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Article

Secrecy Analysis and Error Probability of LIS-aided Communication Systems under Nakagami-m Fading

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Abstract: In this work, we derive the spectral efficiency, secrecy outage probability, and bit error rate of a communication system assisted by a large intelligent surface (LIS). We consider a single-antenna user and an array of antennas at the transmitter side and the possibility of a direct link between transmitter and receiver. Additionally, there is a single-antenna eavesdropper with a direct link to the transmitter, which is modeled as a Nakagami-m distributed fading coefficient. The channels from transmitter to the LIS and from the LIS to the user may or may not have the line-of-sight (LoS) and are modeled by the Nakagami- m distribution. Moreover, we assume that the LIS elements perform non-ideal phase cancellation leading to a residual phase error that assumes a Von Mises distribution. We show that the resulting channel can be accurately approximated by a Gamma distribution whose parameters are analytically estimated using the moments of the equivalent signal-to-noise ratio. We also provide an upper bound for the error probability for M-QAM modulations. With the derived formulas, we analyze the effect of the strength of the LoS link by varying the Nakagami parameter, m.

Keywords: Large intelligent surfaces; 6G; bit error probability; Nakagami fading; Von Mises distribution

0. Introduction

Intelligent surfaces are a promising technology for beyond fifth-generation (B5G) systems, given the number of papers that emphasize their advantages [1–3]. Whether compared to relays [4,5] or even when used to enhance the power of millimeter wave technologies [6,7], the characteristic of reflecting signals with extreme precision with almost no power consumption, significantly reduces interference, improves the received signal-to-noise ratio and energy efficiency, mainly when the direct path between transmitter and destination is weak and the reflecting surfaces are really large, composed of many reflectors.

Also known as large reflecting surfaces, they have recently been studied as a solution for different modulation schemes and communication channels. Their performance metrics show their significant potential for mobile communications. For example, Yang et al. [8] proposed a transmission protocol to reduce the channel estimation overhead when adjacent cells share the same reflection coefficients. Optimization methods are used to allocate the transmit power and maximize the achievable rate in an orthogonal frequency division multiplexing (OFDM) scheme under frequency-selective channels.

Basar, in [9], presented a mathematical framework to obtain the signal-to-noise ratio and derive the symbol error probability of a LIS-aided communication system, with or without knowledge of the channel phases. The author also proposed an access point sending signals directly to the users aided by a LIS system.

In [7], Wymeersch et al. emphasized that although there are already other techniques for high frequencies (0.1 to 1 THz), these technologies are limited by multipath

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propagation and obstacles presented in the environment. In this case, LIS can control the physical propagation environment, decrease energy consumption and simplify location and mapping systems, creating a LoS path between transmitter and receiver.

In [6], the authors presented solutions for the adjustment of the LIS elements' phases, which optimizes the channel capacity and the precoder applied at the transmitter side. Elbir et al. [10] developed a deep learning framework to obtain the channel state information (CSI) in a massive multiuser MIMO system aided by a LIS. The authors estimated the composite channel and the direct path for each user through a convolutional neural network whose input are the received pilot signals. Lin et al. [11] performed channel estimation by applying Lagrange multipliers and a dual ascent-based scheme iteratively. They also found a closed-form solution for Cramer-Rao lower bounds and proposed a method that improves the accuracy of the classical least-square method.

Taha et al. [12] presented an energy-efficient architecture where all the LIS's elements are passive except for a few distributed active elements that are arranged in a non-uniform manner. The reflector array applies deep learning models to obtain the optimal matrices of phase shifts.

Although a LIS is usually a panel of reflectors physically organized in planar shapes, Hu et al. [13] proposed alternative structures with a three-dimensional spatial configuration with spherical surfaces. In addition to a broader coverage, they have a more straightforward positioning system when compared to the conventional planar arrays.

In far-field communications, LIS must be large to be competitive with classic massive MIMO systems and to compensate for multipath propagation and electromagnetic interference. Besides that, the optimization of the phase shifts associated with each element of the LIS is a great challenge. Therefore, in [14], Najafi et al. proposed an optimization method based on the physical modeling of the propagation and clusterization of a thousand reflectors into small subsets, also known as tiles. Based on concepts from radar communications, they modeled the impact of each tile on the overall channel, calculated the associated electric and magnetic fields, and showed that it is possible to optimize the operation of the LIS to maximize some quality of service (QoS) criteria.

On the other hand, Garcia et al. [15] focused on near-field environments. They establish a relation between the array size and the Fresnel zones and state that the punctual approximation of the scattering characterization presented dependence with the second and third-order moments of the distance. On the contrary, for far-field, the dependence is given for the fourth power. Kishk et al. [16] employed some stochastic geometry tools to analyze the effect of the large-scale deployment of LIS on the performance of cellular networks in the presence of blockages surfaces. They established a relation between the density of LIS panels and blockages.

In addition to the works related to optimal estimation and power control in transmission systems aided by LIS, it has become a trend to compute the channel's capacity in the face of eavesdroppers. The question to answer is "Does such a system offer the physical layer security that prevents an intruder from receiving a signal not intended for him?" The secrecy outage probability metric can be used to answer this question since it consists of the probability that the instantaneous secrecy capacity be less or equal to a given capacity threshold.

For the case of Gaussian distributed channels and considering parameters such as the distances between devices and the number of LIS elements, Yang et al. derived closed-form expression for SOP. Trigui et al. assumed a more realistic model in which there are errors caused by phase quantization. By leveraging Fox's H transforms, they obtained exact SOP expression under the assumption of many reconfigurable elements of LIS and channels distributed according to the Rayleigh distribution.

On the other hand, Ai et al. demonstrated the potential of improving secrecy with LIS aid under different scenarios where a passive eavesdropper is attempting to retrieve the transmitted information: a vehicular-to-vehicular and a vehicular-to-infrastructure.

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Makarfi et al. showed how the source power, eavesdropper distance, the number of LIS elements, the source-to-relay distance, and the secrecy threshold affect the secrecy capacity and SOP when the vehicular source uses a LIS as an access point.

In this paper, we derive the secrecy outage probability (SOP), Bit Error Probability (BEP) and Spectral Efficiency of a system in which the base station is equipped with K antennas and transmit to a single user aided by large intelligent surface with N reflectors. We assume that the fading channels are Nakagami-m distributed. We also assume a non-ideal phase cancellation leading to a phase error which is modelled by a Von Misses distribution with concentration parameter κ.

The paper is organized as follows: the Section 1 presents the system model and the initial equations that based our formulation of the problem while the Section 2 presents the closed-form expressions for spectral efficiency, BER, its upper bound and SOP. Finally, Section 3 demonstrates the validity of the proposed analytical expressions through Monte Carlo simulations.

1. System Model

As shown in Fig. 1, we consider a base station (BS), the source, equipped with an antenna array of K antennas transmitting the same signal to a unique single antenna user, the destination. Additionally, a large intelligent surface system with N reflecting elements aids the system. Both channels BS to LIS and LIS to user are modeled by the Nakagami-m distribution. There is a direct link between the user and the BS and other, between an eavesdropper and the BS whose channel is also Nakagami-m distributed. The signal that arrives at the destination antenna is given by

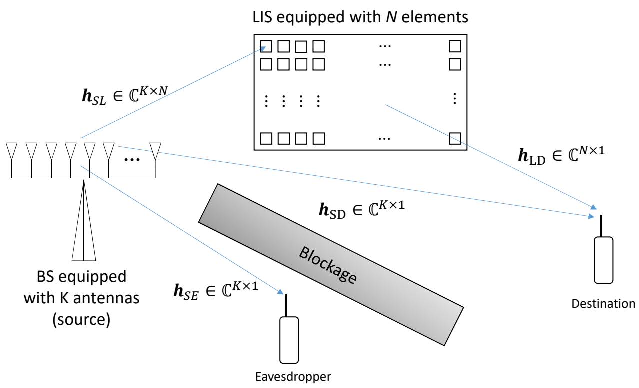

Figure 1. System model with eavesdropper link.

$$r = \left(\mathbf{h}_{SL}^{H}\mathbf{\Phi}^{H}\mathbf{h}_{LD}^{H} + \mathbf{h}_{SD}^{H}\right)\mathbf{\Psi} + \eta, \tag{1}$$

where $\mathbf{h}_{SL} \in \mathbb{C}^{N\times 1}$ is the link between the source and the LIS, $\mathbf{h}_{LD} \in \mathbb{C}^{K\times N}$ is the link between the LIS and the destination and $\mathbf{h}_{SD} \in \mathbb{C}^{K\times 1}$ is the direct link between the source and the destination. The term $\mathbf{\Phi} \in \mathbb{C}^{N\times N}$ is a diagonal matrix, whose elements are the phase shifts $e^{-j\phi_1} \ldots e^{-j\phi_N}$ applied by the LIS to the incident electromagnetic waves. The LIS's phases, ϕ_n $\forall n$, are assumed continuous in the interval of 0 to 2π radians. The term $\mathbf{\Psi} = \mathbf{v}s$ represents the precoded signal, where the data symbol is $s \sim \mathcal{CN}(0,1)$ and the optimal precoding vector is applied by BS, according to the MRT criterion, i.e,

$$\mathbf{v} = \frac{\mathbf{h}^{H}}{||\mathbf{h}||}. \tag{2}$$

Finally, the term $\eta \sim \mathcal{CN}(0,1)$ is additive white Gaussian noise (AWGN) with zero mean and unit variance. We suppose there is no LoS in the direct link and that it is modeled as complex normal random variable, with zero mean and variance σ_{SD}^2. Additionally, the magnitude of the channels $\mathbf{h}_i = |\mathbf{h}_i|e^{j\phi_i}$ with $i \in \{SL, LD\}$ are Nakagami-m distributed with probability density function (PDF) given by

$$f_X(x) = \frac{2m_i^{m_i}}{\Gamma(m_i)\Omega_i^{m_i}} x^{2m_i-1} e^{-\frac{m_i}{\Omega_i}x^2}. \tag{3}$$

In this work, the parameters m_i and Ω_i refer to the shape and spread of the Nakagami-m PDF, respectively. The distribution of the phases is not specified since, for our model, these phases are not relevant.

