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Connection Between Maxwell's Equations and the Dirac Equation

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Electrically charged particles such as Electrons and Protons carry electric, $\boldsymbol{E}$, and magnetic, $\boldsymbol{B}$, fields. In addition to these fields, Quantum Mechanics (QM) endows these particles with an '*arcane and spooky*' field — the wavefunction, ψ. This wavefunction, ψ, of QM is not only assumed to be separate but distinct from the electromagnetic field, A_μ. We herein upend this view by demonstrating otherwise. That is, we demonstrate that the four components $(\psi_0, \psi_1, \psi_2, \psi_3)$ of the Dirac wavefunction, ψ, can be shown to not only be an intimate, but, a direct function of the electromagnetic field (A_μ) carried by the particle in question — *i.e.*: $\psi = \psi(A_\mu)$. Insofar as unity, depth in our understanding and insight into both Dirac [1, 2] and Maxwell [3]'s equations as major pillars of *Modern Physics*, we believe that this work may very well *inch us one-step-closer* to the truth.

"*Those who are not shocked when they first come across Quantum Theory cannot possibly have understood it.*"

— ***Niels Henrik David Bohr*** **(1885-1962)**

1 Introduction

ATTEMPTS at finding a connection between the Dirac equation [1, 2] and Maxwell's equations of electrodynamics [3] has been an on-going field of research (see *e.g.*, Refs. [4–6]). A similar attempt is made in the present. What makes the present attempt unique and interesting is that the four components of the the Dirac wavefunction are directly identified with the electric and magnetic field components of Maxwell's equations of electrodynamics [3]; or in short, a clear relationship between the Dirac wavefunction and the electromagnetic four vector potential, A_μ, is here established. Ultimately, what this means is that the Dirac equation can be envisaged as being no more than a re-casting of the Maxwell's equations of electrodynamics.

We have arranged this reading as follows: for no more than instructive, self-containment and completeness purposes, in §(2 and 3), we give an exposition of Maxwell [3]'s equations, and, the Dirac [1, 2] equation — respectively. Thereafter in §(4), we transform Maxwell [3]'s equation into a system of three particles satisfying the Klein [7] and Gordon [8] equation (KGE). Now, in-order to link Maxwell [3]'s equations, and, the Dirac [1, 2], a fourth component is needed. For that — in §(5), we not only make clear what is needed in-order to link Maxwell's equation to the Dirac equation, but also a strategy of how this can and will be attained. Finally, in §(6), we give an exposition of the Maxwell-Proca [9–13] theory in which process we obtain the much needed fourth component that allows us to finally link Maxwell's equation to the Dirac equation. Having accomplished our mission, in §(7), we give a general discussion.

2 Maxwell's equations

According to Maxwell [3]'s legendary theory of electrodynamics, the electric, $\boldsymbol{E}$, and magnetic, $c\boldsymbol{B}$, fields generated from a (moving) charge (ϱc) and current density $(\boldsymbol{J})$, these are governed by the following five equations:

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \mu c\,(\varrho c)\,, \tag{1a}$$

$$\boldsymbol{\nabla} \cdot (c\boldsymbol{B}) = 0, \tag{1b}$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{1}{c}\frac{\partial\,(c\boldsymbol{B})}{\partial t} = 0, \tag{1c}$$

$$\boldsymbol{\nabla} \times (c\boldsymbol{B}) - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \mu c\boldsymbol{J}, \tag{1d}$$

$$0 = \frac{1}{c}\frac{\partial\,(\varrho c)}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{J}. \tag{1e}$$

We have written Maxwell [3]'s equations (1*a*-*e*) in such a manner that on the left handside we have the fields $(\boldsymbol{E}, c\boldsymbol{B})$, and on the right, the sources $(\varrho c, \boldsymbol{J})$. In addition to this, notice that, the fields $(\boldsymbol{E}, c\boldsymbol{B})$ and sources $(\varrho c, \boldsymbol{J})$ have the same dimensions — this we have done for our own convenient purposes and this will become clear as we go.

