Article

Constraints on General Relativity Geodesics by a Covariant Geometric Uncertainty Principle

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Abstract: The classical uncertainty principle inequalities were imposed as a mathematical constraint over the general relativity geodesic equation. In this way, the uncertainty principle was reformulated in terms of the proper space-time length element, Planck length and a geodesic-derived scalar, leading to a geometric expression for the uncertainty principle (GeUP). This re-formulation confirmed the necessity for a minimum length for the space-time line element in the geodesic, dependent on a geodesic-derived scalar which made the expression Lorentz-covariant. In agreement with quantum gravity theories, GeUP required the imposition of a perturbation over the background Minkowski metric unrelated to classical gravity. When applied to the Schwarzschild metric, a geodesic exclusion zone was found around the singularity where uncertainty in space-time diverged to infinity.

Keywords: General relativity; Uncertainty principle; Geodesics; Black hole singularity; zero-point energy; Quantum gravity; Planck star

1. Introduction

General Relativity (GR) describes gravitation as a dynamical space-time geometry in a pseudo-Riemannian manifold shaped by energy-momentum densities [1]. Its mathematical framework is highly consistent and valid in any reference frame. However, GR is largely incompatible with quantum mechanics. One key difficulty for unification is that GR world-lines for particles can be defined with infinite precision [1], while this is not allowed in quantum mechanics. The process of measuring position in quantum mechanics introduces uncertainty in momentum and *vice versa* [2]. The momentum/position uncertainty originally proposed by Heisenberg is considered a fundamental principle in nature [3]. This principle is behind many quantum phenomena [4,5]. Other major difficulties in reconciling general relativity with quantum mechanics are the non-renormalizability of GR when formulated as a quantum field theory [7] and the quantum-mechanical violation of the weak equivalence principle [6].

In quantum gravity theories, a limit on the length of the space-time line element is imposed when energy fluctuations alter the space-time metric [8]. These quantum fluctuations may constitute part of the source of a gravitational background state that imposes such a length limit [8]. String theory leads naturally to expressions dependent on a fundamental length [9], from which a generalized uncertainty principle (GUP) associated to Planck length is derived [8,10]:

$$\Delta x \Delta p \geq \frac{\hbar}{2} + \frac{\alpha}{c^3} G \Delta p^2 \tag{1}$$

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Considering GUP in the framework of quantum geometry theory, any accelerating particle in the absence of gravity experiences a gravitational field [8]. This field is the result of a perturbation unrelated to classical gravitation applied over the background Minkowski metric. This approach recovers the generalized uncertainty principle in its p-quadratic form, which depends on the mass of the particle and proper acceleration ($\boldsymbol{A}$) as a constant if expressed in terms of Planck mass [8]:

$$\Delta x \Delta p \geq \frac{\hbar}{2} + \frac{\hbar c^2}{m^2 A^2 \delta s^2} \Delta p^2 \qquad (2)$$

However, the necessity for a minimum space-time length in quantum gravity theories still clashes with relativity, because this fundamental line element must be Lorentz-invariant. Hence, adapting the uncertainty principle to Lorentz covariance requires corrections over the GUP canonical momentum/position commutator in Minkowski space, described by [4,11] and shown below:

$$[P^{\mu}, X^{\nu}] = -i\hbar\left(1 + (\varepsilon - \alpha)\gamma^2 P^{\rho} P_{\rho}\right)\eta^{\mu\nu} - i\hbar(\beta + 2\xi)\gamma^2 P^{\mu} P^{\nu} \quad (3)$$

Where α, β, and ξ are dimensionless parameters to adapt the equation to the particular problem.

The uncertainty principle has also been brought up to counteract relativistic singularities. Some GR solutions contain singularities in which space-time adopts infinite curvature, such as in black holes [12-14]. However, the imposition of a minimum allowable space-time length element would be incompatible with a point singularity. Thus, assuming quantized gravity and space-time, the uncertainty principle would be the source of a repulsion force that prevents particles from reaching the singularity. The matter repelled from it would form a "Planck star" [15]. An "equivalent" concept is used in string theory with the "fuzzball" structure [16]. This is important, because although infinite curvature is not problematic as a geometry, a black hole singularity could erase the history of any particle that ends up in it. This irreversible process would contribute to the black hole information paradox [17,18]. It may be nevertheless argued that particles in a black hole singularity differ in proper time. However, the space-time length element is not defined right at the singularity. This is evident at radial coordinate position 0 in the Schwarzschild solution in spherical coordinates:

$$d\tau^2 = -\left(\frac{R - R_s}{R}\right) dt^2 + \frac{R\, dR^2}{R - R_s} + R^2 d\theta^2 + R^2 sin^2\theta d\varphi^2 \quad (4)$$

