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Adaptive fuzzy fault-tolerant control against time-varying faults via a new sliding mode observer method

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Abstract: In this paper, the problem of observer-based adaptive sliding mode control is discussed for nonlinear systems with sensor and actuator faults. The time-varying actuator degradation factor and external disturbance are considered in the system simultaneously. In this study, the original system is described as a new normal system by combining the state vector, sensor faults and external disturbance into a new state vector. For the augmented system, a new sliding mode observer is designed, where a discontinuous term is introduced such that the effects of sensor and actuator faults and external disturbance will be eliminated. In addition, based on a tricky design of the observer, the time-varying actuator degradation factor term is developed in the error system. On the basis of the state estimation, an integral-type adaptive fuzzy sliding mode controller is constructed to ensure the stability of the closed-loop system. Finally, the effectiveness of the proposed control methods can be illustrated with a numerical example.

Keywords: Fault tolerant control, Adaptive fuzzy control, Time-varying actuator faults, Sliding mode control, Sliding mode observer

1. Introduction

In industrial process, actuator and/or sensor always occur various components faults due to unexpected physical constraints and reasons [3,10,13,14]. In order to maintain the reliability of the overall control systems, fault detection and isolation (FDI) and fault-tolerant control (FTC) have been paid to increasing research investigation during the past decade [11,12,15,16]. The design scheme of FDI is to generate a residual signal to judge whether the faults occur and provide a solution to determine the location of the faults [17,18]. However, in practice it is difficult to obtain the exact information of the fault. In this sense, the fault estimate has been developed and become an ideal design basis of FTC [5,19]. In recent years, a great number of fault estimation methods have been reported in the existing literature, for instance, nonlinear observer method, adaptive learning observer method, filter-based estimation method and differential geometry methods, etc [1,2,6].

Among these existing fault estimation approaches, sliding mode observer (SMO)[20–22] refers to one of the most papular nonlinear observer methods, where the fault is reconstructed by the so-called equivalent output error injection principle [1]. In this research forefront, a few fault estimation SMO results have been developed for various systems by the researchers. In [4] a fault estimation SMO was developed for mismatched nonlinear systems with unknown disturbances, where an adaptive law was designed to update the sliding mode gain online. In [9], the authors proposed a cascaded SMO method to cope with the fault estimation problem for the case that the first Markov matrix of the system is of not full rank. In [8] and [7], based on a descriptor system augmentation strategy, the authors proposed a new type of extended SMO approach, which was applied to Itô stochastic systems and Markovian jump systems, respectively.

It should be pointed out that, however, most existing fault estimation results are concerned only on actuator faults or sensor faults. Moreover, most of the reported work has been focused on only additive actuator fault, while multiplicative type actuator fault (also called fault degradation factor) has been received little research attention. In fact, in many practical control systems such as satellite systems, the multiplicative actuator faults may always occur with a time-varying characterization. However, the existing SMO methods in the aforementioned literature cannot be applied directly to solve this design problem due to technique constraints and only additive actuator faults are considered there. It is thus desirable to develop new effective SMO approach to investigate this problem.

In this paper, we are dedicated in the researching of the fault estimation and FTC design problem for continuous-time nonlinear system, where sensor fault, external disturbance, time-varying multiplicative actuator faults and unknown nonlinearity are considered simultaneously in a unified framework. A new type of SMO based on system augmentation scheme is developed for the investigated plant. The designed observer can estimate state vector, sensor faults and external disturbance which thus possesses a more extensive estimation performance compared to traditional SMO method. Moreover, due to the tricky structure of the observer, the time-varying actuator degradation factor in the derived error system can be eliminated. Based on the state estimation of the SMO, an adaptive integral-type sliding control law is designed to ensure the asymptotic stability of the overall fault control systems, where an adaptive fuzzy updating law is involved with the controller gain to approximate the unknown nonlinearity of the plant. Finally, a simulation example is given to verify the effectiveness of the proposed FTC methods.

Notation: The n-dimensional Euclidean space is defined by $\mathbf{R}^n$ denotes. The set of all $m \times n$ real matrices is represented as $\mathbf{R}^{m\times n}$. Positive-definite (negative definite) matrix A is defined by$A > 0$ (< 0). An identity matrix is defined by I_n (n is the dimension of matrix I). diag$\{...\}$ denotes a block diagonal matrix.

2. Problem Formulation and Preliminaries

2.1. Problem statement

Consider the following uncertainty non-linear system subject to time-varying actuator fault, sensor fault and external disturbance

$$\begin{aligned} \dot{x}(t) =& Ax(t) + B(\rho(t)u(t) + f_a(t)) + Ef(x) + D_x d(t) \\ y(t) =& Cx(t) + D_s f_s(t) + D_y d(t). \end{aligned} \tag{1}$$

where $x(t) \in \mathbf{R}^n$, $u(t) \in \mathbf{R}^m$, $f_a(t) \in \mathbf{R}^m$, $f(x) \in \mathbf{R}^{n_f}$, $d(t) \in \mathbf{R}^{n_d}$, $y(t) \in \mathbf{R}^p$, $f_s(t) \in \mathbf{R}^q$, $\rho(t) = \text{diag}\{\rho_1(t), \rho_2(t), ..., \rho_h(t)\}$, $\rho_h(t)(h = 1, 2, ..., m)$ represent the immeasurable system state, control input, unknown stuck actuator fault, unknown smooth non-linear function, unknown external disturbance, measurable output, unknown sensor fault, unknown time-varying actuator efficiency factor, hth actuator, respectively. It is assumed that $0 \le \underline{\rho}_h \le \rho_h(t) \le \bar{\rho}_h \le 1$, for $h = 1, 2, ..., m$, where $\underline{\rho}_h$ and $\bar{\rho}_h$ are the known constants. Then, defining that $\text{diag}\{\bar{\rho}_1, \bar{\rho}_2, ..., \bar{\rho}_h\} = \bar{\rho}$, $\text{diag}\{\underline{\rho}_1, \underline{\rho}_2, ..., \underline{\rho}_h\} = \underline{\rho}$, the following cases of hth actuator failure are considered:

$$\begin{cases} \textit{Case } 1: & \rho_h(t) = 1, & \text{the } h\text{th actuator has no fault;} \\ \textit{Case } 2: & \rho_h(t) = 0, & \text{the } h\text{th actuator is outage;} \\ \textit{Case } 3: & \rho_h(t) \in (0,\ 1), & \text{the } h\text{th actuator is partial loss of effectiveness;} \\ \textit{Case } 4: & f_a(t) \neq 0, & \text{the } h\text{th actuator undergoes stuck fault.} \end{cases}$$

For the given system matrices A, B, C, D_s, D_x, D_y, E, the matrix E is supposed to satisfy that $E = BB_f$ in this paper. Without loss of generality, we suppose that the pair (A, B) is controllable, and the pair (A, C) is observable. In order to study the problem of the redundancy actuator fault, we assume that rank$(B) = l \leq m$. Thus, we have $B = B_1 B_2$, where $B_1 \in \mathbf{R}^{n\times l}$ and $B_2 \in \mathbf{R}^{l\times m}$. Then, the state equation of the original system (1) can be rewritten as

$$\dot{x}(t) = Ax(t) + B_1 B_2(\rho(t)u(t) + f_a(t)) + Ef(x) + D_x d(t). \tag{2}$$

Remark 1. *Different from the existing results of the simultaneous actuator fault and sensor fault in [23], the fault problem in this paper will be more complex. The time-varying actuator fault including loss of effectiveness fault, outage fault and stuck fault, combined with bias sensor fault are first studied simultaneously. Due to more general character of actuator fault, the traditional observer-based controllers are unable to provide the desired estimation and control performance, this is also the difficulty in FTC design.*

In this paper, we give the following assumptions.

