

Article

On the operational utility of measures of EEG integrated information

David Darmon¹ , Tomas Watanabe² Christopher Cellucci³ and Paul E. Rapp^{4*}

¹ Department of Mathematics, Monmouth University, West Long Branch, New Jersey, 07764, USA; david.m.darmon@gmail.com

² Vagalume LLC. Palo Alto, California, USA; tomas@vagalume.cloud

³ Aquinas LLC, Berwyn, Pennsylvania, USA 3; cellucci@gmail.com

⁴ Department of Military and Emergency Medicine, Uniformed Services University, Bethesda, Maryland, USA; paul.rapp@usuhs.edu

* Correspondence: paul.rapp@usuhs.edu

Abstract: Multichannel EEGs were obtained from healthy participants in the eyes-closed no-task condition (where the alpha component is typically abolished). EEG dynamics in the two conditions were quantified with two related binary Lempel-Ziv measures of the first principal component and with three measures of integrated information including the more recently proposed integrated synergy. Both integrated information and integrated synergy with model order $p = 1$ had greater values in the eyes closed condition. If the model order of integrated synergy was determined with the Bayesian Information Criterion, this pattern was reversed, and in common with other measures, integrated synergy was greater in the eyes open condition. Eyes open versus eyes closed separation was quantified by calculation of the between-condition effect size. Lempel-Ziv complexity of the first principal component showed greater separation than the measures of integrated information. The performance of the integrated information measures investigated here when distinguishing between indisputably different physiological states encourages caution when advocating for their use as measures of consciousness.

Keywords: integrated information theory; Lempel-Ziv complexity; multichannel EEGs; Electroencephalography; measures of consciousness

1. Introduction

Integrated information measures of multichannel EEGs have attracted attention, in part, because it has been suggested that they might quantify consciousness [1]. It should be noted, however, that others have argued against this suggestion [2], [3]. Several variant measures of integrated information have been considered [4]. Mediano, et al [5] have compared six candidate measures with computationally generated data. The quantification of consciousness raises several deep questions including the definition of consciousness itself. We investigate here an objective criterion of the utility of measures of multichannel EEGs: how effective are these measures in discriminating between incontestably different cognitive/physiological states?

In this study we assessed multichannel EEG measures by comparing values obtained in the no-task eyes open and no-task eyes closed condition. One of the most consistent properties of the EEG is alpha blocking discovered by Berger in 1924 [6] and confirmed by Adrian and Matthews in 1934 [7]. In most individuals, but certainly not all, a very prominent alpha rhythm (8-13 Hz) is observed in the eyes closed condition. This alpha rhythm typically disappears immediately when the eyes are opened. Examples of eyes open contra eyes closed spectra are presented in Hartoyo, et al. [8] and Liley and Muthukumaraswamy [9].

A study comparing ten measures calculated from multichannel EEGs in the eyes open and eyes closed state was published by Rapp, et al. [10]. The present study follows

the same pattern and incorporates a more recently proposed measure of integrated information theory, integrated synergy. Additionally, this study extends the earlier study by including calculations performed with signals (both eyes open and eyes closed) after the alpha band had been removed with a digital filter. The efficacy of these measures to discriminate between physiological states is quantified by calculating the eyes open versus eyes closed effect size for both alpha-present and alpha-absent signals.

2. Participants

EEG recordings were obtained from thirteen healthy adult participants. Prior to testing participants gave written informed consent to participate in this study. The study reported here was approved by the Uniformed Services University Human Research Protections Program Office: Protocol DBS.2020.251. Participants were not paid or compensated for their participation. All study procedures were conducted in accordance with human participant protections regulations required by ethical laws and regulations set forth by the Declaration of Helsinki and the Common Rule.

3. Data

Free-running, no-task, monopolar EEG signals referenced to linked earlobes were obtained in two conditions, eyes closed and eyes open, from F_Z, C_Z, P_Z, O_Z, F₃, F₄, C₃, C₄, P₃ and P₄ using an Electrocap. Bipolar recordings of vertical and horizontal eye movements were recorded from electrode sites above and below the right eye and from near the outer canthi of each eye. Artifact corrupted records were removed from the analyses. All EEG impedances were less than 5 KOhm. Signals were amplified, Gain=18000. Signals were digitized at 1024 Hz using a twelve-bit digitizer. Continuous artifact-free records were obtained from each subject in the two conditions. Ten thousand point records were used in these calculations. As reported in the introduction, an objective of the study was to determine the effect of alpha content on the resulting dynamical measures. All signals were initially passband filtered with cutoff settings at 1 Hz and 200 Hz. A second set of signals was obtained by additionally filtering these signals again with a Butterworth filter and an 8 Hz to 13 Hz stopband.

4. Measures

Five measures were used in this study. The first was constructed using Lempel-Ziv complexity [11]. Let $(V_1^m, V_2^m, \dots, V_{10000}^m)$ denote the mean normalized time series of the m -th channel ($m=1, \dots, 10$). These vectors become columns in a $10,000 \times 10$ matrix.

$$A = \begin{bmatrix} V_1^1 & \dots & V_1^{10} \\ \vdots & \ddots & \vdots \\ V_{10000}^1 & \dots & V_{10000}^{10} \end{bmatrix} = V \cdot D \cdot U^T \quad (1)$$

where $V \cdot D \cdot U^T$ is the singular value decomposition of A . The singular value decomposition was calculated using the Golub-Reinsch algorithm [12], [13]. D is the diagonal matrix of singular values $D = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_{10})$ where we introduce the convention $\lambda_j \geq \lambda_{j+1}$ for all j , and U is the corresponding orthogonal transformation. For these data, the first principal component carries more than 70% of the multichannel signal's variance [14].

