


## Article

## A TIME-SYMMETRIC RESOLUTION OF THE EINSTEIN'S BOXES PARADOX

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**Abstract:** The Einstein's Boxes paradox was developed by Einstein, de Broglie, Heisenberg, and others to demonstrate the incompleteness of the Copenhagen Formulation of quantum mechanics. I explain the paradox using the Copenhagen Formulation. I then show how a Time-Symmetric Formulation of quantum mechanics resolves the paradox in the way envisioned by Einstein and de Broglie. Finally, I describe an experiment that can distinguish between these two formulations.

**Keywords:** quantum foundations; time-symmetric; Einstein's boxes; Einstein–Podolsky–Rosen (EPR)

## 1. Introduction

A grand challenge of modern physics is to resolve the conceptual paradoxes in the foundations of quantum mechanics [1]. Some of these paradoxes concern nonlocality and completeness. For example, Einstein believed the Copenhagen Formulation [CF] of quantum mechanics was incomplete. He presented a thought experiment (later known as "Einstein's Bubble") explaining his reasoning at the 1927 Solvay conference [2]. In this experiment, an incident particle's wavefunction diffracts at a pinhole in a flat screen and then spreads to all parts of a hemispherical screen capable of detecting the wavefunction. The wavefunction is then detected at one point on the hemispherical screen, implying the wavefunction everywhere else vanished instantaneously. Einstein believed this instantaneous wavefunction collapse violated the special theory of relativity, and the wavefunction must have been localized at the point of detection immediately before the detection occurred. Since the CF does not describe the wavefunction localization before detection, it must be an incomplete theory. In an earlier paper, I analyzed a one-dimensional version of this thought experiment with a Time-Symmetric Formulation (TSF) of quantum mechanics [3], showing that the TSF did not need wavefunction collapse to explain the experimental results.

Einstein, de Broglie, Heisenberg, and others later modified Einstein's original thought experiment to emphasize the nonlocal action-at-a-distance effects. In the modified experiment, the particle's wavefunction was localized in two boxes which were separated by a spacelike interval. This modified thought experiment became known as "Einstein's Boxes." Norsen wrote an excellent analysis of the history and significance of the Einstein's Boxes thought experiment using the CF [4].

Time-symmetric explanations of quantum behavior predate the discovery of the Schrödinger equation [5] and have been developed many times over the past century [6–33]. The TSF used in this paper has been described in detail and compared to other TSF's before [3,31–33]. Note in particular that the TSF used in this paper is significantly different than the Two-State Vector Formalism (TSVF) [12,23,24]. First, the TSVF postulates that a *quantum particle* is completely described by two state vectors, written as  $\langle\phi|$   $|\psi\rangle$ , where  $|\psi\rangle$  is a retarded state vector satisfying the initial boundary conditions while  $\langle\phi|$  is an advanced state vector satisfying the final boundary conditions. In contrast, the TSF postulates that the *transition* of a quantum particle is completely described by a complex transition amplitude density  $\phi^*\psi$ , defined as the algebraic product of the two wavefunctions. Second, the TSVF postulates that wavefunctions collapse upon measurement, while the TSF has no collapse postulate. The particular TSF used in this paper is a type IIB model, in the classification system of Wharton and Argaman [34].

Section 2 explains the paradox associated with the CF of the Einstein's Boxes thought experiment, as described by de Broglie. Section 3 reviews a CF numerical model of



**Citation:** Heaney, M.B. Title.

*Preprints* 2021, 1, 0. <https://doi.org/>

Received:

Accepted:

Published:

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the thought experiment which does not resolve the paradox. Section 4 describes a TSF numerical model of the thought experiment which resolves the paradox. Section 5 discusses conclusions and implications.