As is well known, the $(\boldsymbol{E}, c\boldsymbol{B})$-fields can be expressed in terms of the electromagnetic four vector potential, A_μ,

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as follows:

$$\boldsymbol{E} = -\boldsymbol{\nabla} A_0 - \frac{\partial \boldsymbol{A}}{\partial t}, \tag{2a}$$

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}. \tag{2b}$$

This electromagnetic four vector potential, $A_\mu = (A_0, \boldsymbol{A})$, satisfies the Lorenz [14] gauge condition:

$$\partial^\mu A_\mu = \frac{1}{c}\frac{\partial A_0}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{A} = 0. \tag{3}$$

In tensor form, Maxwell [3]'s equations are such that:

$$\partial^\mu F_{\mu\nu} = \mu c J_\mu, \tag{4a}$$

$$\partial_\alpha F_{\mu\nu} + \partial_\nu F_{\alpha\mu} + \partial_\mu F_{\nu\alpha} = 0, \tag{4b}$$

where:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{5}$$

and written in full:

$$[F_{\mu\nu}] = \begin{bmatrix} 0 & -E_1 & -E_2 & -E_3 \\ +E_1 & 0 & -cB_3 & +cB_2 \\ +E_2 & +cB_3 & 0 & -cB_1 \\ +E_3 & -cB_2 & -cB_1 & 0 \end{bmatrix}, \tag{6}$$

and: $J_\mu = (\varrho c, \boldsymbol{J})$, is the four current with: ϱ, and: $\boldsymbol{J}$, being the electrical charge and current densities respectively.

Written in terms of electromagnetic four vector potential, $A_\mu = (A_0, \boldsymbol{A})$, under the Lorenz [14] gauge condition [Eq. (3)], the source coupled Maxwell [3]'s equation [*i.e.*, Eq. (1a,d)] reduce to the following equation:

$$\Box A_\nu = \mu c J_\mu, \tag{7}$$

where:

$$\Box = \boldsymbol{\nabla}^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}, \tag{8}$$

is the *D'Alembert operator*. All the above presented is standard material that is readily found in textbooks and this is also true for what we shall present in the next section. As stated, we are doing this for no more than instructive, self-containment and completeness purposes.

3 Dirac equation

For a particle whose rest-mass and wave-function are m_0 and ψ respectively, its Dirac equation is given by:

$$[\imath\hbar\gamma^\mu\partial_\mu - m_0 c]\,\psi = 0, \tag{9}$$

where:

$$\gamma^0 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \tag{10}$$

are the 4×4 Dirac gamma matrices (I_2 and 0 are the 2×2 identity and null matrices respectively) and ψ is the four component Dirac wave-function, $\hbar$ is the normalized Planck constant, c is the speed of light in vacuum, $\imath = \sqrt{-1}$ and:

$$\psi = \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}, \tag{11}$$

is the Dirac 4×1 four component wavefunction. Throughout this reading — *unless otherwise specified* — the Greek indices will be understood to mean: $\mu, \nu, ... = 0, 1, 2, 3$; and the lower case English alphabet indices: $i, j, k ... = 1, 2, 3$.

As is well know, '*squaring*' the Dirac equation results in the KGE: $\Box\psi = (m_0 c/\hbar)^2\psi$. In full vector form, this KGE is such that:

$$\Box \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = \left(\frac{m_0 c}{\hbar}\right)^2 \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}. \tag{12}$$

In §(5 and 6), we shall make use of the full vector form of the KGE [*i.e.*, Eq. (12)].