The first measure is constructed as follows. The first principal component was then partitioned into a binary symbol sequence about the median, and the Lempel-Ziv complexity was calculated [11]; pseudocode is given in Appendix A of [14].

The second measure is nearly identical to the first. In this case the mean normalized time series of each channel was also normalized against the channel's standard deviation before constructing matrix A .

The third measure is one of the earliest measures of central nervous system information integration proposed by Tononi, et al. [15]. It is constructed by comparing the

degree of integration of k -dimensional subsystems with the degree of integration of the N -dimensional parent system. $\text{Corr}(\mathbf{X}_j^k)$ is the j -th instance of a $k \times k$ correlation matrix formed by using k of the N channels. Tononi, et al. [15] define the integration of this subsystem as

$$I(\mathbf{X}_j^k) = -\frac{1}{2} \ln(\det(\text{Corr}(\mathbf{X}_j^k))) \quad (2)$$

The average integration of k -dimensional subsystem is denoted by $\langle I(\mathbf{X}_j^k) \rangle$. The system integration C_N , the third measure for this study is determined by comparing the integration of the N -dimensional system $I(\mathbf{X}^N)$ against the integration of subsystems of k channels.

$$C_N = \sum_{k=1}^N \left\{ \left(\frac{k-1}{N-1} \right) I(\mathbf{X}^N) - \langle I(\mathbf{X}_j^k) \rangle \right\} \quad (3)$$

We note that Equation 4 of Tononi, et al. [15] uses k/N as the scaling factor of $I(\mathbf{X}^N)$. van Putten and Stam [16] argue that $(k-1)/(N-1)$ rather than k/N is the appropriate scaling factor and is used here. Pseudocode for C_N is given in [10]. That paper also outlines difficulties with this definition of integrated information. The expression $-\frac{1}{2} \ln[\text{Corr}(\mathbf{X}_j^k)]$ has a singularity of infinite integration if two channels are completely correlated. Perfect correlation will not occur with biological data, but highly correlated signals can be observed in high density EEG montages. Notably, in the calculations with simulation data in van Putten and Stam [16], a noise term was added to the simulations to produce computationally stable examples. A numerically stable alternative definition of integration based on the Morgerra covariance complexity [17] was identified in [10].

The fourth measure examined in this study is another measure of integrated information. As noted in the Introduction, several versions have been considered. Within a broad conceptual structure, a system is deemed to be "complex" if it balances integration (portions of the system work together) and segregation (portions of the system work in isolation). Various Integrated Information Theory measures seek to quantify the "whole"-ness versus the "part"-ness of the system. This is implemented by quantifying the information that the current state of the system has about its past state and comparing this information in the fully integrated system against a system partitioned to have the weakest informational links between partition elements; that is, the system partitioned to have the weakest possible integration. The latter is called the Minimum Information Partition. Broadly stated, Integration Information Theory has been presented in three versions: Version 1 [18], Version 2 [1] and Version 3 [19]. Version 3 was formulated for discrete systems and since our present objective is the analysis of continuous EEG signals, we focused on Version 2 implementations. Mediano, Seth and Barrett [5] compared six measures of integrated information. In simulations on Gaussian vector autoregressive processes, they obtained best performance with three measures: integrated synergy, ψ , decoder-based integrated information, Φ^* , and causal density, CD. Causal density is the average of the conditional transfer entropies between each pair of components of the system, and it could therefore be argued that it falls outside the domain of Integrated Information Theory. Between ψ and Φ^* , ψ is easier to compute for Gaussian processes. We therefore selected integrated synergy [20] for incorporation into this study. A concise mathematical description of integrated synergy is given in the Supplement of this paper.

Measure 5 is again integrated synergy. In this case, however, the model order of the underlying Gaussian autoregressive process is not fixed at $p = 1$ as in [5] but is determined for each multichannel data set by the Bayesian Information Criterion, BIC. Identified orders were between 3 and 6.

5. Results

Five measures were obtained in two behavioral conditions (eyes open and eyes closed) for two signal configurations (alpha band present and alpha band removed). An initial examination of eyes open versus eyes closed differences suggested that results

from one participant were markedly different. A systematic investigation was undertaken, and it was established that results from one participant met a standard outlier criterion, one and one half times the interquartile range, for six of the ten difference scores (five measures obtained with alpha present and the same five measures obtained with alpha removed produced ten measures). This data set was removed from the analysis. Results reported here were obtained using data from the remaining twelve participants.