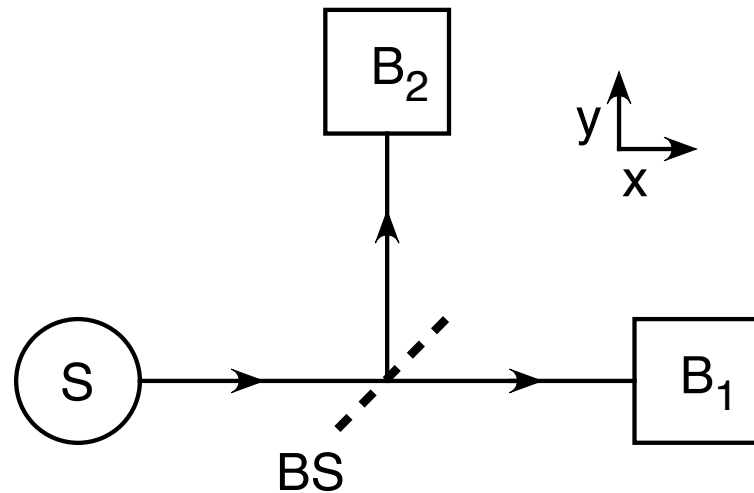
## 2. The Einstein's Boxes Paradox

De Broglie explained the Einstein's Boxes paradox as follows [35]:

Suppose a particle is enclosed in a box  $B$  with impermeable walls. The associated wave  $\psi$  is confined to the box and cannot leave it. The usual interpretation asserts that the particle is "potentially" present in the whole of the box  $B$ , with a probability  $|\psi|^2$  at each point. Let us suppose that by some process or other, for example, by inserting a partition into the box, the box  $B$  is divided into two separate parts  $B_1$  and  $B_2$  and that  $B_1$  and  $B_2$  are then transported to two very distant places, for example to Paris and Tokyo. The particle, which has not yet appeared, thus remains potentially present in the assembly of the two boxes and its wavefunction  $\psi$  consists of two parts, one of which,  $\psi_1$ , is located in  $B_1$  and the other,  $\psi_2$ , in  $B_2$ . The wave function is thus of the form  $\psi = c_1\psi_1 + c_2\psi_2$ , where  $|c_1|^2 + |c_2|^2 = 1$ .

The probability laws of [the Copenhagen Formulation] now tell us that if an experiment is carried out in box  $B_1$  in Paris, which will enable the presence of the particle to be revealed in this box, the probability of this experiment giving a positive result is  $|c_1|^2$ , while the probability of it giving a negative result is  $|c_2|^2$ . According to the usual interpretation, this would have the following significance: because the particle is present in the assembly of the two boxes prior to the observable localization, it would be immediately localized in box  $B_1$  in the case of a positive result in Paris. This does not seem to me to be acceptable. The only reasonable interpretation appears to me to be that prior to the observable localization in  $B_1$ , we know that the particle was in one of the two boxes  $B_1$  and  $B_2$ , but we do not know in which one, and the probabilities considered in the usual wave mechanics are the consequence of this partial ignorance. If we show that the particle is in box  $B_1$ , it implies simply that it was already there prior to localization. Thus, we now return to the clear classical concept of probability, which springs from our partial ignorance of the true situation. But, if this point of view is accepted, the description of the particle given by  $\psi$ , though leading to a perfectly *exact* description of probabilities, does not give us a *complete* description of the physical reality, because the particle must have been localized prior to the observation which revealed it, and the wave function  $\psi$  gives no information about this.

We might note here how the usual interpretation leads to a paradox in the case of experiments with a negative result. Suppose that the particle is charged, and that in the box  $B_2$  in Tokyo a device has been installed which enables the whole of the charged particle located in the box to be drained off and in so doing to establish an observable localization. Now, if nothing is observed, this negative result will signify that the particle is not in box  $B_2$  and it is thus in box  $B_1$  in Paris. But this can reasonably signify only one thing: the particle was already in Paris in box  $B_1$  prior to the drainage experiment made in Tokyo in box  $B_2$ . Every other interpretation is absurd. How can we imagine that the simple fact of having observed *nothing* in Tokyo has been able to promote the localization of the particle at a distance of many thousands of miles away?



**Figure 1.** The modified Einstein's Boxes thought experiment. The source  $S$  can emit single particle wavefunctions on command. Each wavefunction travels to the balanced beam-splitter  $BS$ , then to box  $B_1$  and box  $B_2$ . The two boxes are separated by a spacelike interval.

