard to evaluate the impact the practice would have on marginal
127 producer behavior. As the practices influence farmers’ behavior at the margin, wider implications
128 would be expected at a regional or basin level.

129 Economists model producer behavior primarily as pursuing a business objective: maximizing
130 profits or delivering a level of output at the minimum cost. Despite a multitude of other objectives,
131 including cultural ones, the assumption of profit maximization is used because it predicts economic
132 behavior reasonably well, particularly at some level of aggregation [28]. The decision regarding how
133 input use, such as irrigation water, is determined “at the margin”, meaning the decision is made based
134 on whether the treatment is expected to return a higher benefit than the cost of applying it. Figure
135 4 illustrates the concept with respect to water use: apply irrigation water until the benefit of the last
136 unit applied equals its cost (marginal cost = marginal revenue). The response of crop yields to the
137 amount of irrigation water applied depends on how much of other inputs have been used on the
138 field (notably, fertilizer). However, because irrigation events occur after most of the other inputs have
139 already been applied, it is acceptable to model crop yield response to water as a single-input function.
140 The equations in Figure 4 reflect how plot and field level results can be incorporated into an economic
141 behavior model: if the innovation affects yields, production costs, or crop prices; then we can expect
142 that it will affect farmers’ economic behavior.

143 With the insights of how agricultural innovations may affect producer behavior, the next step
144 would be to assess how the adoption of the innovation at the region or basin level would affect aquifer
145 levels or environmental outcomes. Examples of agricultural research that could be incorporated in this
146 framework abound. Plot level research on improved irrigation systems and technologies, and better

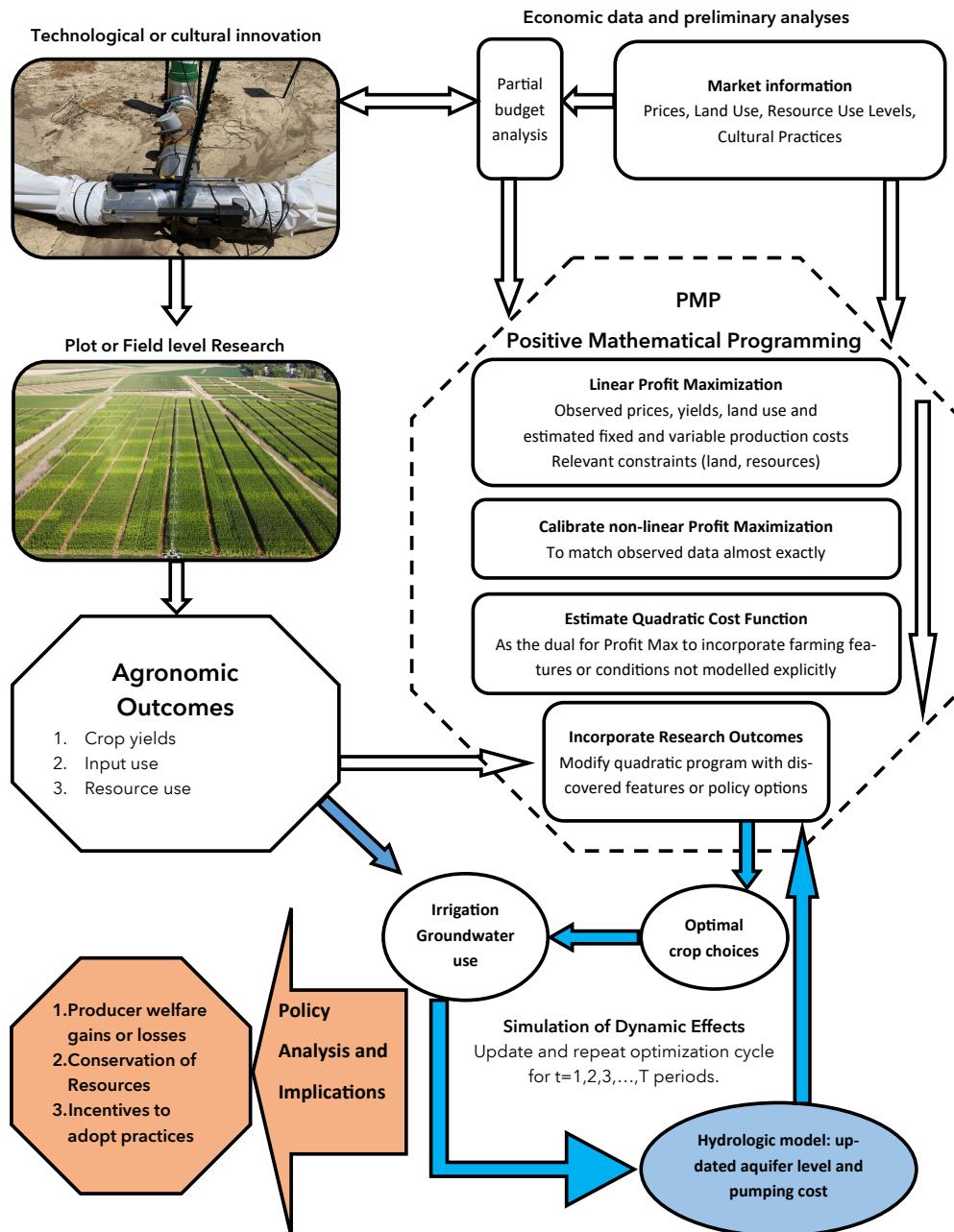


Figure 3. Diagram depicting multidisciplinary research using Positive Mathematical Programming (PMP) to integrate plot level research to basin-wide models and drawing policy implications.

