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Article

Global stability of delayed ecosystem via impulsive differential inequality and Minimax principle

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Abstract: This paper reports applying Minimax principle and impulsive differential inequality to derive the existence of multiple stationary solutions and the global stability of a positive stationary solution for a delayed feedback Gilpin-Ayala competition model with impulsive disturbance. The conclusion obtained in this paper reduces the conservatism of the algorithm compared with the known literature, for the impulsive disturbance is not limited to impulsive control.

Keywords: Minimax principle; linear approximation theory; ecosystem; steady state solution

1. Introduction

It is well known that Gilpin-Ayala competition model (GACM) has been hotly discussed (see [1-7]) due to its importance in simulating two or more competing biological populations in nature. Since diffusion is an essential characteristic of most biological populations, Ling Bai and Ke Wang began to investigate the global stability of reaction-diffusion Gilpin-Ayala ecosystem under Neumann zero boundary value in 2005 (see [8]), and obtained good results. Actually Neumann zero boundary value means that the populations do not migrate beyond the biosphere boundary. However, many animal populations are at the edge of the biosphere, where the population density is usually zero, which is not reflected by Neumann zero boundary value. And hence Dirichlet zero boundary value was considered in recent literature ([6,7]). Note that impulse control is employed to make the GACM stable globally in [6,7], but this paper involves the impulsive disturbance, which is not limited to impulsive control. Minimax principle will be employed to derive the existence of multiple stationary solutions, which improve the method of Mountain Pass Lemma in [6]. On the other hand, the newly-obtained stability criterion will reduce the conservatism of the algorithm compared with the known literature, for the impulsive disturbance is not limited to impulsive control.

2. Preparatory knowledge

Consider the following reaction-diffusion Gilpin-Ayala competition model (RDGACM) with delayed feedback under Dirichlet boundary value

$$\begin{cases} \dfrac{\partial u_1}{\partial t} =d_1\Delta u_1 + u_1(b_1 - a_{11}u_1^{\theta_1} - a_{12}u_2) - k_1(u_1 - u_1(t-\tau_1,x)), \quad t\geqslant 0,\ x\in\Omega, \\ \dfrac{\partial u_2}{\partial t} =d_2\Delta u_2 + u_2(b_2 - a_{21}u_1 - a_{22}u_2^{\theta_2}) - k_2(u_2 - u_2(t-\tau_2,x)), \quad t\geqslant 0,\ x\in\Omega, \\ u_1(t,x) =u_2(t,x) = 0, \quad t\geqslant 0,\ x\in\partial\Omega, \end{cases} \tag{2.1}$$

equipped with the initial value

$$u_1(s,x) = \xi_1(x),\ \ u_2(s,x) = \xi_2(x),\ \ s\in[0,\tau_0],\ \ x\in\Omega, \tag{2.2}$$

where Ω is a bounded domain in $\mathbb{R}^N(1\leqslant N\leqslant 3)$ with smooth boundary $\partial\Omega$, time delays $\tau_1,\tau_2\in[0,\tau_0]$, and the meaning of symbols and variables is the same as that of [7].

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Assume that
(A1) For $i = 1, 2$, set $0 < \theta_i = \frac{\hat{\theta}_i}{\check{\theta}_i} < 1$ with $\hat{\theta}_i$ being an even number, and $\check{\theta}_i$ being an odd number.
(A2) For $i = 1, 2$, there exist positive constants $M_i > 0$ such that

$$0 \leqslant u_i \leqslant M_i.$$

(A3) For $i = 1, 2$, $|\nabla u_i(t, x)|$ is bounded for all $x \in \Omega$.

Due to the limited natural resources, it is reasonable to assume in (A2) that each population density is limited. Besides, the limited natural resources imply that the boundedness assumption of (A3) is suitable to the real state of nature.

Lemma 2.1. (see, e.g. [11]) Let $J \in C^1(H_0^1(\Omega), \mathbb{R}^1)$. If there is an upper boundness of J in $H_0^1(\Omega)$, and J satisfies the (PS) condition, then $c = \sup\limits_{v \in H_0^1(\Omega)} J(v)$ is a critical value of J.

Here, the so-called (PS) condition may be found in [9, Definition 2].

