Article

Ontology of a wavefunction from the perspective of an invariant proper time

Salim Yasmineh [1,*]

[1] University of Paris*; sayasmineh@gmail.com*
[*] Correspondence: sayasmineh@gmail.com

Abstract: All the arguments of a wavefunction are defined at the same instant implying a notion of simultaneity. In a somewhat related matter, certain phenomena in quantum mechanics seem to have non-local causal relations. Both concepts are in contradiction with special relativity. We propose to define the wavefunction with respect to the invariant proper time of special relativity instead of standard time. Moreover, we shall adopt the original idea of Schrodinger suggesting that the wavefunction represents an ontological cloud-like object that we shall call 'individual fabric' that has a finite density amplitude vanishing at infinity. Consequently, measurement can be assimilated to a confining potential that triggers an inherent non-local mechanism within the individual fabric. It is formalised by multiplying the wavefunction with a localising gaussian as in the GRW theory but in a deterministic manner.

Keywords: proper time; non-locality; simultaneity; wavefunction; measurement; ontology

1. Introduction

As clearly explained by Travis Norsen in his book "Foundations of Quantum Mechanics" [1] there are three main interconnected problems in Quantum Mechanics: measurement, relativity (non-locality and simultaneity) and ontology of a wavefunction.

Quantum measurement can be highlighted by the process of measuring a physical property of a microscopic system by a measuring instrument. Suppose the quantum mechanical wavefunction describing the microscopic system is in a superposition of eigenstates of an operator corresponding to the physical property that is being measured. Then, contrary to real observations, the Schrodinger equation evolution implies that the measuring instrument should be in a macroscopic superposition of many distinct states. Standard quantum mechanics (Copenhagen interpretation) solves this problem of macroscopic superposition by introducing Born's Rule. This rule postulates that a wavefunction evolves deterministically in accordance with Schrodinger's equation, except during measurement when instead it collapses with a certain probability measure to one particular eigenstate. The measurement problem consists in understanding this paradoxical transition from a deterministically evolving spread-out wavefunction into a sudden probabilistic localisation.

The relativity problem concerns two main interrelated features. The first relates to the relativity of simultaneity which is a consequence of Lorentz invariance and the second relates to the notion of non-locality. In quantum mechanics, all the arguments of a wavefunction are defined at the same instant requiring thus, a notion of absolute simultaneity. In particular, a quantum system for example made up of a pair of entangled particles behaves in such a manner that the quantum state of one particle cannot be described independently of the state of the other. Standard quantum mechanics postulates that neither one of both particles has a determinate state until it is measured. As both particles are correlated, it is necessary that when the state of one particle is measured the second particle should 'simultaneously' acquire a determinate state. However, the laws of physics are invariant under Lorentz transformations. There is no meaning of 'simultaneity'

independently of any frame of reference and there should be no preferred frame of refence [2, 3]. On the other hand, the non-locality problem can be resumed by Einstein's argument that if quantum mechanics is complete then, the collapse of the wave function is a dynamical process, which conflicts with relativistic locality. This quantum phenomenon was first introduced as a thought experiment in the EPR paper [4] and it was later discovered that it can be experimentally testable by using Bell's inequality [5, 6]. There have been numerous experiments such as Aspect's experiment [7] that proved the validity of quantum entanglement and hence of non-local causal connections. Thus, the notions of non-locality and simultaneity are inferred by quantum mechanics while being forbidden by the postulates of relativity.

The ontological problem concerns the fact that for more than one particle, the wavefunction is defined on a high dimensional configuration space and not on a physical space and thus, cannot represent a physically real field. To have a physical ontology, the wavefunction should be the representation of some sort of a real physical entity.

In addition to the Copenhagen interpretation, there are several candidate theories that propose solutions to at least some of the above problems. The main current theories comprise Everett many-world theory, de Broglie Bohm pilot-wave theory [8, 9] and GRW spontaneous collapse theory [10]. This paper is mainly based on the last two theories.

The de Broglie Bohm theory is very promising and is discussed in detail by Jean Bricmont in his book "Making Sense of Quantum Mechanics" [11] as well as in Norsen [1]. The basic idea behind this theory is that a corpuscle such as an electron has always a well determined position on a definite trajectory through physical space. However, its movement is influenced by an associated wavefunction giving rise to wave-like properties. Thus, according to this theory, an electron is a particle 'and' a wave. The de Broglie Bohm theory solves the measurement problem and accounts for non-locality. However, it does not solve the problem of simultaneity. Indeed, for a multi-particle system, the theory explicitly formulates the dependence of a particle's evolution at a given instant on the positions of all the other particles at the same instant. Moreover, the ontology of the wavefunction which is defined in configuration space remains puzzling.

The GRW spontaneous collapse theory [10] is discussed in Norsen [1]. It modifies Schrodinger's equation with stochastic terms that has the effect of making a wavefunction obey Schrodinger's equation most of the time, except for exceedingly rare and random instants when it instead undergoes a spontaneous collapse. GRW solves the measurement problem, accounts for non-locality and gives a physical explanation to the wavefunction. Nevertheless, it does not solve the problem of simultaneity and seems to be ad hoc.

In this paper we propose to make a certain sense out of the above problems mainly by defining the wavefunction with respect to proper time in accordance with special relativity.

2. Invariant spacetime structure

We propose to consider the evolution of a particle from the perspective of proper time τ, which is the only real physical time, by using the hyperbolic spacetime structure inside a light cone associated to the particle. Specifically, we shall use the formalism of Minkowski spacetime [12] as defined in a geometrical manner by Eric Gourgoulhon in his book 'Special Relativity in General frames' [13].

Minkowski spacetime $\mathcal{E}$ is defined as an affine space of dimension 4 on R endowed with a bilinear metric tensor g defined in an underlying vector space E, of signature $(+, -, -, -)$. In the vector space E, the set C composed of the zero vector and all null vectors form a light cone C composed of two sheets C^+ and C^- defining the future and past light cones, respectively.

Given the above defined spacetime $\mathcal{E}$ and given an arbitrary point $O \in \mathcal{E}$, a family of affine subspaces $(S_\tau)_{\tau \in R}$ is defined such that each subspace S_τ corresponds to the set of points of $\mathcal{E}$ that can be connected to O by a time-like vector $\overrightarrow{OM}$ of modulus τ, where $\tau \in R$:

$$S_\tau = \{M \in \mathcal{E}, \ \overrightarrow{OM}.\overrightarrow{OM} = -\tau^2 < 0\} \qquad (2.1) \qquad\qquad (1)$$

We are henceforth, interested in physical systems that follow time-like or null world-lines and shall not consider the set of space-like vectors. In the spacetime $(\mathcal{E}, g)$, a point $M \in \mathcal{E}$ is said to belong to the subspace S_τ iff $\overrightarrow{OM}.\overrightarrow{OM} = -\tau^2$. Each set of points S_τ is constituted of two subsets or two sheets S_τ^+ and S_τ^- belonging to the interiors of the future C$^+$ and past C$^-$ light cones, respectively:

$$S_\tau^+ = \{M \in S_\tau, \ \tau \geq 0\} \quad (2.2) \qquad\qquad (2)$$

$$S_\tau^- = \{M \in S_\tau, \ \tau < 0\} \quad (2.3) \qquad\qquad (3)$$

Let (x^0, x^1, x^2, x^3) be the coordinates of $M \in S_\tau$ in the affine frame defined by an origin O and a basis (e_α), then $\overrightarrow{OM}.\overrightarrow{OM} = -\tau^2$ can be expressed as follows:

$$-(x^0)^2 + (x^1)^2 + (x^2)^2 + (x^3)^2 = -\tau^2 \quad (2.4) \qquad\qquad (4)$$

where $x^o = t, \ x^1 = x/c, \ x^2 = y/c, \ x^3 = z/c$.