Assumption 1. *The stuck actuator fault $f_a(t)$, bias sensor fault $f_s(t)$ and external disturbance $d(t)$ are supposed to satisfy that*

$$\begin{aligned} &\|f_a(t)\| \leq f_{a1}, \|\dot{f}_s(t)\| \leq f_{s1}, \|f_s(t)\| \leq f_{s2}, \\ &\|\dot{d}(t)\| \leq d_1, \|d(t)\| \leq d_2 \end{aligned} \tag{3}$$

where f_{a1}, f_{s1} ,f_{s2}, d_1, d_2 are unknown scalars.

Assumption 2. *[8] It is assumed that the actuators satisfy the redundancy condition: rank(B_2) = rank$(B_2\rho(t)) = l$.*

Assumption 3. *The system matrix dimensions satisfy: rank(B_1) = rank$(CB_1) = l$, and a scalar σ can be found such that*

$$rank\begin{bmatrix} \sigma I + A & D_x \\ C & D_y \end{bmatrix} = n + n_d. \tag{4}$$

Remark 2. *Compared to the traditional methods in ([24]), the Assumption 2 will relax the restriction that the norm bound of the external disturbance, stuck actuator fault, and bias sensor fault, which will be applicable to a larger class of practical systems.*

2.2. Fuzzy logic systems

The fuzzy IF–THEN rules of FLS are given as follows:

$$\begin{aligned} R^i:\ &\text{If } x_1(t) \text{ is } F_{1i} \text{ and } x_2(t) \text{ is } F_{2i}, \cdots, \text{ and } x_n(t) \text{ is } F_{ni}, \\ &\text{then } \bar{y}(t) \text{ is } G_i \end{aligned}$$

where $x(t) = [x_1(t),\ x_2(t), ..., x_n(t)]^T$ and $\bar{y}(t)$ represent the input and output of the FLS, respectively. $F_{\iota i}$ and $G_{\iota i}$ are fuzzy sets $(\iota = 1, 2, ..., n)$. $i = 1, 2, \cdots, N$ (N is the number of the fuzzy rules).Obviously, the FLS can be represented as follows

$$\bar{y}(x) = \frac{\sum_{i=1}^{N} \bar{y}_i(\prod_{\iota=1}^{n} \mu_{F_{\iota i}}(x_\iota))}{\sum_{i=1}^{N}(\prod_{\iota=1}^{n} \mu_{F_{\iota i}}(x_\iota))} \tag{5}$$

where $\mu_{F_{ii}}(x(t))$ is the membership functions, and $\bar{y}_i$ is the point at which $\mu_{G_i} = \max\{\mu_{G_i}\}$, and it is assumed that $\mu_{G_i}(\bar{y}_i) = 1$. Define the following fuzzy basis functions

$$\varphi_i(x) = \frac{\prod_{\iota=1}^{n} \mu_{F_{\iota i}}(x_\iota)}{\sum_{i=1}^{N} \prod_{\iota=1}^{n} \mu_{F_{\iota i}}(x_\iota)}, \quad i = 1, 2, ..., N. \tag{6}$$

Denoting $\theta = [\bar{y}_1,\ \bar{y}_2, ..., \bar{y}_N]^T$ and $\varphi(x) = [\varphi_1(x),\ \varphi_2(x), ..., \varphi_N(x)]^T$. Then, (5) can be rewritten as

$$\bar{y}(x) = \theta^T \varphi(x). \tag{7}$$

Lemma 1. *([25]) For any continuous function $f(x)$ defined over a compact set Ω and any given positive constant δ_0, there exists θ such that*

$$\sup_{x \in \Omega} |f(x) - \theta^T \varphi(x)| \leq \delta_0. \tag{8}$$

Since $x(t)$ is not measurable, the function $f(x)$ can be represented by the following FLSs:

$$\begin{aligned} f(x) &= \theta^T \varphi(x) + \delta_f(t) \\ &= \theta^T \varphi(\hat{x}) + \theta^T (\varphi(x) - \varphi(\hat{x})) + \delta_f(t) \end{aligned} \tag{9}$$

where $\delta_f(t)$ is the approximation error. Then, the reconstruction error $\delta(t)$ can be obtained

$$\delta(t) = \theta^T (\varphi(x) - \varphi(\hat{x})) + \delta_f(t). \tag{10}$$

In general, $\delta(t)$ is assumed to be bounded with

$$\|\delta(t)\| \leq \bar{\delta}, \tag{11}$$

where $\bar{\delta} > 0$ is an unknown constant.

To design the adaptive law for the unknown vector θ, we suppose that $\theta > 0$ throughout the paper, which is not lose the generality, and also used in the FTC problems of fuzzy logical systems ([25]).

3. Main Results

3.1. Observer Design

Consider the following augmented system:

$$\begin{aligned} \dot{\bar{x}}(t) &= \bar{A}\bar{x}(t) + \bar{B}_1 B_2 \rho(t) u(t) + \bar{B}_1 B_2 B_f f(x) + \bar{D}\bar{\omega}(t) \\ y(t) &= \bar{C}\bar{x}(t) \end{aligned} \tag{12}$$

where

$$\bar{x}(t) = \begin{bmatrix} x(t) \\ d(t) \\ D_s f_s(t) \end{bmatrix}, \bar{\omega}(t) = \begin{bmatrix} f_a(t) \\ \sigma d(t) + \dot{d}(t) \\ \sigma f_s(t) + \dot{f}_s(t) \end{bmatrix},$$

$$\bar{A} = \begin{bmatrix} A & D_x & 0 \\ 0 & -\sigma I_{nd} & 0 \\ 0 & 0 & -\sigma I_p \end{bmatrix}, \bar{B}_1 = \begin{bmatrix} B_1 \\ 0 \\ 0 \end{bmatrix},$$

$$\bar{D} = \begin{bmatrix} B & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & D_s \end{bmatrix}, \bar{C} = \begin{bmatrix} C & D_y & I_p \end{bmatrix},$$

$$\bar{n} = n + n_d + p, \ 0 < \sigma < 1. \tag{13}$$

From assumption 3, we have

$$\text{rank}(\bar{C}\bar{B}_1) = \text{rank}(CB_1) = l. \tag{14}$$

Hence, $\bar{C}\bar{B}_1$ is fully column-rank. Then we define that

$$\begin{aligned} H =& \bar{B}_1((\bar{C}\bar{B}_1)^T\bar{C}\bar{B}_1)^{-1}(\bar{C}\bar{B}_1)^T \\ &+ \zeta[I - \bar{C}\bar{B}_1((\bar{C}\bar{B}_1)^T\bar{C}\bar{B}_1)^{-1}(\bar{C}\bar{B}_1)^T], \end{aligned} \tag{15}$$

where $\zeta \in \mathbf{R}^{\bar{n}\times p}$ is a free matrix to be selected. Before the design of fault-tolerant observer, we define the following matrices,

$$\begin{aligned} A_0 &= \bar{A} - H\bar{C}\bar{A}, \ L_2 = (A_0 - L_1\bar{C})H, \\ L_s &= (I - H\bar{C})\bar{D}, \ L = L_1 + L_2 \end{aligned} \tag{16}$$

where $L_1 \in \mathbf{R}^{\bar{n}\times p}$ is the gain matrix to be design later. Now we introduce the following lemma for the existence of L_1, which will be used in the observer design.