Example of a nonlinear-plateau yield response to irrigation

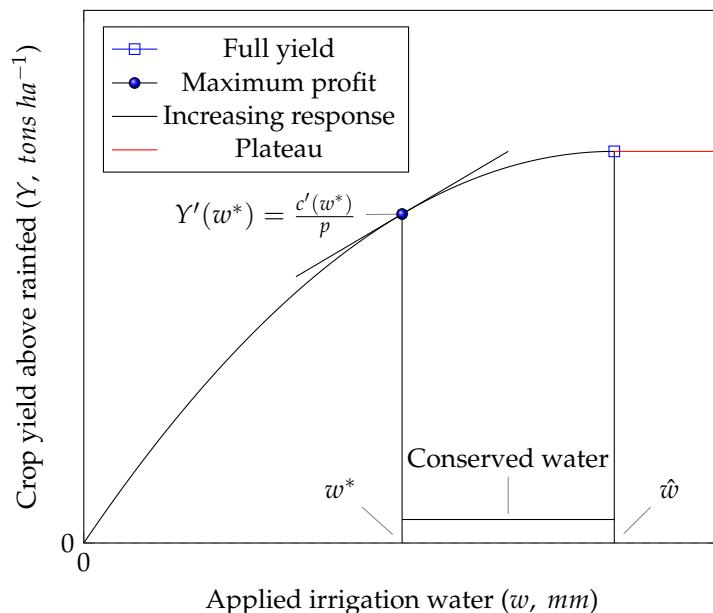


Figure 4. Illustration of the relationship between crop yield, applied irrigation water, and profits.

147 agronomic management practices such as row spacing, cover crops, conservation tillage, and skip row
 148 irrigation are prime candidates.

149 The irrigation technologies that are available to the producers in the LMRB for increasing furrow
 150 irrigation application efficiency and irrigation water use efficiency include computer-hole-selection
 151 (PHAUSET: Pipe Hole and Universal Crown Elevation Tool or Pipe Planner), surge valves, soil
 152 moisture sensors, tailwater recovery systems and recycling the runoff to reuse for irrigation, and
 153 sprinkler irrigation systems [9,11]. The soil moisture sensors, PNAUCET, and surge valves have been
 154 shown to improve in irrigation application efficiency of furrow irrigation systems. However, the
 155 application efficiency of the sprinkler systems is higher than the furrow irrigation systems. But there is
 156 little information available on the comparison of water savings with a sprinkler irrigation system and
 157 a furrow irrigation system in which water conservation practices have been adopted to increase water
 158 use and application efficiencies (eg: computer-hole-selection and moisture sensors). Adopting sprinkler
 159 irrigation systems could potentially increase water savings while increasing irrigation application
 160 efficiency and profits by reducing the costs of irrigation events.

161 Among conservation tillage practices, the use of strip tillage can reduce evaporation losses of
 162 water as it only disturbs 25 percent of the plow layer and allows retention of residues on the surface.
 163 Strip till shank can also break hardpans and reduce subsoil compaction. Retention of crop residues
 164 on the surface and reduction in subsoil compaction can allow better water infiltration in the soil, less
 165 runoff loss, and improve water availability for plant roots which can increase water use efficiency by
 166 plants.

167 Skip row irrigation is another practice followed by some farmers on clay-textured soils in the
 168 MS Delta. Every other row is irrigated in the skip row irrigation strategy to save water and increase
 169 irrigation water use efficiency. Reducing the amount of water applied will result in lower fuel costs
 170 and higher net returns.

171 Cover crops can help with water conservation and improving soil health. Additionally, this
 172 practice can also increase water infiltration in soil, reduces evaporation losses, increase soil water
 173 holding capacity, reduces runoff and nutrient losses, and can increase nitrogen supply to the succeeding
 174 crop. Cover crops can reduce soil crusting and compaction, which are major constraints for crop
 175 production in the MS delta area. All these benefits of cover crops can reduce reliance on MRVAA for

176 irrigation water needs. Improvements in irrigation water use efficiency with the use of cover crops
 177 have been reported by DeLaune *et al.* [29], Currie and Klocke [30].

178 *2.2. Positive Mathematical Programming*

179 Data on individual farm or farmer crop choices, practices, input or resource use, crop yields, and
 180 cost structures is generally unavailable in Mississippi but observed at the county level. Consequently,
 181 the ability of the PMP methodology to model micro-economic behavior capable of reproducing the
 182 activity levels at the county level of aggregation is well suited to bridge the interdisciplinary and data
 183 availability barriers to basin-wide implications of agricultural experimental outcomes (see Figure 3).