The above equation (2.4) is a three-dimensional hyperboloid of the two sheets S_τ^+ and S_τ^- spanned by the free extremities of the time-like vectors $\overrightarrow{OM}$. Hereafter, we shall only consider the positive branches S_τ^+ from which the upper index '+' shall be dropped to be simply denoted S_τ.

The modulus τ of the time-like vector $\overrightarrow{OM}$ is the proper time of the physical system. It generates a family of affine subspaces $(S_\tau)_{\tau \in R}$ defined by (2.1). This family is constituted of three-dimensional hyperboloids associated with the real coordinate $\tau \geq 0$ corresponding to a future-directed proper time τ.

The sheet S_τ of each hyperboloid forms a 'spatial-hypersurface' that we shall simply call 'slice' associated with proper time $\tau \in R$. All points on any given slice S_τ are associated with the same proper time τ which is indeed invariant to all observers from the perspective of any inertial frame of reference.

Without any loss of generality, we will refer hereafter to a two-dimensional spacetime. Let $(O; x, t)$ be an orthonormal frame of reference defined by the origin O, a spatial x-axis and a temporal t-axis. We take c=1, then the light cone is composed of the lines X and Y (where X is defined by $t = x$ and Y is defined by $t = -x$) inside which are piled a family of simple hyperbolas $(S_\tau)_{\tau \in R}$. Let (x, t) be the coordinates of a point M on a given hyperbola $(M \in S_\tau)$, then similarly to equation (2.5), $\overrightarrow{OM}.\overrightarrow{OM} = -\tau^2$ can be expressed as follows:

$$t^2 - x^2 = \tau^2 \quad (2.5) \qquad\qquad (5)$$

For each proper time τ, the free extremity M of the vector $\overrightarrow{OM}$ (of modulus τ) spans the hyperbolic-slice S_τ. The hyperbolic slice S_τ is a piecewise twice continuously differentiable curve of Minkowski spacetime $(\mathcal{E}_I, g)$ composed of a set of hyperbolic points (u, τ). The hyperbolic coordinate u represents the orientation of a straight line (or a ray R_u) passing through the origin. All points (u, τ) on the same ray R_u share the same hyperbolic coordinate u. On the other hand, all points (u, τ) on the same slice S_τ share the same invariant proper time coordinate τ.

Thus, the rays $(R_u)_u$ and hyperbolic slices $(S_\tau)_\tau$ define a hyperbolic frame of reference $(O; R_u, S_\tau)$ where a given hyperbolic point (u, τ) is the intersection between the corresponding ray R_u and slice S_τ.

The hyperbolic slice S_τ can be parametrized by a bijective function φ from the points on the real axis R into the points on that slice S_τ (i.e. $: R \to S_\tau$) such that any point u on the slice S_τ is given by $u = \varphi(\lambda) \equiv u(\lambda)$. The parameter λ can be chosen to be the standard time coordinate t or the standard space coordinate x.

The mapping between a point (x, t) in the orthonormal frame of reference $(O; x, t)$ and a hyperbolic point (u, τ) in the hyperbolic frame of reference $(O; R_u, S_\tau)$ is:

$$u = ln\sqrt{\frac{t+x}{t-x}} \quad \text{and} \quad \tau = \sqrt{t^2 - x^2} \qquad (2.6) \qquad\qquad (6)$$

The inverse mapping of (2.11) can be expressed as follows:

$$x = \tau\, sinh\, u \quad \text{and} \quad t = \tau\, cosh\, u \qquad (2.7) \qquad\qquad (7)$$

As each slice S_τ is associated with a corresponding proper time τ, then, all points u belonging to that slice S_τ are said to be 'simultaneous'. That is, each slice S_τ is a class of simultaneity made up of a set of points u_τ that are associated to the same proper time τ.

The passage from one slice S_{τ_1} into a subsequent slice S_{τ_2} represents the 'transition' from a first proper time τ_1 to a consequent proper time τ_2. Proper time provides thus, an invariant time ordering of the set of slices.

We note that the passage from one slice S_{τ_1} into a subsequent slice S_{τ_2} expands the standard spatial separation between any two points. Indeed, let (u_a, τ_1) and (u_b, τ_1) be two different points at an initial proper time τ_1, and (u_a, τ_2) and (u_b, τ_2) be the corresponding points at a subsequent proper time τ_2, where u_a and u_b represent two different rays. As $\tau_2 > \tau_1$, then, the standard spatial separation $\tau_2\, sinh\,(u_b - u_a)$ at the later proper time τ_2 is greater than the standard spatial separation $\tau_1\, sinh\,(u_b - u_a)$ at the previous proper times τ_1.

3. Wavefunction and its evolution through proper time

In view of the above, we propose to define the wavefunction (or state vector) in a Hilbert space associated to a corresponding slice S_τ. The wavefunction's arguments are associated to the hyperbolic points (τ, u) of the corresponding slice S_τ, and are thus, invariant with respect to τ. The coordinates of each hyperbolic point (τ, u) can be considered as the module τ and argument u of a vector or a complex number $u_\tau = \tau e^{iu}$. The wavefunction that we shall call 'invariant wavefunction' evolves according to the progression of subsequent slices.

Each slice S_τ can be associated to a Hilbert Space $\mathcal{H}$. The slice S_τ represents a position basis $\{|u_\tau\rangle\}$ of the associated Hilbert Space $\mathcal{H}$ with elements $|u_\tau\rangle$ labelled by a continuous hyperbolic variable u_τ normalised using the Dirac δ-function:

$$\langle u'_\tau | u_\tau \rangle = \delta(u'_\tau - u_\tau) \qquad (3.1) \qquad\qquad (8)$$

An invariant unit state vector $|\psi(\tau)\rangle$ in the Hilbert space $\mathcal{H}$ associated to the slice S_τ can then be expanded as an integral in function of the base elements $|u_\tau\rangle$ as follows:

$$|\psi(\tau)\rangle = \int du_\tau\ \psi(u_\tau)\, |u_\tau\rangle \qquad (3.2) \qquad\qquad (9)$$

The right-hand side of the above equation (3.2) is defined by a line integral along the piecewise smooth curve representing the slice S_τ. It describes the state vector $|\psi(\tau)\rangle$ of a physical system as a superposition of position basis elements $|u_\tau\rangle$ each one of which corresponds to a definite hyperbolic coordinate u of the slice S_τ at a given proper time instant τ. The expanding coefficients or 'weights' $\psi(u_\tau)$ represent a complex valued invariant wavefunction.