The values obtained from each measure in each condition are presented in the Supplement. Of more immediate interest are the differences observed in the eyes open versus eyes closed condition (Table 1). Difference scores (eyes open – eyes closed) are given in the next table. Cases where the complexity of eyes closed is greater than eyes open are highlighted in red. In the case of C_N the observation that eyes closed values are greater than eyes open values is consistent with van Putten and Stam [16], van Cappellen van Walsum, et al. [21], and Rapp, et al. [10]. This ordering for C_N was not, however, observed in Trujillo, et al. [22]. This divergence of observation is addressed in the discussion. The two measures using Lempel-Ziv complexity have greater values in the eyes open condition. When computed with model order equal to one, integrated synergy also has a greater value in the eyes closed condition, but this pattern is reversed when the Bayesian Information Criterion is used to identify an appropriate value of model order. For p chosen by BIC, the results are consistent with Lempel-Ziv as used here and with nine measures of the 2005 study; integrated synergy is greater in the eyes open condition when p is chosen by BIC.

Table 1: Difference Values: Eyes Open - Eyes Closed Signals containing the alpha component.

	Eyes Open - Eyes Closed Mean \pm StDev
Binary Lempel Ziv Signals mean normalized	63.4167 \pm 36.9360
Binary Lempel Ziv Signals normalized by mean and by standard deviation	51.2500 \pm 31.5873
C_N Tononi, et al. [15] Equation 4	-0.6469 \pm 0.3973
ψ $p = 1$ Mediano, et al. [5] Equation 23	-0.8556 \pm 0.5906
ψ p via BIC Mediano, et al. [5] Equation 23	0.2013 \pm 0.1944

Table 2: Difference Values: Eyes Open - Eyes Closed Signals with the alpha component removed.

	Eyes Open - Eyes Closed Mean \pm StDev
Binary Lempel Ziv Signals mean normalized	42.7500 \pm 30.8460
Binary Lempel Ziv Signals normalized by mean and by standard deviation	39.6667 \pm 28.2081
C_N Tononi, et al. [15] Equation 4	-0.2102 \pm 0.3658
$\psi p = 1$ Mediano, et al. [5] Equation 23	-0.6755 \pm 0.4900
ψp via BIC Mediano, et al. [5] Equation 23	0.0679 \pm 0.2093

The differences between eyes open and eyes closed scores were quantified by calculation of the corresponding effect sizes. A standard estimator (difference normalized against the standard deviation) and a robust estimator [23] were calculated. For both estimators 10,000 bootstrap samples were used to construct 95% confidence intervals of the effect sizes using a bias-corrected and adjusted (BCa) confidence interval [24]. The results obtained with the standard estimator follow in Tables 3 and 4. Effect sizes obtained with the robust estimator are in the Supplement. As before, results where the eyes closed values are greater than the corresponding eyes open values are highlighted in red.

Table 3: Effect Size: Eyes Open - Eyes Closed Signals containing the alpha component, standard estimator.

Measure	Estimated Effect Size	95% Confidence Interval
Binary Lempel Ziv Signals mean normalized	1.63	(1.17, 2.26)
Binary Lempel Ziv Signals normalized by mean and by standard deviation	1.56	(1.134, 2.27)
C_N Tononi, et al. [15] Equation 4	-1.60	(-2.53, -1.12)
$\psi p = 1$ Mediano, et al. [5] Equation 23	-1.40	(-2.13, -0.90)
ψp via BIC Mediano, et al. [5] Equation 23	0.99	(0.13, 1.93)

Table 4: Effect Size: Eyes Open - Eyes Closed Signals with the alpha component removed, standard estimator.

Measure	Estimated Effect Size	95% Confidence Interval
Binary Lempel Ziv Signals mean normalized	1.33	(0.72, 2.05)
Binary Lempel Ziv Signals normalized by mean and by standard deviation	1.35	(0.65, 2.52)
C_N Tononi, et al. [15] Equation 4	-0.57	(-1.33, -0.05)
$\psi p = 1$ Mediano, et al. [5] Equation 23	-1.34	(-2.35, -0.75)
ψp via BIC Mediano, et al. [5] Equation 23	0.31	(0.32, 0.90)

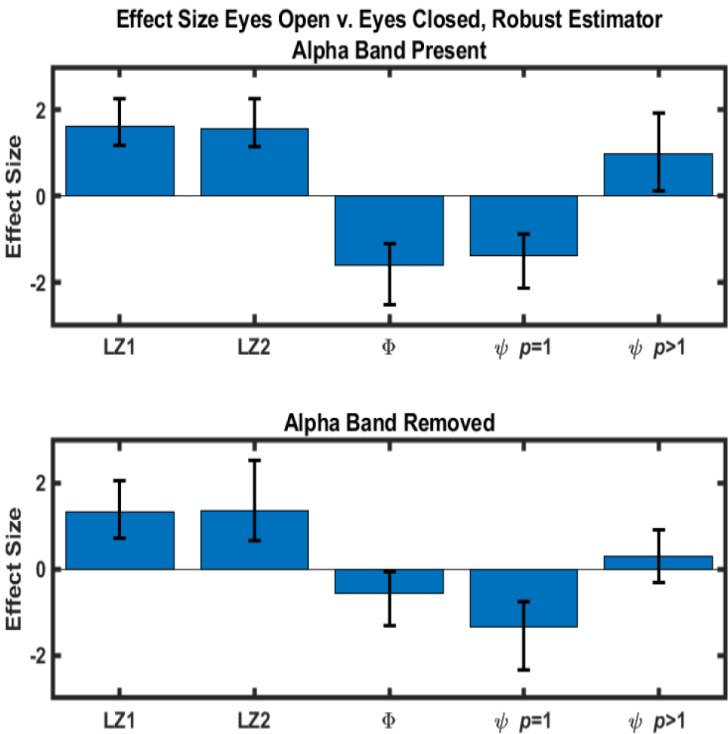


Figure 1. Eyes open versus eyes closed effect size as quantified by the standard estimator for five measures. Confidence intervals were determined with a bias-corrected and adjusted bootstrap.
(a) Effect sizes calculated with signals that contain the alpha component
(b) Effect sizes calculated after the alpha component had been removed with an 8 Hz to 13 Hz stopband filter