Lemma 2. *The pair $(A_0, \bar{C})$ is detectable, if there exists a matrix ζ such that $(I - H\bar{C})$ is invertible.*

Proof. Since $(I - H\bar{C})$ can be invertible through selecting an appropriate matrix ζ, the matrix $\begin{bmatrix} I - H\bar{C} & sH \\ 0 & I_p \end{bmatrix}$ is of full column rank for $\forall\, s \in \mathbf{R}^+$. Then, it can be obtained that

$$\begin{aligned} \text{rank}\begin{bmatrix} sI - A_0 \\ \bar{C} \end{bmatrix} &= \text{rank}\left(\begin{bmatrix} I - H\bar{C} & sH \\ 0 & I_p \end{bmatrix}\begin{bmatrix} sI - \bar{A} \\ \bar{C} \end{bmatrix}\right) \\ &= \text{rank}\begin{bmatrix} sI - \bar{A} \\ \bar{C} \end{bmatrix} \\ &= \text{rank}\begin{bmatrix} sI - A & -D_x & 0 \\ 0 & (s+\sigma)I_d & 0 \\ 0 & 0 & (s+\sigma)I_p \\ C & D_y & I_p \end{bmatrix} \\ &= \text{rank}\begin{bmatrix} sI + A & D_x \\ 0 & (s+\sigma)I_d \\ C & D_y \end{bmatrix} + p. \end{aligned} \tag{17}$$

Since the pair (A, C) is detectable, when $s \neq -\sigma$, it is obvious that

$$\begin{aligned} &\text{rank}\begin{bmatrix} sI+A & D_x \\ 0 & (s+\sigma)I_d \\ C & D_y \end{bmatrix} \\ =&\text{rank}\begin{bmatrix} sI+A \\ C \end{bmatrix} + n_d = n + n_d. \end{aligned} \tag{18}$$

When $s = -\sigma$, the following equation holds from Assumption 3

$$\begin{aligned} &\text{rank}\begin{bmatrix} sI+A & D_x \\ 0 & (s+\sigma)I_d \\ C & D_y \end{bmatrix} \\ =&\text{rank}\begin{bmatrix} -\sigma I+A & D_x \\ C & D_y \end{bmatrix} = n + n_d. \end{aligned} \tag{19}$$

Summarizing the analysis above, we have

$$\text{rank}\begin{bmatrix} sI - A_0 \\ \bar{C} \end{bmatrix} = n + n_d + p = \bar{n}. \tag{20}$$

Consequently, the pair $(A_0, \bar{C})$ is detectable. It completes the proof.

Then, the following sliding mode observer for system (12) is developed,

$$\begin{aligned} \dot{z}(t) &= (A_0 - L_1\bar{C})z(t) + Ly(t) + L_s u_s(t) \\ \hat{\bar{x}}(t) &= z(t) + Hy(t) \end{aligned} \tag{21}$$

where $z(t) \in \mathbf{R}^{\bar{n}}$; $\hat{\bar{x}}(t) = [\hat{x}(t),\ \hat{d}(t),\ D_s\hat{f}_s(t)]^T$ is the estimation of $\bar{x}(t)$; $u_s(t)\mathbf{R}^{\bar{n}}$ is the discontinue input to be designed; the matrices $A_0,\ L_1,\ L,\ l_s,\ H$ are defined in (16). Then, we have

$$\begin{aligned} \dot{\hat{\bar{x}}}(t) =&(A_0 - L_1\bar{C})z(t) + Ly(t) + L_s u_s(t) + H\bar{C}\dot{\bar{x}} \\ =&(A_0 - L_1\bar{C})\hat{\bar{x}}(t) - (A_0 - L_1\bar{C})Hy(t) + L_1 y(t) \\ &+ L_2 y(t) + L_s u_s(t) + H\bar{C}\dot{\bar{x}} \\ =&(A_0 - L_1\bar{C})\hat{\bar{x}}(t) + L_1\bar{C}\bar{x}(t) + L_s u_s(t) + H\bar{C}\dot{\bar{x}}. \end{aligned} \tag{22}$$

The augment system (12) can be rewritten as

$$\begin{aligned} \dot{\bar{x}}(t) =&(A_0 - L_1\bar{C})\bar{x}(t) + \bar{B}_1 B_2 \rho(t) u(t) + \bar{B}_1 B_2 B_f f(x) \\ &+ H\bar{C}\bar{A}\bar{x}(t) + L_1\bar{C}\bar{x}(t) + \bar{D}\bar{\omega}(t) \\ =&(A_0 - L_1\bar{C})\bar{x}(t) + L_s\bar{\omega}(t) + L_1\bar{C}\bar{x}(t) + H\bar{C}\bar{D}\bar{\omega}(t) \\ &+ H\bar{C}(\bar{B}_1 B_2 B_f f(x) + \bar{A}\bar{x}(t) + \bar{B}_1 B_2 \rho(t) u(t)) \\ =&(A_0 - L_1\bar{C})\bar{x}(t) + L_s\bar{\omega}(t) + L_1\bar{C}\bar{x}(t) + H\bar{C}\dot{\bar{x}}. \end{aligned} \tag{23}$$

Define that $\bar{e}(t) = \hat{\bar{x}}(t) - \bar{x}(t)$, we have

$$\dot{\bar{e}}(t) = (A_0 - L_1\bar{C})\bar{e}(t) + (I - H\bar{C})\bar{D}(u_s(t) - \bar{\omega}(t)). \tag{24}$$

Remark 3. *It can be seen that the effect of the time-varying actuator degradation has been removed in the error dynamics (24) by using of an interesting matrix parameter design of H. This will help us to employ the sliding mode observer (SMO) technology to obtain the estimation of the system state $\hat{x}(t)$.*

Since the constants f_{a1}, f_{s1} , f_{s2}, d_1, d_2 are unknown in Assumption 1, we introduce a positive constant ψ such that

$$f_{a1} + f_{s1} + \sigma f_{s2} + d_1 + \sigma d_2 \leq \psi \tag{25}$$

where σ is given in (13). It can be seen that ψ is also unknown in (25), so we will substitute the estimation $\hat{\psi}(t)$ for ψ in the observer design, and the adaptive law for ψ is presented,

$$\dot{\hat{\psi}}(t) = c_e \|s_e(t)\|, \ \ \hat{\psi}(0) \geq 0 \tag{26}$$

where $s_e(t)$ is defined in (27), and c_e is the adaptive gain parameter.

Now the sliding mode is defined as follows:

$$s_e(t) = \bar{D}^T(I - H\bar{C})^T P\bar{e}(t) \tag{27}$$

where $s_e(t) \in \mathbf{R}^{\bar{n}}$, and $P > 0$ is the Lyapunov matrix such that

$$\bar{D}^T(I - H\bar{C})^T P = R\bar{C} \tag{28}$$

where the parameter matrix $R \in \mathbf{R}^{(m+n_d+q)\times p}$ is to be determined. Then, we design the discontinuous input $u_s(t)$ as follows

$$u_s(t) = -(\varepsilon + \hat{\psi}(t))sgn(s_e(t)). \tag{29}$$

where ε is a positive constant designed later.

3.2. Controller Design

Let $u(t) = B_2^T\tilde{u}(t)$, we have

$$\begin{aligned}\dot{x}(t) =& Ax(t) + B_1B_2\rho(t)B_2^T\tilde{u}(t) + B_1B_2f_a(t)\\ &+ Ef(x) + Dd(t).\end{aligned} \tag{30}$$

In the following part a Lemma is presented.