The left-hand side $|\psi(\tau)\rangle$ belongs to Hilbert space $\mathcal{H}$ and represents the vector sum or resultant of the decomposed position states. As all the superposed arguments are defined at the same invariant proper time instant τ, then there is a sense in calculating their resultant.

The invariant wavefunction $\psi(u_\tau) \equiv \psi(u, \tau)$ describes the quantum state of a physical system at a specific proper time τ. It is considered to be governed by a gradual evolution through proper time according to a unitary transformation that can be expressed by the following equation of motion:

$$i\hbar\big(\psi(u,\tau+\delta\tau)-\psi(u,\tau)\big)=g(E)\delta\tau\psi(u,\tau) \qquad (3.3) \qquad\qquad (10)$$

where $g(E)$ is a function of the physical system's energy. The invariant wavefunction at each proper time τ may be considered as a single bloc and thus, its evolution depends only on proper time τ. The dependency on u can be dropped from the above equation (3.3) which simply becomes:

$$i\hbar\big(\psi(\tau+\delta\tau)-\psi(\tau)\big)=g(E)\delta\tau\psi(\tau) \qquad (3.4) \qquad\qquad (11)$$

Posing $\psi(\tau+\delta\tau)-\psi(\tau)=\delta\psi(\tau)$, then, equation (3.4) can be expressed as follows:

$$i\hbar\delta\psi(\tau)=g(E)\delta\tau\psi(\tau) \qquad (3.5) \qquad\qquad (12)$$

For a physical system moving within the upper cone, proper time is positive whereas, it is negative for a physical system moving within the lower cone. However, the equation of motion should not be affected by one or the other direction of proper time and thus, energy which should be conserved on either direction of proper time should not differentiate between both movements. This implies that the function of energy $g(E)$ should rather depend on energy square E^2. But $g(E^2)$ should have the dimension of energy and not energy square and thus, we divide by an energy constant of the physical system, which naturally is mc^2 where m is the mass of the physical system.

On the other hand, equation (3.5) should account also for antiparticles which by convention have negative energy. Thus, the function $g(E)$ in equation (3.5) should be of the form:

$$g(E)=\pm(f(E^2)/mc^2) \qquad (3.6) \qquad\qquad (13)$$

where the plus/minus sign accounts for particles and antiparticles.

By injecting (3.6) into (3.5), we obtain:

$$i\hbar\delta\psi(\tau)=\pm(f(E^2)/mc^2)\delta\tau\psi(\tau) \qquad (3.7) \qquad\qquad (14)$$

Equation (3.7) can be expressed as a derivative with respect to proper time τ, as follows:

$$i\hbar\frac{\partial\psi(\tau)}{\partial\tau}=\pm(f(E^2)/mc^2)\psi(\tau) \qquad (3.8) \qquad\qquad (15)$$

The exact form of the equation can be verified by comparing it with the standard Schrodinger's equation. Nevertheless, we shall take the simplest form and let $f(E^2)=E^2$, and thus, equation (3.8) becomes:

$$i\hbar\frac{\partial\psi(\tau)}{\partial\tau}=\pm(E^2/mc^2)\psi(\tau) \qquad (3.9) \qquad\qquad (16)$$

Equation (3.9) shall be called 'invariant wavefunction equation' and its solution is:

$$\psi(\tau)=\psi_0 e^{\pm i(E^2/\hbar mc^2)\tau} \qquad (3.10) \qquad\qquad (17)$$

$$\text{where } \psi_0=\psi(0). \qquad\qquad (18)$$

This solution shows that an increase in proper time τ changes the phase of the proper wavefunction at a proper-time rate $E^2/\hbar mc^2$.

To recover the standard equation, we express the proper-time differential $\delta\tau$ in the invariant wavefunction equation (3.9) in terms of the standard-time differential δt, as follows:

$$i\hbar \frac{\delta \psi(\tau)}{\delta t} = \pm (E^2/mc^2)\frac{\delta \tau}{\delta t}\psi(\tau) \quad (3.11) \qquad (19)$$

Using the relation (2.6) between proper time τ and the standard coordinates x and t, the differential $\delta \tau$ can be expressed as follows:

$$\delta\tau = \delta t \sqrt{1 - (\delta x/\delta t)^2/c^2} = \delta t\sqrt{1 - v^2/c^2} \qquad (3.12) \qquad (20)$$

Noting that:

$$\frac{\delta\tau}{\delta t} = \sqrt{1 - v^2/c^2} = \frac{mc^2}{E} \qquad (3.13) \qquad (21)$$

Introducing the expression (3.13) into equation (3.11), we finally obtain:

$$i\hbar\frac{\partial \psi(x,t)}{\partial t} = \pm E\psi(x,t) \quad (3.14) \qquad (22)$$

Equation (3.14) describes the evolution of the wavefunction in standard spacetime and is equivalent to the invariant wavefunction equation (3.11). When the right-hand side is positive, we get an equation similar to Schrodinger's equation except that E represents a relativistic energy.

In particular, the energy E in the non-relativistic limit can be expressed as follows:

$$E = \sqrt{m^2 c^4 + P^2 c^2} \approx mc^2\sqrt{1 + v^2/c^2} \approx mc^2 + mv^2/2 \qquad (3.15) \qquad (23)$$

We thus obtain the energy of a free particle knowing that the term mc^2 is a constant corresponding to the rest energy that has no consequences for the evolution of the particle. This shows that Schrodinger's equation is a non-relativistic limit to the invariant wavefunction equation (3.9) for particles (i.e. not for antiparticles).

On the other hand, equation (3.14) can be written as:

$$\left(i\hbar\frac{\partial}{\partial t} \pm E\right)\psi(x,t) = 0 \qquad (3.16) \qquad (24)$$

$$\left(i\hbar\frac{\partial}{\partial t} + \sqrt{m^2 c^4 + P^2 c^2}\right)\left(i\hbar\frac{\partial}{\partial t} - \sqrt{m^2 c^4 + P^2 c^2}\right)\psi(x,t) = 0 \quad (3.17) \qquad (25)$$

$$\left(-\hbar^2\frac{\partial^2}{\partial t^2} - m^2 c^4 - P^2 c^2\right)\psi(x,t) = 0 \qquad (3.18) \qquad (26)$$

Using the momentum operator, equation (3.18) simply becomes Klein-Gordon equation:

$$-\hbar^2\frac{\partial^2 \psi(x,t)}{\partial t^2} = (-\hbar^2 c^2 \nabla^2 + m^2 c^4)\psi(x,t) \qquad (3.19) \qquad (27)$$

This shows that the the invariant wavefunction equation (3.9) is equivalent to Klein-Gordon equation.