Where $\boldsymbol{R}$, $\boldsymbol{\theta}$ and $\boldsymbol{\varphi}$ stand for radial, polar and azimuthal coordinates, respectively, in a Lorentzian metric signature {- + + +}, and $\boldsymbol{Rs}$ refers to the Schwarzschild radius. When approaching the singularity, the time component of the metric ($\boldsymbol{g_{00}}$) diverges to infinity while the radial component ($\boldsymbol{g_{RR}}$) becomes 0. From $\boldsymbol{Rs}$ to the interior of the black hole, the signs of these metric components interchange, with the radial-dependent metric behaving as time rather than space. Despite these considerations, the singularity remains at R position 0 where the length of the space-time line element is undefined.

In this manuscript, the classical principle of uncertainty was reformulated in terms of geometric parameters to impose a mathematical constraint on the geodesic equation. The requirement for a minimum space-time length element was confirmed in this re-formulation, as well as the need of imposing a perturbation over the background Minkowski metric to comply with the uncertainty principle. When applied to geodesics in the

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Schwarzschild metric, the presence of an exclusion zone around the singularity was also confirmed.

2. Derivation of a relativistic tensor expression for the classical uncertainty principle inequalities

For simplicity, c and the mass of the particle were both set to 1. Tensor notation was used throughout the paper, which includes representation of generalized contravariant coordinates as X^{μ}. For clarity, the temporal coordinate X^0 was represented as t in some specific cases.

The classical uncertainty principle is represented by two separate inequalities.

$$|\Delta \boldsymbol{p}||\Delta \boldsymbol{x}| \geq \frac{\hbar}{2} \quad ; \; |\Delta \boldsymbol{E}||\Delta \boldsymbol{x}^0| \geq \frac{\hbar}{2} \qquad (5)$$

Where $\Delta \boldsymbol{p}$ represents the change in magnitude of non-relativistic momentum parametrized by coordinate time, and $\Delta \boldsymbol{x}$ the change in magnitude of the position 3-vector. These two inequalities can be written in tensor notation, following these identities:

$$|\Delta \boldsymbol{p}||\Delta \boldsymbol{x}| = \left|\sqrt{\Delta p^m \Delta p_m \, \Delta x_m \Delta x^m}\right| = |\Delta p_m \Delta x^m|$$

$$\Delta E \equiv \Delta P_0 \quad ; \; |\Delta E||\Delta x^0| = \left|\sqrt{\Delta P^0 \Delta P_0 \, \Delta x_0 \Delta x^0}\right| = |\Delta P_0 \Delta x^0| \qquad (6)$$

In units of c set to 1, energy is identified with the temporal component of the relativistic 4-momentum vector, which is parametrized by proper time "τ". From now on, the relativistic momentum will be represented by capital P. Inequalities (5) then take the following form in tensor notation:

$$|\Delta p_m \Delta X^m| \geq \frac{\hbar}{2} \quad ; \; |\Delta P_0 \Delta X^0| \geq \frac{\hbar}{2} \qquad (7)$$

$$m \in \{1, 2, 3\}$$

To parametrize non-relativistic momentum by proper time, the gamma factor (γ) is introduced as defined below. γ is also equivalent to the ratio of the total energy of a particle (E) by its mass energy. The same correction was included in the energy-time inequality so that both can be merged.

$$\gamma = \frac{dt}{d\tau} \equiv \frac{E}{m}$$

$$\left|\frac{1}{\gamma} \Delta P_m \Delta X^m\right| \geq \frac{\hbar}{2} \quad ; \quad \left|\frac{1}{\gamma} \Delta P_0 \Delta X^0\right| \geq \frac{\hbar}{2\gamma} \qquad (8)$$

Adding inequalities (8) we obtain:

$$|\Delta P_m \Delta X^m| + |\Delta P_0 \Delta X^0| \geq (1 + \gamma)\frac{\hbar}{2} \qquad (9)$$

γ is generally 1 for non-relativistic particles, which will lead to another classical expression for the non-relativistic uncertainty principle:

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$$|\Delta P_m \Delta X^m| + |\Delta P_0 \Delta X^0| \geq \hbar \tag{10}$$

Assuming that the uncertainty principle must also apply to differential changes in momentum and position, the merged inequality can be re-written in terms of changes in momentum and position 4-vectors as follows:

$$|dP^m dX_m| + |dP_0 dX^0| \geq \frac{\hbar}{2} \tag{11}$$

For simplification, the relativistic correction term $(1+\gamma)$ shown in inequality (9) was removed as it can be easily incorporated when needed. We then re-expressed the inequality in terms of Planck length:

$$|dP^m dX_m| + |dP_0 dX^0| \geq \frac{\ell_p^2}{2G} \tag{12}$$

The terms in the inequality were re-expressed as rates of change in momentum and position with proper time as follows:

$$\left|\frac{dP^m}{d\tau} d\tau \frac{dX_m}{d\tau} d\tau\right| + \left|\frac{dP^0}{d\tau} d\tau \frac{dX_0}{d\tau} d\tau\right| \geq \frac{\ell_p^2}{2G} \tag{13}$$

Which leads to:

$$\left|\frac{dP^m}{d\tau} U_m\, d\tau^2\right| + \left|\frac{dP^0}{d\tau} U_0\, d\tau^2\right| \geq \frac{\ell_p^2}{2G} \tag{14}$$

The uncertainty principle is thus reformulated as an uncertainty relationship between proper space-time length, 4-momentum change and proper velocity (U_μ). It has to be noted that the inequality is undefined for null space-time length.

3. Derivation of a covariant geometric form of the uncertainty principle

The reformulated uncertainty principle in inequality (14) can be imposed over the geodesic equation. Geodesic trajectories can be identified with proper momentum change in an interval of space-time length element:

$$\frac{dU^\mu}{d\tau} = -\Gamma^{\ \mu}_{\alpha\beta}\, U^\alpha U^\beta \equiv \frac{dP^\mu}{d\tau} \tag{15}$$

$\mu, \alpha, \beta \in \{0, 1, 2, 3\}$

With Christoffel symbols calculated from the pseudo-Riemannian metric tensor ($g_{\mu\nu}$) that defines the space-time metric as shown below:

$$\Gamma^{\ \mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\lambda}\left(\partial_\beta g_{\lambda\alpha} + \partial_\alpha g_{\beta\lambda} - \partial_\lambda g_{\alpha\beta}\right) \tag{16}$$

$\mu, \alpha, \beta, \lambda \in \{0, 1, 2, 3\}$

The geodesic equation (equation 15) can be incorporated in inequality (14) as follows:

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$$\left|U_m\Gamma^{\ m}_{\alpha\beta}\, U^{\alpha}U^{\beta}\ \ d\tau^2\right| + \left|U_0\Gamma^{\ 0}_{\alpha\beta}\, U^{\ \alpha}U^{\beta}\ \ d\tau^2\right| \geq \frac{\ell_p^2}{2G} \qquad (17)$$

In this way, the uncertainty principle is re-defined as the product of the interval of the space-time line element with a scalar derived from the geodesic trajectory (geometric scalar, G_{geo}):

$$G_{geo} \equiv 2G\left|U_m\Gamma^{\ m}_{\alpha\beta}\, U^{\alpha}U^{\beta}\right| + 2G\left|U_0\Gamma^{\ 0}_{\alpha\beta}\, U^{\alpha}U^{\beta}\right|$$

$$\left|G_{geo}\, d\tau^2\right| \geq \ell_p^2 \qquad (18)$$

This geometric form of the uncertainty principle imposes a Lorentz-covariant minimum length for proper space-time distance through the geodesic scalar. This inequality also allows a degree of uncertainty in the geodesic trajectory.

4. Geometric uncertainty principle in Minkowski space

GeUP was applied for a particle at rest in classical Minkowski space with a {- + + +} metric signature. The Minkowski metric tensor will be denoted as $\eta_{\mu\nu}$. As this metric has null Christoffel connectors, the geodesic scalar is 0. Then, inequality (18) represents a contradiction unless Planck length is considered 0 in the non-quantum limit**:**

$$0 \geq \ell_p^2\ ? \qquad (19)$$

To meet the GeUP condition, the Minkowski metric has to deviate from flat space, for example by introducing a differential perturbation to the metric (classically denoted as $h_{\mu\nu}$). The resulting space-time metric tensor $g_{\mu\nu}$ will be the sum of both metrics.