Lemma 3. *For the non-singular matrix $B_2\rho(t)B_2^T$ in (30), a positive scalar μ can be found such that $B_2\rho(t)B_2^T \geq \mu I_l$.*

Proof. Based on Assumption 3, we have

$$\text{rank}(\rho(t)) \geq \text{rank}(B_2\rho(t)) = l \tag{31}$$

that is, $m(m \geq l)$ actuators do not surfer outage. Without loss of generality, the first l actuators are assumed to kept from outage, and $\rho_o(t)$, $\rho_a(t) \in \mathbf{R}^{m\times m}$ are defined as follows

$$\begin{aligned}\rho_o(t) &= \text{diag}\{\rho_1^{1/2}(t), \rho_2^{1/2}(t), ..., \rho_l^{1/2}(t), 0, ...0\},\\ \rho_a(t) &= \text{diag}\{\rho_1^{1/2}(t), \rho_2^{1/2}(t), ..., \rho_l^{1/2}(t), \rho_1^{1/2}(t), ...\rho_1^{1/2}(t)\},\end{aligned} \tag{32}$$

where $0 < \rho_h(t) \leq 1$ with $h = 1, 2, ..., l$. So we have

$$\text{rank}(B_2\rho_o(t)) = \text{rank}(B_2\rho_o(t)\rho_a(t)) = \text{rank}(B_2\rho(t)) = l. \tag{33}$$

Obviously,

$$\text{rank}(B_2\rho(t)B_2^T) = \text{rank}(B_2\rho_o(t)\rho_o(t)B_2^T) = l. \tag{34}$$

Consequently, the matrix $B_2\rho(t)B_2^T$ is invertible. Then, we have

$$B_2\rho(t)B_2^T - B_2\underline{\rho}B_2^T = B_2(\rho(t) - \underline{\rho})B_2^T \geq 0$$
$$B_2\underline{\rho}B_2^T - \mu I_l \geq 0 \tag{35}$$

where $\mu = \lambda_{\min}(B_2\underline{\rho}B_2^T)$. Hence, we have

$$B_2\rho(t)B_2^T \geq \mu I_l. \tag{36}$$

It completes the proof. Then, the following integral sliding surface is constructed

$$s(t) = Fy_c(t) + \int_0^t K\hat{x}(t)d(t) \tag{37}$$

where

$$y_c(t) = y(t) - D_s\hat{f}_s(t) - D_y\hat{d}(t),$$
$$F = ((CB_1)^T CB_1)^{-1}(CB_1)^T \tag{38}$$

and $K \in \mathbf{R}^{l\times n}$ is designed later. $D_s\hat{f}_s(t)$ and $\hat{d}(t)$ can obtained in the observer (21) that

$$D_s\hat{f}_s(t) = [0,\ 0,\ I_p]\hat{\bar{x}}(t),\ \ \hat{d}(t) = [0,\ I_{nd},\ 0]\hat{\bar{x}}(t). \tag{39}$$

Denoting $e_{fs}(t) = \hat{f}_s(t) - f_s(t)$, $e_d(t) = \hat{d}(t) - d(t)$, we have

$$\begin{aligned}\dot{s}(t) =& FC\dot{x}(t) - FD_s\dot{e}_{fs}(t) - FD_y\dot{e}_d(t) + K\hat{x}(t)\\ =& FC(Ax(t) + B_1B_2\rho(t)B_2\tilde{u}(t) + B_1B_2f_a(t) + Ef(x))\\ &+ FCD_xd(t) - FD_s\dot{e}_{fs}(t) - FD_y\dot{e}_d(t) + K\hat{x}(t)\\ =& FCAx(t) + B_2\rho(t)B_2\tilde{u}(t) + B_2f_a(t) + B_2B_ff(x)\\ &+ FCD_xd(t) - FD_s\dot{e}_{fs}(t) - FD_y\dot{e}_d(t) + K\hat{x}(t).\end{aligned} \tag{40}$$

It can be seen that $B_2\rho(t)B_2^T$ is invertible according to lemma 1. Therefore, the equivalent control law in the sliding mode can be obtained from $\dot{s}(t) = 0$ that

$$\begin{aligned}\tilde{u}_{eq}(t) =& -(B_2\rho(t)B_2^T)^{-1}[FCAx(t) + B_2f_a(t) + B_2B_ff(t)\\ &+ FCD_xd(t) - FD_s\dot{e}_{fs}(t) - FD_y\dot{e}_d(t) + K\hat{x}(t)].\end{aligned} \tag{41}$$

Substituting (41) into (30), the sliding mode dynamics can be obtained as follows

$$\begin{aligned}\dot{x}(t) =& (A - B_1FCA - B_1K)x(t) - B_1KI_e\bar{e}(t)\\ &+ B_1F(D_s\dot{e}_{fs}(t) + D_y\dot{e}_d(t)) + (D_x - B_1FCD_x)d(t)\\ =& (A_a - B_1K)x(t) - B_1KI_e\bar{e}(t) + B_\Phi\Phi(t)\end{aligned} \tag{42}$$

where $A_a = A - B_1FCA$, $I_e = \left[\ I_n \quad 0\ \right]$, $B_\Phi = \left[\ B_1FD_s \quad B_1FD_y \quad D_x - B_1FCD_x\ \right]$, $\Phi(t) = \left[\ \dot{e}_{fs}^T(t) \quad \dot{e}_d^T(t) \quad d^T(t)\ \right]^T$.

According to Assumption 2, it can bee shown that $\dot{e}_{fs}(t)$, $\dot{e}_d(t)$ are bounded, and they will both converge to 0. Besides, the disturbance $d(t)$ is also been assumed in the sense of L_2 norm in (1). Therefore, we assume $\Phi(t) \in L_2[0,\ \infty]$.

In the following theorem, the stability condition for the overall closed-loop system is given.

Theorem 1. *Given a positive scalar γ, the closed-loop system (24) and (42) is robust stable with an H_∞ performance γ, that is $\|x(t)\|^2 + \|\bar{e}(t)\|^2 \leq \gamma^2\|\Phi(t)\|^2$, if there exist symmetric positive* *definite matrices $P \in \mathbf{R}^{\bar{n}\times\bar{n}}$, $Q \in \mathbf{R}^{n\times n}$, matrices $X \in \mathbf{R}^{n\times n}$, $Y \in \mathbf{R}^{\bar{n}\times p}$, $R \in \mathbf{R}^{(m+q)\times p}$ such that*

$$\Omega = \begin{bmatrix} \Omega_{11} + I & -XI_e & QB_\Phi \\ * & \Omega_{22} + I & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{43}$$

$$D^T(I - H\bar{C})^T P = R\bar{C} \tag{44}$$

where

$$\begin{aligned} \Omega_{11} &= QA_a + A_a^T Q - X - X^T, \\ \Omega_{22} &= PA_0 + A_0^T P - Y\bar{C} - \bar{C}^T Y^T. \end{aligned} \tag{45}$$

The proportional gain L_1 in (24) and K in (42) can calculated as

$$L_1 = P^{-1}Y, \;\; K = (B_1^T B_1)^{-1} B_1^T Q^{-1} X. \tag{46}$$

Proof. First, we define the error variable $\tilde{\psi}(t) = \hat{\psi}(t) - \psi$, where ψ and $\hat{\psi}(t)$ are defined in (25) and (26), respectively. Choose the following Lyapunov function,

$$V(t) = V_x(t) + V_e(t) + V_\psi(t) \tag{47}$$

where

$$V_x(t) = x^T(t)Qx(t), \;\; V_e(t) = \bar{e}^T P\bar{e}, \;\; V_\psi(t) = \frac{\tilde{\psi}^2(t)}{c_e}. \tag{48}$$

Then, we have

$$\begin{aligned} \dot{V}_x(t) =& x^T(t)[(A_a - B_1K)^T Q + Q(A_a - B_1K)]x(t) \\ & - 2x^T(t)QB_1KI_e\bar{e}(t) + 2x^T(t)Q\bar{B}_\Phi\Phi(t) \\ \dot{V}_e(t) =& \bar{e}^T(t)[P(A_0 - L_1\bar{C}) + (A_0 - L_1\bar{C})^T P]\bar{e}(t) \\ & + 2\bar{e}^T(t)P(I - H\bar{C})\bar{D}(u_s(t) - \bar{f}(t)) \\ \dot{V}_\psi(t) =& \frac{2\tilde{\psi}(t)\dot{\tilde{\psi}}(t)}{c_e}. \end{aligned} \tag{49}$$