4. Ontology of the invariant wavefunction

An invariant wavefunction $\psi(u_\tau) \equiv \psi(u,\tau)$ describes the quantum state of a physical system and has two main properties: The first property is the fact that all the arguments of the invariant wavefunction are associated to the hyperbolic points (τ, u) of a corresponding slice S_τ, and are thus, defined at a specific proper time instant τ which is invariant for all observers. The second property is the fact that its evolution through proper time is unitary, i.e., its density $|\psi(u,\tau)|^2$ is conserved at each proper time instant τ

whatever is the spacetime interval between any two arguments. These first and second properties imply that the distribution of density throughout all the arguments u_τ of the invariant wavefunction forms a holistic bloc having an inseparable identity. Thus, the invariant wavefunction $\psi(u, \tau)$ can be considered as a single 'quantum event' describing the quantum state of a physical system at a specific proper time τ.

Moreover, as explained by Tim Maudlin in his book 'Philosophy of Physics: Quantum Theory [14], the wavefunction accounts faithfully for the interference pattern (for example, in a two-slit experiment) and it is reasonable to consider that it represents some real physical features of a physical system. The characteristics and behaviour of the wavefunction should then reflect those of the physical system.

Indeed, Schrodinger originally suggested that a particle can be assimilated to a 'cloud' that continuously fills the entire space and whose density is given by the square of the wavefunction. This idea has been abandoned by Schrodinger mainly because the cloud keeps diffusing and does not seem to correspond to the relatively sharp macroscopic world [1]. Moreover, all the points of the cloud are defined at the same instant which contradicts the relative simultaneity principle of special relativity.

Nevertheless, this idea has been reintroduced in the GRW theory [10, 1] and we shall hereafter, adopt this concept in relation to the invariant wavefunction which is invariant for all observers. The physical system is thus considered as a kind of a continuously spread-out cloud that we shall call 'individual fabric' which makes part of the spacetime fabric itself. Each individual fabric has its proper individual identity (or individuality) and has a finite 'density amplitude' that vanishes at infinity reflecting the effects and properties of the corresponding invariant wavefunction.

In particular, the individual fabric's density $\rho(u_\tau)$ at each hyperbolic point u_τ is equal to the square of the invariant wavefunction at that point:

$$\rho(u_\tau) = |\psi(u, \tau)|^2 \qquad (4.1) \tag{28}$$

The dispersion and probabilities of the invariant wavefunction reflect then, a real density distribution within the corresponding individual fabric.

For example, an invariant wavefunction whose density amplitude is defined by a normal distribution would be the result of a normal distribution of the individual fabric's density itself. The dispersion of the invariant wavefunction would represent the width of a central region where the individual fabric's density is concentrated, the median would be the individual fabric's centre of density, while the tails would represent rarefied peripheral density regions. The region where the density is concentrated interacts the most with its environment and shall be called the 'useful part' of the individual fabric whereas, the rarefied density regions have little or no interactions with their environment.

More generally, the proper wavefunction $\psi(u_{\tau 1}, u_{\tau 2}, \dots u_{\tau k}, \tau)$ of a set of physical systems composed for example, of k particles is defined in a high-dimensional configuration space on $R^{3k} \times R$, codifying their different positions and identities.

We will extrapolate Schrodinger's original idea of defining the individual density for each particle separately, and then construct the global density by 'summing' all individual densities. Thus, each particle corresponds to an individual fabric whose points $(u_{\tau j}, \tau)$ are defined in a four-dimensional physical spacetime $R^3 \times R$ such that the density distribution of each individual fabric continuously fills the entire space. All individual fabrics are isomorphic and thus, their density distributions can be projected on the same four-dimensional physical spacetime $R^3 \times R$. Consequently, the multi-particle invariant wavefunction $\psi(u_{\tau 1}, u_{\tau 2}, \dots u_{\tau k}, \tau)$ can be perceived as representing a 'global fabric' defined in the four-dimensional physical spacetime $R^3 \times R$ and whose density distribution corresponds to the sum of all the density distributions of its constituent individual fabrics. Concretely, the global density $\rho(u_\tau)$ of the global fabric representing the k-particle-system is the sum of all the individual densities $\rho_1(u_\tau), \dots, \rho_i(u_\tau), \dots, \rho_k(u_\tau)$ of the corresponding individual fabrics representing the k different particles, where the i-th density $\rho_i(u_\tau)$ of the i-th individual fabric being the integral of the square of the invariant wavefunction over

the coordinates associated with all the other individual fabrics. We note that here, the k-particle-system is unambiguously defined at a specific proper time instant τ whatever is the distance between the different centres of density and does not contradict relativity.

The global fabric is thus, defined by a global density distribution $\rho(u_\tau)$ over a set of points (u, τ) in a four-dimensional spacetime $R^3 \times R$ where each point u_τ represents a common position of all the k-individual positions.

However, the global density distribution $\rho(u_\tau)$ gives only a partial representation of the global fabric. Indeed, by making the above summing operation we lose the individual identity of each individual fabric knowing that these individual identities also determine the way they interact with each other to form the global fabric. Therefore, to conserve the individual identities of the different individual fabrics, the global fabric should be represented by the multi-particle invariant wavefunction $\psi(u_{\tau 1}, u_{\tau 2}, \dots u_{\tau k}, \tau)$ which is a mathematical representation that conserves the individual identities of all individual fabrics while indirectly defining the global density $\rho(u_\tau)$.

In other words, the multi-particle invariant wavefunction $\psi(u_{\tau 1}, u_{\tau 2}, \dots u_{\tau k}, \tau)$ gives a consistent definition of the global fabric as it determines its global density $\rho(u_\tau)$ according to Schrodinger's summing operation on all individual fabrics' densities while conserving their individual identities. We note that the multi-particle invariant wavefunction $\psi(u_{\tau 1}, u_{\tau 2}, \dots u_{\tau k}, \tau)$ determines also whether the merging or summing operation is a simple 'juxtaposition' or a 'combination' of the different density distributions. Indeed, a non-factorable invariant wavefunction would mean that the individual fabrics are combined together, i.e., they are intermingled or entangled to each other as if formed of a single fabric (having a common global identity). On the other hand, a factorable invariant wavefunction would mean that the individual fabrics are only juxtaposed to each other without being entangled.

5. Contracting function

The invariant wavefunction representing a free particle (i.e., not an anti-particle) corresponds to an individual fabric that evolves according to the invariant wavefunction equation (3.9) as an indivisible holistic bloc within the upper light cone and thus, its evolution is local and consistent with special relativity. During this evolution, the invariant wavefunction spreads out in standard space in a smooth manner as a consequence of the passage from one slice S_{τ_1} into a subsequent slice S_{τ_2}. This spreading out suggests that the individual fabric's density should be stretching out (i.e. becoming more diluted) while evolving through proper time.