We next consider the statistical significance of the difference between the eyes open and eyes closed conditions. The small number of participants in the study argued against a significance test that assumed a normal distribution. The Wilcoxon signed ranks test was used to assess the statistical significance of the differences obtained in

the eyes open versus the eyes closed condition with a two-sided test ($p < .05$). The null hypothesis (measures obtained in the eyes open condition are indistinguishable from measures obtained in the eyes open condition) was rejected in eight of the ten cases (five measures considered with and without alpha band content giving ten measures). The two measures that failed to reject the null hypothesis were C_N with alpha content removed and integrated synergy, ψ model order $p = 1$ also for the case where the alpha band was removed.

Nonparametric correlations between measures were quantified with Kendall's tau. The results shown in Tables 5 and 6 were calculated combining both eyes closed and eyes open data in the alpha band present and alpha band absent conditions. Calculations of Kendall's tau obtained separately with eyes closed and eyes open data are in the Supplement. As seen in the table, the results are largely unremarkable. In all cases, the correlation decreases with the removal of alpha. The two variants of Lempel-Ziv complexity are highly correlated. Measure 4, which is ψ , $p = 1$ is negatively correlated with ψ , p via BIC which is expected since on average the eyes open versus eyes closed relationship is reversed in the two cases.

Table 5: Kendall tau correlation, Alpha component present

	LZ1	LZ2	C_N	ψ $p = 1$	ψ p via BIC
LZ1	1.0000	0.8436	-0.4218	-0.3636	0.4000
LZ2		1.0000	-0.4058	-0.4130	0.3406
C_N			1.0000	0.2246	-0.3351
ψ $p = 1$				1.0000	-0.4203
ψ p via BIC					1.0000

Table 6: Difference Values: Eyes Open - Eyes Closed Signals with the alpha component removed.

	LZ1	LZ2	C_N	ψ $p = 1$	ψ p via BIC
LZ1	1.0000	0.6374	-0.0873	-0.1673	0.1164
LZ2		1.0000	-0.2190	-0.1971	0.1679
C_N			1.0000	0.1522	-0.0435
ψ $p = 1$				1.0000	-0.2971
ψ p via BIC					1.0000

6. Discussion

Four principal conclusions follow from the computational results. First, in the case of integrated synergy, identifying an appropriate model order is an essential element of the analysis. Comparison of $p = 1$ and p via BIC results showed that eyes open minus eyes closed values change sign when p via BIC is used and that the p via BIC results align with other measures of complexity. In addition to the calculations presented here, comparisons should be made to the results in [10]. Of the ten measures examined in that study, the complexity was greater in the eyes open condition in nine measures. The only exception was the 1994 measure of integrated information, C_N [15]. It is concluded that statistically responsible determination of model order for integrated synergy does not simply result in modest quantitative differences. Qualitative differences central to assessing the underlying cognitive processes are obtained.

The signs of the C_N eyes-open versus eyes-closed difference merits further consideration. In addition to [10], van Putten and Stam [16] examining EEGs and van Cappellen

van Walsum, et al. [21] examining MEGs found C_N greater in the eyes-closed condition. Van Putten and Stam [16] also found that C_N increased in neurological disorders where consciousness was severely compromised (generalized seizures and severe postanoxic encephalopathy). van Cappellen van Walsum, et al. [21] examined MEG records obtained from patients with a probable diagnosis of Alzheimer's dementia according to NINCS-ADRDA criteria [25]. They found C_N higher in Alzheimer's disease as compared to controls in 2-4 Hz and 4-8 Hz frequency bands. They concluded: "The hypothesis of Tononi, et al. (1994) that the neural complexity decreases in AD patients has to be rejected."

The integrated information results presented by Trujillo, et al. [22] diverge from those presented here and by previous investigators. Trujillo, et al. found that integrated information was greater in the eyes open condition. Trujillo, et al. analyzed signals that had been bandpassed to the theta/alpha range (4-13 Hz) and to the beta range (14-30 Hz). Additionally, the signals used in Trujillo, et al. were transformed to a normal distribution using a procedure published in van Albada and Robinson [26]. When Trujillo, et al. calculated integrated information with their data in the absence of this transformation, a greater value was obtained in the eyes closed condition as was found in our calculations of integrated information and in calculations of integrated synergy with model order $p = 1$. Using a model order for integrated synergy determined by the Bayesian Information Criterion produced results consistent with Trujillo, et al. It seems possible that the results reported by van Putten and Stam using EEGs obtained from neurological patients and the results reported by van Cappellen van Walsum et al. with MEGs obtained from demented participants might be revised if the data were first normal-transformed and/or analyzed with integrated synergy and model order determined by a model selection criterion.

Second, while the correlation between the two Lempel-Ziv measures is, as expected, high, the correlation among the integrated information measures is low.