4 Charged particle systems

We shall here assume that charged particles such as Electrons, Protons *etc* are described by Maxwell [3] equations [*i.e.*, Eq. (1)]. Further, we shall assume that the electrical charge distribution in space of these particles is such that:

$$\boldsymbol{\nabla}\varrho = 0. \tag{13}$$

With these assumptions in place, we shall proceed to take the ***curl*** of Eq. (1*c*) and (1*d*), and third — applying the vector identity:

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{V} = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{V}) - \boldsymbol{\nabla}^2\boldsymbol{V}, \tag{14}$$

and lastly — making use of Eq. (1*a*) and (1*b*), one can show that:

$$\Box \boldsymbol{E} = \mu c \frac{\partial \boldsymbol{J}}{c \partial t}, \quad (15a)$$
$$\Box (c\boldsymbol{B}) = -\mu c^2 \boldsymbol{\nabla} \times \boldsymbol{J}. \quad (15b)$$

From these two equations (15a,b), we shall generate KGEs for the $(\boldsymbol{E}, c\boldsymbol{B})$-fields.

4.1 Klein-Gordon equation

As stated above, our aim here is to convert the two Eqs. (15) into Klein-Gordon type equations for the $(\boldsymbol{E}, c\boldsymbol{B})$-fields. For this to be so, it is necessary to assume that the electrical field and currents produced by these charged particles as they travel in their local bound space are related by the following relation:

$$\frac{1}{c}\frac{\partial \boldsymbol{J}}{\partial t} = \left(\frac{\sigma}{c\tau}\right)\boldsymbol{E}, \quad (16)$$

where: σ, is the conductance [with the usual SI Units: Ω^{-1}, *i.e.*, per Ohm] of space associated with the particle in question and τ is some constant parameter with the dimensions of time. In the case of Electrons moving through a conductor, τ, is the usual mean time between collisions of the Electron with the lattice.

Now, in-order for us to obtain the KGE from Eqs. (15), we realise that if the current density where to be related to the $\boldsymbol{E}$-field as follows:

$$\boldsymbol{J} = \frac{\sigma}{c\tau}\int_0^\tau \boldsymbol{E}\, cdt = \frac{1}{\mu c}\left(\frac{m_0 c}{\hbar}\right)^2 \int_0^\tau \boldsymbol{E}\, cdt. \quad (17)$$

Substituting Eq. (17) into the two field Eqs. (15), we obtain:

$$\Box \boldsymbol{E} = \left(\frac{m_0 c}{\hbar}\right)^2 \boldsymbol{E}, \quad (18a)$$
$$\Box (c\boldsymbol{B}) = \left(\frac{m_0 c}{\hbar}\right)^2 (c\boldsymbol{B}). \quad (18b)$$

These Eqs. (18a,b), are indeed the KGE for the $(\boldsymbol{E}, c\boldsymbol{B})$-fields where:

$$m_0 = \sqrt{\frac{\mu\sigma\hbar^2}{c^2\tau}}, \quad (19)$$

is the nonzero mass of the particle in question and: $\hbar$ is Planck's normalised constant. This system of equations [*i.e.*, Eq. (18)] is not what we have in mind for the KGE that we want. For example, we want a KGE that a complex wavefunction that combines the $(\boldsymbol{E}, c\boldsymbol{B})$-fields into a single and unified system. We are going to seek such an equation in §(4.1.1).

Before we move on to seek the KGE that we desire, it is important to notice that if we are to take the ***curl*** of Eq. (18), we obtain:

$$\boldsymbol{\nabla} \times \boldsymbol{J} = -\left(\frac{\sigma}{c\tau}\right) c\boldsymbol{B}, \quad (20)$$

and this Eq. (20) together with Eq. (17) are the London [15] system of equations used to explain the Meissner effect [16] in superconductivity. We are not going to explore this interesting observation in the present reading — perhaps in a future reading, we will do that. For the time being, we will keep our focus on what we hope to achieve herein and this is to demonstrate that Maxwell [3]'s equations can indeed be recast into the Dirac equation where the four components $(\psi_0, \psi_1, \psi_2, \psi_3)$ of the Dirac wavefunction are expressed in-terms of the electromagnetic four vector potential, A_μ.