Lemma 2.2. ([7, Theorem 3.1]) Set $u^*(x) = (u_1^*(x), u_2^*(x))^T$. Suppose that the condition (A2) holds, and $0 < \theta_i < 1$ for $i = 1, 2$. Moreover, if there exists a positive constant $c_* > 0$ such that

$$0 \leqslant h(u^*(x)) \leqslant c_* D\mathfrak{H}, \tag{2.3}$$

then there are at least a positive bounded equlibrium solution $u^*(x)$ for the RDAGCM (2.1), where $\mathfrak{H} = (1, 1)^T$, $h(u) = (h_1(u_1, u_2), h_2(u_1, u_2))^T$ with $u = (u_1, u_2)^T$ and

$$h_1(u_1, u_2) = u_1(b_1 - a_{11}u_1^{\theta_1} - a_{12}u_2), \quad h_2(u_1, u_2) = u_2(b_2 - a_{21}u_1 - a_{22}u_2^{\theta_2}),$$

$$D = \begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix} > 0.$$

The conditions of Lemma 2.2 guarantee the existence of a positive stationary solution $(u_1^*(x), u_2^*(x))$ for the delayed feedback system (2.1). Set

$$\begin{cases} U_1 = u_1 - u_1^*(x) \\ U_2 = u_2 - u_2^*(x), \end{cases}$$

and the stationary solution $(u_1^*(x), u_2^*(x))$ of the system (2.1) corresponds to the zero solution $(0, 0)^T$ of the following system:

$$\begin{cases} \frac{\partial U_1}{\partial t} = d_1 \Delta U_1 + b_1 U_1 - \Phi_1(U_1, U_2) - k_1[U_1 - U_1(t - \tau_1, x)], & t \geqslant 0,\ x \in \Omega, \\ \frac{\partial U_2}{\partial t} = d_2 \Delta U_2 + b_2 U_2 - \Phi_2(U_1, U_2) - k_2[U_2 - U_2(t - \tau_2, x)], & t \geqslant 0,\ x \in \Omega, \\ U_1(t,x) = U_2(t,x) = 0, \quad t \geqslant 0,\ x \in \partial\Omega, \end{cases}$$

or

$$\begin{cases} \frac{\partial U_1}{\partial t} = d_1 \Delta U_1 + (b_1 - k_1) U_1 - \Phi_1(U_1, U_2) + k_1 U_1(t - \tau_1, x), & t \geqslant 0,\ x \in \Omega, \\ \frac{\partial U_2}{\partial t} = d_2 \Delta U_2 + (b_2 - k_2) U_2 - \Phi_2(U_1, U_2) + k_2 U_2(t - \tau_2, x), & t \geqslant 0,\ x \in \Omega, \\ U_1(t,x) = U_2(t,x) = 0, \quad t \geqslant 0,\ x \in \partial\Omega, \end{cases} \tag{2.4}$$

where we denote $U = (U_1, U_2)^T$, and

$$\begin{aligned} \Phi_1(U) &= (U_1 + u_1^*(x))[a_{11}(U_1 + u_1^*(x))^{\theta_1} + a_{12}(U_2 + u_2^*(x))] - u_1^*(x)(a_{11}u_1^*(x)^{\theta_1} + a_{12}u_2^*(x)), \\ \Phi_2(U) &= (U_2 + u_2^*(x))[a_{21}(U_1 + u_1^*(x)) + a_{22}(U_2 + u_2^*(x))^{\theta_2}] - u_2^*(x)(a_{21}u_1^*(x) + a_{22}u_2^*(x)^{\theta_2}). \end{aligned} \tag{2.5}$$

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The following system is the system (2.4) in form of vector-matrix:

$$\begin{cases} \frac{\partial U}{\partial t} = D\Delta U + (B-K)U - \Phi(U) + KU(t-\tau,x), \quad t \geqslant 0, \ x \in \Omega, \\ U(t,x) = 0, \quad t \geqslant 0, \ x \in \partial\Omega, \end{cases} \tag{2.6}$$

where $U = (U_1, U_2)^T$, $U(t-\tau,x) = (U(t-\tau_1,x), U(t-\tau_2,x))^T$, $\Phi(U) = (\Phi_1(U), \Phi_2(U))^T$ and

$$D = \begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix}, \ B = \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix}, \ K = \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}. \tag{2.7}$$

Considering the impulse disturbance on (2.6), one can get the following system

$$\begin{cases} \frac{\partial U}{\partial t} = D\Delta U + (B-K)U - \Phi(U) + KU(t-\tau,x), \quad t \geqslant 0, \ t \neq t_k, \ x \in \Omega, \\ U(t_k^+, x) = A_k U(t_k^-, x), \quad k = 1, 2\cdots \\ U(t,x) = 0, \quad t \geqslant 0, \ x \in \partial\Omega, \end{cases} \tag{2.8}$$

where $U_i(t_k^+, x) = U_i(t_k, x)$ for all $i = 1, 2$, $k = 1, 2, \cdots$.