Since $\dot{\tilde{\psi}}(t) = \dot{\hat{\psi}}(t)$, it can be derived from the adaptive law (26) and (27) that,

$$\begin{aligned} & 2\bar{e}^T(t)P(I - H\bar{C})\bar{D}(u_s(t) - \bar{f}(t)) + \frac{2\tilde{\psi}(t)\dot{\hat{\psi}}(t)}{c_e} \\ \leq & -2s_e^T(t)(\varepsilon + \hat{\psi}(t))\mathrm{sgn}(s_e(t)) + 2\|s_e(t)\|\|\bar{f}(t)\| + \frac{2\tilde{\psi}(t)\dot{\hat{\psi}}(t)}{c_e} \\ \leq & -2\|s_e(t)\|(\varepsilon + \hat{\psi}(t)) + \frac{2\tilde{\psi}(t)\dot{\hat{\psi}}(t)}{c_e} \\ & + 2\|s_e(t)\|(f_{a1} + f_{s1} + \sigma f_{s2} + d_1 + \sigma d_2) \\ \leq & -2\varepsilon\|s_e(t)\| - 2\|s_e(t)\|\tilde{\psi}(t) + \frac{2\tilde{\psi}(t)\dot{\hat{\psi}}(t)}{c_e} \\ \leq & -2\varepsilon\|s_e(t)\|. \end{aligned} \tag{50}$$

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Let $QB_1K = X$ and $PL_1 = Y$, when $\Phi(t) = 0$, after some algebraic manipulation, it can be obtained from (49) and (50) that

$$\begin{aligned}\dot{V}(t) =& \dot{V}_x(t) + \dot{V}_e(t) + \dot{V}_\psi(t) \\ \leq & x^T(t)[Q(A_a - B_1K) + (A_a - B_1K)^TQ]x(t) \\ & - 2x^T(t)QB_1KI_e\bar{e}(t) + \bar{e}^T(t)[P(A_0 - L_1\bar{C}) \\ & + (A_0 - L_1\bar{C})^TP]\bar{e}(t) - \varepsilon\|s_e(t)\| \\ \leq & \begin{bmatrix} x(t) \\ \bar{e}(t) \end{bmatrix}^T \begin{bmatrix} \Omega_{11} & -XI_e \\ * & \Omega_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ \bar{e}(t) \end{bmatrix}. \end{aligned} \tag{51}$$

If we can obtain the feasible solutions to (43), then it can be concluded that $\dot{V}(t) < 0$ in (51). Therefore, system (24) and (42) is asymptotically stable when $\Phi(t) = 0$.

Now we will consider the H_∞ performance under zero initial conditions that,

$$\begin{aligned} J =& \int_0^\infty (x^T(t)x(t) + \bar{e}^T(t)\bar{e}(t) - \gamma^2\Phi^T(t)\Phi(t))dt \\ \leq & \int_0^\infty (x^T(t)x(t) + \bar{e}^T(t)\bar{e}(t) - \gamma^2\Phi^T(t)\Phi(t) \\ & + \dot{V}(t))dt - \int_0^\infty \dot{V}(t)dt \\ \leq & \int_0^\infty (x^T(t)x(t) + \bar{e}^T(t)\bar{e}(t) - \gamma^2\Phi^T(t)\Phi(t) \\ & + \dot{V}(t))dt - V(\infty) + V(0) \\ \leq & \int_0^\infty (x^T(t)x(t) + \bar{e}^T(t)\bar{e}(t) - \gamma^2\Phi^T(t)\Phi(t) + \dot{V}(t))dt. \end{aligned} \tag{52}$$

From (49) and (50), we have

$$\begin{aligned} & x^T(t)x(t) + \bar{e}^T(t)\bar{e}(t) - \gamma^2\Phi^T(t)\Phi(t) + \dot{V}(t) \\ \leq & [\ x^T(t) \quad \bar{e}^T(t) \quad \Phi^T(t)\ ]\Omega[\ x^T(t) \quad \bar{e}^T(t) \quad \Phi^T(t)\ ]^T \\ < & 0 \end{aligned} \tag{53}$$

where Ω is defined in (43). From (52) and (53), it can be obtained that $J < 0$, and the H_∞ performance has been established.

Since B_1 is of full column rank, $B_1^TB_1$ is nonsingular. Hence, we have $K = (B_1^TB_1)^{-1}B_1^TQ^{-1}X$. It completes the proof.

Remark 4. *It is obviously that there is linear matrix equality in theorem 1, the LMI toolbox can not be used directly. According to the algorithm in [25], (44) can be taken as*

$$\begin{aligned} & \mathrm{Trace}[(D^T(I - H\bar{C})^TP - R\bar{C})^T(D^T(I - H\bar{C})^TP - R\bar{C})] \\ & = 0. \end{aligned}$$

Thus, the following inequality can be obtained

$$(D^T(I - H\bar{C})^TP - R\bar{C})^T(D^T(I - H\bar{C})^TP - R\bar{C}) < \eta I_{\bar{n}} \tag{54}$$

where η_i is a parameter to be designed. By the Schur complement, it is derived that

$$\begin{bmatrix} -\eta I_{\bar{n}} & (D^T(I - H\bar{C})^TP - R\bar{C})^T \\ * & -I_{m+n_d+q} \end{bmatrix} < 0. \tag{55}$$

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Then, the following minimization problem is equivalent to Theorem 1

$$\begin{aligned} &\min \eta \\ &\text{subject to (43) and (55)} \end{aligned} \quad (56)$$

which can be solved by the LMI toolbox in Matlab directly.

3.3. Reachability analysis of sliding motion

In the following part, the reachability of the sliding surfaces $s(t)$ in (37) will be analyzed.

Before designing the sliding mode control law $u(t)$, we present the following adaptive laws.

$$\begin{aligned} \dot{\hat{\theta}}_h(t) &= c_{\theta h}\|s(t)\|\|B_2 B_f\|\varphi_h(\hat{x}),\ \hat{\theta}(0) \geq 0,\ h = 1, 2, ..., N, \\ \dot{\hat{\bar{\delta}}}(t) &= c_\delta\|s(t)\|\|B_2 B_f\|,\ \hat{\bar{\delta}}(0) \geq 0, \\ \dot{\hat{\xi}}(t) &= c_\xi\|s(t)\|,\ \hat{\xi}(0) \geq 0, \end{aligned} \quad (57)$$

where $c_{\theta h}$, c_δ, c_ξ are the positive adaptive gains to be designed, and $\hat{\xi}(t)$ is the estimation of ξ such that

$$\begin{aligned} &\|B_2 f_a(t)\| + \|FCD_x d(t)\| + \|FD_s \dot{f}_s(t)\| \\ &+\|FD_y \dot{d}(t)\| + \|FCAe_x(t)\| \leq \xi. \end{aligned} \quad (58)$$

Obviously, we have $\hat{\theta}_h(t)$, $\hat{\bar{\delta}}(t)$, $\hat{\xi}(t) \geq 0$. The sliding mode law $\tilde{u}(t)$ is designed as

$$\begin{aligned} \tilde{u}(t) = &-\frac{1}{\mu}(\eta + \zeta(t) + \hat{\xi}(t) + \sum_{h=1}^{N}\hat{\theta}_h(t)\varphi(\hat{x}(t)) \\ &+\hat{\bar{\delta}}(t))sgn(s(t)), \end{aligned} \quad (59)$$

where $\eta > 0$ will be designed later,

$$\zeta(t) = \|FCA\hat{x}(t)\| + \|K\hat{x}(t)\| + \|FD_s\dot{\hat{f}}_s(t)\| + \|FD_y\dot{\hat{d}}(t)\|. \quad (60)$$

By analyze the reachability of sliding motion, we have the following theorem.