The overall channel, including the LIS and the antenna array, can be defined as

$$\mathbf{h} = \mathbf{h}_{SL}^H \mathbf{\Phi}^H \mathbf{h}_{LD}^H + \mathbf{h}_{SD}^H. \tag{4}$$

We can rewrite (4) in scalar form as

$$h_k = \sum_{i=1}^{N} |h_{ki}^{LD}| \left|h_i^{SL}\right| e^{j\left(\phi_i - \phi_i^{SL} - \phi_{ki}^{LD}\right)} + h_k{}^{SD},\ k \in \mathbb{N} \tag{5}$$

Perfect phase cancelling occurs when $\phi_i - \phi_i^{SL} - \phi_{ki}^{LD} = 0$. Therefore, in this scenario we have that $\phi_i = \phi_i^{SL} + \phi_{ki}^{LD}$. However, the task of removing the overall channel phase is unfeasible and some residual phase noise is left behind, in this case $\phi_i - \phi_i^{SL} - \phi_{ki}^{LD} = \theta_{ki}$, where θ_{ki} is the phase noise, which, in this work, is modeled as a Von Mises random variable with concentration parameter κ.

Therefore we rewrite the overall channel as

$$h_k = \sum_{i=1}^{N} |h_{ki}^{LD}| \left|h_i^{SL}\right| e^{j\theta_{ki}} + h_k^{SD}. \tag{6}$$

We hope that there will be no phase errors in the best case analysis, but this situation is entirely unfeasible. However, it is possible to estimate an optimal phase adjustment matrix that will provide a performance as good as possible, so we expect that, on average, the phase errors be zero.

We use the zero mean Von Mises circular distribution to model the phases of each antenna's fading coefficients [17]. It has nonzero support in the interval $-\pi$ and π and a concentration parameter κ associated with the quality of the phase adjustment promoted by the LIS and the efficiency of the channel estimation method.

The moment-generating function (MGF) of the Von Mises distribution will be useful since the phase adjustments are represented by a complex exponential. With the MGF, we can calculate the statistical moments associated with the channel coefficients.

Let X be a Von Mises random variable, therefore its MGF is given by $\varphi_p = E\left[e^{-jpX}\right] = \alpha_p + j\beta_p$. Since the zero mean Von Mises distribution is symmetric about zero, then the imaginary part of the MGF $\beta_p = E[\sin pX] = 0$, and the real part is $\alpha_p = \frac{I_p(\kappa)}{I_0(\kappa)}$, where $I_p(\kappa)$ is the modified Bessel function of first kind and order p.

Considering that the precoder is the normalized hermitian of the overall channel, therefore the SNR of the desired link is

$$\gamma_D = \left(\mathbf{h}_{SL}^H \mathbf{\Phi}^H \mathbf{h}_{LD}^H + \mathbf{h}_{SD}^H\right)\mathbf{v} = \|\mathbf{h}\|^2. \tag{7}$$

Assuming, as an approximation, that γ_D is Gamma distributed, then, its statistical moments, α and β, can be estimated as

$$\alpha = \frac{\mathbb{E}[\gamma_D]^2}{\text{var}(\gamma_D)},\ \ \beta = \frac{\mathbb{E}[\gamma_D]}{\text{var}(\gamma_D)}, \tag{8}$$

where α and β are the shape and rate parameters, respectively.

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The assumption that the distribution of γ_D is Gamma distributed can be assessed using the Hellinger distance. According to Beran [18], the Hellinger distance between two arbitrary discrete probability distributions p_k and q_k can be obtained as

$$D_{HL} = \frac{1}{\sqrt{2}} \sqrt{\sum_{k=0}^{N_p-1} (\sqrt{p_k} - \sqrt{q_k})^2}, \tag{9}$$

where N_p is the number of samples available to calculate the distance. The Hellinger distance is limited in the interval $0 \leq D_{HL} \leq 1$ and can be considered as an absolute metric.

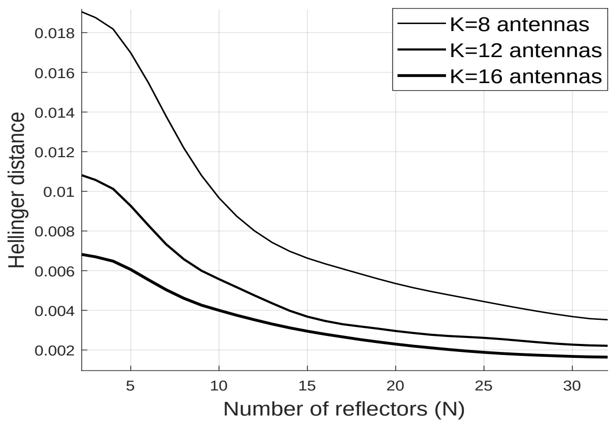

Figure 3. Hellinger distance.

In Fig. 3, we can see a Monte Carlo simulation of the distribution in comparison with the expected Gamma PDF. We consider 10^6 iterations, unit variance for all channels, Von Mises concentration parameter $\kappa = 2$ and the Nakagami parameter $m = 2$. We can clearly notice that the Hellinger distance decreases when N and K increase. In the last case, for $K = 16$, the decrease is even more pronounced. Therefore, this accurate approximation motivates us to further formulate the problem.

2. Problem Formulation

Knowing that the SNR can be approximated by a Gamma random variable, we derive closed-form expressions for spectral efficiency, BER and SOP in the next subsections.

2.1. Spectral Efficiency

The average spectral efficiency of the system can be defined as

$$C = \mathbb{E}[\log_2(1+\gamma)] = \int_0^\infty \log_2(1+\gamma) \frac{\beta^\alpha}{\Gamma(\alpha)} \gamma^{\alpha-1} e^{-\beta\gamma} d\gamma, \tag{10}$$

whose approximated solution is given by (11); $\psi^{(0)}(.)$ is the digamma function, $\Gamma(.)$ is the gamma function, $\Gamma(.,.)$ is the incomplete gamma function and ${}_2F_2(a,b;c,d;e)$ is the generalized hypergeometric function.

$$C = \frac{(-1)^{-\alpha}\beta}{\log(2)\Gamma(\alpha)} \left[(-1)^\alpha \left(\Gamma(\alpha-1)\, {}_2F_2(1,1;2,2-\alpha;\beta) + \frac{\Gamma(\alpha)(\psi^{(0)}(\alpha) - \log(\beta))}{\beta} \right) + \pi\beta \csc(\pi\alpha)(\Gamma(\alpha) - \Gamma(\alpha,-\beta)) \right] \tag{11}$$

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2.2. Bit Error Probability

The error probability for the M-QAM modulation can be approximately obtained by [19]

$$P_e^{QAM}(\gamma) = 1 - \left(1 - 2\left(1 - \frac{1}{\sqrt{\mathcal{M}}}\right)Q\left[\sqrt{\frac{3\gamma \log_2 \mathcal{M}}{(\mathcal{M}-1)}}\right]\right)^2. \tag{12}$$

Assuming, as an approximation, that γ is Gamma distributed, the mean bit error probability $\bar{P}_e^{QAM}$ can be calculated by (13).

$$\bar{P}_e^{QAM}(\gamma) = \int_0^\infty P_e^{QAM}(\gamma v) f_{\|\mathbf{w}\|^2}(v)dv \tag{13}$$

Ferreira et al. [20] derived a close upper bound for the mean error probability of an M-QAM schema under Gamma fading, by using the approximation given by (14).

$$\bar{P}_e^{QAM}(\gamma) \approx \frac{4}{\log_2 M} Q\left(\sqrt{\frac{3\gamma \log_2 M}{M-1}}\right). \tag{14}$$

The Chernoff bound, using the approximation

$$Q(x) \leq \frac{1}{2} e^{-\frac{1}{2}x^2},$$

obtained by Ferreira et al. [20], is given as

$$\bar{P}_e^{QAM}(\gamma) < \frac{1.38629\left(\frac{2.16404\gamma \log(M)}{(M-1)\beta} + 1\right)^{-\alpha}}{\log(M)}, \tag{15}$$

which is close to the exact solution.