4.1.1 Deterministic 3-vector KGE

Now, we move onto a part of what we seek and desire. To that end, one must realise that, for so long as the vectors $(\boldsymbol{E}, c\boldsymbol{B})$ are real vectors, we can always define the *Riemann-Weber-Silberstein* (RWS) vector function $\boldsymbol{\Psi}$ (see *e.g.*, Refs. [17–19]) which is such that:

$$\boldsymbol{\Psi} = \sqrt{\frac{\varepsilon}{2\epsilon}}\left(\boldsymbol{E} + \imath c\boldsymbol{B}\right), \quad (21)$$

where: ε, is the permittivity of space, and: ϵ, is in *Classical Physics*, the usual energy density of the $(\boldsymbol{E}, c\boldsymbol{B})$-fields, *i.e.*:

$$\epsilon = \frac{1}{2}\varepsilon \boldsymbol{E}^2 + \frac{1}{2}\frac{\boldsymbol{B}^2}{\mu} = \frac{1}{2}\varepsilon\left[\boldsymbol{E}^2 + (c\boldsymbol{B})^2\right]. \quad (22)$$

From the RWS vector function, $\boldsymbol{\Psi}$, we certainly and surely can write Eq. (18), in much more condensed and succinct manner as:

$$\Box\boldsymbol{\Psi} = \left(\frac{m_0 c}{\hbar}\right)^2 \boldsymbol{\Psi}. \quad (23)$$

The imaginary number: $\imath = \sqrt{-1}$, in Eq. (21) has been in-cooperated in-order to keep separate (and separable), the $(\boldsymbol{E}, c\boldsymbol{B})$-fields. In this way, one can easily recover the Maxwell [3]'s Eq. (2) from the resulting recasting by way of decomposing the real and imaginary parts of the recast set of equations.

In expanded form, the RWS-vector, $\boldsymbol{\Psi}$, is such that:

$$\boldsymbol{\Psi} = \sum_{k=1}^{3}\psi_k\hat{\boldsymbol{e}}_k = \psi_1\hat{\boldsymbol{e}}_1 + \psi_2\hat{\boldsymbol{e}}_2 + \psi_3\hat{\boldsymbol{e}}_3, \quad (24)$$

where: $\hat{\boldsymbol{e}}_1, \hat{\boldsymbol{e}}_2, \hat{\boldsymbol{e}}_3$, are the unit vectors along the xyz-axis respectively, and the '*space components*' (ψ_1, ψ_2, ψ_3), of

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the Dirac wavefunction, can now — *via*, Eq. (21), be related to the $(\boldsymbol{E}, \imath c\boldsymbol{B})$-fields as follows:

$$\psi_k = \sqrt{\frac{\varepsilon}{2\epsilon}}\,(E_k + \imath c B_k)\,, \tag{25}$$

and these vector components can be written in the *de Brogile-Bohm* polar form [20–23] as:

$$\psi_k = R_k e^{\imath S_k/\hbar}, \qquad \ldots \qquad k = 1, 2, 3; \tag{26}$$

where: (R_k, S_k) are well behaved (smooth, continuous, integrable and differentiable) real valued functions — with, $|R_k|^2$, being the xyz-components of the quantum probability amplitude function — while the, S_k's, are the xyz-components of the phase of the particle. From the foregoing, these functions (R_k, S_k) are defined as follows:

$$R_k, \quad = \quad \sqrt{\frac{\varepsilon\,(E_k^2 + c^2 B_k^2)}{2\epsilon}}, \tag{27a}$$

$$S_k \quad = \quad \hbar \tan^{-1}\left(\frac{cB_k}{E_k}\right). \tag{27b}$$

About these fields: ψ_k, *i.e.*, as they are defined in Eq. (21, 22 and 25), notice that their norms are such that:

$$0 \quad < \quad \psi_k^* \psi_k \quad < \quad 1, \tag{28a}$$

$$\boldsymbol{\Psi}^* \cdot \boldsymbol{\Psi} \quad = \quad 1. \tag{28b}$$

If, as is the case in Quantum Mechanics (QM), these norms are to represent the quantum probability measure, then, Eq. (28*a*) should be telling us that if the individual components, ψ_k are particles, they are not fully-fledged particles while Eq. (28*b*), is telling us that only the full set of the ψ_k's form a fully-fledged particle system.