Theorem 2. *If there exist matrices $0 < P^T = P \in \mathbf{R}^{\bar{n}\times\bar{n}}$, $0 < Q^T = Q \in \mathbf{R}^{n\times n}$, and matrices $R \in \mathbf{R}^{(m+n_d+q)\times p}$, $X \in \mathbf{R}^{n\times n}$, $Y \in \mathbf{R}^{\bar{n}\times p}$, such that (43)– (44) hold. Based on the input $u(t)$ defined in (59), the system state of (42) can be driven onto the sliding surface $s(t) = 0$ in finite time.*

Proof. First, denoting that

$$\begin{aligned} \tilde{\theta}_h(t) &= \hat{\theta}_h(t) - \theta_h,\ h = 1, 2, ..., N, \\ \tilde{\xi}(t) &= \hat{\xi}(t) - \xi,\ \tilde{\bar{\delta}}(t) = \hat{\bar{\delta}}(t) - \bar{\delta}. \end{aligned} \quad (61)$$

Then, we define that

$$V_0(t) = V_s(t) + V_\xi(t) + V_\theta(t) + V_\delta(t) \quad (62)$$

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where

$$V_s(t) = \frac{1}{2}s^T(t)s(t),\ V_\theta(t) = \sum_{h=1}^{N}\frac{\tilde{\theta}_h^2(t)}{2c_{\theta h}},$$
$$V_{\tilde{\xi}}(t) = \frac{\tilde{\xi}^2(t)}{2c_{\tilde{\xi}}},\ V_\delta(t) = \frac{\tilde{\delta}^2(t)}{2c_\delta} \tag{63}$$

We have

$$\begin{aligned}
&\dot{V}_s(t) = s^T(t)\dot{s}(t)\\
&= s^T(t)[FCAx(t) + B_2d(t) + B_2\rho(t)B_2^T\tilde{u}(t) + K\hat{x}(t)]\\
&\le \|s(t)\|[\|FCAx(t)\| + \|B_2\|\|d(t)\| + \|K\hat{x}(t)\|]\\
&\quad - s^T(t)B_2\rho(t)B_2^T\varphi(t)sgn(s(t))\\
&\le \|s(t)\|[\|FCAx(t)\| + \|B_2\|\|d(t)\| + \|K\hat{x}(t)\|]\\
&\quad - s^T(t)B_2\underline{\rho}B_2^T\varphi(t)sgn(s(t))\\
&\le \|s(t)\|[\|FCAx(t)\| + \|B_2\|\|d(t)\| + \|K\hat{x}(t)\|] - \mu|s(t)|\varphi(t)\\
&\le \|s(t)\|[\|FCA\hat{x}(t)\| + \|FCAe_x(t)\| + \|B_2\|\|d(t)\| + \|K\hat{x}(t)\|]\\
&\quad - |s(t)|(\|FCA\hat{x}(t)\| + \|FCAB_n\|\frac{\epsilon(t)}{\sqrt{\lambda_{\min}(P)}}\\
&\quad + \|B\|\bar{d} + \|K\hat{x}(t)\| + \eta)\\
&\le -\eta\|s(t)\|.
\end{aligned} \tag{64}$$

The proof is completed.

Remark 5. *Specifically, when the unknown actuator efficiency factor is constant as $\rho(t) = \rho$, the estimation of the ρ can be given in the proposed methods, and the stabilization of the closed-loop system can be also guaranteed simultaneously.*

Now, the adaptive law for ρ_h is given by

$$\begin{aligned}
\dot{\hat{\rho}}_h(t) &= \text{Proj}_{[\underline{\rho}_h,\ \bar{\rho}_h]}\{L_h(t)\}\\
&= \begin{cases} 0, & \text{if } \hat{\rho}_h(t) = \underline{\rho}_h, \text{ and } L_h(t) \le 0\\ & \text{or } \hat{\rho}_h(t) = \bar{\rho}_h, \text{ and } L_h(t) \ge 0\\ L_h(t), & \text{otherwise}\end{cases}
\end{aligned} \tag{65}$$

where

$$L_h(t) = c_h s^T(t)B_2^{(h)}(B_2^{(h)})^T\tilde{u}(t) \tag{66}$$

where B_2^h is the hth column of B_2. The SMC law $\tilde{u}(t)$ is designed in (59).

Theorem 3. *If there exist symmetric positive definite matrices $P \in \mathbf{R}^{\bar{n}\times\bar{n}}$, $Q \in \mathbf{R}^{n\times n}$, matrices $R \in \mathbf{R}^{(m+q)\times p}$, $X \in \mathbf{R}^{n\times n}$, $Y \in \mathbf{R}^{\bar{n}\times p}$, such that (43) and (44) hold. Under the control input $u(t)$ in (59), the trajectory $x(t)$ of the closed-loop system (42) will be driven onto the sliding surface $s(t) = 0$ in finite time.*

Proof. Define that

$$\tilde{\rho}_h(t) = \hat{\rho}_h(t) - \rho_h,\ \ V_s = 0.5s^T(t)s(t) + \sum_{h=1}^{m}\frac{\tilde{\rho}_h^2(t)}{c_h}. \tag{67}$$

Then, we have

$$\begin{aligned}\dot{V}_s(t) =& s^T(t)\dot{s}(t) + \sum_{h=1}^{m} \frac{\tilde{\rho}_h(t)\dot{\hat{\rho}}_h(t)}{c_h} \\ \leq& \|s(t)\|[\|FCAx(t)\| + \|B_2\|\|d(t)\| + \|K\hat{x}(t)\|] \\ & - \frac{1}{\mu}s^T(t)B_2\rho B_2^T\varphi_1(t)sgn(s(t)) + \sum_{h=1}^{m} \frac{\tilde{\rho}_h(t)\dot{\hat{\rho}}_h(t)}{c_h} \\ \leq& \|s(t)\|[\|FCAx(t)\| + \|B_2\|\|d(t)\| + \|K\hat{x}(t)\|] \\ & - \frac{1}{\mu}s^T(t)B_2\hat{\rho}(t)B_2^T\varphi_1(t)sgn(s(t)) \\ & + \sum_{h=1}^{m} \frac{\tilde{\rho}_h(t)\dot{\hat{\rho}}_h(t)}{c_h} - s^T(t)B_2\tilde{\rho}(t)B_2^T\tilde{u}(t) \\ \leq& -\eta\|s(t)\| - \sum_{h=1}^{m} \frac{\tilde{\rho}_h(t)L_h}{c_h} + \sum_{h=1}^{m} \frac{\tilde{\rho}_h(t)\dot{\hat{\rho}}_h(t)}{c_h} \\ \leq& -\eta\|s(t)\|. \end{aligned} \tag{68}$$

The proof is completed.

4. Simulation examples

In this section, a numerical example is given and the correctness of the theorem is verified. Consider the uncertainty non-linear system subject to time-varying actuator fault, sensor fault and external disturbance as form (1), where

$$\begin{aligned} A &= \begin{bmatrix} -1 & 1 \\ -8 & -1 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ -5 & -4 \end{bmatrix}, \\ C &= \begin{bmatrix} 0.14 & 0.1 \\ -1 & 2 \end{bmatrix}, B_1 = \begin{bmatrix} -1 & 0 \\ -5 & -4 \end{bmatrix}, \\ B_2 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D_s = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, B_f = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \\ D_x &= \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}, D_y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, E = BB_f = \begin{bmatrix} -1 \\ -13 \end{bmatrix} \end{aligned}$$

with $n = 2$, $m = 2$, $p = 2$, $l = 2$, $q = 1$, $n_d = 2$, $\sigma = 0.2$, $\bar{n} = n + n_d = 4$. It can be checked that (A, B) is controllable and (A, C) is observable. Let $f(x) = sin(x_1(t))$, $\sigma = 0.2$, $f_a(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ denote the stuck actuator fault.