184 The PMP-based dynamic simulation process is to:

- 185 1. use observed county-level data to formulate a constrained linear profit maximization model in
 186 which resource and input use as well as other resource, environmental or policy limitations are
 187 represented as constraints and the choice variable is crop acreage;
- 188 2. reformulate the problem as a nonlinear constrained optimization problem that calibrates almost
 189 exactly to the observed levels;
- 190 3. calibrate a quadratic function to capture desired production features (e.g.; water use) not included
 191 in the data or modelled explicitly;
- 192 4. implement a quadratic program including the estimated cost function as part of the objective
 193 function;
- 194 5. solve a dynamic model iteratively by updating aquifer levels based on periodic solutions to the
 195 quadratic program to produce the optimal land and water use choices.

196 **The first step** consists in using observed data to obtain the shadow prices on land use acres by
 197 solving the following problem for the observed period:

$$\max_{x_{rj}} \pi = \sum_r \sum_j (p_{rj} \times y_{rj} - c_{rj}) \times x_{rj}; \quad (1)$$

$$\text{s.t.} \quad \sum_j x_{rj} \leq A_r = \sum_j a_{rj} \quad \forall r; \quad (2)$$

$$a_{rj} - \epsilon \leq x_{rj} \leq a_{rj} + \epsilon \quad \forall r, j; \quad (3)$$

198 where p_{rj} indicates the price of commodity j in region r at the time of the observed data; y_{rj} indicates
 199 the observed yield level; c_{rj} is the per-acre production costs; x_{rj} is the choice variable for crop land
 200 allocation and a_{rj} is the observed acreage for each crop; and $\epsilon > \approx 0$ is a small perturbation on the
 201 observed acreage to produce calibrating shadow prices. Additional subscripts can be used to represent
 202 different production systems for which data is observed (e.g. different irrigation systems) or if only
 203 one region is analyzed, the r subscript can be used for that purpose. Crop prices are generally available
 204 from United States Department of Agriculture's Economics, Statistics and Market Information System
 205 (USDA ESMIS) for specific elevators; acreage and average yield data is available from USDA NASS at
 206 the county level; and per acre cost of production by crop and production system are usually available
 207 via Crop Planning Budgets from the Extension Service at Land Grant Universities– in our case, [the Department of Agricultural Economics at Mississippi State University](#).

208 The Lagrangean and first order conditions for the problem for each region at the initial state are:

$$\mathcal{L}_{0r} = \sum_j (p_j \times y_{0j} - c_{0j}) \times x_j + \lambda \left(A - \sum_j x_j \right) + \sum_j \mu_j (a_j + \epsilon - x_j); \quad (4)$$

$$\frac{\partial \mathcal{L}_r}{\partial x_j} = p_j \times y_{0j} - c_{0j} - \lambda - \mu_j = 0, \quad \forall j; \quad (5)$$

$$\frac{\partial \mathcal{L}_r}{\partial \lambda} = A - \sum_j x_j = 0; \quad (6)$$

$$\mu_j (a_j + \epsilon - x_j) = 0, \quad \forall j; \quad (7)$$

210 for which the solutions x_j^* would be very close to the observed levels a_j by construction.

211 For the second step, a cost function $C(w_{rj}, x_{rj}; \alpha_{rj}, \gamma_{rj}, \delta_{rj})$ to replace c_{rj} in equation (1) is estimated
212 to incorporate additional desired features—i.e., water use, w_j . Additionally, we would be interested in
213 calibrating a crop yield function $Y_j(\cdot)$ that captures the crop's response to irrigation water application
214 (or other inputs of interest) such that $Y_j(w_{rj}) = y_{rj}$ at the observed levels in the initial period.

215 A function that captures crop yield response to irrigation water applied can be specified as
proposed by Martin *et al.* [31] and calibrated to reflect observed yields and water use [14,18]:

$$Y_j(w_{rj}) = Y_{mrj} + (Y_{frj} - Y_{mrj}) \left[1 - \left(1 - \frac{w_{rj}}{GIR_{rj}} \right)^{-IE_{rj}} \right]; \quad (8)$$

216 where Y_{mrj} is the minimum crop yield before irrigation water is applied; Y_{frj} is the fully-watered
217 yield; GIR_{rj} is the crop's gross irrigation water requirement to achieve fully watered yield (given
218 observed seasonal weather); and IE_{rj} is the irrigation application efficiency. This function is estimated
219 to reflect the initial observed levels of yield and water use.

220 The arguments for the function $Y_j(w_{rj})$ is the first instance in which results from the plot or
221 field level research can be introduced. Practices that affect minimum yields (for example dryland),
222 fully-watered yields, irrigation efficiency or irrigation requirements can be incorporated in this
223 formulation. In fact, the entire yield response function can be supplied by agronomic or plant
224 physiology modeling as a component of the program.