4.1.2 Non-deterministic 3-vector KGE

As they are given in Eq. (26), the wavefunctions, ψ_k, are fully deterministic. We can introduce quantum indeterminacy into these wavefunctions by adding random variable quantum probability coefficients: $w_k \in \mathbb{C}$, as follows:

$$\psi_k = w_k \sqrt{\frac{\varepsilon}{2\epsilon}}\,(E_k + \imath c B_k) = R_k e^{\imath S_k/\hbar}. \tag{29}$$

These random variable quantum probability coefficients: $w_k \in \mathbb{C}$, are such that:

$$0 < |w_k|^2 < 1. \tag{30}$$

In this new setting, the phases: S_k, are deterministic and are still defined as in the case of the deterministic KGE [*i.e.*: in Eq. (27*b*), respectively], while the R-functions are (as a result of the introduction of the w-functions) now non-deterministic as they are now given by:

$$R_k = |w_k| \sqrt{\frac{\varepsilon\,(E_k^2 + c^2 B_k^2)}{2\epsilon}}. \tag{31}$$

Written in the usual parlance of QM, the normalization conditions of the wavefunction(s) now become:

$$0 \quad < \quad \langle \psi_k | \, | \psi_k \rangle \quad < \quad 1, \tag{32a}$$

$$\langle \boldsymbol{\Psi} | \, | \boldsymbol{\Psi} \rangle \quad = \quad 1. \tag{32b}$$

These normalization conditions have the same meaning as that given for Eq. (28).

5 Desideratum

As things stand at this present moment of our journey, what we thus far have managed to do with Maxwell [3]'s equations is to show that one can harness three wavefunctions (ψ_1, ψ_2, ψ_3) that satisfy the same KGE that is satisfied by the four components $(\psi_0, \psi_1, \psi_2, \psi_3)$ of the Dirac equation. In-order for us to complete our journey of directly linking Maxwell [3]'s equations of electrodynamics with the Dirac [1, 2] equation, we need to obtain — from within the confines and domains of Maxwell [3]'s system of equations, a way to generate the fourth component, ψ_0. That is to say, what we have thus far is that:

$$\Box \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = \left(\frac{m_0 c}{\hbar}\right)^2 \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}, \tag{33}$$

where this equation is the same as Eq. (23), *albeit*, with the three wavefunctions (ψ_1, ψ_2, ψ_3) now written in column vector form. In-order to obtain a four component KGE as Eq. (12), what we need is a fourth equation, namely:

$$\Box \psi_0 = \left(\frac{m_0 c}{\hbar}\right)^2 \psi_0. \tag{34}$$

The logical, natural and sensible condition is that we are not to postulate it into existence because we need it, but we must harness it from within the framework of Maxwell [3]'s equations of electrodynamics. In-order for us to achieve this, let us have a look at the Maxwell-Proca theory and from its domains, seek this much needed Eq. (34).

6 Maxwell-Proca theory

In-order to generate our desired fourth component wavefunction, ψ_0, we are going to make use of the Maxwell-Proca [9–13] theory of massive electrodynamics, *i.e.*:

$$\partial^\mu F_{\mu\nu} + \kappa^2 A_\nu = \mu c J_\mu. \quad (35)$$

In its most basic form, Maxwell [3]'s theory is believed and is assumed to describe massless particles and this Eq. (35) is no more than Maxwell [3]'s original theory that has been supplemented with a mass term: $\kappa^2 A_\nu$. This modification is usually used in theories that seek to endow the Photon with a nonzero mass. In the present case, we are using this theory to endow charged particles with a nonzero mass and the reason for this is simple: we know that the Electron and Proton are massive particles, so, if we are to come-up with a theory that hopes or purports to describe these particles, it — *at least* — must contain a nonzero mass term.