2.3. Secrecy Outage Probability

Considering that an eavesdropper has access to the signal provided by the source and according to [21], the secrecy capacity associated with the two fading channels can be obtained as

$$C = \begin{cases} \ln(1+\gamma_D) - \ln(1+\gamma_E) & \gamma_D > \gamma_E \\ 0 & \gamma_D \leq \gamma_E \end{cases}, \tag{16}$$

where γ_E is the SNR of the link between the source and the eavesdropper. Therefore, the SOP is defined as the probability that the instantaneous secrecy capacity, C, be less than or equal to a given capacity threshold, $\ln(1+\gamma_{th})$, which is expressed as

$$\text{SOP} = Pr\left[\ln\frac{(1+\gamma_D)}{(1+\gamma_E)} \leq \ln(1+\gamma_{th})\right] = \int_0^\infty \int_0^{(1+\gamma_E)(1+\gamma_D)-1} f_{\gamma_E}(w) f_{\gamma_D}(u) du dw, \tag{17}$$

where $Pr[.]$ denotes the probability of a random event.

Considering a Nakagami-m distributed eavesdropper channel, the SOP can be obtained as (18). Its solution is derived as follows:

$$\text{SOP} = \int_0^\infty \int_0^{(\gamma_{th}+1)(x+1)-1} \frac{\beta^\alpha (2m^m x^{2m-1}) y^{\alpha-1} \exp(-\beta y) \exp\left(-\frac{mx^2}{\Omega}\right)}{\Gamma(\alpha)(\Omega^m \Gamma(m))} dy dx \tag{18}$$

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Solving the first integral, the remaining expression can be written as

$$\mathrm{SOP} = \int_0^\infty \frac{2e^{-\frac{mx^2}{\Omega}} m^m x^{2m-1} \Omega^{-m} [\Gamma(\alpha) - \Gamma(\alpha, \beta(x + \gamma_{th}(1+x)))]}{\Gamma(m)\Gamma(\alpha)} dx, \tag{19}$$

whose term $\Gamma(\alpha) - \Gamma(\alpha, \beta(x + \gamma_{th}(1+x)))$ can be rewritten as function of the lower incomplete gamma function as

$$\gamma(s, x) \triangleq \int_0^x t^{s-1} e^{-t} dt. \tag{20}$$

Representing the exponential in terms of power series, we have that the incomplete Gamma function can be written as

$$\int_0^x t^{s-1} e^{-t} dt = \int_0^x \sum_{k=0}^\infty (-1)^k \frac{t^{s+k-1}}{k!} dt = x^s \sum_{k=0}^\infty \frac{(-x)^k}{k!(s+k)}.$$

Applying the expansion (21) in (18), the SOP can be rewritten as (2.3), where ${}_p\tilde{F}_q$ is the regularized ${}_pF_q$ hypergeometric function. The expression given in (2.3) is a infinite sum, however we have found that the error by considering only the first term is very small, as will be shown in the numerical results section.

$$\mathrm{SOP} = \sum_{k=0}^{\infty} \frac{(-1)^k \beta^{\alpha+k} \gamma_{th}^{\alpha+k}}{\Gamma(\alpha)\Gamma(k+1)} \times \left(\frac{\pi m^m 2^{-\alpha-k} v^{-2m} \Omega^{-m} \Gamma\left(m+\frac{1}{2}\right) \csc(\pi(\alpha+k+2m))\, {}_2\tilde{F}_2\left(m, m+\frac{1}{2}; \frac{1}{2}(k+2m+\alpha+1), \frac{1}{2}(k+2m+\alpha+2); -\frac{m}{v^2\Omega}\right)}{\Gamma(-k-\alpha+1)} \right.$$
$$+ \frac{\pi^{3/2} \Omega^{\frac{\alpha+k}{2}} m^{\frac{1}{2}(-\alpha-k)} v^{\alpha+k}}{2\Gamma(m)} \left(\frac{2\csc\left(\frac{1}{2}\pi(\alpha+k+2m)\right) {}_2\tilde{F}_2\left(\frac{1}{2}(-k-\alpha), \frac{1}{2}(-k-\alpha+1); \frac{1}{2}, \frac{1}{2}(-k-2m-\alpha+2); -\frac{m}{v^2\Omega}\right)}{\alpha+k} \right.$$
$$\left.\left. - \frac{\sqrt{m} \sec\left(\frac{1}{2}\pi(\alpha+k+2m)\right) {}_2\tilde{F}_2\left(\frac{1}{2}(-k-\alpha+1), \frac{1}{2}(-k-\alpha+2); \frac{3}{2}, \frac{1}{2}(-k-2m-\alpha+3); -\frac{m}{v^2\Omega}\right)}{v\sqrt{\Omega}} \right)\right), \text{where } v = \frac{1+\gamma_{th}}{\gamma_{th}}.$$

3. Numerical Results

This section analyzes the accuracy of the proposed approximations and discusses the improvements in capacity provided by LISs. In unspecified cases, we adopt, by default, the Nakagami-m shape parameters $m_{SL} = m_{LD} = m = 2$, the spread parameters $\Omega_{SL} = \Omega_{LD} = \Omega$ were chosen to make the variances $\sigma_{SL}^2 = \sigma_{LD}^2 = 1$, the Von Mises concentration parameter $\kappa = 2$, $K = 16$ antennas at the source, the size of the M-QAM constellation is $M = 16$, and the number of iterations is 10^6 for each Monte Carlo simulation.

In Fig. 4, we compare the simulated and theoretical BERs considering the Von Mises and uniformly distributed phase errors. The theoretical BER is obtained assuming that the overall fading channel has a Gamma distribution. Note that the larger the number of reflectors, N, the smaller the error probability for any SNR value. When the phase errors are uniformly distributed ($\kappa = 0$) the error probability is higher than in the Von Mises scenario. This result shows the importance of estimating the phases and channel gains properly and choosing the optimization method to find the best LIS phase-shifts. Uniformly distributed phase noise indicates that the algorithm has equally likely chances to present large phase errors (close to $\pm\pi$) or small phase errors (close to zero). That implies greater bit error probabilities, which can be compensated only with a

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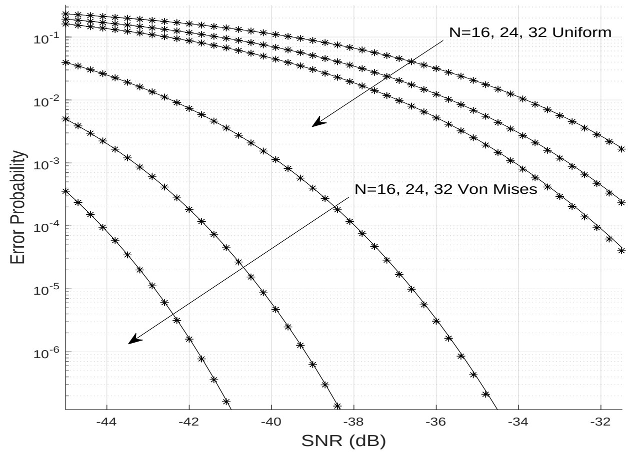

Figure 4. Bit error probability varying the SNR

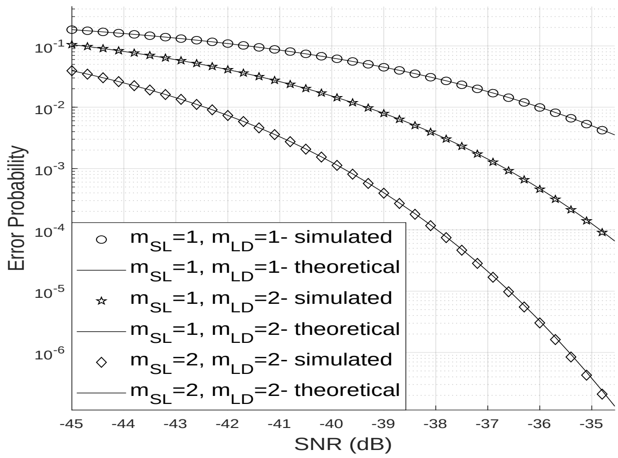

Figure 5. Bit error probability varying the LoS strength

large number of antennas at the transmitter or with a large number of reflectors at the LIS.

Large reflecting surfaces can produce a LoS link between the transmitter and the user even in a far-field Rayleigh fading channel. However, in a near-field scenario, a stronger LoS link (higher Nakagami m parameter) implies a lower probability of error, as we can see in Figure 5.

The upper bound (15) for the error probability proposed by Ferreira et al. [20] is very close to the bit error rate as shown Figure 6, even when the fading coefficients are Nakagami-m distributed.

We consider two scenarios for the computation of the spectral efficiency, the first one has uniformly distributed phase noise, and the second one has a Von Mises distributed phase, as we can see in Figure 7. Note that the efficiency is higher for the case in which the phase errors have a Von Mises distribution. As the probability distribution function of the phase noise is close to zero, the spectral efficiency is consequently more significant. We can also notice that the efficiency increases with the number of antennas at the transmitter and/or the number of reflectors at the LIS. Besides, it is also noticeable that the formula proposed in this work fits the simulation very closely.