Third, the examination of differences in the eyes open and eyes closed condition failed to meet statistical significance for Measure 3, C_N , and Measure 5, integrated synergy with p via BIC, when the alpha band content is removed from the EEG.

Fourth, an examination of eyes open versus eyes closed effect sizes also calls the operational utility of integrated synergy into question. In a comparison of effects sizes, the Lempel-Ziv complexity of the first principal component of a multichannel EEG was more effective than integrated information or integrated synergy. The quantification of consciousness, a stated objective of integrated information theory, is a profoundly complex problem. We consider here an arguably more pedestrian but answerable question: is this measure useful? A first primitive criterion of usefulness of a measure would be, whether it effectively discriminates between different cognitive states, where it should be recalled that alpha blocking, the spontaneous disappearance of the alpha rhythm when the eyes open, is not a subtle phenomenon (examples comparing spectra are presented in [9] and in [8]). The performance of the integrated information measures investigated here when distinguishing between indisputably different physiological states encourages caution when advocating for their use as measures of consciousness.

Author Contributions: Conceptualization: PER, DD and CJC, Methodology: PER, DD and CJC, Software: DD, CJC and PER Data Acquisition: TAAW, Formal Analysis: DD and PER, Writing - Original Draft Presentation: PER, Writing - Review and Editing: DD, CJC and TAAW, Project Administration: PER. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: The study reported here was approved by the Uniformed Services University's Human Research Protections Program Office: Protocol DBS.202.251. participants were not paid or compensated for their participation. All study procedures were conducted in accordance with human participant protections regulations as set forth by the Declaration of Helsinki and the Common Rule.

Informed Consent Statement: Informed consent was obtained from all participants involved in the study.

Data Availability Statement: Requests for data should be directed to the corresponding author. Availability is subject to Department of Defense policies concerning human research data

Conflicts of Interest: The authors declare no conflict of interest. The opinions and assertions contained herein are those of the authors and do not necessarily reflect the official policy or position of the Uniformed Services University, the Department of Defense or the Henry M Jackson Foundation for the Advancement of Military Medicine.

Appendix A. Integrated Synergy

Let \mathcal{P} be a particular partition of the state vector $\mathbf{V} = (V_1, \dots, V_d)$ into r state vectors $(\mathbf{M}^{(1)}, \mathbf{M}^{(2)}, \dots, \mathbf{M}^{(r)})$. Then the integrated synergy ψ_1 is defined as the decrease in the mutual information between the most-recent past and present of the system when using a partition of the most-recent past that maximizes a certain notion of non-redundant information between the partition elements and the present,

$$\psi_1 = I[\mathbf{V}_{t-1} \wedge \mathbf{V}_t] - \max_{\mathcal{P}} I_{\cup} \left[\left(\mathbf{M}_{t-1}^{(1)}, \dots, \mathbf{M}_{t-1}^{(r)} \right) \wedge \mathbf{V}_t \right], \quad (\text{A1})$$

where $I_{\cup} \left[\left(\mathbf{M}_{t-1}^{(1)}, \dots, \mathbf{M}_{t-1}^{(r)} \right) \wedge \mathbf{V}_t \right]$ is the intersection information between a particular partition of \mathbf{V}_{t-1} and \mathbf{V}_t (Mediano, et al.). The intersection information is defined via an inclusion-exclusion principle as

$$I_{\cup} \left[\left(\mathbf{M}_{t-1}^{(1)}, \dots, \mathbf{M}_{t-1}^{(r)} \right) \wedge \mathbf{V}_t \right] = \sum_{\mathcal{S} \in \mathcal{P}^+ (\{\mathbf{M}^{(1)}, \dots, \mathbf{M}^{(r)}\})} (-1)^{|\mathcal{S}|+1} I_{\cap} \left[\left(\mathbf{S}_{t-1}^{(1)}, \dots, \mathbf{S}_{t-1}^{(|\mathcal{S}|)} \right) \wedge \mathbf{V}_t \right]. \quad (\text{A2})$$

where $\mathcal{P}^+ (\{\mathbf{M}^{(1)}, \dots, \mathbf{M}^{(r)}\})$ is the power set of $\{\mathbf{M}^{(1)}, \dots, \mathbf{M}^{(r)}\}$ excluding the empty set and $I_{\cap} \left[\left(\mathbf{S}_{t-1}^{(1)}, \dots, \mathbf{S}_{t-1}^{(|\mathcal{S}|)} \right) \wedge \mathbf{V}_t \right]$ is the intersection information, i.e. the redundant information, between a subset \mathcal{S} of the partition and \mathbf{V}_t (Griffith).

For a linear Gaussian VAR, the union information reduces to the maximum over the mutual informations between the present state and each single element of the partition

$$I_{\cup} \left[\left(\mathbf{M}_{t-1}^{(1)}, \dots, \mathbf{M}_{t-1}^{(r)} \right) \wedge \mathbf{V}_t \right] = \max_k I \left[\mathbf{M}_{t-1}^{(k)} \wedge \mathbf{V}_t \right]. \quad (\text{A3})$$

(Olbrich, Bertschinger, and Rauh).

As in (Mediano, et al.), we restrict our analysis to equal-sized bipartitions of the state vector. This is done to avoid bias due to differences in integrated information between different-sized partitions.