However, the introduction of an external potential such as a confining potential $V(u, \tau)$ that interacts with the useful part of the individual fabric modifies the energy of the system changing thus, the form of the invariant wavefunction equation (3.9). The confining potential $V(u, \tau)$ acts on the useful part of the individual fabric which in its turn reacts by undergoing an abrupt redistribution and more precisely, an abrupt contraction of its density (i.e. becomes more concentrated) in compliance to the newly introduced potential.

The abrupt redistribution of the individual fabric's density due to the abrupt introduction of the external potential may be formalised by multiplying the invariant wavefunction by a contracting function $\varphi(u_\tau)$ whose form depends on the external potential, as follows:

$$\psi^+(u_\tau, \tau) = \varphi(u_\tau)\psi^-(u_\tau, \tau) \qquad (5.1) \qquad\qquad (29)$$

where $\psi^+(u_\tau, \tau)$ represents the subsequent invariant wavefunction resulting from the action of the external potential on the antecedent invariant wavefunction $\psi^-(u_\tau, \tau)$.

Equation (5.1) is a non-dynamic state equation wherein, antecedent and subsequent invariant wavefunctions $\psi^-(u_\tau, \tau)$ and $\psi^+(u_\tau, \tau)$ are both defined at the same proper time instant τ. Indeed, the reconfiguration in the distribution of density is instantaneous (or quasi-instantaneous) and inherent to the individual fabric. In particular, the individual

fabric is a single indivisible object represented by a single event and a variation in its density distribution is a variation of its state or property and not the result of any displacement. This implies a non-local connectivity within the individual fabric which does not experience any displacement with respect to the spacetime fabric and thus, does not contradict special relativity.

We note that in response to a variation in the external confining potential, the contracting function $\varphi(u_\tau)$ may in general, be defined by a contracting Gaussian $g(u_\tau)$ applied to the zone of interaction of the invariant wavefunction and whose dispersion depends on the form of the external confining potential.

The narrower is the confinement potential, the smaller is the wavefunction's dispersion and thus, the more concentrated is the fabric's dense region. However, in accordance with the uncertainty principle, the individual fabric's density cannot be concentrated beyond an infinitesimally small minimal localised region. In this later case, the contracting Gaussian $g(u_\tau)$ becomes a localizing Gaussian $g_U(u_\tau)$ similar to that in the GRW theory, except that it is not a random spontaneous collapse but rather a deterministic response to the external potential as will be described in more details in sections 7 and 8. The localizing Gaussian $g_U(u_\tau)$ has a minimal dispersion ε consistent with the uncertainty principle and is centred at a point $u_\tau = U$ around which the contraction takes place. Thus, unlike the GRW theory, the point U is not random and depends on the region of interaction between the wavefunction and the external potential that triggered the contraction. On the other hand, this point U is not necessarily the centre of density.

6. Non-relativistic limit

The arguments of a standard non-relativistic wavefunction $\psi(r, t)$ cannot be considered as simultaneous and even less as a single event. Nevertheless, the standard wavefunction $\psi(r, t)$ is considered here as an approximation of the invariant wavefunction $\psi(\tau)$ which in revenge, is defined without ambiguity at a given invariant proper time instant τ and is considered as a single quantum event.

For simplicity, we shall consider the evolution of a particle in a two-dimensional space through standard time t, defined by a standard space-time coordinate system $(O; x, y, t)$. In the non-relativistic limit, each slice S_τ becomes a corresponding 'quasi-plane' L_t parallel to the (x, y) plane.

The invariant state vector $|\psi(\tau)\rangle$ becomes a standard state vector $|\psi(t)\rangle$ in function of standard temporal and spatial coordinates (x, y, t), as follows:

$$|\psi(t)\rangle = \int dx\, dy\, \psi(x, y, t)\, |x, y\rangle \qquad (6.1) \qquad\qquad (30)$$

The evolution of the wavefunction $\psi(x, y, t)$ is thus, governed by Schrodinger's equation:

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi \qquad (6.2) \qquad\qquad (31)$$

The wavefunction $\psi(x, y, t)$ of a free particle spreads out in time and its dispersion σ keeps growing. However, the introduction of a potential $V(x, y)$ is a discontinuous process that triggers an internal mechanism making the wavefunction consistent with the imposed potential. This internal mechanism is defined by a contracting operator L corresponding to the multiplication of the wavefunction by a contracting function $\varphi(x, y)$ that depends on the potential $V(x, y)$. For example, in the case of an abrupt barrier potential $V(x, y)$ such as a barrier with an opening, the contracting function $\varphi(x, y)$ is an almost symmetrical image $S(V)$ of the barrier potential $V(x, y)$ around its mean value $\bar{V}$, with a normalising factor $1/M$, defined as follows:

$$\varphi(x, y) = S(V)/M \qquad (6.3) \qquad\qquad (32)$$

The subsequent wavefunction $\psi^+(x, y, t)$ resulting from the abrupt introduction of the barrier potential $V(x, y)$ is then equal to the contracting function multiplied by the antecedent wavefunction $\psi^-(x, y, t)$ at the instant of the introduction of the barrier potential $V(x, y)$, as follows:

$$\psi^+(x, y, t) = \varphi(x, y)\psi^-(x, y, t) \qquad (6.4) \tag{33}$$

The multiplication of the wavefunction $\psi^-(x, y, t)$ by the normalised contraction function $\varphi(x, y)$ concentrates most of its density within a certain region whose extension depends on the barrier potential.

Indeed, when we describe for example a particle in a box with a square potential or a harmonic oscillator representing a particle in a quadratic potential, we consider that the wavefunction vanishes at the edges. In other words, we implicitly multiply the wavefunction of a free particle by a contracting function that limits its extension to a region within the barrier potential.

7. Application: Evolution of a particle through a slit

Let a particle coming out of a source and travelling towards a barrier with a slit and passing through the slit before impacting a screen. Let the source be at the origin O of a standard space-time coordinate system $(O; x, y, z, t)$ such that the particle is travelling in the y-direction towards the barrier, the screen and the barrier being both parallel to the (x, z) plane, and the barrier's slit being along the z-direction. For simplicity, we will neglect the z-direction and restrict the analysis in a two-dimensional space with respect to the system $(O; x, y, t)$.

The evolution of the particle can be described according to a first step from the source to the barrier, a second step through the slit, and a third step from the slit to the screen.