Definition 1. $(u_1^*(x), u_2^*(x))^T$ is said to be globally exponentially stable under impulsive disturbances if the zero solution of the system (2.8) is globally exponentially stable.

Lemma 2.3 (see [12]). Consider the following differential inequality:

$$\begin{cases} D^+ v(t) & \leq -av(t) + b[v(t)]_\tau, \ t \neq t_k \\ v(t_k) & \leq a_k v(t_k^-) + b_k [v(t_k^-)]_\tau, \end{cases}$$

where $v(t) \geq 0$, $[v(t_k)]_\tau = \sup\limits_{t-\tau \leq s \leq t} v(s)$, $[v(t_k^-)]_\tau = \sup\limits_{t-\tau \leq s < t} v(s)$ and $v(t)$ is continuous except $t_k, k = 1, 2, \cdots$, where it has jump discontinuities. The sequence t_k satisfies $0 = t_0 < t_1 < t_2 < \cdots < t_k < t_{k+1} < \cdots$, and $\lim\limits_{k\to\infty} t_k = \infty$. Suppose that

(1) $a > b \geq 0$;

(2)$t_k - t_{k-1} > \delta\tau$, where $\delta > 1$, and there exist constants $\gamma > 0$, $M > 0$ such that

$$\rho_1 \rho_2 \cdots \rho_{k+1} e^{k\lambda\tau} \leqslant M e^{\gamma t_k}, \tag{2.9}$$

where $\rho_i = max\{1, a_i + b_i e^{\lambda\tau}\}$, $\lambda > 0$ is the unique solution of equation $\lambda = a - be^{\lambda\tau}$;
then

$$v(t) \leqslant M[v(0)]_\tau e^{-(\lambda-\gamma)t}.$$

In addition, if $\theta = \sup\limits_{k\in Z}\{1, a_k + b_k e^{\lambda\tau}\}$, then

$$v(t) \leqslant \theta[v(0)]_\tau e^{-(\lambda - \frac{\ln(\theta e^{\lambda\tau})}{\delta\tau})t}, \quad t \geq 0.$$

Notations : Denote by λ_1 the first positive eigenvalue of the operator $-\Delta$ in the Sobolev space $H_0^1(\Omega)$ equipped with the norm $\|v\| = \sqrt{\int_\Omega |\nabla v|^2 dx}$ for any $v(x) \in H_0^1(\Omega)$. Denote by $E(\lambda_1)$ the eigenfunction space of λ_1. Denote by $\varphi_1(x) > 0$ the positive eigenfunction corresponding to $E(\lambda_1)$ with $\|\varphi_1(x)\| = 1$. Besides, I represents the identity matrix. Denote by $\lambda_{\max}(A)$ the maximum eigenvalue of symmetric matrix A, and by $\lambda_{\min}(A)$ the minimum eigenvalue of symmetric matrix A.

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3. Main results

Theorem 3.1. Suppose the conditions (A1)-(A3) and (2.3) hold, and if the following conditions are satisfied:

$$b_1 < d_1\lambda_1 \tag{3.1}$$

$$b_2 < d_2\lambda_1 \tag{3.2}$$

then the system (2.1) owns multiple stationary solutions, including the positive solution $(u_1^*(x), u_2^*(x))^T$.

Proof. To complete the proof of Theorem 3.1, the author needs to do it step by step.

Step 1. Under the condition (3.1), there are at least a stationary solution $(\alpha_*(x), 0)$ for the system (2.1).