1. Observer Design: In the first step, the fault-tolerant observer is designed. Given the following matrices

$$H = \begin{bmatrix} 5.2763 & 0.8476 \\ 2.6131 & 0.9185 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, A_0 = \begin{bmatrix} 3.3368 & 3.3318 & 3.2731 & -0.9444 & 1.0552 & 0.1695 \\ 1.6441 & 1.6511 & 1.6281 & -0.3620 & 0.5226 & 0.1837 \\ 0 & 0 & -0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.2 \end{bmatrix}$$

$$L_s = \begin{bmatrix} 1.11396 & 8.8916 & -5.2763 & -0.8476 & -11.4002 \\ 0.5457 & 4.3937 & -2.6131 & -0.9185 & -6.1448 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} -257676 & -160190 \\ -202696 & -128852 \\ -36008 & -24683 \\ 87374 & 57837 \\ 87818 & 57410 \\ 43079 & 28139 \end{bmatrix}, L = \begin{bmatrix} 5323 & 54247 \\ 4321 & 42672 \\ 855 & 7580 \\ -1972 & 18394 \\ -1946 & 18488 \\ -953 & 9069 \end{bmatrix}$$

2. Design of controller $u_s(t)$: Next, we design the sliding mode function (27). According to 24 and the adaptive gain $c_e = 0.1$, the discontinuous input $u_s(t)$ is given by

$$s_e(t) = \begin{bmatrix} -2.3389 & 9.0417 & 14.3679 & 3.8213 & 14.1294 & 4.0084 \\ -29.7330 & 95.3811 & 116.8905 & 43.4552 & 116.2150 & 43.5985 \\ 3.6878 & -1.8811 & 19.6842 & -1.7707 & 18.8433 & -5.1520 \\ -15.3572 & 26.7646 & -14.2582 & 18.1904 & -14.3375 & 14.4901 \\ -22.5776 & 52.2656 & 19.5038 & 25.2978 & 24.8239 & 25.2804 \end{bmatrix} \bar{e}(t)$$

$$\dot{\hat{\psi}}(t) = 0.1\|s_e(t)\|$$

$$u_s(t) = -\big(0.2 + \hat{\psi}(t)\big)\mathrm{sgn}(s_e(t))$$

3. Design of controller: The state feedback gain matrix K as

$$K = \begin{bmatrix} -3005 & 401 \\ 4762 & -1179 \end{bmatrix}$$

and the matrix F is

$$F = \begin{bmatrix} -5.2631 & 0.2631 \\ 0 & -1.2500 \end{bmatrix}$$

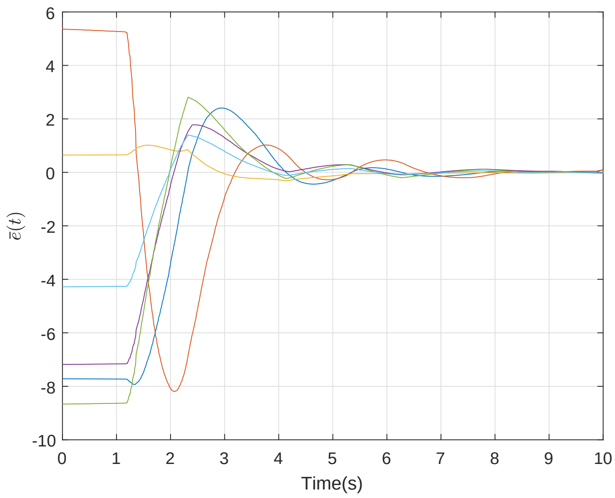

Figure 1. Trajectories of $\bar{e}(t)$

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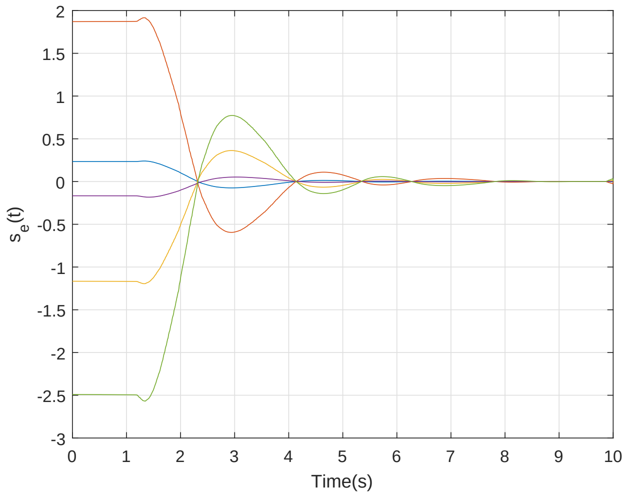

Figure 2. Trajectories of $s_e(t)$

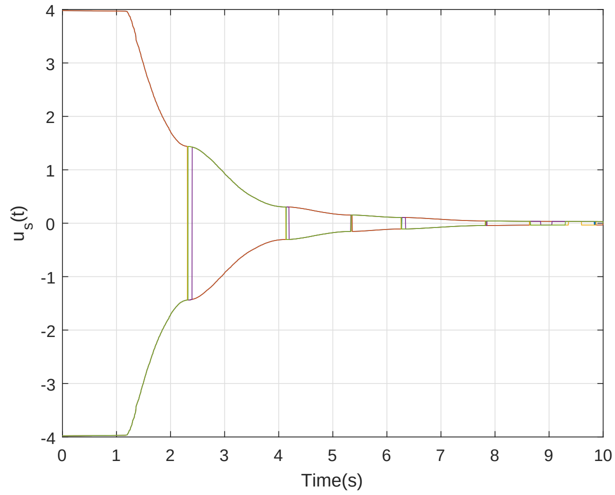

Figure 3. The discontinuous input $u_s(t)$

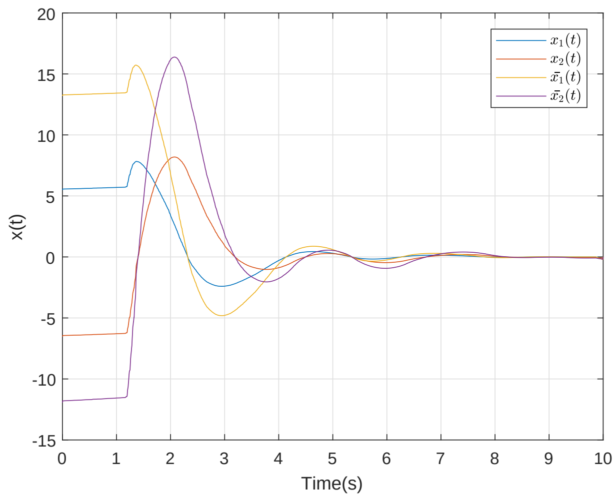

Figure 4. $x(t)$ and its estimation

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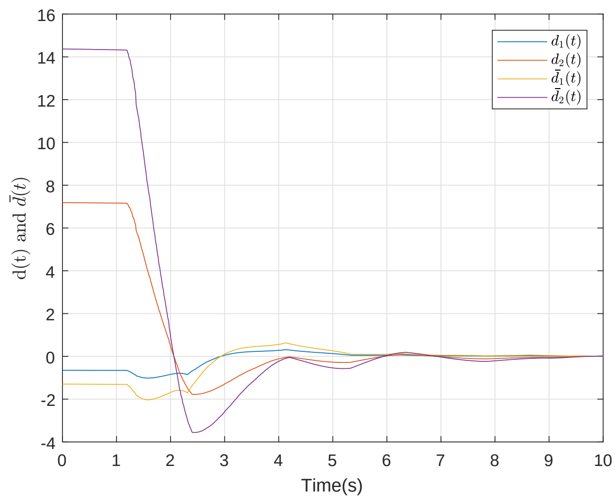