Appendix B. Order- p Integrated Synergy

The integrated synergy (A1) implicitly assumes that the stochastic process governing the state variable is an order-1 Markov process. That is, it assumes that knowledge of the most recent past is sufficient to predict the future of the time series. There is no reason to assume this a priori, especially for time series derived from electrophysiology. In principle, one could allow for the entire past of the time series to be necessary for optimal prediction, i.e. if the process is a linear stochastic process. In this section, we define this infinite-order integrated synergy as well as an order- p compromise.

Let $\mathbf{Z}_{a:b} = (\mathbf{Z}_a, \mathbf{Z}_{a+1}, \dots, \mathbf{Z}_{b-1})$ be the random vector containing the state vector from time points a to $b-1$. Then the infinite order integrated synergy ψ_{∞} is defined as

$$\psi_{\infty} = I[\mathbf{V}_{-\infty:t} \wedge \mathbf{V}_t] - \max_{\mathcal{P}} I_{\cup} \left[\left(\mathbf{M}_{-\infty:t}^{(1)}, \dots, \mathbf{M}_{-\infty:t}^{(r)} \right) \wedge \mathbf{V}_t \right]. \quad (\text{A4})$$

In practice, with finite data available to estimate the time series model, finite orders are necessary. Truncating at sufficiently large lag p into the past gives the order- p integrated synergy ψ_p ,

$$\psi_p = I[\mathbf{V}_{t-p:t} \wedge \mathbf{V}_t] - \max_p I_{\cup} \left[\left(\mathbf{M}_{t-p:t}^{(1)}, \dots, \mathbf{M}_{t-p:t}^{(r)} \right) \wedge \mathbf{V}_t \right], \quad (\text{A5})$$

where all quantities are defined as in the order-1 case with the inclusion of additional lags.

In general, a vector stochastic process that is VAR(p) overall need not be VAR(p) in any of its components (Lütkepohl), and thus infinite orders may be necessary for the $I_{\cup} \left[\left(\mathbf{M}_{t-p:t}^{(1)}, \dots, \mathbf{M}_{t-p:t}^{(r)} \right) \wedge \mathbf{V}_t \right]$ term. While we do not pursue this avenue here, one approach would be to take sufficiently many lags into the past based on the spectral radius of the coefficient matrix from the VAR model.

Appendix C. Estimating Order-1 and Order- p Integrated Synergy

In all cases, we estimate the integrated synergy using a plug-in estimate from a linear Gaussian VAR model fit to the data using ordinary least squares, using (A3) or its order- p generalization to compute the union information in (A1) or (A5). For the order- p integrated synergy, the model order p is chosen using Schwarz's Bayesian Information Criterion assuming a linear Gaussian VAR model (Lütkepohl) with a maximum possible model order of $p_{\max} = 60$. See the Section 4.1 of (Mediano, et al.) for more details on estimating the linear Gaussian VAR and computing its associated information-theoretic properties.

References

1. Griffith, V. A principled infotheoretic Φ -like measure. arXiv: 1401.0978v3 [cs.IT]
2. Lütkepohl, H. New Introduction to Multiple Time Series Analysis., **2005**, New York, NY: Springer Science and Business Media.
3. Mediano, P.A.M., Seth, A.K. and Barrett, A.B. Measuring integrated information: comparison of candidate measures in theory and simulation. *Entropy*. **2019**, 21, 77
4. Olbrich, E. Bertschinger, N. and Rauh, J. Information decomposition and synergy. *Entropy* **2015**, 17, 3501-3517.
5. Tononi, G., Sporns, O. and Edelman, G. A measure for brain complexity: Relating functional segregation and integration in the nervous system. *Proceedings National Academy of Science (USA)* **1994**, 91(11), 5033-5037.

**Table S1. Mean and Standard Deviation
Signals containing the alpha component**

Measure	Eyes Closed Mean \pm StDev	Eyes Open Mean \pm StDev
Binary Lempel Ziv Signals mean normalized	174.250 ± 32.811	237.667 ± 47.483
Binary Lempel Ziv Signals normalized by mean and by standard deviation	188.250 ± 34.489	239.500 ± 38.339
C_N Tononi, et al. 1994 Equation 4	5.5095 ± 0.659	4.863 ± 0.657
$\psi p = 1$ Mediano, et al. 2019 Equation 23	6.415 ± 0.942	5.559 ± 1.017
ψp via BIC Mediano, et al. Equation 23	0.2535 ± 0.0843	0.4548 ± 0.1569

**Table S2. Mean and Standard Deviation
Signals with the alpha component removed**

Measure	Eyes Closed Mean \pm StDev	Eyes Open Mean \pm StDev
Binary Lempel Ziv Signals mean normalized	230.667 ± 47.266	273.417 ± 46.800
Binary Lempel Ziv Signals normalized by mean and by standard deviation	227.250 ± 38.248	266.917 ± 34.935
C_N Tononi, et al. 1994 Equation 4	4.8328 ± 0.760	4.623 ± 0.646
$\psi p = 1$ Mediano, et al. 2019 Equation 23	5.687 ± 0.802	5.012 ± 0.847
ψp via BIC Mediano, et al. Equation 23	0.3717 ± 0.1922	0.4396 ± 0.1591

Table S3. Effect Size: Eyes Open - Eyes Closed
Signals containing the alpha component, robust estimator