In Fig. 8, we can see the secrecy outage probability for a Nakagami$-m$ eavesdropper link with $\Omega = 1$ and $m = 1.4$. In this result, we truncated the sum up to the index 1000 and performed 10^6 iterations of the Monte Carlo method to generate the Gamma distributed random variables with parameters α and β calculated from the channel characteristics, Von Mises parameter $\kappa = 2$, $K = 2$ antennas, unity variance and Nakagami-m fading distribution for all channels between the antennas, the LIS and the user. As we

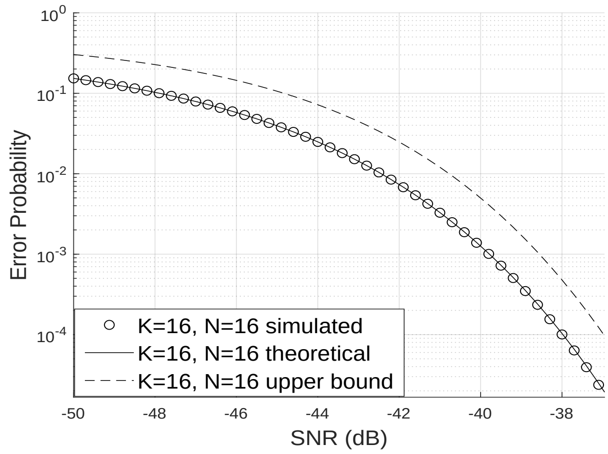

Figure 6. Bit error probability upper bound

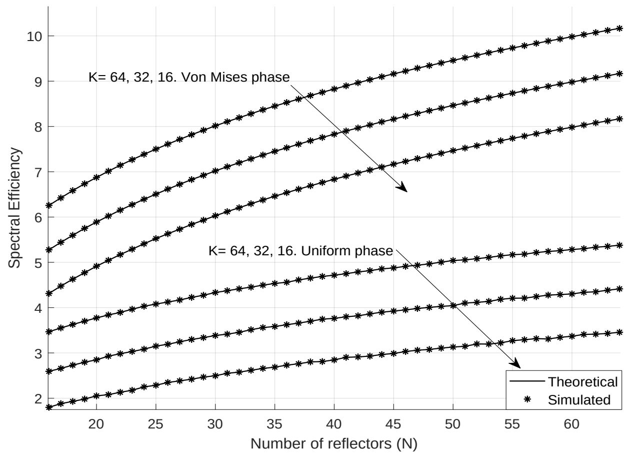

Figure 7. Spectral Efficiency varying M and N

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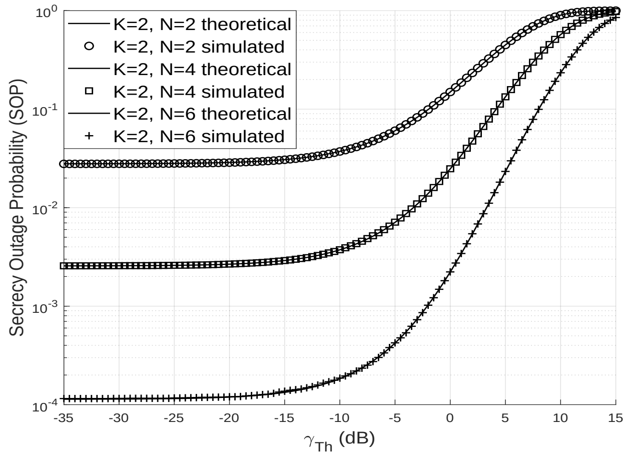

Figure 8. Secrecy Outage Probability (SOP) for $K = 2$ antennas and different number of reflectors (N)

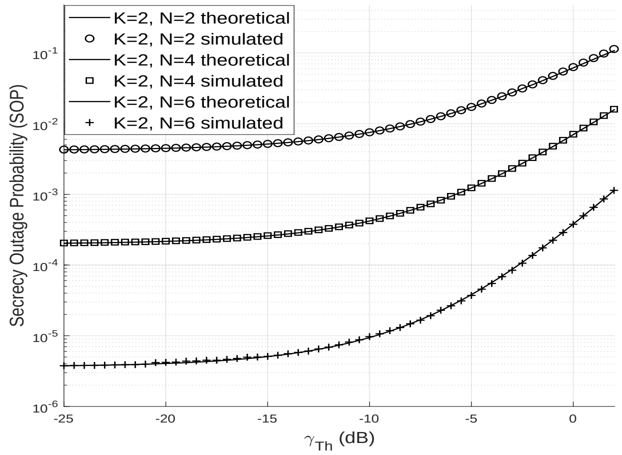

Figure 9. First order approximation of the Secrecy Outage Probability (SOP) for $K = 2$ antennas and different number of reflectors (N)

can see, the greater the number of reflectors, the greater will be the SOP. Moreover, the greater is the SNR, the greater will be the SOP.

The first order approximation of the SOP, considering that the Nakagami-m parameters are $m = 2.5$ and $\Omega = 0.1$ for all the channels in the system model, is also close to the simulated result as shown in 9.

4. Final Considerations

This work presented an in-depth analysis of the performance of systems aided by large intelligent surfaces considering the existence of an eavesdropper links in a generic scenario that contemplates channels with and without LoS links, by means of the Nakagami-m distribution, and channels with or without a direct link to the transmitter and the user. We derived very accurate analytical expressions to compute the secrecy outage probability, bit error probability, and secrecy capacity, in addition to reasonable approximations for estimating the equivalent channel parameters based on the central limit theorem.

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Appendix D.1 Expected value of each fading coefficient

Since the expected value is a linear operator, then

$$\mathbb{E}[h_k] = \mathbb{E}\left[\sum_{i=1}^{N} |h_{ki}^{LD}|\left|h_i^{SL}\right|e^{j\theta_{ki}} + h_k^{SD}\right] = N\mathbb{E}\left[\left|h_{ki}^{LD}h_i^{SL}\right|e^{j\theta_{ki}}\right] + \mathbb{E}\left[h_k^{SD}\right]. \tag{A21}$$

Since the channels are independent and identically distributed, $\mathbb{E}\left[h_k^{SD}\right] = 0$, and $\mathbb{E}\left[e^{j\theta_{ki}}\right] = \alpha_1$, thus we have that

$$\mathbb{E}[h_k] = N\,\mu_{LD} \times \mu_{SL} \times \alpha_1, \tag{A22}$$

where

$$\mu_{LD} = \mathbb{E}\left[\left|h_{ki}^{LD}\right|\right] = \frac{\Gamma(m_{LD}+0.5)}{\Gamma(m_{LD})}\left(\frac{\Omega_{LD}}{m_{LD}}\right)^{\frac{1}{2}}, \tag{A23}$$

and

$$\mu_{SL} = \mathbb{E}\left[\left|h_i^{SL}\right|\right] = \frac{\Gamma(m_{SL}+0.5)}{\Gamma(m_{SL})}\left(\frac{\Omega_{SL}}{m_{SL}}\right)^{\frac{1}{2}}, \tag{A24}$$

are the expected values of each Nakagami-m channel.

To obtain the variance of the overall channel fading coefficient, we need the mean of $c_k = Re\{h_k\}$ and $s_k = Im\{h_k\}$, i.e., the in-phase and quadrature components of the fading coefficient, respectively. The in-phase component can be written as

$$c_k = \sum_{i=1}^{N}\left|h_{ki}^{LD}\right|\left|h_i^{SL}\right|\cos\theta_{ki} + Re\{h_k^{SD}\}, \tag{A25}$$

while the quadrature component is

$$s_k = \sum_{i=1}^{N}\left|h_{ki}^{LD}\right|\left|h_i^{SL}\right|\sin\theta_{ki} + Im\{h_k^{SD}\}. \tag{A26}$$

Then, the expected value of c_k is

$$\mathbb{E}[c_k] = \sum_{i=1}^{N}\mathbb{E}\left[\left|h_{ki}^{LD}\right|\left|h_i^{SL}\right|\cos\theta_{ki}\right] + \mathbb{E}\left[Re\{h_k^{SD}\}\right]. \tag{A27}$$

Since $\mathbb{E}\left[Re\{h_k^{SD}\}\right] = 0$ and all the summation terms are independent, we have that

$$\mathbb{E}[c_k] = N \times \mathbb{E}\left[\left|h_{ki}^{LD}\right|\right]\mathbb{E}\left[\left|h_i^{SL}\right|\right]\mathbb{E}[\cos\theta_{ki}]N \times \mu_{LD}\,\mu_{SL}\,\alpha_1,$$

as well as $\mathbb{E}[h_k]$.

In its turn, the expected value of s_k is given by

$$\mathbb{E}[s_k] = \sum_{i=1}^{N}\mathbb{E}\left[\left|h_{ki}^{LD}\right|\left|h_i^{SL}\right|\sin\theta_{ki}\right] + \mathbb{E}\left[Im\{h_k^{SD}\}\right]. \tag{A28}$$

Since $\mathbb{E}\left[Im\{h_k^{SD}\}\right] = 0$ and all the summation terms are independent,

$$\mathbb{E}[s_k] = N \times \mathbb{E}\left[\left|h_{ki}^{LD}\right|\right]\mathbb{E}\left[\left|h_i^{SL}\right|\right]\mathbb{E}[\sin\theta_{ki}] = 0,$$

since $\mathbb{E}[\sin\theta_{ki}] = 0$.