The idea that we are going to use originates from our on-going work [24–27] on massive Photons. We are not going to give an exposition of it here but go straight to the idea that we want. First, we shall write Eq. (35) in-terms of the electromagnetic four vector potential, A_ν, as follows:

$$\Box A_\nu - \partial_\nu \partial^\mu A_\mu + \kappa^2 A_\nu = \mu c J_\mu. \quad (36)$$

Next, we shall introduce the following violation (see Refs. [24–27]) of the Lorenz [14] gauge:

$$\partial^\mu A_\mu = \frac{1}{c}\frac{\partial A_0}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{A} = \kappa \psi_0 \neq 0, \quad (37)$$

where of course, ψ_0, is our sought for fourth component of the Dirac wavefunction. With, the wavefunction no in-place, we want to the corresponding KGE for it. For, this, we introduce yet another gauge condition, namely:

$$\partial_\nu \partial^\mu A_\mu = \kappa^2 A_\nu, \quad (38)$$

so that resultantly, the source couple Maxwell-Proca equations reduce to the usual massless source coupled field equation [*i.e.*, Eq. (7)] under the Lorenz [14] gauge [*i.e.*, Eq. (3)]. Taking the four divergence on both sides of Eq. (38) [and remembering to take into account Eq. (37)], we now obtain the desired KGE for, ψ_0, *i.e.*:

$$\Box \psi_0 = \kappa^2 \psi_0. \quad (39)$$

In-order for Eq. (39) to be identical to our sought for Eq. (34), we must have that*:

$$\kappa = \pm \frac{m_0 c}{\hbar} = \pm \frac{\mu\sigma}{\tau}. \quad (40)$$

This Eq. (40) completes our mission as this allows us to obtain our desired KGE for, ψ_0.

*__NB:__ κ, is assumed to be such that: $\partial^\mu \kappa \equiv 0 \equiv \partial_\mu \kappa$. That is to say, κ is independent of both space and time.

7 Discussion

At any rate imaginable, we have herein demonstrated that the usual four components (ψ_0, ψ_k) of the Dirac wavefunction, ψ, can be shown to not only be an intimate, but, a direct function of the electromagnetic field (A_μ) that define the particle carrying this field. In summary, these four components are defined in terms of, A_μ, as follows:

$$\psi_0 = w_0 \left(\frac{\partial^\mu A_\mu}{\kappa}\right) = R_0 e^{\imath S_0/\hbar}, \quad (41a)$$

$$\psi_k = w_1 \sqrt{\frac{\varepsilon}{2\epsilon}} \left(E_k + \imath c B_k\right) = R_k e^{\imath S_k/\hbar}. \quad (41b)$$

Since according to Eq. (2), the $(\boldsymbol{E}, c\boldsymbol{B})$-fields are defined in-terms of A_μ, it follows that as claimed: $\psi = \psi(A_\mu)$.

For these four components (ψ_0, ψ_k), we have herein set that they satisfy the following normalization conditions:

$$0 < \langle\psi_0|\,|\psi_0\rangle < 1, \text{ and, } 0 < \langle\psi_k|\,|\psi_k\rangle < 1, \quad (42)$$

the meaning of which is that, these components (ψ_0, ψ_k) are not fully-fledged and independent particles. It is only when they are acting and interaction together as four components that they form a fully-fledged particle and this is expressed by requiring that they meet the full normalization condition of QM, namely:

$$\langle\psi|\,|\psi\rangle = 1. \quad (43)$$

In-closing, we must say that: insofar as unity, depth in our understanding and insight into both Maxwell [3]'s equations and the Dirac [1,2] equation as one of the two major pillars of *Modern Physics*, we believe that this rather subtle demonstration, may very well *inch us one-step-closer* to the truth of '*What is the meaning of the Dirac equation?*'. We shall not say anything further but leave the reader to excogitate and ponder on the meaning of what is presented herein.

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