7.1. First step from the source to the barrier

Suppose that initially, at t=0, just downstream of the source, the particle corresponds to a free particle represented by a Gaussian wavefunction. In particular, the wavefunction's transverse profile (i.e. in the x-direction) is defined as follows:

$$\psi(x, 0) = \frac{1}{(2\pi\sigma^2)^{1/4}} e^{-\frac{x^2}{4\sigma^2}} \qquad (7.1) \tag{34}$$

The wavefunction's transverse profile [1, 11] which is a solution of Schrodinger's equation at an instant t between the source and the barrier, is:

$$\psi(x, t) = \frac{N\sigma}{\sqrt{\sigma^2 + i\hbar t/2m}} e^{-x^2/4(\sigma^2 + i\hbar t/2m)} \qquad (7.2) \tag{35}$$

The amplitude square of the wavefunction $\psi(x, t)$ is a Gaussian [1, 11] defined as follows:

$$|\psi(x, t)|^2 = \frac{N^2}{\sqrt{1 + \hbar^2 t^2/4m^2\sigma^2}} e^{-x^2/2\sigma^2(1 + \hbar^2 t^2/4m^2\sigma^2)} \qquad (7.3) \tag{36}$$

where the dispersion at an instant t, is given by:

$$\sigma^2(t) = \sigma^2 + (\hbar t/2m\sigma)^2 \qquad (7.4) \tag{37}$$

For simplicity, we shall consider that the transverse profile of the wavefunction at an instant t can be approximated by the square root of the Gaussian square-amplitude given by (7.3). Thus, the transverse profile of the wavefunction at an instant t, can be expressed by the following Gaussian wavefunction:

$$\psi(x,t) \approx A(t)e^{-\frac{x^2}{4\sigma^2(t)}} \qquad (7.5)$$

(38)

$$\text{where } A(t) = \frac{N}{(1+\hbar^2 t^2/4m^2\sigma^2)^{1/4}} \qquad (7.6)$$

(39)

7.2. Second step at the barrier and through the slit

Suppose the wavefunction of the free particle interacts with the barrier, at a certain instant t_s. The slit potential can be regarded as an abrupt square potential whose transverse profile is defined as follows:

$$V(x) = \begin{cases} \varepsilon \sim 0 & for \ |x| < a \\ V_0 & otherwise \end{cases} \qquad (7.7)$$

(40)

where a is the width of the slit and $V_0 \gg 1$. That is, the slit potential is almost equal to zero within a small diameter a and almost infinite elsewhere.

The slit potential has the effect of a contracting function $\varphi(x)$ that concentrates the density of the wavefunction within the width of the slit and dilutes it outside the slit. To be consistent with the principle that a wavefunction should vanish at the edges of regions of infinite potential energy [15], the transverse contracting function $\varphi(x)$ should be a quasi-symmetrical image $S(V)$ of the transvers potential $V(x)$ with a normalising factor $1/M$, defined as follows:

$$\varphi(x) \sim \begin{cases} V_0/M & for \ |x| < a \\ \varepsilon/M & for \ |x| > a \end{cases} \qquad (7.8)$$

(41)

On the other hand, to avoid the discontinuity at the sharp edges (i.e. at $|x| = a$), the transverse contracting function $\varphi(x)$ can be approximated by a transverse normalised contracting Gaussian $g(x)$ centred at $x = 0$, with a dispersion equals to the slit's width a, and with a normalizing factor G:

$$g(x) \approx Ge^{-\frac{x^2}{4a^2}} \qquad (7.9)$$

(42)

Suppose, the useful part (i.e. dense region) of the individual fabric comes into contact with the slit barrier at an instant t_s, where according to equation (7.5), the transverse profile of its wavefunction is defined by a normalized gaussian $\psi^-(x,y,t_s)$ having a dispersion σ much larger than the slit's width a, as follows:

$$\psi^-(x,y,t_s) = A(t_s)e^{-\frac{x^2}{4\sigma^2(t_s)}} \qquad (7.10)$$

(43)

The interaction of the individual fabric with the slit barrier instantaneously transforms the normalized gaussian $\psi^-(x,y,t_s)$ into another normalised gaussian $\psi^+(x,y,t_s)$ presenting a dispersion equals to the slit's width a, as follows:

$$\psi^+(x,y,t_s) = g(x)\psi^-(x,y,t_s) = Ge^{-\frac{x^2}{4a^2}}A(t_s)e^{-\frac{x^2}{4\sigma^2(t_s)}} \approx B(t_s)e^{-\frac{x^2}{4a^2(t_s)}} \qquad (7.11)$$

(44)

where $B(t_s) = G \times A(t_s)$ and $\sigma \gg a$.

It should be noted that the whole individual fabric represented by the wavefunction crosses the slit barrier. The individual fabric's useful part passes through the slit while all other regions are so rarified that they do not interact with anything else, passe unhindered across the barrier itself.

7.3. Third step from the slit to the screen

Downstream of the slit, the wavefunction starts at an instant t_d, as in the first step by having a Gaussian transverse profile given by:

$$\psi(x, t_d) = B(t_d)e^{-\frac{x^2}{4a^2(t_d)}} \qquad (7.12) \qquad\qquad (45)$$

It evolves by spreading out according to Schrodinger's equation such that its dispersion increases which, at any instant t, between the slit and the screen, shall be noted $b(t)$, where $b^2(t) > a^2(t_d)$.

Suppose that at an instant t_m, the useful part of the individual fabric reaches the screen. At this instant t_m, the transverse profile of the wavefunction is a normalized gaussian $\psi^-(x, y, t_m)$ presenting a dispersion $b(t_m)$, and having the following form:

$$\psi^-(x, y, t_m) = C(t_m)e^{-\frac{x^2}{4b^2(t_m)}} \qquad (7.13) \qquad\qquad (46)$$

The screen is a barrier that can be assimilated to a constant potential $V(x, y) = V_0 \gg 1$ whereas, the zone of impact on the screen can be assimilated to an infinitesimal well. Then, at the instant t_m of impact, the potential of the screen is equal to V_0 everywhere except within the zone of contact with the useful part of the wavefunction. This zone of contact can be represented by an infinitesimal Gaussian well of width $\Delta_x \approx 2\varepsilon$ such that its associated contracting function is a localising Gaussian having an infinitesimally small and minimal dispersion ε around a certain point (X_0, Z_0) within the zone of contact.

In particular, the transverse profile of the contracting function at the transverse point X_0 within the zone of contact can be expressed by a localising gaussian, as follows:

$$g_{X_0}(x) = \frac{1}{(2\pi\varepsilon^2)^{1/4}}e^{-\frac{(x-X_0)^2}{4\varepsilon^2}} \qquad (7.14) \qquad\qquad (47)$$

where the dispersion ε of the localising Gaussian is much smaller than the dispersion $b(t_m)$ of the wavefunction $\psi^-(x, y, t_m)$.

The localising gaussian instantaneously reduces the dispersion of the wavefunction to ε. Indeed, the transverse profile of the antecedent wavefunction $\psi^-(x, y, t_m)$ having a dispersion $b(t_m)$ much larger than the zone of contact 2ε, is instantaneously transformed into a subsequent normalised gaussian $\psi^+(x, y, t_m)$ presenting a dispersion equals to the radius ε of the zone of contact, as follows:

$$\psi(x, y, t_m) = D(t_m)e^{-\frac{(x-X_0)^2}{4\varepsilon^2}} \qquad (7.15) \qquad\qquad (48)$$

Thus, the impact of the wavefunction's useful part on the screen contracts this region to a localised condensed extension 2ε, making it to behave as a corpuscle and in particular, as a particle in an extremely narrow well.