Measure	Estimated Effect Size	95% Confidence Interval
Binary Lempel Ziv Signals mean normalized	1.37	(1.025, 2.665)
Binary Lempel Ziv Signals normalized by mean and by standard deviation	1.14	(0.881, 2.280)
C_N Tononi, et al. 1994 Equation 4	-1.48	(-4.121, -0.829)
$\psi p = 1$ Mediano, et al. 2019 Equation 23	-0.75	(-1.537, -0.501)
ψp via BIC Mediano, et al. 2019 Equation 23	0.79	(0.233, 2.003)

Table S4. Effect Size: Eyes Open - Eyes Closed
Signals with the alpha component removed, robust estimator

Measure	Estimated Effect Size	95% Confidence Interval
Binary Lempel Ziv Signals mean normalized	0.89	(0.269, 2.030)
Binary Lempel Ziv Signals normalized by mean and by standard deviation	1.63	(0.412, 6.783)
C_N Tononi, et al. 1994 Equation 4	-0.28	(-1.035, 1.401)
$\psi p = 1$ Mediano, et al. 2019 Equation 23	-1.16	(-3.106, -0.344)
ψp via BIC Mediano, et al. 2019 Equation 23	-0.03	(-1.015, 2.241)

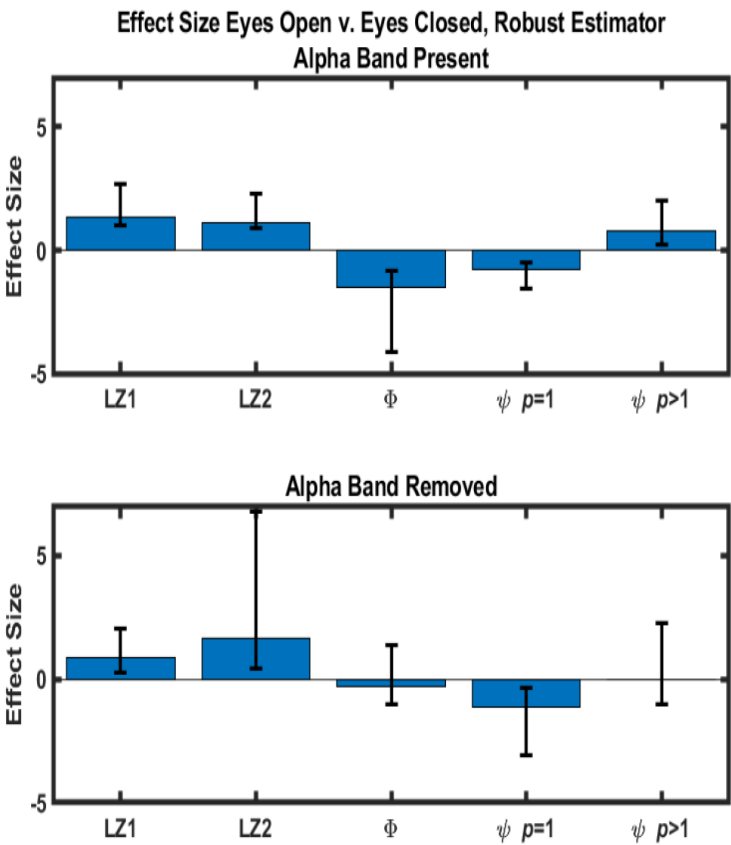


Figure S1. Eyes open versus eyes closed effect size as quantified by the robust estimator for five measures. Confidence intervals were determined with a bias-correct and adjusted bootstrap. (A). Effect sizes calculated with signals that contain the alpha component. (B). Effect sizes calculated after the alpha component had been removed with an 8 Hz to 13 Hz stopband filter.

Table S5. Kendall’s tau correlation
Eyes Closed, Alpha Component Present

	LZ1	LZ2	Φ	$\psi p = 1$	ψp via BIC
LZ1	1.0000	0.7273	-0.2424	-0.0909	0.2424
LZ2		1.0000	-0.3333	-0.3636	0.2121
Φ			1.0000	0.2424	-0.1515
$\psi p = 1$				1.0000	-0.5455
ψp via BIC					1.0000

**Table S6. Kendall's tau correlation
Eyes Closed, Alpha Component Removed**

	LZ1	LZ2	Φ	$\psi p = 1$	ψp via BIC
LZ1	1.0000	0.6364	-0.1818	0.0909	0.1818
LZ2		1.0000	-0.3030	0.0303	0.1212
Φ			1.0000	-0.1212	0.0303
$\psi p = 1$				1.0000	-0.4242
ψp via BIC					1.0000

**Table S7. Kendall's tau correlation
Eyes Open, Alpha Component Present**

	LZ1	LZ2	Φ	$\psi p = 1$	ψp via BIC
LZ1	1.0000	0.9313	-0.0763	-0.4428	0.1679
LZ2		1.0000	-0.0606	-0.3939	0.0909
Φ			1.0000	0.0000	-0.2424
$\psi p = 1$				1.0000	-0.0909
ψp via BIC					1.0000

**Table S8. Kendall's tau correlation
Eyes Open, Alpha Component Removed**

	LZ1	LZ2	Φ	$\psi p = 1$	ψp via BIC
LZ1	1.0000	0.5198	-0.0153	-0.2595	-0.1069
LZ2		1.0000	-0.0313	-0.2189	-0.0938
Φ			1.0000	-0.2121	-0.0606
$\psi p = 1$				1.0000	-0.1212
ψp via BIC					1.0000