Appendix D.2 Variance of the in-phase and quadrature components of each fading coefficient

The variance of the in-phase component is written as

$$\mathrm{var}(c_k) = \mathrm{var}\left(\sum_{i=1}^{N}\left|h_{ki}^{LD}\right|\left|h_{i}^{SL}\right|\cos\theta_{ki} + Re\{h_k^{SD}\}\right). \tag{A29}$$

Since $\mathrm{var}\left(h_k^{SD}\right) = \sigma_{SD}^2$, the summation terms and $Re\{h_k^{SD}\}$ are independent and the h_k^{SD} coefficient is zero mean, then

$$\mathrm{var}(c_k) = N \times \mathrm{var}\left(\left|h_{ki}^{LD}\right|\left|h_{i}^{SL}\right|\cos\theta_{ki}\right) + \frac{\sigma_{SD}^2}{2}. \tag{A30}$$

Next, we need to evaluate the variance of the term $|h_{ki}^{LD}||h_i^{SL}|\cos\theta_{ki}$, considering that the variance of the product of two random variables X and Y is $\mathrm{var}(XY) = \mathrm{var}(X)\mathrm{var}(Y) + \mathrm{var}(X)\mathrm{E}[Y]^2 + \mathrm{var}(Y)\mathrm{E}[X]^2$ and the phase noise is independent of the fading magnitudes. So we have that

$$\begin{aligned}\mathrm{var}\left(\left|h_{ki}^{LD}\right|\left|h_{i}^{SL}\right|\cos\theta_{ki}\right) = \mathrm{var}\left(\left|h_{ki}^{LD}\right|\left|h_{i}^{SL}\right|\right)\mathrm{var}(\cos\theta_{ki}) + \dots \\ \mathrm{var}\left(\left|h_{ki}^{LD}\right|\left|h_{i}^{SL}\right|\right)(\mathbb{E}[\cos\theta_{ki}])^2 + \mathrm{var}(\cos\theta_{ki})\left(\mathbb{E}\left[\left|h_{ki}^{LD}\right|\left|h_{i}^{SL}\right|\right]\right)^2.\end{aligned} \tag{A31}$$

Since $|h_{ki}^{LD}|$ and $|h_i^{SL}|$ are independents,

$$\begin{aligned}\mathrm{var}\left(\left|h_{ki}^{LD}\right|\left|h_{i}^{SL}\right|\right) = \mathrm{var}\left(\left|h_{ki}^{LD}\right|\right)\mathrm{var}\left(\left|h_{i}^{SL}\right|\right) + \mathrm{var}\left(\left|h_{ki}^{LD}\right|\right)\left(\mathbb{E}\left[\left|h_{i}^{SL}\right|\right]\right)^2 + \dots \\ \mathrm{var}\left(\left|h_{i}^{SL}\right|\right)\left(\mathbb{E}\left[\left|h_{ki}^{LD}\right|\right]\right)^2\end{aligned} \tag{A32}$$

and

$$\mathbb{E}\left[\left|h_{ki}^{LD}\right|\left|h_{i}^{SL}\right|\right] = \mu_{LD} \times \mu_{SL}. \tag{A33}$$

Next, considering that $\mathrm{var}(|h_i^{SL}|) = \sigma_{SL}^2$ and $\mathrm{var}(|h_{ki}^{LD}|) = \sigma_{LD}^2$, then

$$\mathrm{var}\left(\left|h_{ki}^{LD}\right|\left|h_{i}^{SL}\right|\right) = \sigma_{LD}^2\sigma_{SL}^2 + \sigma_{LD}^2\mu_{SL}^2 + \sigma_{SL}^2\mu_{LD}^2. \tag{A34}$$

By using (A34), we can evaluate (A31) as

$$\begin{aligned}\mathrm{var}\left(\left|h_{ki}^{LD}\right|\left|h_{i}^{SL}\right|\cos\theta_{ki}\right) = \\ \left(\sigma_{LD}^2\sigma_{SL}^2 + \sigma_{LD}^2\mu_{SL}^2 + \sigma_{SL}^2\mu_{LD}^2\right) \times \frac{1}{2}(1+\alpha_2) + \\ \left(\sigma_{LD}^2\sigma_{SL}^2 + \sigma_{LD}^2\mu_{SL}^2 + \sigma_{SL}^2\mu_{LD}^2\right)\alpha_1^2 + \\ \frac{1}{2}(1+\alpha_2) \times \mu_{SL}^2\mu_{LD}^2,\end{aligned} \tag{A35}$$

which can be rewritten as

$$\begin{aligned}\mathrm{var}\left(\left|h_{ki}^{LD}\right|\left|h_{i}^{SL}\right|\cos\theta_{ki}\right) = \frac{1}{2}(1+\alpha_2) \times \mu_{SL}^2\mu_{LD}^2 + \dots \\ \frac{1}{2}\left(\sigma_{LD}^2\sigma_{SL}^2 + \sigma_{LD}^2\mu_{SL}^2 + \sigma_{SL}^2\mu_{LD}^2\right)\left(1+\alpha_2+2\alpha_1^2\right).\end{aligned} \tag{A36}$$

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Therefore, the variance of the in-phase component is

$$\mathrm{var}(c_k) = \frac{\sigma_{SD}^2}{2} + \frac{N}{2} \times \Big[(1+\alpha_2) \times \mu_{SL}^2\mu_{LD}^2 + \dots \\ \left(\sigma_{LD}^2\sigma_{SL}^2 + \sigma_{LD}^2\mu_{SL}^2 + \sigma_{SL}^2\mu_{LD}^2\right)\left(1+\alpha_2+2\alpha_1^2\right)\Big]. \quad \text{(A37)}$$

On other hand, the variance of the quadrature component of each fading coefficient is given by

$$\mathrm{var}(s_k) = \mathrm{var}\left(\sum_{i=1}^{N}\left|h_{ki}^{LD}\right|\left|h_i^{SL}\right|\sin\theta_{ki} + Im\{h_k^{SD}\}\right). \quad \text{(A38)}$$

Considering that $\mathrm{var}(h_k^{SD}) = \sigma_{SD}^2$, the summation terms and $Im\{h_k^{SD}\}$ are independent and the term h_k^{SD} is zero mean, we can show that

$$\mathrm{var}(s_k) = N \times \mathrm{var}\left(\left|h_{ki}^{LD}\right|\left|h_i^{SL}\right|\sin\theta_{ki}\right) + \frac{\sigma_{SD}^2}{2}. \quad \text{(A39)}$$

Next, we have that

$$\mathrm{var}\left(\left|h_{ki}^{LD}\right|\left|h_i^{SL}\right|\sin\theta_{ki}\right) = \mathrm{var}\left(\left|h_{ki}^{LD}\right|\left|h_i^{SL}\right|\right)\mathrm{var}(\sin\theta_{ki}) + \dots \\ \mathrm{var}\left(\left|h_{ki}^{LD}\right|\left|h_i^{SL}\right|\right)(\mathbb{E}[\sin\theta_{ki}])^2 + \mathrm{var}(\sin\theta_{ki})\left(\mathbb{E}\left[\left|h_{ki}^{LD}\right|\left|h_i^{SL}\right|\right]\right)^2. \quad \text{(A40)}$$

Since $\mathbb{E}[\sin\theta_{ki}] = 0$, we can apply (A34) and obtain

$$\mathrm{var}\left(\left|h_{ki}^{LD}\right|\left|h_i^{SL}\right|\sin\theta_{ki}\right) = \frac{1}{2}(1-\alpha_2)\mu_{SL}^2\mu_{LD}^2 + \dots \\ \frac{1}{2}(1-\alpha_2)\left(\sigma_{LD}^2\sigma_{SL}^2 + \sigma_{LD}^2\mu_{SL}^2 + \sigma_{SL}^2\mu_{LD}^2\right), \quad \text{(A41)}$$

which can be rewritten as

$$\mathrm{var}\left(\left|h_{ki}^{LD}\right|\left|h_i^{SL}\right|\sin\theta_{ki}\right) = \frac{1}{2}(1-\alpha_2)\left(\sigma_{LD}^2\sigma_{SL}^2 + \sigma_{LD}^2\mu_{SL}^2 + \sigma_{SL}^2\mu_{LD}^2 + \mu_{SL}^2\mu_{LD}^2\right). \quad \text{(A42)}$$

Therefore, the variance of the quadrature component is expressed by

$$\mathrm{var}(s_k) = \frac{\sigma_{SD}^2}{2} + \frac{N(1-\alpha_2)(\sigma_{LD}^2\sigma_{SL}^2 + \sigma_{LD}^2\mu_{SL}^2 + \sigma_{SL}^2\mu_{LD}^2 + \mu_{SL}^2\mu_{LD}^2)}{2}. \quad \text{(A43)}$$

255 *Appendix D.3 Expected value of γ_D*

All the fading coefficients are independent and identically distributed and $\mathbb{E}[\gamma_D] = \mathbb{E}\left[\|\mathbf{h}\|^2\right]$. Therefore

$$\mathbb{E}[\gamma_D] = \mathbb{E}\left[\sum_{k=1}^{K}|h_k|^2\right] = K \times \mathbb{E}\left[|h_k|^2\right]. \quad \text{(A44)}$$