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad (20)$$

$\mu, \nu \in \{0, 1, 2, 3\}$

In this example, the perturbation will depend only on the temporal coordinate to fulfil conditions of spatial homogeneity and isometry. For a particle at rest only the temporal component of its proper velocity will be non-zero, and the only relevant component of the metric tensor for the calculations will correspond to:

$$g_{00} = \eta_{00} + h_{00} = -1 - \varepsilon(t) \qquad (21)$$

We define $\varepsilon(t)$ as the time-dependent perturbation function corresponding to h_{00}, to avoid confusion with Planck constant. Likewise, for a particle at rest only the X^0 coordinate (or t) will contribute to proper time. The inequality (18) takes the following form in terms of Planck constant:

$$\left|-2U_0\Gamma^{\ 0}_{00}\, U^0U^0\ \ (-1-\varepsilon)\ dt^2\right| \geq \hbar \qquad (22)$$

The calculation of the Christoffel symbol is straightforward because only g_{00} has a dependency in the X^0 coordinate (equation 21).

$$\Gamma^{\ 0}_{00} = \frac{1}{2}g^{00}(\partial_0 g_{00}) = \frac{-1}{2(1+\varepsilon)}\partial_0(-1-\varepsilon) = \frac{\partial_0\varepsilon}{2(1+\varepsilon)} \qquad (23)$$

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After calculating the term (23), the inequality (22) can be solved as follows:

$$\left|-2U_0U^0\,U^0\,\frac{\partial_0\varepsilon}{2(1+\varepsilon)}\,\,(-1-\varepsilon)\,dt^2\right| \geq \hbar$$

$$\left|-U^0\,\partial_0\varepsilon\,\,\,dt^2\right| \geq \hbar \qquad (24)$$

This inequality can be re-expressed by simplifying the derivative of the perturbation field by multiplication with dt, and re-introducing the mass of the particle in the temporal component of the 4-momentum vector which replaces U^0:

$$\left|P^0\,d\,\varepsilon\,\,\,dt\right| \geq \hbar \qquad (25)$$

The classical expression for the time-energy uncertainty principle is recovered, with $\mathbf{P^0}$ corresponding to the energy of the particle. Thus, we can determine the particle´s energy in a given interval of geodesic time $\mathbf{dt}$ to be $\mathbf{P^0 d\varepsilon}$, with $\mathbf{d\varepsilon}$ denoting the accuracy on the measurement. High-precision determination of $\mathbf{P^0 d\varepsilon}$ implies long intervals of time according to inequality (25). Likewise, measurements over increasingly precise intervals of geodesic time correspond to increased fluctuations ($\mathbf{d\varepsilon}$) in the energy of the particle. These fluctuations of the $\boldsymbol{\varepsilon}\,(\mathbf{t})$ function will alter the background space-time metric (equation 21). In this expression the relativistic factor omitted in inequality (9) will be re-introduced with a value of 2 leading to:

$$\left|P^0\,\,d\,\varepsilon\,\,\,dt\right| \geq 2\hbar \quad ; \quad |d\varepsilon\,| \geq \left|\frac{2\hbar}{P^0\,dt}\right| \qquad (26)$$

We can express the inequality as an equation for the lowest value that would agree with GeUP. For an illustrative example, the differential in the $\varepsilon(t)$ will be approximated to an interval.

$$\varepsilon = \varepsilon_0 + \frac{2\hbar}{P^0\,dt} \qquad (27)$$

The initial value for the perturbation field can be chosen as 0 (no correction over the Minkowski metric). This leads to an expression for the components of the metric with a minimum allowed perturbation of the metric as follows:

$$g_{00} = -1 - \frac{2\hbar}{P^0 dt} \qquad (28)$$

$\mathrm{m} \in \{1, 2, 3\}$

In this case, the metric perturbation over the Minkowski metric background is unrelated to classical gravitation, but it is a function of the accuracy in the interval of coordinate time in the geodesic trajectory. Minkowski space-time is recovered at the non-quantum limit ($\hbar \rightarrow \mathbf{0}$).