### 3. The Conventional Formulation of Einstein's Boxes

de Broglie's version of Einstein's boxes is experimentally impractical. We will use Heisenberg's more practical version [36] shown in figure 1. The Conventional Formulation (CF) postulates that a single free particle wavefunction with mass  $m$  is completely described by a wavefunction  $\psi(\vec{r}, t)$  which satisfies the initial conditions and evolves in time according to the Schrödinger equation:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi. \quad (1)$$

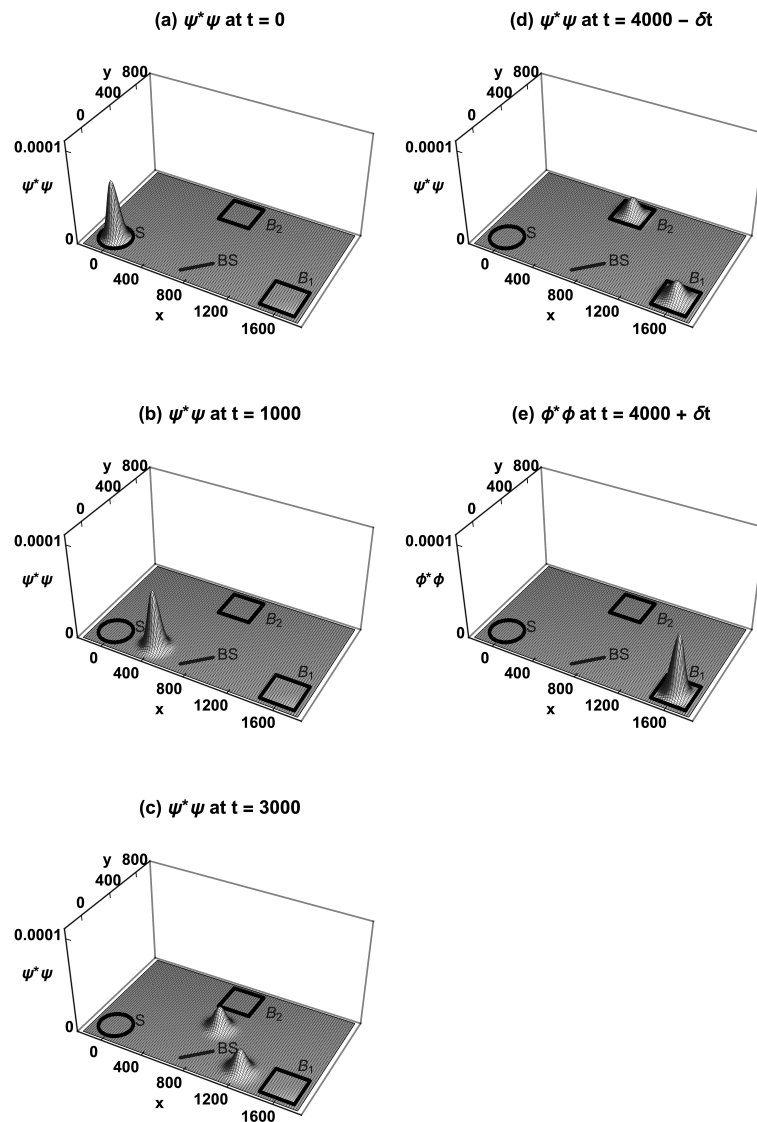
We will use units where  $\hbar = 1$  and assume the wavefunction  $\psi(\vec{r}, t)$  is a traveling Gaussian with an initial standard deviation  $\sigma = 50$ , initial momentum  $k_x = 0.4$ , and mass  $m = 1$ . We will also assume that each box contains a detector whose eigenstate is the same Gaussian as that emitted by the source. The CF assumes that a single-particle wavefunction emitted from source  $S$  will travel to beam-splitter  $BS$  where half of it will pass through  $BS$  and continue to box  $B_1$  while the other half will be reflected from  $BS$  and travel to box  $B_2$ . Let us assume the two halves reach the boxes at the same time.