Let $(\alpha(x), 0)^T$ be a stationary solution of the system (2.1), satisfying

$$d_1\Delta\alpha(x) + \alpha(x)(b_1 - a_{11}\alpha(x)^{\theta_1} - a_{12}\cdot 0) = 0, \quad x\in\Omega; \quad \alpha(x)|_{\partial\Omega} = 0, \tag{3.3}$$

whose functional is

$$\Psi(\alpha) = \frac{1}{2}\int_\Omega |\nabla\alpha(x)|^2dx - \frac{b_1}{2d_1}\int_\Omega |\alpha(x)|^2dx + \frac{a_{11}}{(2+\theta_1)d_1}\int_\Omega \alpha(x)^{2+\theta_1}dx, \tag{3.4}$$

It is obvious that $\Psi(0) = 0$ and $\Psi \in C^1(H_0^1(\Omega), \mathbb{R}^1)$, and then a critical point of the functional Ψ is corresponding to the solution of the equation (3.3).

Next, the authorclaim that Ψ satisfies the (PS) condition.

Indeed, if there exists $\{\alpha_n\} \subset H_0^1(\Omega)$, satisfying $\Psi(\alpha_n) \to a \in R^1$ and $\|\Psi'(\alpha_n)\|_{(H_0^1(\Omega))^*} \to 0$, it means that when n is big enough,

$$\psi(\alpha_n) = \frac{1}{2}\int_\Omega |\nabla\alpha_n(x)|^2dx - \frac{b_1}{2d_1}\int_\Omega |\alpha_n(x)|^2dx + \frac{a_{11}}{(2+\theta_1)d_1}\int_\Omega \alpha_n(x)^{2+\theta_1}dx = a + o(1), \tag{3.5}$$

which together with (A1) and poincare inequality means

$$\frac{1}{2}(1 - \frac{b_1}{d_1\lambda_1})\int_\Omega |\nabla\alpha_n(x)|^2dx \leqslant a + o(1) \leqslant a + |1+a|. \tag{3.6}$$

(3.6) implies the boundedness of $\{\alpha_n\}$ in the Sobolev space $H_0^1(\Omega)$. Further, obviously there exist two positive numbers $c_1, c_2 > 0$ big enough such that

$$|\frac{b_1}{d_1}\alpha(x) - \frac{a_{11}}{d_1}\alpha(x)^{1+\theta_1}| < c_1 + c_2|\alpha(x)|^3, \quad \forall (x,\alpha)\in\Omega\times\mathbb{R}^1, \ \Omega\subset R^N(1\leqslant N\leqslant 3),$$

which means the Caratheodory condition is satisfied. Employing the methods used in the proof of [9, Statement 2] or [10, Theorem 1] results in the existence of a convergent subsequence of the bounded sequence $\{\alpha_n\}$ in the Sobolev space $H_0^1(\Omega)$, and hence the (PS) condition is satisfied.

Next, the author claims that there is an upper boundedness for Ψ.

In fact, (A1) and (A2) yields

$$\begin{aligned}\Psi(\alpha) =& \frac{1}{2}\int_\Omega |\nabla\alpha(x)|^2dx - \frac{b_1}{2d_1}\int_\Omega |\alpha(x)|^2dx + \frac{a_{11}}{(2+\theta_1)d_1}\int_\Omega \alpha(x)^{2+\theta_1}dx \\ \leqslant& \frac{1}{2}\int_\Omega |\nabla\alpha(x)|^2dx + \frac{a_{11}}{(2+\theta_1)d_1}M_1^{2+\theta_1}mes(\Omega),\end{aligned}$$

which together with (A3) means that there exists an upper boundedness for Ψ.

According to Lemma 2.1, there exists $\alpha_*(x)$ such that

$$J(\alpha_*(x)) = \sup_{v \in H_0^1(\Omega)} J(v)$$

and $(\alpha_*(x), 0)^T$ is a stationary solution of the system (2.1).

Step 2. The author claims that the system (2.1) owns multiple stationary solutions, including the positive solution.