It is known [15] that for a particle within a well of width $\Delta_x \approx 2\varepsilon$, its momentum in the ground state (n=1) is of the order:

$$\hbar k = \hbar\pi/2\varepsilon \qquad (7.16) \qquad\qquad (49)$$

where

$$k = \sqrt{\frac{2mE}{\hbar^2}} \qquad (7.17)$$

(50)

Therefore, the radius of the well or the dispersion ε of the contracting Gaussian should be of the order:

$$\varepsilon \sim \frac{\pi\hbar}{\sqrt{8mE}} = \frac{h}{\sqrt{2mE}} \qquad (7.18)$$

(51)

The above expression (7.18) gives an estimate of the localised condensed extension 2ε. We note that the uncertainty in momentum is $\Delta_p \approx \hbar\pi/\varepsilon$ and the uncertainty in position is $\Delta_x \approx 2\varepsilon$ and thus, their product $\Delta_x\Delta_p \approx 2\hbar\pi$ is consistent with the uncertainty principle.

By injecting the parameters relative to an electron having for example an energy E of 600 eV in the above formula (7.18), we get a dispersion ε of the order of 10^{-11}m.

The impact of the individual fabric's useful part with the screen produces thus, a complete transition of the wavefunction's density from a stretched state to a localised state. This is similar to the localising function in the GRW theory except that it is not a random spontaneous collapse but rather a response to the interaction between the wavefunction and the screen.

We note that if the barrier in the second step had two slits centred at $x = a$ and $x = -a$, forming a transverse double-square potential, then the induced contracting function $\varphi(x)$ would have concentrated the density of the wavefunction within the widths of both slits. The transverse contracting function $\varphi(x)$ can be approximated by a double-Gaussian centred at $x = a$ and $x = -a$, each one of which presenting a dispersion equals to the slit's width a. The transverse profile of the wavefunction just downstream of the double-slit screen would thus, be a superposition of two Gaussians centred at $x = a$ and $x = -a$. As the two Gaussians representing the distribution of the fabric's density spread out and overlap, an interference pattern would be created.

8. Measurement

In view of the above, measurement may not be the right term to use because the outcome of a measurement is not a pre-existing value. In particular, the measurement of a position of a physical system is a transformation like all others, the only special thing about it is that it transforms a pre-existing fabric having a stretched density into a fabric that has a highly localised density. This transformation is triggered by the introduction of a confining potential. Measurement can thus, be formalised by multiplying the wavefunction $\psi^-(r,t)$ with a localising gaussian $\varphi_{r_0}(r,t)$, as follows:

$$\psi^+(r,t) = \varphi_{r_0}(r,t)\psi^-(r,t) \qquad (8.1)$$

(52)

Consequently, the individual fabric has a wave-like behaviour when most of its density is in a stretched state and a particle-like behaviour when most of its density is in a minimally contracted or localised state.

Moreover, Schrodinger's equation is always applicable, and discontinuity arises only because the process of interaction or measurement modifies the potential in Schrodinger's equation.

In general, the wavefunction $\psi(r_1, r_2, \ldots r_k, t)$ of a set of k particles represents a global fabric composed of a combination (i.e. entanglement) or a juxtaposition (i.e. non-entanglement) of the individual fabrics of the different particles.

In a similar manner to the GRW theorem [10] and as explained in detail for a two-particle system in [1], if the wavefunction is in the non-entangled (i.e. factorizable) state, a localisation of a particular dense region of an individual fabric does not affect the other dense regions. A localising gaussian $\varphi_{r_0}(r_j, t)$ acts only on a corresponding individual

wavefunction $\psi(r_j, t)$ without affecting the other individual wavefunctions. However, if the wavefunction is in the entangled (i.e. non-factorizable) state, a localisation of a particular dense region within the global fabric affects all other dense regions. A localising gaussian $\varphi_{r_0}(r_j, t)$ relative to any dense region acts holistically on the global fabric and thus on all its dense regions.

We note that in a macroscopic object, each particle's fabric is subject to a confining potential corresponding to a resultant action created by all other particles' fabrics. The interaction between the different particles has the effect of confining the dense part of each fabric represented by its wavefunction within the confining potential and thus, a macroscopic object is always in a localized state.

9. Relation with de Broglie Bohm theorem (Fabric's centre of density)

Using Ehrenfest theory relative to the expectation values $\langle x \rangle$ and $\langle p \rangle$ of the position and momentum operators respectively, we have:

$$d\langle x(t) \rangle / dt = \langle p(t) \rangle / m \qquad (9.1)$$

In this context, the parameter $X(t) \equiv \langle x(t) \rangle$ at a given instant t, can be considered as a point representing the centre of density of the fabric at that instant. The parameter $P(t) \equiv \langle p(t) \rangle$ can then be considered as the momentum of the fabric's centre of density $X(t)$.

De Broglie's formula can be used to relate the momentum $P(t)$ of the fabric's centre of density to a corresponding wave number $k(t)$, as follows:

$$(53)$$

$$dX(t)/dt = P(t)/m = \hbar k(t)/m \qquad (9.2)$$

More generally, the de Broglie Bohm formalism may be used to express the dynamics of the fabric's centre of density. Let the wavefunction $\psi(x, t)$ be expressed in polar form in function of amplitude $R(x, t)$ and phase $S(x, t)$, as follows:

$$\psi(x, t) = R(x, t) e^{iS(x,t)} \qquad (9.3) \qquad\qquad (54)$$

Then, in a similar manner to the de Broglie Bohm theorem, the variation of the fabric's centre of density $X(t)$ can be related to the phase $S(x, t)$ of the wavefunction, according to the following equation:

$$\frac{d}{dt}X(t) = \frac{\hbar}{m}\frac{\partial S(x,\ t)}{\partial x}\bigg|_{x=X(t)} \qquad (9.4) \qquad\qquad (55)$$

The gradient of the phase $\partial S(x,\ t)/\partial x$ is evaluated at the actual location of the fabric's centre of density $X(t)$.

The difference with de Broglie Bohm theorem is that here, the parameter $X(t)$ does not represent an actual position of a particle but simply a centre of density of the fabric represented by the wavefunction. This centre of density is not necessarily located in the densest region of the fabric. For example, in the case of a wavefunction moving across two slits, the dense region is distributed between two zones corresponding to the locations of these two slits and thus, the centre of density is somewhere in between. Consequently, the zone of interaction during a measurement is also not necessarily that where the centre of density is located.

For a free particle, the individual fabric's centre of density may be interpreted as the centre of a sphere-like space-time fabric whose density diminishes at infinity. The fabric's centre of density can be assimilated to a point particle as in point mechanics.

It should be noted that the centre of density is only a manner to represent the distribution of density within a fabric akin to the centre of mass in classical physics and is not

a hidden variable. On the other hand, the variation of the centre of density according to equation (9.4) describes a non-local inherent transformation within the individual fabric and does not imply a displacement of the individual fabric.

In the case of a global fabric composed of different individual fabrics, the variation of the centre of density of one individual fabric affects all other individual fabrics if they are combined and has no effect if juxtaposed.