225 Next, a cost function can be formulated as a linear function of the inputs and acreage [1,14,18]:

$$C(w_{rj}, x_{rj}; \alpha_{rj}, \gamma_{rj}, \delta_{rj}) = (w_{rj} - wo_{rj})\delta_{rj} + \alpha_{rj} + 0.5\gamma_{rj}x_{rj}; \quad (9)$$

226 where wo_{rj} is the initially observed rate of irrigation water application per acre. At the initial
observation levels, the function collapses to

$$C(wo_{rj}, x_{rj}; \alpha_{rj}, \gamma_{rj}, \delta_{rj}) = \alpha_{rj} + 0.5\gamma_{rj}x_{rj} = co_{rj}. \quad (10)$$

227 The nonlinear program is now expressed as follows for the calibration problem:

$$\max_{x_{rj}, w_{rj}} \pi_r = \sum_j (p_j \times Y_j(w_j) - C(w_j, x_j; \alpha_j, \gamma_j, \delta_j)) \times x_j; \quad (11)$$

228 and first order conditions:

$$\frac{\partial \pi_r}{\partial x_j} = p_j \times Y_j(w_{rj}) - C(w_{rj}, x_{rj}; \alpha_{rj}, \gamma_{rj}, \delta_{rj}) = 0, \quad \forall j; \quad (12)$$

$$\frac{\partial \pi_r}{\partial w_j} = p_j \times \frac{\partial Y_j(w_{rj})}{\partial w_{rj}} - \frac{\partial C(w_{rj}, x_{rj}; \alpha_{rj}, \gamma_{rj}, \delta_{rj})}{\partial w_{rj}} = 0, \quad \forall j. \quad (13)$$

229 The third step consists in combining the conditions from the two previous steps to match the
initial observed levels of the variables of interest. From equations (5) and (12) we obtain:

$$\alpha_j + \gamma_j a_j = co_j + \mu_j; \quad (14)$$

230 and equation 10 is a second equality which can be used to solve for the two calibrating parameters
231 (α_j, γ_j) since the value of the shadow prices (λ, μ_j) were obtained from the original program. The
232 solutions are:

$$\alpha_j = 2 \frac{\mu_j}{x_j^*}; \quad \text{and} \quad (15)$$

$$\gamma_j = co_j - \mu_j. \quad (16)$$

The remaining calibrating parameter, δ_j , can be found from equation (9) and first order condition (13) by taking the derivative of the yield response function $Y_j(w_j)$ specified in equation (8) :

$$\delta_j = p_j \left(\frac{Yf_j - Ym_j}{IE_j \times GIR_j} \right) \left(1 - \frac{wo_j}{GIR_j} \right)^{(IE^{-1}-1)}. \quad (17)$$

The fourth step consists in preparing the cost function to adjust based on updated aquifer status. In this case, the pumping lift affects the pumping costs at time t [18]:

$$\Theta_t = \theta_{et} \times 0.114 \times \frac{TDH_t}{EF_t}; \quad (18)$$

where θ_{et} is the price per unit of energy source e ; TDH_t is total dynamic head at time t ; and EF_t is energy efficiency of source e . TDH is the sum of pumping lift L_t , which depends on aquifer levels at the end of period $t-1$; and pumping head which converts the irrigation system pressurization requirement to feet of additional lift.

The resulting cost function takes the following form:

$$C(w_{jt}, x_{jt}) = (w_{jt} - wo_j)(\delta_j + \Theta_t) + \alpha_j + 0.5\gamma_j x_{jt}. \quad (19)$$

A similar approach can be followed to study the effect of changing costs of other inputs or resources.

The final step consists in simulating the effects over time by the following aquifer equation of motion:

$$Lift_t = Lift_{t-1} + \frac{\sum_j w_{jt} \times x_{jt} - R}{A_s}; \quad (20)$$

where R is the rate of net natural recharge of the aquifer and A_s is the area in the region that overlays the aquifer times the aquifer specific yield. This aquifer formulation can be interpreted as a "localized" aquifer impact on the areas covered by the crops considered in the program. The change in lift distance over time is the amount of aquifer depletion (positive difference) or replenishment (negative change).

A word of caution with respect to PMP is that simulations should not be over very long time horizons because the calibration procedure seeks to fit results to the original conditions as much as possible. Over long periods of time, farmers can adapt in ways that make the original period observations become less relevant.

3. Illustrative example: improved soybean dryland yields in Sunflower County, MS.

To illustrate the methodology, we present a case study based on a hypothetical plot-level research that shows a 33 percent improvement in dryland soybean yields that do not involve changes in production costs relative to baseline conditions. Most agronomic studies do not include an economic analysis of this type of result and few include only the partial budget analysis for the practice that tends to indicate how dryland soybean farmers would benefit from the practice. However, the PMP framework is able to expand the impact of the effect more systemically. For instance, an impact on irrigated soybean is easily detectable via equation (17). The yield improvement level is applied on the dynamic simulation state to both dryland soybean yields and to the minimum yield, Ym_{soy} , levels for soybean.

3.1. Sunflower County, MS

To setup the model, we start with baseline information available from publicly accessible sources. County-level parameters are summarized in tables 1 and 2. It fully overlies an acute depression of

Table 1. Model parameters for Sunflower County, MS.