Figure 2 shows how the wavefunction's CF probability density  $\psi^* \psi$  evolves over time, assuming the initial condition is localization in source  $S$ . At  $t = 0$ ,  $\psi^* \psi$  is localized inside the source  $S$ . At  $t = 1000$ ,  $\psi^* \psi$  is traveling toward the beam-splitter  $BS$ . At  $t = 3000$ ,  $\psi^* \psi$  has been split in half by the beam-splitter, and the two halves are traveling towards box  $B_1$  and box  $B_2$ . At  $t = 4000 - \delta t$ , half of  $\psi^* \psi$  arrives at box  $B_1$  while the other half arrives at box  $B_2$ . Upon a measurement at box  $B_2$  at  $t = 4000$ , the CF postulates that in 50% of the runs the half-wavefunction in box  $B_2$  collapses to zero while simultaneously the half-wavefunction in box  $B_1$  collapses to a full wavefunction  $\phi(\vec{r}, t)$ , which we will assume has the same shape and size as the initial wavefunction. De Broglie believed this prediction of the CF was absurd: "How can we imagine that the simple fact of having observed *nothing* in [box  $B_2$ ] has been able to promote the localization of the particle [in box  $B_1$ ] at a distance of many thousands of miles away?"

The CF assumes the probability  $P_c$  for the collapse in Box  $B_1$  is  $P_c = A_c^* A_c$ , where the subscript  $c$  denotes the CF, and the CF amplitude  $A_c$  for the collapse is:

$$A_c = \int_{-\infty}^{\infty} \phi^*(x, y, 4000) \frac{1}{\sqrt{2}} \psi(x, y, 4000) dx dy, \quad (2)$$

where  $t = 4000$  is the time of wavefunction collapse, and the "quantum" factor  $\frac{1}{\sqrt{2}}$  accounts for the initial wavefunction  $\psi(x, y, t)$  being split in half when it reaches box  $B_1$ . Plugging in numbers gives a collapse probability  $P_c = 0.43$ .