Considering that $\mathbb{E}\left[|h_k|^2\right] = \mathbb{E}[c_k^2 + s_k^2] = \mathbb{E}[c_k^2] + \mathbb{E}[s_k^2]$, then

$$\mathbb{E}\left[c_k^2\right] = \mathrm{var}(c_k) + (\mathbb{E}[c_k])^2,$$

and by using (A37) and (A43), we can show that

$$\mathbb{E}\left[c_k^2\right] = \frac{\sigma_{SD}^2}{2} + N^2 \times \mu_{LD}^2\, \mu_{SL}^2\, \alpha_1^2 + \frac{N}{2} \times \Big[(1+\alpha_2) \times \mu_{SL}^2 \mu_{LD}^2 + \ldots \\ \left(\sigma_{LD}^2\sigma_{SL}^2 + \sigma_{LD}^2\mu_{SL}^2 + \sigma_{SL}^2\mu_{LD}^2\right)\left(1+\alpha_2+2\alpha_1^2\right)\Big]. \quad \text{(A45)}$$

For the quadrature component, consider that $\mathbb{E}[s_k] = 0$, and, consequently $\mathbb{E}\left[s_k^2\right] = \mathrm{var}(s_k)$. Then,

$$\mathbb{E}\left[s_k^2\right] = \frac{\sigma_{SD}^2}{2} + \frac{N(1-\alpha_2)(\sigma_{LD}^2\sigma_{SL}^2 + \sigma_{LD}^2\mu_{SL}^2 + \sigma_{SL}^2\mu_{LD}^2 + \mu_{SL}^2\mu_{LD}^2)}{2}. \quad \text{(A46)}$$

and the mean value of the overall fading coefficient magnitude is given by

$$\mathbb{E}[\gamma_D] = \frac{K}{2} \times \Big(2\sigma_{SD}^2 + 2N^2 \times \mu_{LD}^2\, \mu_{SL}^2\, \alpha_1^2 N\Big[(1+\alpha_2) \times \mu_{SL}^2\mu_{LD}^2 + \ldots \\ \left(\sigma_{LD}^2\sigma_{SL}^2 + \sigma_{LD}^2\mu_{SL}^2 + \sigma_{SL}^2\mu_{LD}^2\right)\left(1+\alpha_2+2\alpha_1^2\right)\Big] + \ldots \\ N(1-\alpha_2)\left(\sigma_{LD}^2\sigma_{SL}^2 + \sigma_{LD}^2\mu_{SL}^2 + \sigma_{SL}^2\mu_{LD}^2 + \mu_{SL}^2\mu_{LD}^2\right)\Big). \quad \text{(A47)}$$

256 *Appendix D.4 Variance of* γ_D

257 Let $Z_i = |h_i|^2$, then

$$\mathrm{var}(\gamma_D) = \mathrm{var}\left(\|\mathbf{h}\|^2\right) = \mathrm{var}\left(\sum_{i=1}^{M} Z_i\right). \quad \text{(A48)}$$

258 Therefore, the variance of the sum of the terms Z_i is given by

$$\mathrm{var}\left(\sum_{i=1}^{M} Z_i\right) = \sum_{i=1}^{M} \mathrm{var}(Z_i) + 2 \sum_{1\le i<k\le M} \mathrm{cov}(Z_i, Z_k), \quad \text{(A49)}$$

whose magnitudes are equally distributed. Therefore,

$$\mathrm{var}\left(\sum_{i=1}^{M} Z_i\right) = M\, \mathrm{var}(Z_i) + M(M-1)\ \mathrm{cov}(Z_i, Z_k). \quad \text{(A50)}$$

The covariance can be obtained by

$$\mathrm{cov}(Z_i, Z_k) = \mathbb{E}[Z_i Z_k] - \mathbb{E}[Z_i]\mathbb{E}[Z_k], \quad \text{(A51)}$$

where

$$\mathbb{E}[Z_i Z_k] = \mathbb{E}\left[c_i^2 c_k^2\right] + 2\mathbb{E}\left[c_i^2 s_k^2\right] + \mathbb{E}\left[s_i^2 s_k^2\right]. \quad \text{(A52)}$$

The expected value of the product of two different in-phase coefficients can be written as

$$\mathbb{E}[c_i c_k] = \mathbb{E}\Bigg[\left(\sum_{l=1}^{N} \left|h_{il}^{LD}\right|\left|h_l^{SL}\right| \cos\theta_{il} + Re\{h_i^{SD}\}\right) \times \\ \left(\sum_{m=1}^{N} \left|h_{km}^{LD}\right|\left|h_m^{SL}\right| \cos\theta_{km} + Re\{h_k^{SD}\}\right)\Bigg]. \quad \text{(A53)}$$

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By expanding the product, we have that

$$\mathbb{E}[c_i c_k] = \mathbb{E}\left[Re\{h_i^{SD}\}Re\{h_k^{SD}\}\right] + \sum_{l=1}^{N}\sum_{m=1}^{N}\mathbb{E}\left[\left|h_{il}^{LD}\right|\left|h_{km}^{LD}\right|\left|h_{l}^{SL}\right|\left|h_{m}^{SL}\right|\cos\theta_{il}\cos\theta_{km}\right], \tag{A54}$$

where the independent terms can be separated as

$$\mathbb{E}\left[\left|h_{il}^{LD}\right|\left|h_{km}^{LD}\right|\left|h_{l}^{SL}\right|\left|h_{m}^{SL}\right|\cos\theta_{il}\cos\theta_{km}\right] = \mathbb{E}\left[\left|h_{km}^{LD}\right|\right]^2\mathbb{E}\left[\left|h_{m}^{SL}\right|\right]^2\mathbb{E}[\cos\theta_{km}]^2 = \mu_{LD}^2\mu_{SL}^2\alpha_1^2\forall l \neq m, \tag{A55}$$

and

$$\mathbb{E}\left[\left|h_{il}^{LD}\right|\left|h_{km}^{LD}\right|\left|h_{l}^{SL}\right|\left|h_{m}^{SL}\right|\cos\theta_{il}\cos\theta_{km}\right] = \mathbb{E}\left[\left|h_{km}^{LD}\right|\right]^2\mathbb{E}\left[\left|h_{m}^{SL}\right|^2\right]\mathbb{E}[\cos\theta_{im}]\mathbb{E}[\cos\theta_{km}] = \mu_{LD}^2\left(\sigma_{SL}^2+\mu_{SL}^2\right)\alpha_1^2\forall i \neq k, l = m, \tag{A56}$$

where the term $\mathbb{E}\left[\left|h_{m}^{SL}\right|^2\right] = \sigma_{SL}^2 + \mu_{SL}^2$ and the variance of the Nakagami-m distributed term is

$$\sigma_{SL}^2 = \Omega_{SL}\left(1 - \frac{1}{m_{SL}}\left(\frac{\Gamma(m_{SL}+0.5)}{\Gamma(m_{SL})}\right)^2\right). \tag{A57}$$

For $i = k$, we have that

$$\mathbb{E}\left[\left|h_{il}^{LD}\right|\left|h_{km}^{LD}\right|\left|h_{l}^{SL}\right|\left|h_{m}^{SL}\right|\cos\theta_{il}\cos\theta_{km}\right] = \mathbb{E}\left[\left|h_{km}^{LD}\right|^2\right]\mathbb{E}\left[\left|h_{m}^{SL}\right|^2\right]\mathbb{E}\left[\cos^2\theta_{km}\right] = \frac{1}{2}\left(\sigma_{LD}^2+\mu_{LD}^2\right)\left(\sigma_{SL}^2+\mu_{SL}^2\right)(1+\alpha_2)\forall l = m, i = k, \tag{A58}$$

where

$$\sigma_{LD}^2 = \Omega_{LD}\left(1 - \frac{1}{m_{LD}}\left(\frac{\Gamma(m_{LD}+0.5)}{\Gamma(m_{LD})}\right)^2\right). \tag{A59}$$

To compute the variance according to (A52), we need to obtain the term $\mathbb{E}\left[c_i^2 c_k^2\right]$.The terms are approximately correlated Gaussian random variables by the central limit theorem (CLT), for large values of N, therefore

$$\mathbb{E}\left[c_i^2 c_k^2\right] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2 y^2 f_{c_i,c_k}(x,y)dxdy, \tag{A60}$$

where $f_{c_i,c_k}(x,y)$ is the joint distribution of the two correlated Gaussian variables c_i and c_k. The result of the integral is

$$\mathbb{E}[c_i^2 c_k^2] = \mu_{c_k}^4 + 2\mu_{c_k}^2(1+2\rho_{c_i,c_k})\sigma_{c_k}^2 + \left(1+2\rho_{c_i,c_k}^2\right)\sigma_{c_k}^4, \tag{A61}$$

where $\mu_{c_k} = \mathbb{E}[c_k]$, $\sigma_{c_k}^2 = \mathrm{var}(c_k)$, and since $\mathrm{var}(c_i) = \mathrm{var}(c_k)$ and $\mathbb{E}[c_i] = \mathbb{E}[c_k]$, the correlation coefficient ρ_{c_i,c_k} can be obtained as

$$\rho_{c_i,c_k} = \frac{\mathbb{E}[c_i c_k] - \mu_{c_k}^2}{\mathrm{var}(c_k)}, \tag{A62}$$

259 where $\mathbb{E}[c_i c_k]$ can be calculated by (A63) in the next page.