Component	Parameter	Value
Aquifer	Surface elevation (FASL)	118
	Initial water table elev. (FASL)	77.91
	Aquifer base elevation (FASL)	-18.49
	Net recharge (R , acre-ft)	231,802
	Acres x specific yield (A_S)	89,344
Crop mix	Soybean share	77%
	Corn share	12%
	Rice share	4%
	Cotton share	7%
Irrigation	Application efficiency (IE)	0.54
Discount	Rate	0.03

255 the MRVAA water table¹ that has drawn concern from producers as well as federal and state agencies
 256 [32]. Because of concerns about MRVAA depletion, Mississippi Governor Phil Bryant established the
 257 Governor's Delta Sustainable Water Resources Task Force in November of 2011 to ensure the future
 258 sustainability of water resources in the Delta[33].

259 Sunflower County, MS, is in the center of the Delta area of Mississippi (red contour in fig. 2). The
 260 row-crop agriculture in the county is widely representative of the Delta. Consequently, the area is ideal
 261 for a representative agent type of model such as this, as it is big enough to draw conclusions about the
 262 aquifer but small enough that a simplified aquifer model is capable of capturing its most important
 263 dynamics [34].

264 Table 2 summarizes the selected variables in the model for Sunflower County, MS. USDA NASS
 265 data for 2017 is the latest available so we match the rest of the data to observations for that year.
 266 Price and cost information was obtained from the Mississippi State University, [2017 Delta Crop](#)
 267 [Planning Budgets](#). Crop acreage and average yields were obtained from USDS NASS [35]. Information
 268 on minimum and maximum yields was obtained from expert opinion and from Mississippi State
 269 University various variety trials in 2017. Average irrigation water use by crop was calculated from
 270 Mississippi Department of Environmental Quality's (MDEQ) voluntary well metering program and
 271 verified with information from experimental on-farm NCAAR data. Average irrigation efficiency was
 272 based on Bryant *et al.* [9], and Spencer *et al.* [11]. Parameters to calculate gross irrigation requirements
 273 (GIR) were obtained from Tang *et al.* [36].

274 The calibrated problem was modified, and the results simulated over 20 years and compared to
 275 the baseline results. The results of the calibrated problem updated only for aquifer depletion is called
 276 the "calibrated" scenario and the modified program to reflect the increase in dryland soybean yields is
 277 called the "shock" scenario.

278 3.2. Results and discussion for an illustrative example

279 The dynamic simulation is run under the two scenarios over 20 years. The "calibrated" scenario
 280 is the modified program that includes the ability to update the status of the aquifer which affects
 281 pumping lifts over time which in turn affects costs. The "shock" scenario is also modified to update
 282 pumping lift but also incorporates an improvement in the level of dryland soybean yields (affecting
 283 minimum yield as well). Table 3 summarizes select results by crop.

1 The area is referred to colloquially, and by USGS [32] as the "cone of depression;" a potentially confusing misnomer as a cone of depression occurs at any well actively pumping.

Table 2. Summary of observed and estimated parameters for Sunflower County, MS.

Crop	Irrigation	Min. yield	Full-water yield	Average yield	Water use (ft/acre)	Cost (\$/acre)	Acres
Corn	Furrow Dryland	114 bu/a	280 bu/a	220 bu/a	0.83	680	27,857
				170 bu/a		585	8,343
Soybean	Furrow Dryland	26 bu/a	82 bu/a	77 bu/a	1.16	498	158,144
				57 bu/a		404	76,356
Cotton	Furrow Dryland	1090 lb/a	1800 lb/a	1479 lb/a	0.5	924	16,958
				1261 lb/a		833	3,747
Rice	Flood	99 bu/a	253 bu/a	228 bu/a	2.7	817	13,830

Table 3. Salient Positive Mathematical Programming results simulated for 20 years, by crop and practice.

Crop	Irrigation	Acres		Water use (acre-ft)		Profits (\$/year)	
		year 1	year 20	year 1	year 20	year 1	year 20
Corn/calib.	Furrow	27,873	27,620	23,135	22,789	22.8M	22.5M
	Dryland	8,343	8,343	0	0	5.3M	5.3M
Corn/shock	Furrow	23,752	23,775	19,715	19,757	19.4M	19.4M
	Dryland	4,995	4,971	0	0	3.19M	3.18M
Soybean/calib.	Furrow	158,142	157,490	184,077	182,783	117.2M	116.6M
	Dryland	76,356	76,356	0	0	43.8M	43.8M
Soybean/shock	Furrow	144,668	144,707	168,393	168,536	107.2M	107.3M
	Dryland	109,167	109,094	0	0	83.2M	83.2M
Cotton/calib.	Furrow	16,913	16,592	8,457	8,235	16.4M	16.1M
	Dryland	3,747	5,110	0	0	3.1M	4.3M
Cotton/shock	Furrow	9,811	9,827	4,905	4920	9.5M	9.5M
	Dryland	≈ 0	≈ 0	0	0	0	0
Rice/calib.	Flood	13,859	13,723	37,420	36,799	14.9M	14.8M
Rice/shock	Flood	12,841	12,861	34,670	34,772	13.9M	13.9M

284 As expected, dryland soybean acreage and profitability increase with the shock. This result is
 285 the limit of the typical economic analysis of agronomic research. However, PMP allows to identify
 286 additional implications with respect to the calibrated baseline. The increase in soybean dryland acreage
 287 comes at the expense not only of the irrigated soybean acreage, but also from all other crops including
 288 virtually eliminating dryland cotton cultivation.