Firstly, the condition (2.3) and Lemma 2.2 guarantee the existence of a positive stationary solution for the system (2.1). Secondly, zero solution $(0,0)^T$ is obviously another stationary solution for the system (2.1). Next, $(\alpha_*(x), 0)^T$ is the third stationary solution thanks to Step 1. In fact, the continuity of $\varphi_1(x)$ yields

$$J(\alpha_*(x)) = \sup_{v \in H_0^1(\Omega)} J(v) \geqslant J(\varphi_1) \geqslant \frac{a_{11}}{(2+\theta_1)d_1} \int_\Omega \varphi_1(x)^{2+\theta_1} dx > 0,$$

which means that $(\alpha_*(x), 0)^T$ is a nontrivial stationary solution for the system (2.1). Finally, one can similarly prove that there exists a nontrivial stationary solution $(0, \beta_*(x))^T$ for the system (2.1). □

Theorem 3.2. Suppose that all the conditions of Theorem 3.1 are satisfied. Assume, in addition,

(B1) there exist three positive constants p_m, p_M, ε, and a positive definite diagonal matrix $P = diag(p_1, p_1) > 0$ such that the following LMI conditions hold :

$$2\lambda_1 PD - 2P(B-K) - p_M\Theta - \varepsilon PK > 0 \tag{3.7}$$

$$P < p_M I \tag{3.8}$$

$$p_m I < P \tag{3.9}$$

where

$$\Theta = \begin{pmatrix} 2\Big(a_{11}(1+\theta_1)(2M_1)^{\theta_1} + a_{12}M_2\Big) & a_{12}M_1 + a_{21}M_2 \\ * & 2\Big(a_{22}(1+\theta_2)(2M_2)^{\theta_2} + a_{21}M_1\Big) \end{pmatrix}$$

(B2) $a > b \geqslant 0$, where $a = \frac{\lambda_{\min}\Big(2\lambda_1 PD - 2P(B-K) - p_M\Theta - \varepsilon PK\Big)}{p_M}$, $b = \frac{\lambda_{\max}(K)}{\varepsilon}$

(B3) there exists a constant $\delta > 1$ such that $\inf_{k \in Z}(t_k - t_{k-1}) > \delta\tau$ and $\lambda > \frac{\ln(\rho e^{\lambda\tau})}{\delta\tau}$, where $\rho = \sup_{j \in \mathbb{Z}}\{1, a_j + b_j e^{\lambda\tau}\}$ with $a_j = \frac{\lambda_{\max}(A_j^T P A_j)}{p_m}$ and $b_j \equiv 0$, and $\lambda > 0$ is the unique solution of the equation $\lambda = a - be^{\lambda\tau}$.

then the zero solution of the system (2.8) is globally exponentially stable with convergence rate $\frac{1}{2}(\lambda - \frac{\ln(\rho e^{\lambda\tau})}{\delta\tau})$, and $(u_1^*(x), u_2^*(x))^T$ is said to be globally exponentially stable under impulsive disturbances with convergence rate $\frac{1}{2}(\lambda - \frac{\ln(\rho e^{\lambda\tau})}{\delta\tau})$.

Proof. Consider the following Lyapunov function:

$$V(t) = \int_\Omega U^T(t,x) P U(t,x) dx = \int_\Omega |U(t,x)|^T P |U(t,x)| dx$$

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then for $t \geqslant 0$, $t \neq t_k$, the Poincare inequality yields

$$\begin{aligned} D^+V =& 2\int_\Omega U^T P\Big(D\Delta U + (B-K)U - \Phi(U) + KU(t-\tau,x)\Big)dx \\ \leqslant& \int_\Omega U^T P\Big(-2\lambda_1 D + 2(B-K)\Big)Udx + \int_\Omega \Big(-2U^T P\Phi(U) + 2U^T PKU(t-\tau,x)\Big)dx \\ \leqslant& \int_\Omega |U|^T P\Big(-2\lambda_1 D + 2(B-K)\Big)|U|dx + \int_\Omega \Big(2|U|^T P|\Phi(U)| + 2|U|^T PK|U(t-\tau,x)|\Big)dx \end{aligned} \tag{3.10}$$

On the other hand, it follows from (2.5) that $\Phi_1(0,0) = 0 = \Phi_2(0,0)$, and

$$\Phi_1(0,U_2) = a_{12}u_1^*(x)(U_2 + u_2^*(x)) - a_{12}u_1^*(x)u_2^*(x) = a_{12}u_1^*(x)U_2 \tag{3.11}$$

and hence differential mean value theorem and (A2) yield

$$\begin{aligned} &|\Phi_1(U)| = |\Phi_1(U) - \Phi_1(0)| \leqslant |\Phi_1(U_1,U_2) - \Phi_1(0,U_2)| + |\Phi_1(0,U_2) - \Phi_1(0,0)| \\ \leqslant& \Big(a_{11}(1+\theta_1)(2M_1)^{\theta_1} + a_{12}M_2\Big)|U_1| + a_{12}M_1|U_2|. \end{aligned} \tag{3.12}$$