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$$\mathbb{E}[c_i c_k] = \begin{cases} N(N-1)\left(\mu_{LD}^2 \mu_{SL}^2 \alpha_1^2\right) + N\left(\mu_{LD}^2\left(\sigma_{SL}^2 + \mu_{SL}^2\right)\alpha_1^2\right) & i \neq k \\ N(N-1)\left(\mu_{LD}^2 \mu_{SL}^2 \alpha_1^2\right) + \frac{N}{2}\left(\left(\sigma_{LD}^2 + \mu_{LD}^2\right)\left(\sigma_{SL}^2 + \mu_{SL}^2\right)(1+\alpha_2)\right) + \frac{1}{2}\sigma_{SD}^2 & i = k \end{cases} \tag{A63}$$

Since the moments of c_k were previously calculated, ρ_{c_i,c_k} can be obtained by (A64).

$$\rho_{c_i,c_k} = \frac{N(N-1)\left(\mu_{LD}^2 \mu_{SL}^2 \alpha_1^2\right) + N\left(\mu_{LD}^2\left(\sigma_{SL}^2 + \mu_{SL}^2\right)\alpha_1^2\right) - \left(N\,\mu_{LD}\,\mu_{SL}\,\alpha_1\right)^2}{\frac{\sigma_{SD}^2}{2} + \frac{N}{2}\left[(1+\alpha_2)\times\mu_{SL}^2\mu_{LD}^2 + \left(\sigma_{LD}^2\sigma_{SL}^2 + \sigma_{LD}^2\mu_{SL}^2 + \sigma_{SL}^2\mu_{LD}^2\right)\left(1+\alpha_2+2\alpha_1^2\right)\right]} \quad i \neq k. \tag{A64}$$

In its turn, applying (A64) for the correlation coefficient ρ_{c_i,c_k} in the definition of $\mathbb{E}[c_i^2 c_k^2]$ in (A61) we finally can compute (A65).

$$\mathbb{E}[c_i^2 c_k^2] = \begin{cases} a_1^4\left(1 - \frac{4}{N}\right) + a_1^2\left(\sigma_{SD}^2 + a_4 + N a_2 a_3 - \frac{2}{N} + 4a_5\right)2a_5 + \frac{1}{4}\left(\sigma_{SD}^2 + a_4 + N a_2 a_3\right)^2 & i \neq k \\ a_1^4 + 3a_1^2\left(\sigma_{SD}^2 + a_4 + N a_2 a_3\right) + \frac{3}{4}\left(\sigma_{SD}^2 + a_4 + N a_2 a_3\right) & i = k \end{cases} \tag{A65}$$

where $a_1 = N\mu_{SL}\mu_{LD}\alpha_1$, $a_2 = \sigma_{LD}^2\sigma_{SL}^2 + \sigma_{LD}^2\mu_{SL}^2 + \sigma_{SL}^2\mu_{LD}^2$, $a_3 = 1 + \alpha_2 + 2\alpha_1^2$, $a_4 = N\mu_{SL}^2\mu_{LD}^2(1+\alpha_2)$, $a_5 = N\mu_{LD}^2\alpha_1^2\left(\sigma_{SL}^2 + \mu_{SL}^2\right)$

The term $\mathbb{E}[c_i^2 s_k^2]$ depends of two uncorrelated but not independent random variables c_i and s_k, the correlation is zero and to calculated the correlation of the squared product we need to expand the very definition of the two terms as shown in (A66).

$$\mathbb{E}\left[c_i^2 s_k^2\right] = \mathbb{E}\left[\left(\sum_{l=1}^{N}\left|h_{il}^{LD}\right|\left|h_l^{SL}\right|\cos\theta_{il} + Re\{h_i^{SD}\}\right)^2 \times \left(\sum_{m=1}^{N}\left|h_{km}^{LD}\right|\left|h_m^{SL}\right|\sin\theta_{km} + Im\{h_k^{SD}\}\right)^2\right] \tag{A66}$$

Expanding the product in (A66) we have (A67),

$$\mathbb{E}\left[\left(\sum_{l=1}^{N}\left|h_{il}^{LD}\right|\left|h_l^{SL}\right|\cos\theta_{il}\right)^2\left(\sum_{m=1}^{N}\left|h_{km}^{LD}\right|\left|h_m^{SL}\right|\sin\theta_{km}\right)^2\right] = \mathbb{E}\left[\sum_{l=1}^{N}\sum_{t=1}^{N}\sum_{d=1}^{N}\sum_{m=1}^{N}\left|h_{il}^{LD}\right|\left|h_{it}^{LD}\right|\left|h_{kd}^{LD}\right|\left|h_{km}^{LD}\right|\left|h_l^{SL}\right|\left|h_m^{SL}\right|\left|h_d^{SL}\right|\left|h_m^{SL}\right|\cos\theta_{il}\cos\theta_{it}\sin\theta_{kd}\sin\theta_{km}\right] \tag{A67}$$

where the general and simplified result given in Ferreira et al. [20] as shown (A68).

$$\mathbb{E}\left[c_i^2 s_k^2\right] = \mathbb{E}\left[\left(\sum_{l=1}^{N}\left|h_{il}^{LD}\right|\left|h_l^{SL}\right|\cos\theta_{il}\right)^2\left(\sum_{m=1}^{N}\left|h_{km}^{LD}\right|\left|h_m^{SL}\right|\sin\theta_{km}\right)^2\right]\ldots + \mathbb{E}\left[\left(\sum_{l=1}^{N}\left|h_{il}^{LD}\right|\left|h_l^{SL}\right|\cos\theta_{il}\right)^2\right]\frac{\sigma_{SD}^2}{2} + \mathbb{E}\left[\left(\sum_{m=1}^{N}\left|h_{km}^{LD}\right|\left|h_m^{SL}\right|\sin\theta_{km}\right)^2\right]\frac{\sigma_{SD}^2}{2} + \frac{\sigma_{SD}^4}{4} \tag{A68}$$

The in-phase term is

$$\mathbb{E}\left[\left(\sum_{l=1}^{N}\left|h_{il}^{LD}\right|\left|h_{l}^{SL}\right|\cos\theta_{il}\right)^{2}\right]=\mathbb{E}\left[\sum_{l=1}^{N}\sum_{t=1}^{N}\left|h_{il}^{LD}\right|\left|h_{it}^{LD}\right|\left|h_{l}^{SL}\right|\left|h_{m}^{SL}\right|\cos\theta_{il}\cos\theta_{it}\right]= \frac{N}{2}\chi_{LD}\chi_{SL}(1+\alpha_2)+N(N-1)\mu_{LD}^2\mu_{SL}^2\alpha_1^2 \quad \text{(A69)}$$

267 and the quadrature term is

$$\mathbb{E}\left[\left(\sum_{m=1}^{N}\left|h_{km}^{LD}\right|\left|h_{m}^{SL}\right|\sin\theta_{km}\right)^{2}\right]=\mathbb{E}\left[\sum_{d=1}^{N}\sum_{m=1}^{N}\left|h_{kd}^{LD}\right|\left|h_{km}^{LD}\right|\left|h_{d}^{SL}\right|\left|h_{m}^{SL}\right|\sin\theta_{kd}\sin\theta_{km}\right]= N(\chi_{LD}\chi_{SL}\frac{1}{2}(1-\alpha_2)) \quad \text{(A70)}$$

where

$$\chi_{LD}=\mathbb{E}\left[\left|h_{il}^{LD}\right|^{2}\right]=\Omega_{LD} \quad \text{(A71)}$$

$$\chi_{SL}=\mathbb{E}\left[\left|h_{d}^{SL}\right|^{2}\right]=\Omega_{SL} \quad \text{(A72)}$$

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$$\xi_{LD}=\mathbb{E}\left[\left|h_{il}^{LD}\right|^{3}\right]=\frac{\Gamma(1.5+m_{LD})}{\left(\frac{m_{LD}}{\Omega_{LD}}\right)\Gamma(m_{LD})} \quad \text{(A74)}$$

$$\xi_{SL}=\mathbb{E}\left[\left|h_{d}^{SL}\right|^{3}\right]=\frac{\Gamma(1.5+m_{SL})}{\left(\frac{m_{SL}}{\Omega_{SL}}\right)\Gamma(m_{SL})} \quad \text{(A75)}$$

$$\tau_{LD}=\mathbb{E}\left[\left|h_{il}^{LD}\right|^{4}\right]=\frac{(1+m_{LD})\Omega_{LD}^2}{m_{LD}} \quad \text{(A76)}$$

$$\tau_{SL}=\mathbb{E}\left[\left|h_{d}^{SL}\right|^{4}\right]=\frac{(1+m_{SL})\Omega_{SL}^2}{m_{SL}} \quad \text{(A77)}$$

To obtain the term $\mathbb{E}[s_i^2 s_k^2]$, we claim that s_i and s_k are uncorrelated and zero mean, therefore

$$\mathbb{E}[s_i^2 s_k^2]=\begin{cases}\left(\mathbb{E}\left[s_i^2\right]\right)^2 & i\neq k\\ \mathbb{E}\left[s_k^4\right]=3(\mathrm{var}(s_k))^2 & i=k\end{cases} \quad \text{(A78)}$$

269 and by substituting the variances, we find (A79).