289 An actual analysis of the idiosyncrasies of cotton production would caution against this
 290 implication due to the level of specialization involved in cotton production which would make
 291 it hard for a cotton farmer to immediately convert to another row crop. Notice that in the calibrated
 292 scenario, the program allocates more acreage to dryland cotton (see year 1 vs. year 20 land allocation).

293 With the significant increase in profitability of dryland soybean, the corresponding increased land
 294 allocation to its cultivation result in a net replenishment of the localized aquifer (see table 4). This
 295 aquifer replenishment allows a sustainable increase in all the irrigated acreage over time, although
 296 never reaching those under the calibrated scenario.

297 The other important extension of the analysis is with respect to the aggregate results that allow to
 298 draw insights at regional or basin-wide scales. Table 4 summarizes the aggregate producer welfare
 299 results expressed as the net present value (NPV) of the sum of the stream of profits under the two
 300 scenarios. The NPV is calculated using a discount factor that incorporates the current FSA Loan rate
 301 for Farm Ownership loans of 3 percent.

302 The yield shock introduced produces almost \$200 million more in producer welfare while reducing
 303 aggregate water use by over 400k acre-ft. The health of the aquifer is substantially better under the
 304 shock scenario which results in a slightly replenished aquifer. The implications for sustainability are
 305 important as they indicate a substantial amount of sustainable available water to expand irrigated
 306 agriculture (remember that the program constrains the total acreage to the initially observed). The
 307 aquifer level presents a difference of over 6.4 ft between the two scenarios after 20 years. Given the
 308 improvement in both producer welfare and aquifer levels, research to improve dryland yields and
 309 provide incentives for conversion to dryland varieties appear as an attractive target for public policy
 310 and funds.

Table 4. Farmer welfare, aggregate water use and localized change in groundwater levels (in 20 years).

Scenario	Net present value of farm profits	Aggregate water use (acre-ft)	Change in aquifer level (ft)
Calibrated scenario	\$3.42 billion	5 million	4.5 ft decrease
Yield shock scenario	\$3.62 billion	4.6 million	0.9 ft increase

311 4. Conclusion

312 Positive Mathematical Programming offers the ability to integrate compartmentalized disciplinary
 313 research to produce deeper insights on the effects and repercussions experimental plot or field level
 314 research can have on regional or basin wide producer welfare and natural resource conditions. The
 315 typical economic analysis of agronomic research is limited to the partial budget analysis associated
 316 with implementing an experimental practice. PMP includes and extends the analysis by showing
 317 implications on the wider agricultural system including input and resource use allocations across
 318 crops and practices.

319 We present a clear step-by-step guide to implement the methodology employing straight-forward
 320 mathematical optimization techniques and including ways in which the programs can be modified to
 321 incorporate unobserved features of interest. The application of this methodology would make highly
 322 disciplinary research more relevant across disciplines and to various stakeholders who could more
 323 easily assess the implications of the agricultural experimental practices proposed and the eventual
 324 technology transfer as producers adopt them.

325 A caveat of PMP is that the resulting programs, by design, try to produce allocations that mimic
 326 as much as possible those observed in the initial period on which the program is calibrated. But as
 327 evidenced by the hypothetical case presented, the directions of change are readily identified.

328 The procedure described in section 2.2 can be implemented in any quantitative or statistical
 329 analysis software. The results for the example presented were produced using *MatLab's linprog* and
 330 *quadprog* optimization tools.

331 **Author Contributions:** Conceptualization, N.Q.; methodology, N.Q.; validation, N.Q., G.K., G.S. and D.G.; formal
 332 analysis, N.Q.; investigation, N.Q., G.K., G.S., S.H., D.G. and C.D.; resources, C.D. and J.K.; writing—original draft
 333 preparation, N.Q., G.K., G.S., C.D.; writing—review and editing, N.Q., G.K., G.S., C.D., S.H., D.G. and J.K.; funding
 334 acquisition, C.D. and J.K.

335 **Funding:** This publication is a contribution of the National Center for Alluvial Aquifer Research and the
 336 Mississippi Agricultural and Forestry Experiment Station. This material is based upon work that is funded jointly
 337 by the Agricultural Research Service, United States Department of Agriculture, under Cooperative Agreement
 338 number 58-6001-7-001.