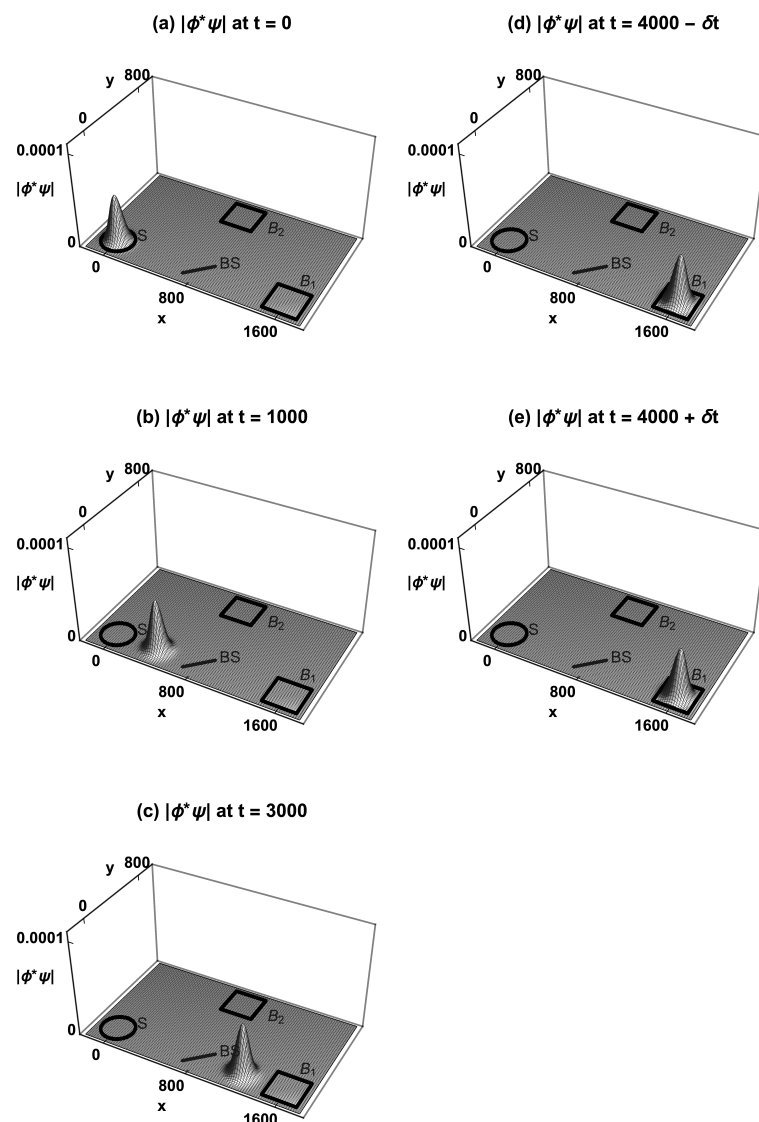


**Figure 2.** The Conventional Formulation (CF) explanation of the Einstein's Boxes experiment, with a single particle wavefunction emitted from source S. (a) The probability density  $\psi^* \psi$  is localized inside S. (b)  $\psi^* \psi$  has left S and is traveling towards the beam-splitter BS. (c)  $\psi^* \psi$  has been split in half by BS, and the two halves are traveling towards boxes  $B_1$  and  $B_2$ . (d) The two halves arrive at  $B_1$  and  $B_2$ . (e) a measurement at either  $B_1$  or  $B_2$  at  $t = 4000$  causes either  $\psi$  to collapse to zero in  $B_2$  and to a full wavefunction in  $B_1$  (shown), or  $\psi$  to collapse to zero in  $B_1$  and to a full wavefunction in  $B_2$  (not shown).

#### 4. The Time-Symmetric Formulation of Einstein's Boxes

The TSF postulates that quantum mechanics is a theory about *transitions*, described by the transition amplitude density  $\phi^* \psi$ , where  $\psi$  is a wavefunction that obeys the Schrödinger equation and satisfies the initial conditions, while  $\phi$  is a wavefunction that obeys the Schrödinger equation and satisfies the final conditions. As in the TSF,  $\psi$  can be interpreted as a retarded wave from the past initial conditions, and  $\phi^*$  can be interpreted as an advanced wave from the future final conditions [3]. We will assume the same wavefunctions  $\psi(\vec{r}, t)$  and  $\phi(\vec{r}, t)$  as in the CF above.

An electron (e.g.) can be absorbed by a few molecules in a detector. The number of few-molecule sites in a detector is orders of magnitude larger than the number of square-centimeter sites in a detector. This makes it overwhelmingly more likely that the electron will be absorbed in an area localized to a few square nanometers than much larger areas. This could explain why the transition amplitude density refocuses to a localized area at the detector. Let us then assume the final conditions are either a transition amplitude density localized in box  $B_1$  or a transition amplitude density localized in box  $B_2$ . The question of why the transition amplitude density localizes to one or the other box is left unanswered.



**Figure 3.** The Time-Symmetric Formulation (TSF) explanation of the Einstein's Boxes experiment, with a single particle transition amplitude density emitted from source  $S$  and detected at box  $B_1$ . (a) The absolute value of the transition amplitude density  $|\phi^*\psi|$  is localized inside  $S$ . (b)  $|\phi^*\psi|$  has left  $S$  and is traveling towards the beam-splitter  $BS$ . (c)  $|\phi^*\psi|$  has passed through  $BS$  and is traveling towards box  $B_1$ .  $|\phi^*\psi|$  is zero on the path from  $BS$  to  $B_2$  because  $\phi^*$  is zero on this path. (d)  $|\phi^*\psi|$  arrives at  $B_1$ . (e) Measurements at  $t = 4000$  show a transition amplitude density in  $B_1$  and no transition amplitude density in  $B_2$ . With equal probability, the final condition could have been localization in box  $B_2$ . Transition amplitude density collapse never occurs.  $|\phi^*\psi|$  is normalized.

Figure 3 shows the TSF explanation of the Einstein's Boxes thought experiment, assuming the final condition is a particle transition amplitude density localized in box  $B_1$ .



At  $t = 0$ ,  $|\phi^*\psi|$  is localized inside the source  $S$ . At  $t = 1000$ ,  $|\phi^*\psi|$  is traveling toward the beam-splitter  $BS$ . At  $t = 3000$ ,  $|\phi^*\psi|$  has passed through the beam-splitter, and is traveling towards box  $B_1$ .  $|\phi^*\psi|$  is zero on the path from  $BS$  to  $B_2$  because  $\phi^*$  is zero on this path. At  $t = 4000 - \delta t$ ,  $|\phi^*\psi|$  arrives at box  $B_1$ . Upon a measurement at box  $B_2$  at  $t = 4000$ , no **particle transition amplitude density** is found. Upon a measurement at box  $B_1$  at  $t = 4000$ , one particle's transition amplitude density is found. The one-particle transition amplitude density was localized inside box  $B_1$  before the measurement was made.