Figure 5. $d(t)$ and its estimation

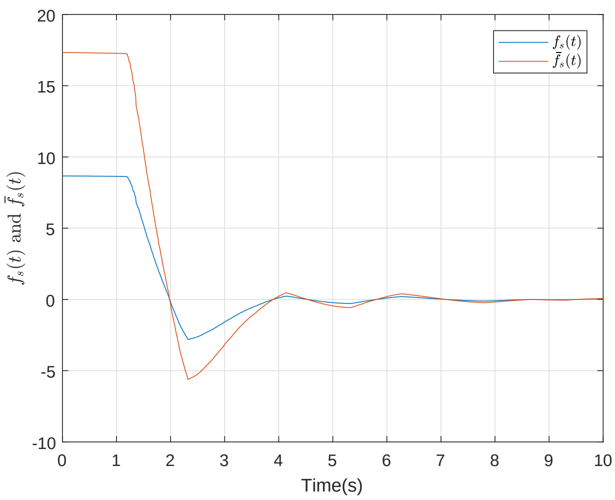

Figure 6. $f_s(t)$ and its estimation

The simulation results for system (2) are shown in Figures 2–6 below. The trajectory of error vector $\bar{e}(t)$ is shown in Figure 1, the trajectories of output error sliding surface $s_e(t)$ and discontinuous term $u_s(t)$ are shown in Figures 2–3, respectively. The comparisons of state vector $x(t)$, external disturbance $d(t)$ and sensor fault $f_s(t)$ and their estimations are illustrated in Figures 4, 5, and 6, respectively. It can be seen that the proposed FTC approach an ensure the asymptotically stability of the closed-loop fault system.

5. Conclusions

In this paper, the adaptive fuzzy FTC problem has been addressed for a class of nonlinear systems with actuator fault, sensor fault and external disturbance. By augmenting the original plant into a normal system, a new SMO is designed to obtain the estimation of the state vectors and faults information. Based on the state estimation, an integral-type SMC strategy is developed to stabilize the closed-loop fault system. Future work will focus on extending the designed methods to more complicated systems such as switched system and stochastic systems.

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References

1. Zhen Li, Xin Chen, Adaptive actuator fault compensation and disturbance rejection scheme for spacecraft. International Journal of Control, Automation and Systems, 357(9): 5380–5401, 2021.
2. Casavola, Alessandro, Famularo, Domenico, Franzè Giuseppe, A robust deconvolution scheme for fault detection and isolation of uncertain linear systems: an LMI approach. Automatica, 41(8): 1463–1472, 2005.
3. Gao Zhiwei, Ding Steven X, State and Disturbance Estimator for Time-Delay Systems With Application to Fault Estimation and Signal Compensation. IEEE Transactions on Signal Processing, 55(12): 5541–5551, 2007.
4. Huang Sunan N, Tan Kok Kiang, Fault detection, isolation, and accommodation control in robotic systems. IEEE Transactions on Automation Science and Engineering, 5(3): 480–489, 2008.
5. Jiang Bin, Zhang Ke, Shi Peng, Integrated fault estimation and accommodation design for discrete-time Takagi-Sugeno fuzzy systems with actuator faults. IEEE Transactions on Fuzzy Systems, 19(2): 291–304, 2011.
6. Karimi Hamid Reza, Robust synchronization and fault detection of uncertain master-slave systems with mixed time-varying delays and nonlinear perturbations, International Journal of Control, Automation and Systems, 9(4): 671–680, 2011.
7. Li Hongyi, Gao Huijun, Shi Peng, Zhao Xudong, Fault-tolerant control of Markovian jump stochastic systems via the augmented sliding mode observer approach. Automatica, 50(7): 1825–1834, 2014.
8. Liu Ming, Shi Peng, Sensor fault estimation and tolerant control for Itô stochastic systems with a descriptor sliding mode approach. Automatica, 49(5): 1242–1250, 2013.
9. Muhammad Shamrooz Aslam, Rizwan Ullah, Dai Xisheng, Event-triggered scheme for fault detection and isolation of non-linear system with time-varying delay. IET Control Theory and Applications, 14(16): 2429–2438, 2020.
10. Wu Ligang, Ho Daniel WC, Fuzzy filter design for Itô stochastic systems with application to sensor fault detectionIEEE Transactions on Fuzzy Systems. 17(1): 233–242, 2009.

11. Liu Xiuhua,Han Jian, Wei Xinjiang, Zhang Huifeng, Hu Xin, Intermediate variable observer based fault estimation and fault-tolerant control for nonlinear stochastic system with exogenous disturbance, Journal of the Franklin Institute, 357(9): 5380–5401, 2020.
12. Zhang Ke, Jiang Bin, Staroswiecki Marcel, Dynamic output feedback-fault tolerant controller design for Takagi–Sugeno fuzzy systems with actuator faults. IEEE Transactions on Fuzzy Systems, 18(1): 194–201, 2010.
13. Liu Xiaoxu, Gao Zhiwei, Chen Michael, Takagi-Sugeno Fuzzy Model Based Fault Estimation and Signal Compensation With Application to Wind Turbines. IEEE Transactions on Industrial Electronics, 64(7): 5678–5689, 2017.
14. Liu Xiaoxu, Gao Zhiwei, Integrated fault estimation and fault-tolerant control for stochastic systems with Brownian motions. International Journal of Robust and Nonlinear Control, 28(6): 1915–1941, 2018.
15. Wang Xin, Yang Guang-Hong, Fault-Tolerant Consensus Tracking Control for Linear Multiagent Systems Under Switching Directed Network. IEEE Transactions on Cybernetics, 50(5): 1921–1930, 2020.
16. Li Yongming, Sun Kangkang, Tong Shaocheng, Observer-Based Adaptive Fuzzy Fault-Tolerant Optimal Control for SISO Nonlinear Systems. IEEE Transactions on Cybernetics, 49(2): 649–661, 2019.
17. Deng Chao, Yang Guang-Hong, Distributed adaptive fault-tolerant control approach to cooperative output regulation for linear multi-agent systems. Automatica, 103: 62–68, 2019.
18. Tariq Muhammad Faraz, Khan Abdul Qayyum, Abid Muhammad, Mustafa Ghulam, Data-Driven Robust Fault Detection and Isolation of Three-Phase Induction Motor. Automatica, 66(6): 4707–4715, 2019.
19. Farideh Allahverdi, Amin Ramezani, Mehdi Forouzanfar, Sensor fault detection and isolation for a class of uncertain nonlinear system using sliding mode observers. Automatica, 61(2): 219–228, 2020.
20. Yi Xiong, Saif M, Sliding mode observer for nonlinear uncertain systems. IEEE Transaction on Automatic Control, 46(12): 2012-2017, 2001.
21. Apaza-Perez W Alejandro, Moreno Jaime A, Fridman Leonid, Global Sliding Mode Observers for Some Uncertain Mechanical Systems. IEEE Transaction on Automatic Control, 65(3): 1348–1355, 2020.
22. Li Jinghao, Yang Guang-Hong, Fuzzy Descriptor Sliding Mode Observer Design: A Canonical Form-Based Method. IEEE Transaction on Fuzzy Systems, 28(9): 2048–2062, 2020.
23. Qian Moshu,Yi Hui, Zheng ZhongIntegrated fault tolerant attitude control approach for satellite attitude system with sensor faults, Optimal Control Applications and Methods, 41(2): 555–570, 2020.
24. Seema Mallavalli, Afef Fekih, A fault tolerant tracking control for a quadrotor UAV subject to simultaneous actuator faults and exogenous disturbances. International Journal of Control, 99(3): 655–668, 2020.
25. Feng Xuexin, Wang YouqingFault estimation based on sliding mode observer for Takagi-Sugeno fuzzy systems with digital communication constraints, Journal of the Franklin Institute, 357(1): 569–588, 2020.