Acknowledgments: The authors acknowledge and thank Amilcar Vargas, PhD student, Department of Plant and Soil Sciences, Mississippi State University, for plot-level research images.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

ARS	USDA Agricultural Research Service
BMP	Best Management Practice
bu/a	Bushels per acre
C2VSim	California Central Valley Groundwater-Surface Water Simulation Model
CVPM	California Central Valley Production Model
DREC	Mississippi State University Delta Research and Extension Center
ECR	Early Career Researcher
EF	Energy efficiency
ESMIS	USDA Economics, Statistics and Market Information System
ft	Feet
FSA	USDA Farm Service Agency
GIR	Gross irrigation requirement
GMD3	Kansas Groundwater Management District 3
GW	Groundwater
IE	Irrigation water use efficiency
lb/a	Pounds per acre
LMRB	Lower Mississippi River Basin
LP	Linear program
MDEQ	Mississippi Department of Environmental Quality
MRVAA	Mississippi River Valley Alluvial Aquifer
NASS	USDA National Agricultural Statistics Service
NCAAR	National Center for Alluvial Aquifer Research
NPV	Net present value
NRCS	USDA Natural Resources Conservation Service
PMP	Positive Mathematical Programming
SW	Surface water
SWAP	California State-wide Agricultural Production economic model
TDH	Total dynamic head
USA	United States of America
USD	U.S. dollar
USDA	U.S. Department of Agriculture

References

1. Howitt, R.E. Positive Mathematical Programming. *American Journal of Agricultural Economics* **1995**, *77*, 329–342.
2. Yasarer, L.M.; Taylor, J.M.; Rigby, J.R.; Locke, M.A. Trends in land use, irrigation, and streamflow alteration in the Mississippi River Alluvial Plain. *Frontiers in Environmental Science* **2020**.
3. Quintana-Ashwell, N.E.; Gholson, D.M.; Krutz, L.J.; Henry, C.G.; Cooke, T. Adoption of Water-Conserving Irrigation Practices among Row-Crop Growers in Mississippi, USA. *Agronomy* **2020**, *10*. doi:10.3390/agronomy10081083.
4. Quintana-Ashwell, N.E.; Anapalli, S.S.; Pinnamaneni, S.R.; Kaur, G.; Reddy, K.N.; Fisher, D.K. Profitability of twin-row planting and skip-row irrigation in a humid climate. *Agronomy Journal* **in production**.
5. Elliott, J.; Deryng, D.; Müller, C.; Frieler, K.; Konzmann, M.; Gerten, D.; Glotter, M.; Flörke, M.; Wada, Y.; Best, N.; others. Constraints and potentials of future irrigation water availability on agricultural production under climate change. *Proceedings of the National Academy of Sciences* **2014**, *111*, 3239–3244.

359 6. Pinnamaneni, S.R.; Anapalli, S.S.; Reddy, K.N.; Fisher, D.K.; Quintana Ashwell, N.E. Assessing irrigation
360 water use efficiency and economy of twin-row soybean in the Mississippi Delta. *Agronomy Journal* **2020**.

361 7. Pinnamaneni, S.R.; Anapalli, S.S.; Reddy, K.N.; Fisher, D.K. Effects of irrigation and planting geometry on
362 cotton productivity and water use efficiency. *Journal of Cotton Science* **2020**, *24*, 2–96.

363 8. Henry, W.B.; Krutz, L.J. Water in agriculture: Improving corn production practices to minimize climate
364 risk and optimize profitability. *Current Climate Change Reports* **2016**, *2*, 49–54.

365 9. Bryant, C.; Krutz, L.; Falconer, L.; Irby, J.; Henry, C.; Pringle, H.; Henry, M.; Roach, D.; Pickelmann, D.;
366 Atwill, R.; others. Irrigation water management practices that reduce water requirements for Mid-South
367 furrow-irrigated soybean. *Crop, Forage & Turfgrass Management* **2017**, *3*.

368 10. Wood, C.; Krutz, L.; Falconer, L.; Pringle, H.; Henry, B.; Irby, T.; Orlowski, J.; Bryant, C.; Boykin, D.; Atwill,
369 R.; others. Surge irrigation reduces irrigation requirements for soybean on smectitic clay-textured soils.
370 *Crop, Forage & Turfgrass Management* **2017**, *3*.

371 11. Spencer, G.; Krutz, L.; Falconer, L.; Henry, W.; Henry, C.; Larson, E.; Pringle, H.; Bryant, C.; Atwill,
372 R. Irrigation Water Management Technologies for Furrow-Irrigated Corn that Decrease Water Use and
373 Improve Yield and On-Farm Profitability. *Crop, Forage & Turfgrass Management* **2019**, *5*.

374 12. Heckelei, T.; Britz, W. Models based on positive mathematical programming: state of the art and further
375 extensions **2005**.

376 13. Qureshi, M.E.; Whitten, S.M.; Kirby, M.; others. A multi-period positive mathematical programming
377 approach for assessing economic impact of drought in the Murray–Darling Basin, Australia. *Economic
378 Modelling* **2014**, *39*, 293–304.

379 14. Garay-Armoa, P.V. The impact of climate change on the effectiveness of water conservation policies in
380 western Kansas and the Ogallala Aquifer. PhD thesis, 2015.