The TSF assumes the probability  $P_t$  for the transition from localization in Source  $S$  to localization in Box  $B_1$  is  $P_t = \frac{1}{2} A_t^* A_t$ , where the subscript  $t$  denotes the TSF, the "classical" probability factor  $\frac{1}{2}$  accounts for the fact that there are two equally likely possible final states, and the TSF amplitude  $A_t$  for the transition is:

$$A_t = \int_{-\infty}^{\infty} \phi^*(x, y, t) \psi(x, y, t) dx dy, \quad (3)$$

Plugging in numbers gives a TSF transition probability  $P_t = 0.43$ , which is identical to the CF collapse probability result. Note that there is no transition amplitude density collapse in the TSF, so there is no need to specify the time of collapse in the integrand.

## 5. Discussion

To the best of my knowledge, this is the first time a TSF has been shown to resolve the Einstein's boxes paradox. The TSF resolves the paradox in the ways that Einstein and de Broglie envisioned. The transition amplitude density  $\phi^*\psi$  "localises the particle during the propagation [2]" and  $\phi^*\psi$  "was already in Paris in box  $B_1$  prior to the drainage experiment made in Tokyo in box  $B_2$ . [35]" None of the problems associated with wavefunction collapse occur. The TSF appears to give the sought-after exact description of the probabilities and a complete description of the physical reality.

One might wonder if a theory based on transition amplitude densities will be able to reproduce all of the predictions of the CF. In 1932 Dirac showed that all the experimental predictions of the CF of quantum mechanics can be formulated in terms of transition probabilities [37]. The TSF inverts this fact by postulating that quantum mechanics is a theory which experimentally predicts *only* transition probabilities. This implies the TSF has the same predictive power as the CF.

The TSF has the additional benefit of being consistent with the classical explanation of the Einstein's boxes thought experiment. As the size of the "particle" becomes larger and starts behaving more like a classical particle, it will always go to either one box or the other. There is a logical continuity between its behavior in the quantum and classical regimes, in contrast to the CF predictions.

A central issue raised by the Einstein's boxes paradox is the question of which elements of quantum theory should be thought of as elements of reality (ontic), and which elements are merely states of knowledge (epistemic). In my view, the TSF wavefunctions  $\psi$  and  $\phi$  and the transition amplitude density  $\phi^*\psi$  should be thought of as elements of reality. For multiple particles,  $\phi^*\psi$  lives in a higher dimensional configuration spacetime, which should be thought of as the stage for reality [32]. The CF concept of a superposition of paths after the beam-splitter then becomes just a state of knowledge in the TSF: in reality, only one path is taken, we just do not know in advance which one. Since the TSF assumes both the initial and final boundary conditions are always specified, it is a deterministic theory. In analogy with the classical theory of special relativity, the transition amplitude density can be thought of as a quantum worldtube. The higher dimensional configuration spacetime is then the quantum equivalent of Minkowski spacetime.

Finally, the CF predicts a rapid oscillating motion of a free particle's wavefunction in empty space. Schrödinger discovered the possibility of this rapid oscillating motion in 1930, naming it zitterbewegung [38]. This prediction of the CF is inconsistent with Newton's first law, since it implies a free particle's wavefunction does not move with a constant velocity. The TSF predicts zitterbewegung will never occur [3]. Direct measurements of

zitterbewegung are beyond the capability of current technology, but future technological developments should allow measurements to confirm or deny its existence. Given the technology, one possible way to test for zitterbewegung would be to hold an electron in the ground state in a parabolic potential, then turn off the potential while looking for radiation at the zitterbewegung frequency of  $10^{21}\text{s}^{-1}$ . This could distinguish between the CF and the TSF.

**Funding:** This research received no external funding

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** Not applicable.

**Acknowledgments:** I thank Travis Norsen and David A. Fotland for helpful conversations.

**Conflicts of Interest:** The author declares no conflict of interest.

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