**Supplemental Information**

To explain, the equation is a general form of the equations found for most convex polyhedra, therefore, wedges of the concave polyhedra are cut and inverted to calculate volume. The third dimension was equated by extending the face surface until a point on the face can make a 90° angle with the center of the polyhedron. Surface area and volume equations for Dodecadodecahedron, Small-Ditrigonal Icosidodecahedron, and Excavated Dodecahedron are formulated by calculating face area individually or subtracting negative space form the concave polyhedron’s shape as if their facets were filled in.

General SA and V formulas is as follows for Medial/Great Triambic Icosahedron and Medial Rhombic Triacontahedron that needed special derivations:

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| $$SA = 60A\_{f},$$ |  |
| $$V = 20A\_{f}h,$$ |  |
| where, Af, is the area of the “approximated” polyhedron’s face and the h is the height of the extended face from the center point of the solid. |

Associated specialized formulas for Medial Rhombic Triacontahedron:

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| $$SA = 60r^{2}sin30°tan36°,$$ |
| $ V = 10r^{3}sin^{2}30°tan36°$  |
|  |

For Medial/Great Triambic Icosahedron:

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| $$h = rsin 7.761°,$$ |  |
| $$SA = 60r^{2}sin 7.761°tan 36°,$$ |
| $V = 20r^{3}sin27.761°tan36°,$  |
|   |

 where, h, is the height derived from the face of the shapes

Associated specialized formulas for “filled-in facet” shapes: Dodecadodecahedron:

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| $$a = 2b,$$$b={2r}/{\left(\sqrt{3}\left(1+\sqrt{5}\right)\right)}$,$$SA=3a^{2}\sqrt{25+10\sqrt{5}}-60b^{2}\sin(126.86°,)$$$$V= \frac{a^{3}}{4}\left(15+7\sqrt{5}\right)-20b^{3}\sin(126.86°)$$Where a is the formula for twice the side length of a dodecahedron since the dodecadodecahedron’s side derives from it and b, is just half of a for easier trigonometric calculations. First terms in SA and V are derived from the dodecahedron whereas the second are the subtraction of the respective parameters absent from the Dodecadodecahedron. |   |

 Small-Ditrigonal Icosidodecahedron:

 $b= {2r}/{\left(\sqrt{3}\left(1+\sqrt{5}\right)\right)},$

 $m=b·\tan(32°),$

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| $$SA=3a^{2}\sqrt{25+10\sqrt{5}}-60b^{2}\tan(32°,)$$$V= \frac{a^{3}}{4}\left(15+7\sqrt{5}\right)-30\left({2}/{3}\right)m^{3}\sin(116.565°)\sin(58.2825°)\tan( 30°),$ \*follows previous logic\* |

 Excavated Dodecahedron:

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| $$a = {4r}/{\left(\sqrt{3}\left(1+\sqrt{5}\right)\right)}$$ |  |
| $$l = {2r}/{\left(\sqrt{3}\left(1+\sqrt{5}\right)\sin(36°)\right)}$$ |
| $$h= \sqrt{a^{2}-l^{2}}$$ |
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| $$B= \left({4r}/{\left(\sqrt{3}\left(1+\sqrt{5}\right)\right)}\right)∙\left({2r}/{\left(\sqrt{3}\left(1+\sqrt{5}\right)\tan(36°)\right)}\right)$$$$SA=15\sqrt{3}a^{2}$$$V=\left({a^{3}}/{4}\right)\left(15+7\sqrt{5}\right)-20Bh$ Where a, is the dodecahedral side length, l, the length along the inward facet of dodecahedron, and h is the length from the edge of the facets to what would be the side length of the dodecahedron and, B, the base of half the facet that fills to a dodecahedron. | (22) |

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