References

1. Tononi, G., Consciousness as integrated information: a provisional manifesto. *Biological Bulletin* **2008**, 215, 216-242.
2. Searle, J.R., Can information theory explain consciousness? *New York Review of Books* **2013**, 2013-01-10.
3. Aaronson, S., Why I am not an integrated information theorist. 2014 <https://www.scottaaronson.com/blog/?p=1799>.
4. Tegmark, M., Improved measures of integrated information. *PLoS Computational Biology* **2016**, 12(11). e10051
5. Mediano, P.A.M., Seth, A.K. and Barrett, A.B., Measuring integrated information: comparison of candidate measures in theory and simulation. *Entropy* **2019**, 21:77
6. Berger, H., Über das Elektroenkephalogramm des Menschen. *Archiv für Psychiatrie und Nervenkrankheiten*, **1929**, 87, 527-570.
7. Adrian, E.D. and Matthews, B.H.C., The Berger rhythm: potential changes from the occipital lobes of man. *Brain*, **1934**, 57, 355-385.
8. Hartoyo, A., Cadusch, P.J., Liley, D.T.J. and Hicks, D.G. Inferring a simple mechanism for alpha-blocking by fitting a neural population model to EEG spectra. *PLoS Computational Biology* **2020**, 16(4).
9. Liley, D.T.J. and Muthukumaraswamy, D.S. Evidence that alpha blocking is due to increases in system-level oscillatory damping and not neuronal population desynchronization. *NeuroImage* **2020**, 208, 116408.
10. Rapp, P.E., Cellucci, C.J., Watanabe, T.A.A. and Albano, A.M. Quantitative characterization of the complexity of multichannel human EEGs. *International Journal of Bifurcation and Chaos* **2005**, 15, 1737-1744.
11. Lempel, A. and Ziv, J. On the complexity of finite sequences. *IEEE Transactions on Information Theory* **1976**, IT-22, 75-81.
12. Golub, G.H. and Reinsch, C. Singular value decomposition and least squares solutions. *Numerical Mathematics* **1970**, 14, 403-420.
13. Golub, G.H. and Reinsch, C. *Handbook for Automatic Computation. Vol. II. Linear Algebra* Springer Verlag: Heidelberg, Germany **1971**, pp. 134-151

14. Watanabe, T.A.A., Cellucci, C.J., Kohegyi, E., Bashore, T.R., Josiassen, R.C., Greenbaun, N.N. and Rapp, P.E. (2003). The algorithmic complexity of multichannel EEGs is sensitive to changes in behavior. *Psychophysiology* **2003**, 40, 77-97.
15. Tononi, G., Sporns, O. and Edelman, G. A measure for brain complexity: Relating functional segregation and integration in the nervous system. *Proceedings National Academy of Science (USA)* **1994**, 91(11), 5033-5037.
16. van Putten, M.J.A.M. and Stam, C.J. (2001). Application of a neural complexity measure to multichannel EEG. *Physics Letters* **2001**, 281A, 131-141.
17. Morgera, S.D. Information theoretic covariance complexity and its relation to pattern recognition. *IEEE Transactions on Systems, Man and Cybernetics* **1985**, 15, 608-619.
18. Tononi, G. An information integration theory of consciousness. *BMC Neuroscience* **2004**, 5, 42-63.
19. Oizumi, M., Albantakis, L. and Tononi, G. From phenomenology to the mechanisms of consciousness: integrated information theory 3.0. *PLoS Computational Biology* **2014**, 10(5).
20. Griffith, V. A principled infotheoretic φ -like measure. **2014**, arXiv: 1401.0978v3[cs.IT]2014-10-09.
21. van Cappellen van Walsum, A.-M., Pijnenburg, Y.A.L., Berendse, H.W., van Dijk, B.W., Knol, D.L., Sheltens, Ph. and Stam, C.J. A neural complexity measures applied to MEG data in Alzheimer's disease. *Clinical Neurophysiology* **2003**, 114, 1034-1040.
22. Trujillo, L., Stanfield-Wiswell, C.T. and Vela, R.D. The effect of electroencephalogram (EEG) reference on information theoretic measures of the complexity and integration of EEG signals. *Frontiers in Neuroscience* **2017**, 11: 425.
23. Rousseeuw, P.J. and Croux, C. Alternatives to the median absolute deviation. *Journal of the American Statistical Association* **1993**, 88(424), 1273-1283.
24. Efron, B. Better bootstrap confidence intervals. *Journal of the American Statistical Association* **1987**, 82, 171-185.
25. McKhann, G., Drachman, D., Folstein, M., Katzman, R., Price, D. and Stadlan, E.M. and an additional five authors. Clinical diagnosis of Alzheimer's disease: report of the NINCDS-ADRDA Work Group under the auspices of the Department of Health and Human Services Task Force on Alzheimer's disease. *Neurology* **1984**, 34, 939-944.
26. van Albada, S.J. and Robinson, P.A. Transformation of arbitrary distributions to the normal distribution with application to EEG test-retest reliability. *Journal of Neuroscience Methods* **2007**, 161(2), 205-211.