$$\mathbb{E}[s_i^2 s_k^2]=\begin{cases}\frac{1}{4}\left(\sigma_{SD}^2+N(1-\alpha_2)(\sigma_{LD}^2\sigma_{SL}^2+\sigma_{LD}^2\mu_{SL}^2+\sigma_{SL}^2\mu_{LD}^2+\mu_{SL}^2\mu_{LD}^2)\right)^2 & i\neq k\\ \\ \frac{3}{4}\left(\sigma_{SD}^2+N(1-\alpha_2)(\sigma_{LD}^2\sigma_{SL}^2+\sigma_{LD}^2\mu_{SL}^2+\sigma_{SL}^2\mu_{LD}^2+\mu_{SL}^2\mu_{LD}^2)\right)^2 & i=k\end{cases} \quad \text{(A79)}$$

270 Finally, the term $\mathbb{E}[Z_i Z_k]$ can be calculated by using the formulas of $\mathbb{E}[c_i^2 c_k^2]$, $\mathbb{E}[c_i^2 s_k^2]$
271 and $\mathbb{E}[s_i^2 s_k^2]$, resulting in (D.4).

$$
\mathbb{E}[Z_i Z_k] = \left\{ \begin{array}{l}
a_1^4\left(1-\frac{4}{N}\right) + a_1^2(\sigma_{SD}^2 + a_4 + Na_2a_3 - \frac{2}{N} + 4a_5)2a_5 + \frac{1}{4}\left(\sigma_{SD}^2 + a_4 + Na_2a_3\right)^2 + \\
\frac{1}{2}(N^2-N)\left(\chi_{LD}^2\chi_{SL}^2(1-\alpha_2^2) + 4\mu_{LD}^2\chi_{LD}\mu_{SL}\xi_{SL}\alpha_1^2(1-\alpha_2)\right) + \frac{N}{2}\chi_{LD}^2\tau_{SL}(1-\alpha_2^2) \\
+\frac{1}{2}\sigma_{SD}^2\left(N\chi_{LD}\chi_{SL}(1+\alpha_2) + 2(N-1)\mu_{LD}^2\mu_{SL}^2\alpha_1^2 + \chi_{LD}\chi_{SL}(1-\alpha_2)\right) + \\
(N^3-3N^2+2N)\mu_{LD}^2\chi_{LD}\mu_{SL}^2\chi_{SL}\alpha_1^2(1-\alpha_2) + \frac{1}{2}\sigma_{SD}^4 + \\
\frac{1}{4}\left(\sigma_{SD}^2 + N(1-\alpha_2)(\sigma_{LD}^2\sigma_{SL}^2 + \sigma_{LD}^2\mu_{SL}^2 + \sigma_{SL}^2\mu_{LD}^2 + \mu_{SL}^2\mu_{LD}^2)\right)^2 \\
\\
a_1^4 + 3a_1^2(\sigma_{SD}^2 + a_4 + Na_2a_3) + \frac{3}{4}\left(\sigma_{SD}^2 + a_4 + Na_2a_3\right) + \\
\frac{1}{4}(N^2-N)\left(\chi_{LD}^2\chi_{SL}^2(1-\alpha_4) + 4\mu_{LD}\xi_{LD}\mu_{SL}\xi_{SL}(\alpha_1-\alpha_3)\alpha_1\right) + \frac{N}{2}\tau_{LD}\tau_{SL}(1-\alpha_2^2) + \\
(N^3-3N^2+2N)\mu_{LD}^2\chi_{LD}\mu_{SL}^2\chi_{SL}\alpha_1^2(1-\alpha_2) + \frac{N}{2}\sigma_{SD}^2[\chi_{LD}\chi_{SL}(1+\alpha_2) + \\
2(N-1)\mu_{LD}^2\mu_{SL}^2\alpha_1^2 + \chi_{LD}\chi_{SL}(1-\alpha_2)] + \frac{1}{2}\sigma_{SD}^4 + \\
\frac{3}{4}\left(\sigma_{SD}^2 + N(1-\alpha_2)(\sigma_{LD}^2\sigma_{SL}^2 + \sigma_{LD}^2\mu_{SL}^2 + \sigma_{SL}^2\mu_{LD}^2 + \mu_{SL}^2\mu_{LD}^2)\right)^2
\end{array} \right.
$$

(A80)

The covariance $\mathrm{cov}(Z_i Z_k)$ can be calculated by (A81) using the result of (D.4). Since $\mathrm{var}(Z_i) = \mathrm{cov}(Z_i, Z_i)$, the variance of the overall fading coefficient can be easily obtained by (A50) as shown in (D.4).

$$
\mathrm{cov}(Z_i Z_k) = \left\{ \begin{array}{l}
a_1^4\left(1-\frac{4}{N}\right) + a_1^2(\sigma_{SD}^2 + a_4 + Na_2a_3 - \frac{2}{N} + 4a_5)2a_5 + \frac{1}{4}\left(\sigma_{SD}^2 + a_4 + Na_2a_3\right)^2 + \\
\frac{1}{2}(N^2-N)\left(\chi_{LD}^2\chi_{SL}^2(1-\alpha_2^2) + 4\mu_{LD}^2\chi_{LD}\mu_{SL}\xi_{SL}\alpha_1^2(1-\alpha_2)\right) + \frac{N}{2}\chi_{LD}^2\tau_{SL}(1-\alpha_2^2) \\
+\frac{1}{2}\sigma_{SD}^2\left(N\chi_{LD}\chi_{SL}(1+\alpha_2) + 2(N-1)\mu_{LD}^2\mu_{SL}^2\alpha_1^2 + \chi_{LD}\chi_{SL}(1-\alpha_2)\right) + \\
(N^3-3N^2+2N)\mu_{LD}^2\chi_{LD}\mu_{SL}^2\chi_{SL}\alpha_1^2(1-\alpha_2) + \frac{1}{2}\sigma_{SD}^4 + \\
\frac{1}{4}\left(\sigma_{SD}^2 + N(1-\alpha_2)(\sigma_{LD}^2\sigma_{SL}^2 + \sigma_{LD}^2\mu_{SL}^2 + \sigma_{SL}^2\mu_{LD}^2 + \mu_{SL}^2\mu_{LD}^2)\right)^2 \\
-\frac{1}{4} \times \left(2\sigma_{SD}^2 + 2N^2 \times \mu_{LD}^2\ \mu_{SL}^2\ \alpha_1^2 + \right. \\
N\left[(1+\alpha_2) \times \mu_{SL}^2\mu_{LD}^2 + \left(\sigma_{LD}^2\sigma_{SL}^2 + \sigma_{LD}^2\mu_{SL}^2 + \sigma_{SL}^2\mu_{LD}^2\right)(1+\alpha_2+2\alpha_1^2)\right] + \\
\left. N(1-\alpha_2)\left(\sigma_{LD}^2\sigma_{SL}^2 + \sigma_{LD}^2\mu_{SL}^2 + \sigma_{SL}^2\mu_{LD}^2 + \mu_{SL}^2\mu_{LD}^2\right)\right)^2 \\
\\
a_1^4 + 3a_1^2(\sigma_{SD}^2 + a_4 + Na_2a_3) + \frac{3}{4}\left(\sigma_{SD}^2 + a_4 + Na_2a_3\right) + \\
\frac{1}{4}(N^2-N)\left(\chi_{LD}^2\chi_{SL}^2(1-\alpha_4) + 4\mu_{LD}\xi_{LD}\mu_{SL}\xi_{SL}(\alpha_1-\alpha_3)\alpha_1\right) + \frac{N}{2}\tau_{LD}\tau_{SL}(1-\alpha_2^2) + \\
(N^3-3N^2+2N)\mu_{LD}^2\chi_{LD}\mu_{SL}^2\chi_{SL}\alpha_1^2(1-\alpha_2) + \frac{N}{2}\sigma_{SD}^2[\chi_{LD}\chi_{SL}(1+\alpha_2) + \\
2(N-1)\mu_{LD}^2\mu_{SL}^2\alpha_1^2 + \chi_{LD}\chi_{SL}(1-\alpha_2)] + \frac{1}{2}\sigma_{SD}^4 + \\
\frac{3}{4}\left(\sigma_{SD}^2 + N(1-\alpha_2)(\sigma_{LD}^2\sigma_{SL}^2 + \sigma_{LD}^2\mu_{SL}^2 + \sigma_{SL}^2\mu_{LD}^2 + \mu_{SL}^2\mu_{LD}^2)\right)^2 \\
-\frac{1}{4} \times \left(2\sigma_{SD}^2 + 2N^2 \times \mu_{LD}^2\ \mu_{SL}^2\ \alpha_1^2 + \dots \right. \\
N\left[(1+\alpha_2) \times \mu_{SL}^2\mu_{LD}^2 + \dots \left(\sigma_{LD}^2\sigma_{SL}^2 + \sigma_{LD}^2\mu_{SL}^2 + \sigma_{SL}^2\mu_{LD}^2\right)(1+\alpha_2+2\alpha_1^2)\right] + \\
\left. N(1-\alpha_2)\left(\sigma_{LD}^2\sigma_{SL}^2 + \sigma_{LD}^2\mu_{SL}^2 + \sigma_{SL}^2\mu_{LD}^2 + \mu_{SL}^2\mu_{LD}^2\right)\right)^2
\end{array} \right. \tag{A81}
$$

$$
\mathrm{var}(\gamma_D) = M\,\mathrm{var}(Z_i) + M(M-1)\ \mathrm{cov}(Z_i, Z_k)
$$

(A82)

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