381 15. Koundouri, P. Current issues in the economics of groundwater resource management. *Journal of Economic
382 Surveys* **2004**, *18*, 703–740.

383 16. Koundouri, P. Potential for groundwater management: Gisser-Sanchez effect reconsidered. *Water resources
384 research* **2004**, *40*.

385 17. Pulido-Velazquez, M.; Andreu, J.; Sahuquillo, A.; Pulido-Velazquez, D. Hydro-economic river basin
386 modelling: The application of a holistic surface–groundwater model to assess opportunity costs of water
387 use in Spain. *Ecological economics* **2008**, *66*, 51–65.

388 18. Clark, M.K. Effects of high commodity prices on western Kansas crop patterns and the Ogallala aquifer.
389 PhD thesis, Kansas State University, 2009.

390 19. Esteban, E.; Albiac, J. The problem of sustainable groundwater management: the case of La Mancha
391 aquifers, Spain. *Hydrogeology journal* **2012**, *20*, 851–863.

392 20. Aistrup, J.A.; Buletewicz, T.; Kulcsar, L.J.; Peterson, J.M.; Welch, S.M.; Steward, D.R. Conserving the
393 Ogallala Aquifer in southwestern Kansas: from the wells to people, a holistic coupled natural–human
394 model. *Hydrology and Earth System Sciences* **2017**, *21*, 6167–6183.

395 21. MacEwan, D.; Cayar, M.; Taghavi, A.; Mitchell, D.; Hatchett, S.; Howitt, R. Hydroeconomic modeling of
396 sustainable groundwater management. *Water Resources Research* **2017**, *53*, 2384–2403.

397 22. Dale, L.L.; Dogru, E.C.; Brush, C.F.; Kadir, T.N.; Chung, F.I.; Miller, N.L.; Vicuna, S.D. Simulating the
398 impact of drought on California’s Central Valley hydrology, groundwater and cropping. *British Journal of
399 Environment and Climate Change* **2013**, *3*, 271.

400 23. Buletewicz, T.; Allen, A.; Peterson, J.M.; Staggenborg, S.; Welch, S.M.; Steward, D.R. The simple script
401 wrapper for OpenMI: enabling interdisciplinary modeling studies. *Environmental Modelling & Software*
402 **2013**, *39*, 283–294.

403 24. Sobey, A.; Townsend, N.; Metcalf, C.; Bruce, K.; Fazi, F.M. Incorporation of Early Career Researchers within
404 multidisciplinary research at academic institutions. *Research Evaluation* **2013**, *22*, 169–178.

405 25. Parson, E.A. Integrated assessment and environmental policy making: In pursuit of usefulness. *Energy
406 Policy* **1995**, *23*, 463–475.

407 26. Rotmans, J.; Van Asselt, M. Integrated assessment: a growing child on its way to maturity. *Climatic Change*
408 **1996**, *34*, 327–336.

409 27. Adams, K.; Kovacs, K. The adoption rate of efficient irrigation practices and the consequences for aquifer
410 depletion and further adoption. *Journal of Hydrology* **2019**, *571*, 244–253.

411 28. Pindyck, R.; Rubinfeld, D. Microeconomics 7Th Edition, 2009.

412 29. DeLaune, P.; Mubvumba, P.; Ale, S.; Kimura, E. Impact of no-till, cover crop, and irrigation on Cotton yield.
413 *Agricultural Water Management* **2020**, *232*, 106038.

414 30. Currie, R.S.; Klocke, N.L. Impact of a terminated wheat cover crop in irrigated corn on atrazine rates and
415 water use efficiency. *Weed science* **2005**, *53*, 709–716.

416 31. Martin, D.L.; Watts, D.G.; Gilley, J.R. Model and production function for irrigation management. *Journal of*
417 *irrigation and drainage engineering* **1984**, *110*, 149–164.

418 32. Barlow, J.R.; Clark, B.R. *Simulation of water-use conservation scenarios for the Mississippi Delta using an existing*
419 *regional groundwater flow model*; US Department of the Interior, US Geological Survey, 2011.

420 33. Bryant, P. Mississippi Governor Executive Order 1341, 2014.

421 34. Brozović, N.; Sunding, D.L.; Zilberman, D. On the spatial nature of the groundwater pumping externality.
422 *Resource and Energy Economics* **2010**, *32*, 154–164.

423 35. USDA-NASS. Mississippi Soybean County Estimates, National Agricultural Statistics Service, United
424 States Department of Agriculture **2020**.

425 36. Tang, Q.; Feng, G.; Fisher, D.; Zhang, H.; Ouyang, Y.; Adeli, A.; Jenkins, J. Rain water deficit and irrigation
426 demand of major row crops in the Mississippi Delta. *Transactions of the ASABE* **2018**, *61*, 927–935.

427 © 2021 by the authors. Submitted to *Journal Not Specified* for possible open access publication
428 under the terms and conditions of the Creative Commons Attribution (CC BY) license
429 (<http://creativecommons.org/licenses/by/4.0/>).