For example, let a global fabric be composed of two individual fabrics representing two physical systems of masses m_1 and m_2. The wavefunction representing the global fabric is defined in a configuration space $R^2 \times R$, as follows:

$$\psi(x_1, x_2, t) = R(x_1, x_2, t)e^{iS(x_1, x_2, t)} \qquad (9.5) \qquad\qquad (56)$$

The variation of each individual fabric's centre of density $X_k(t)$ can be related to the phase $S(x_1, x_2, t)$ of the wave function $\psi(x_1, x_2, t)$, as follows:

$$\frac{dX_k}{dt} = \frac{\hbar}{m_k} \frac{\partial S(x_1, x_2, t)}{\partial x_k}\bigg|_{x_k = X_k} \qquad (9.6) \qquad\qquad (57)$$

where the phase $S(x_1, x_2, t)$ is evaluated at the corresponding centre of density. If the particles are correlated, then any change δX_1 in the centre of density X_1 of the first individual fabric changes the value of the phase $S(X_1 + \delta X_1, X_2, t)$ which automatically changes the centre of density X_2 of the second individual fabric according to equation (9.6).

However, if the particles are not correlated, the wavefunction $\psi(x_1, x_2, t)$ factorises into a product of independent wavefunctions:

$$\psi(x_1, x_2, t) = R(x_1, x_2, t)e^{iS(x_1, x_2, t)} = \psi_1(x_1, t)\psi_2(x_2, t) \qquad (9.7) \qquad (58)$$

The above relation (9.7) can also be expressed in polar form:

$$\psi(x_1, x_2, t) = R(x_1, x_2, t)e^{iS(x_1, x_2, t)} = R_1(x_1, t)e^{iS_1(x_1, t)}R_2(x_2, t)e^{iS_2(x_2, t)}$$
$$(9.8) \qquad\qquad (59)$$

such that;

$$S(x_1, x_2, t) = S_1(x_1, t) + S_2(x_2, t) \qquad (9.9) \qquad\qquad (60)$$

$$R(x_1, x_2, t) = R_1(x_1, t)R_2(x_2, t) \qquad (9.10) \qquad\qquad (61)$$

Applying de Broglie Bohm law (9.6) to the phase $S(x_1, x_2, t)$ as expressed in relation (9.9), gives two independent equations:

$$\frac{dX_1}{dt} = \frac{\hbar}{m_1} \frac{\partial S_1(x_1, t)}{\partial x_1}\bigg|_{x_1 = X_1} \qquad (9.11) \qquad\qquad (62)$$

$$\frac{dX_2}{dt} = \frac{\hbar}{m_2} \frac{\partial S_2(x_2, t)}{\partial x_2}\bigg|_{x_2 = X_2} \qquad (9.12) \qquad\qquad (63)$$

The centre of density of each individual fabric follows its own law independently from the other one.

10. Discussion

Each individual fabric extends throughout the entire spacetime and thus, all the individual fabrics in the universe may be represented by a single universal wavefunction $\psi(u_{\tau 1}, u_{\tau 2}, \dots u_{\tau k}, \dots, \tau)$ defined in an infinite configuration space. The universal wavefunction $\psi(u_{\tau 1}, u_{\tau 2}, \dots u_{\tau k}, \dots, \tau)$ corresponds to a universal spacetime fabric over a set of points (U_τ, τ) in a four-dimensional spacetime $R^3 \times R$ where each point U_τ represents a common position of all the corresponding individual positions. The combination and/or

juxtaposition of all the individual fabrics define a universal density distribution $\rho(U_\tau)$ over these points (U_τ, τ). Indeed, spacetime is shaped by all the density distributions of all the individual fabrics composing the Universe.

On the other hand, as the density distribution of free particles keep stretching, it may be speculated that dark matter is composed of completely free individual fabrics (i.e., free particles) such that the density distribution of each individual fabric is so rarefied that it does not interact with its environment while the global distribution of all the individual fabrics has a gravitational effect.

11. Conclusion

The quantum state of a physical system can be described by an invariant wavefunction $\psi(u, \tau)$ corresponding to a single quantum event at a specific proper time τ which is invariant for all observers. This single quantum event short-circuits the spacetime intervals between the different arguments. In particular, the wavefunction represents an ontological object consisting of an individual fabric that has its proper individuality and has a finite density amplitude that vanishes at infinity. More generally, the invariant wavefunction of a set of physical systems is defined in a high-dimensional configuration space. However, the density distributions of all these individual fabrics form a global fabric defined in the four-dimensional physical spacetime. The global fabric is composed of a combination or a juxtaposition of individual fabrics each of which is also defined in the four-dimensional physical spacetime and its identity is conserved.

Quantum measurement of the position of a physical system is a transformation caused by the introduction of a confining potential that triggers an instantaneous contraction of the individual (or global) fabric's dense region. It is an inherent non-local transformation within the fabric itself whereas, the movement of the fabric relative to all others, i.e., with respect to the spacetime fabric is local.

References

[1] Travis Norsen in his book "Foundations of Quantum Mechanics", Springer 2017.

[2] Albert Einstein (1905) *Zur Elektrodynamik bewegter Körper* (On the Electrodynamics of Moving Bodies), *Annalen der Physik* 17: 891.

[3] T. Maudlin, Philosophy of Physics: Space and Time (Princeton University Press, Princeton, 2012).

[4] Einstein A, Podolsky B, Rosen N; Podolsky; Rosen (1935). *Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?* *Phys. Rev*. 47(10): 777–780.

[5] J. S. Bell (1964). *On the Einstein-Poldolsky-Rosen paradox*. *Physics*. 1 (3): 195–200.

[6] J. S. Bell (1964). *On the problem of hidden variables in quantum mechanics*. Rev. Modern Physics *38, 447-452 (1966)*.

[7] A. Aspect; P. Grangier & G. Roger (1982). *Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment*: "A New Violation of Bell's Inequalities". *Physical Review Letters*. 49 (2): 91–94.

[8] L. de Broglie. *Remarques sur la théorie de l'onde-pilote*, C.R. Acad. Sci, Paris *233, 641-644 (1951)*.

[9] D. Bohm. *A suggested interpretation of the quantum theory in terms of "hidden variables"*, Parts 1 and 2, Phys, Rev, *89, 166-193 (1952)*.

[10] G.C. Ghirardi, A. Rimini, T. Weber, Unified dynamics for microscopic and macroscopic systems. Rev. D 34, 470 (1986).

[11] J. Bricmont. *Making Sense of Quantum Mechanics*, (Springer 2016).

[12] H. Minkowski (1909) *Raum und Zeit*, Jahresberichte der deutschen Mathematiker-Vereinigung 18, 75.

[13] E. Gourgoulhon *Special Relativity in General Frames* Springer 2013.

[14] T. Maudlin, Philosophy of Physics: Quantum Theory (Princeton University Press, Princeton, 2019).

[15] J. Binney and D. Skinner *The Physics of Quantum Mechanics*, Oxford 2014.