5. Application of the geometric uncertainty principle to the Schwarzschild metric

The GR solution for a gravitational field generated by a point mass corresponds to the Schwarzschild metric. This solution has spherical symmetry and it is usually expressed in spherical coordinates (equation 4). The spherical symmetry allows the selection of physically relevant particle trajectories in proper space-time geodesics with constant polar and azimuthal coordinates. These conditions ensure the following statements:

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$$d\theta = d\varphi = 0 \;\;\rightarrow\;\; U^{\theta} = U^{\varphi} = 0 \;\;;\;\; U_0U^0 + U_RU^R = -1 \quad (29)$$

These geodesics will have contributions from t and R components of proper velocity. Inequality (18) can then be expressed as follows:

$$\left|dR^2 + \frac{g_{00}}{g_{RR}}dt^2\right| \geq \left|\frac{\ell_p^2}{g_{RR}G_{geo}}\right| \quad (30)$$

Introducing the components of the metric tensor, we have

$$\left|dR^2 + \frac{(R-R_s)^2}{R^2}dt^2\right| \geq \left|\frac{(R-R_s)\,\ell_p^2}{R\,G_{geo}}\right| \quad (31)$$

The geodesic scalar is calculated with the contributions of the temporal and radial components of the proper velocity:

$$G_{geo} = 2G\left|U_0\Gamma^{\,0}_{\alpha\beta}U^{\alpha}U^{\beta}\right| + 2G\left|U_R\Gamma^{\,R}_{\alpha\beta}U^{\alpha}U^{\beta}\right| \quad (32)$$

The non-zero Christoffel connectors in the Schwarzschild metric relevant for this solution are the following:

$$\Gamma^{\,0}_{R0} = -\Gamma^{\,R}_{RR} = \frac{R_s}{2R(R-R_s)} \quad ; \quad \Gamma^{\,R}_{00} = \frac{R_s(R-R_s)}{2R^3} \quad (33)$$

Leaving the expression for the geometric scalar as:

$$G_{geo} = 2G\left(\left|U_0\Gamma^{\,0}_{R0}U^RU^0 + U_0\Gamma^{\,0}_{0R}U^0U^R\right| + \left|U_R\Gamma^{\,R}_{00}U^0U^0 + U_R\Gamma^{\,R}_{RR}U^RU^R\right|\right) \quad (34)$$

It is reduced after several operations to:

$$G_{geo} = \frac{GR_sU^R(2U_0U^0+1)}{R(R-R_s)} \quad (35)$$

Introducing (35) into (31) and solving for the square of the length element in the R coordinate:

$$dR^2 \geq \left|\frac{2M}{(2U_0U^0+1)U^R}\left(\frac{R}{R_s}-1\right)^2\ell_p^2 - \left(1-\frac{R_s}{R}\right)^2dt^2\right| \quad (36)$$

The gravitational constant G was replaced by $R_s/2M$ so that M now represents the mass generating the gravitational field. In geodesics close to the singularity at R=0, the uncertainty in the R coordinate (represented by dR^2) diverges to infinity. This implies that a particle getting close to the singularity is highly de-localized. No particle geodesic below the threshold set by inequality (36) would be allowed, thus defining an exclusion zone around the singularity as a function of uncertainty in the t coordinate (Figure 1).

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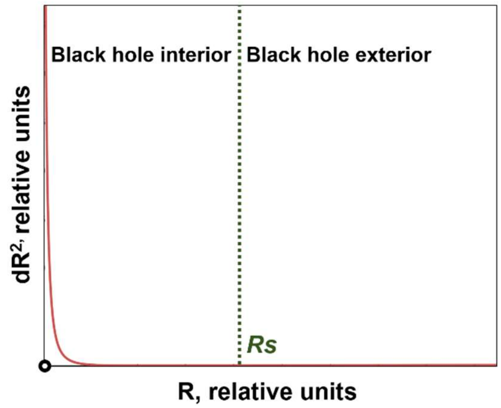

Figure 1. Black hole singularity exclusion zone. The figure represents the function from inequality 36 with relative values for the radial distance and arbitrary coefficients to help visualization. The relative uncertainty in the R coordinate (represented as dR^2) is plotted as a function of R. Close to the singularity at R=0, the uncertainty for allowed geodesics diverges to infinity. Geodesics with dR^2 values below the curve are not allowed, and define a particle exclusion zone in the interior of the black hole.