Similarly,

$$|\Phi_2(U)| \leqslant a_{21}M_2|U_1| + \Big(a_{22}(1+\theta_2)(2M_2)^{\theta_2} + a_{21}M_1\Big)|U_2| \tag{3.13}$$

Thus,

$$\begin{aligned} 2|U|^T P|\Phi(U)| \leqslant& p_M(2|U_1|\cdot|\Phi_1(U)| + 2|U_2|\cdot|\Phi_2(U)|) \\ \leqslant& p_M|U|^T\Theta|U| \end{aligned} \tag{3.14}$$

$$2|U|^T PK|U(t-\tau,x)| \leqslant \varepsilon|U|^T(PK)|U|) + \frac{1}{\varepsilon}\lambda_{\max}(K)|U(t-\tau,x)|^T P|U(t-\tau,x)| \tag{3.15}$$

Combining (3.10)-(3.15) results in

$$\begin{aligned} &D^+V(t) \leqslant \int_\Omega |U|^T P\Big(-2\lambda_1 D + 2(B-K)\Big)|U|dx + \int_\Omega \Big(2|U|^T P|\Phi(U)| + 2|U|^T PK|U(t-\tau,x)|\Big)dx \\ &\leqslant -\frac{\lambda_{\min}\Big(2\lambda_1 PD - 2P(B-K) - p_M\Theta - \varepsilon PK\Big)}{p_M}\int_\Omega |U|^T P|U|dx + \frac{\lambda_{\max}(K)}{\varepsilon}V(t-\tau) \\ &\leqslant -av(t) + b[v(t)]_\tau,\ t \neq t_k. \end{aligned} \tag{3.16}$$

On the other hand, letting $\gamma = \frac{\ln(\rho e^{\lambda\tau})}{\delta\tau}$, one can conclude from Lemma 2.3 that

$$V(t) \leqslant (\rho^2 e^{\lambda\tau})[V(0)]_\tau e^{-(\lambda-\gamma)t}, \quad t \geqslant t_0, \tag{3.17}$$

or equivalently,

$$V(t) \leqslant (\rho^2 e^{\lambda\tau})[V(0)]_\tau e^{-(\lambda - \frac{\ln(\rho e^{\lambda\tau})}{\delta\tau})t}, \quad t \geqslant t_0, \tag{3.18}$$

Indeed,

$$\begin{aligned} V(t_k) =& \int_\Omega U^T(t_k,x)PU(t_k,x)dx \\ \leqslant& \frac{\lambda_{\max}(A_k^T PA_k)}{p_m}\int_\Omega U^T(t_k^-,x)PU(t_k^-,x)dx \\ =& a_k V(t_k^-). \end{aligned}$$

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According the conditions (B1)-(B3), one can see it from Lemma 2.3 that (3.17) and (3.18) holds if the condition (2.9) is verified. In fact, in Lemma 2.3, let $M = \rho^2 e^{\lambda\tau}$, then

$$\begin{aligned} Me^{\gamma t_k} =& (\rho^2 e^{\lambda\tau}) e^{\gamma(t_k - t_0)} \\ \geqslant& (\rho^2 e^{\lambda\tau})(\rho e^{\lambda\tau})^{k-1} \\ =& (\rho^{k+1} e^{k\lambda\tau}), \end{aligned}$$

which means that the condition (2.9) is satisfied, and then Lemma 2.3 makes (3.17) and (3.18) hold. Moreover, (3.18) yields

$$\begin{aligned} p_m \|U\|_{L^2(\Omega)} \leqslant& V(t) \leqslant (\rho^2 e^{\lambda\tau})[V(0)]_\tau e^{-(\lambda - \frac{\ln(\rho e^{\lambda\tau})}{\delta\tau})t} \\ \leqslant& (\rho^2 e^{\lambda\tau}) p_M \|\xi(s,x) - u^*(x)\|_\tau^2 e^{-(\lambda - \frac{\ln(\rho e^{\lambda\tau})}{\delta\tau})t} \quad , \quad t \geqslant t_0, \end{aligned} \tag{3.19}$$

where $\|\xi(s,x) - u^*(x)\|_\tau^2 = \sup\limits_{s\in[-\tau,0]} \int_\Omega [\xi(s,x) - u^*(x)]^T [\xi(s,x) - u^*(x)] dx$ with $\xi = (\xi_1, \xi_2)^T$ and $u^* = (u_1^*, u_2^*)^T$. Obviously, (3.19) completes the proof. □

Remark 1. Theorem 3.2 offers a better stabilization criterion than the previous literature ([6,7]), which reduces the conservatism of the algorithm. In fact, in Theorem 3.2, the impulse condition $\lambda_{\min} A_k$ may not be smaller than 1, which implies that this paper deletes the harsh restrictions on small impulse of the related literature ([6,7]).

4. Numerical examples

Firstly, the following example shows the effectiveness of Theorem 3.1.

Example 4.1. Let $\theta_1 = \frac{2}{3}$, $\theta_2 = \frac{4}{5}$, $b_i = 0.13 + 0.0001i$, $d_i = 0.1 + 0.0001i$, $i = 1,2$, and $\Omega = (0,1) \times (0,1)$. Direct calculation yields that $\lambda_1 = 19.7392$ ([13, Remark 14]), and $b_1 = 0.1301 < 0.1001 \times 19.7392 = d_1\lambda_1$ and $b_2 = 0.1302 < 0.1002 \times 19.7392 = d_2\lambda_1$. Furthermore, set $M_i = 2 + 0.1i$, $i = 1,2$, and $a_{11} = 0.03, a_{12} = 0.02, a_{21} = 0.025, a_{22} = 0.03, k_1 = 0.15, k_2 = 0.12$. An accurate calculation can verify that the condition (2.3) is satisfied if letting $c_* = 100000$. Now one can conclude from Theorem 3.1 that there is a positive stationary solution $(u_1^*(x), u_2^*(x))^T$ and other three stationary solutions for the ecosystem (2.1).

Below, the feasibility of Theorem 3.2 need be verified, too.

Example 4.2. All the data of Example 4.1 are employed in this example, then an accurate calculation yields that

$$\Theta = \begin{pmatrix} 0.3483 & 0.0970 \\ 0.0970 & 0.4583 \end{pmatrix}$$

(B2) $a > b \geqslant 0$. Furthermore, using computer Matlab LMI toolbox to solve LMI condition (3.7)-(3.9) yields the following feasible data:

$$P = \begin{pmatrix} 0.9998 & 0 \\ 0 & 1.0013 \end{pmatrix}, \ \varepsilon = 0.9996, \ p_M = 1.0015, \ p_m = 0.9973.$$

And then a direct calculation obtains $a = 3.3046, b = 0.1501$, and hence $a > b \geqslant 0$. Let $\tau = 0.5$, solving the equality $\lambda = a - be^{\lambda\tau}$ reaches $\lambda = 2.7199$. Set

$$A_j \equiv \begin{pmatrix} 1.0603 & 0 \\ 0 & 1.0783 \end{pmatrix}, \forall j \in \mathbb{Z}, \tag{4.1}$$

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which together the above data derives that $a_j \equiv 1.1674$, and hence $\rho = 1.1674$. Set $\delta = 2$, then an immediate calculation yields $\lambda - \frac{\ln(\rho e^{\lambda\tau})}{\delta\tau} = 1.2052 > 0$, and $\frac{1}{2}[\lambda - \frac{\ln(\rho e^{\lambda\tau})}{\delta\tau}] = 0.6026$. According to Theorem 3.2, the zero solution of the system (2.8) is globally exponentially stable with convergence rate 60.26%.

Remark 2. Example 4.2 illuminates that Theorem 3.2 is less conservative than the related results in the known literature ([6,7]). In fact, (4.1) of Example 4.2 shows that the involved impulse is greater than 1 while the impulse of the previous literature ([6,7]) must be smaller than 1.

5. Conclusions

Compared with the known literature, this paper has double advantages in method and conclusion. On one hand, Employing Minimax principle and impulsive differential inequality improve the methods of [6,7]. For example, in deriving the existence of multiple stationary solutions of RDGACM, the methods involved in Minimax principle is more simpler than those in Mountain Pass Lemma of [6]. Besides, in stabilizing globally the ecosystem, utilizing the impulsive differential inequality makes the impulse range wider. Especially, an impulse range means that people can adjust and manage the ecosystem more flexibly.

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