Then, we considered a special case in which the t coordinate of a particle in the geodesics defined by (36) is known with absolute certainty. Application of this condition provides the minimum possible uncertainty in the R coordinate when approaching the singularity. This implies the following:

$$dt = 0 \tag{37}$$

And the solution to inequality (36) for this condition is:

$$dR^2 \geq \left| \frac{2M}{U^R}\left(\frac{R}{R_s} - 1\right)^2 \ell_p^2 \right| \tag{38}$$

We reach two conclusions. First, it is defined at the singularity (R=0). Second, the uncertainty in the R coordinate of the geodesic decreases as a function of decreasing R down to an allowed minimum value right in the singularity:

$$dR^2(R = 0) \geq \frac{2M}{U^R}\ell_p^2 \tag{39}$$

Although physically unrealistic, a threshold value for the uncertainty in the R coordinate can be calculated below which no geodesic will ever be allowed. An approximate calculation of dR for a stellar mass black hole (without adding the appropriate corrections), gives a value within the range of 10^{-15} to 10^{-16} cm.

5. Discussion

Here the principle of uncertainty was reformulated in terms of covariant geometric parameters. This re-formulation was the result of applying the classical uncertainty principle inequalities over the geodesic equation just as a mathematical constraint. No quantization of space-time or quantum gravitational field were introduced. Hence, it has to be remarked that our current paper does not constitute a theory for quantum gravity, nor it represents a canonical generalization on quantum gravity. However, the inequalities of the uncertainty principle presented in this form yield interesting results that can be interpreted in light of current well-developed quantum gravity theories.

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First, there is an imposition of a minimum length for the space-time line element. This is a common feature of current quantum gravity theories such as loop quantum gravity [19,20], string theory [9,18,21,22] and doubly-special relativity [23]. Therefore, although GeUP is not a quantum gravity theory *per se*, it introduces ambiguity in the space-time trajectory for the geodesics. The space-time distance element is expressed as a relationship with Planck length through a geodesic-derived scalar, which ensures Lorentz covariance.

Secondly, in agreement with results from GUP [8,10], GeUP was incompatible with Minkowski space. Its mathematical constraint over the geodesics imposed a perturbation in the metric unrelated to classical gravity. This incompatibility should be expected in any quantum gravity theory because vacuum energy fluctuations will alter the metric [8]. In the example derived in this manuscript, the perturbation was a function of the interval of coordinate time in the geodesic trajectory, and the energy of the particle. GUP and GeUP enforce a perturbation in the metric of Minkowski space which could be considered part of a gravitational background state. This perturbation alters the classical uncertainty principle in GUP by a factor dependent on the mass-energy and proper acceleration (inequality 2) [8]. GUP has been applied to multiple scenarios such as corrections to black-hole entropy and thermodynamics, and thermodynamics of cosmological models, as extensively reviewed in [24]. However, corrections have also been imposed over its canonical expression to ensure Lorentz invariance [11] (inequality 3). It could be argued whether the metric perturbations caused by the uncertainty principle formulations constitute a background state for the gravitational field. Gravitational background states in GR are highly problematic because of the non-linearity of GR equations. Vacuum energy and dark energy may contribute to such background states and to the expansion of the universe through the cosmological constant Λ. So far, the calculations on the contributions to Λ by different sources do not provide a satisfactory explanation to its small positive value [25].

Thirdly, the uncertainty principle has been proposed to be the source of a repulsion force that avoids black hole singularities. This “uncertainty force” is the basis for the stability of Planck stars [15]. Application of GeUP over the Schwarzschild solution also uncovered a region close to the singularity below which geodesics violate the uncertainty principle. Particles approaching the singularity will have an uncertainty in the R coordinate so large that they would appear to be repelled from the singularity. According to our constraint and after selection of geodesics with constant coordinate time, we calculated the minimum possible uncertainty ($\boldsymbol{dR}$) in the R coordinate for a stellar mass black hole to be of the order of 10^{-15} cm. A rough estimation on the diameter of a Planck star by loop quantum gravity gives a value of about 10^{-10} cm [15,19]. Both calculations provide sizes larger than Planck length by several orders of magnitude.

Concluding, we have re-formulated the classical uncertainty principle to impose a mathematical restriction on GR geodesics. Our equations do not represent a true quantum gravity theory but highlight some of the issues common to all quantum gravity theories: the necessity for a minimum space-time length element that must keep Lorentz invariance, and the need for background quantum gravity states. However, full answers to the several drawbacks will have to be constructed from well-founded quantum gravity theories from canonical principles.

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