Article

Is quantum theory local?

Ghenadie Mardari[1,*]

[1] Open Worlds Research, Baltimore, Maryland, USA
* Correspondence: gmardari@gmail.com.

Abstract: Quantum mechanics is often described as irreducibly non-local. A tacit element of this picture is the assumption that one and the same sub-ensemble of quanta is post-selected in every measurement combination of a Bell test. Yet, this expectation was recently shown to be formally inconsistent with quantum theory (Cetto *et al.*, 2020) and even to be experimentally falsifiable (Mardari 2021). The need to make sense of this development motivated a rigorous conceptual analysis of quantum non-locality, especially as it relates to the basic principles of quantum mechanics. The simple conclusion is that quantum theory and quantum non-locality are fundamentally incompatible. This is not a loophole around the predictions of quantum mechanics, but rather an insight into the essential conditions that make them accurate.

Keywords: quantum entanglement; Bell's theorem; quantum non-locality; quantum realism.

1. Introduction

One of the most controversial aspects of quantum theory is that it makes predictions about "non-real" physical properties. Quantum observables are predicted with extreme precision, but only as part of a measurement context – never as individual qualities that exist objectively by themselves. Yet, assuming this is true, does it follow that quantum behavior is non-local? The Copenhagen school of thought maintains that non-realism automatically entails non-locality. If quantum properties do not exist prior to measurement, then entangled quanta can "choose" to create them in coordinated ways, even at cosmic distances from each other. Thus, loophole-free violations of Bell-type inequalities should be seen as irrefutable proof of non-locality [1-7]. The problem is that quantum observables correspond to various sub-ensembles that need to be post-selected from chaotic beams. The most obvious feature of EPR-type experiments is that raw detections (single counts) have flat distributions, and only post-selected events (coincident counts) exhibit predicted patterns [8-10]. Non-locality is only needed if the same exact sub-ensembles are targeted in incompatible measurements [11]. Though, is it plausible to expect something like that if quantum properties are not real prior to measurement? What if quantum theory predicts Bell violations *because* identical sub-ensembles are impossible to isolate? If so, could it be that quantum theory is fundamentally local?

As a matter of tradition, Bell experiments are presumed to target the same sub-ensembles in every measurement setting. In a way, this is quite intuitive, because Bell violations are expected for maximally entangled quanta. For example, if we measure their polarization, we detect strong correlations. If we measure their position in some image plane, we see the same relationship. Therefore, we should be able to do both types of measurements at the same time, without repercussions. For example, we can isolate two correlated points of detection and use them to post-select a coherent sub-ensemble of quantum pairs, while different polarization measurements are performed in transit (Figure 1, below). The nuance here is that different polarization measurements are non-commuting, while polarization properties and spatial properties do not explicitly constrain each other in this way. Accordingly, it seems natural to assume that detection coordinates are unaffected by polarization measurements. Indeed, a few years ago it would have been unthinkable to suggest otherwise. Nonetheless, a recent paper by Cetto *et al.* [12] has turned this expectation upside down. When the subtleties of spin ½ projections are rigorously analyzed, it turns out that quantum theory predicts Bell violations for incompatible "partitions" of the probability space. In other words, when Alice and Bob choose between different measurement settings, they target different sub-ensembles of quanta from a common input projection. This sounds very strange for a set of maximally entangled quanta, but only if we assume that observed properties correspond to unobserved "real" qualities. (The counterfactual assumption here is that a quantum would have landed on the same pinhole, as if aiming for it from the source, even though it was prepared for observation in a different way.) Previously, it was known that Bell violations are local for imperfect maximal entanglement, but this was only suspected as an experimental loophole (*i.e.*, as a shortcoming of the existing mechanisms for generating entanglement) [13, 14]. It is quite a surprise to discover that maximal entanglement might not be objectively "maximal" at the level of single quanta, at least in the case of non-identical measurements.

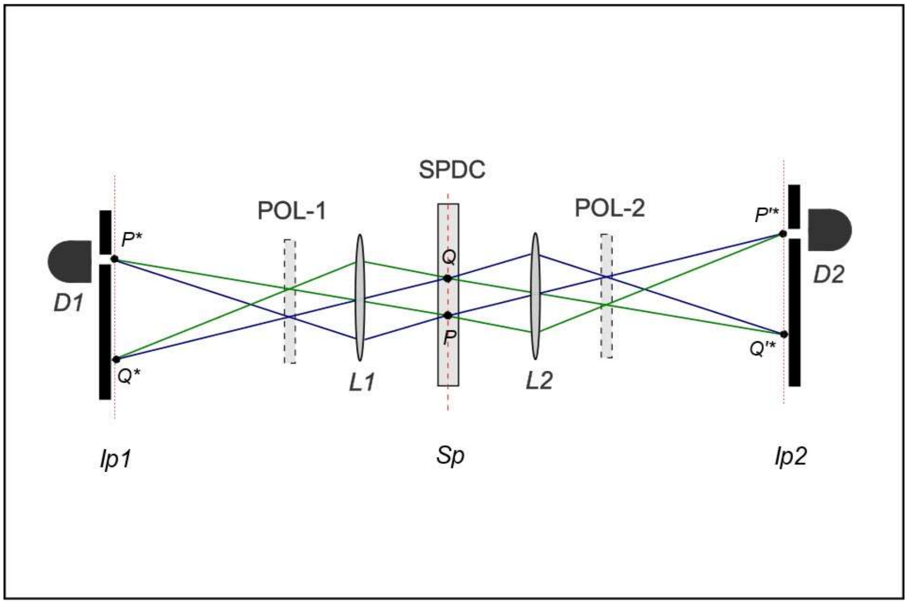

Figure 1. Sub-ensemble post-selection in a typical Bell experiment. Entangled quanta emerge in opposite directions from a non-linear crystal, via spontaneous parametric down-conversion (SPDC). This is a random process, with uncertain points of emission for any given pair. Lens L1 creates an image of the Source plane (*Sp*) in front of detector *D1*. Lens *L2* has the same effect in the Image plane *Ip2*, in front of detector *D2*. As a result, coincident events at *P** and *P′** have a high probability of emerging from the common point of emission *P*, as opposed to point *Q* or any other point in-between. This is demonstrably true, provided measurement devices POL-1 and POL-2 are missing. The question is: what happens when these devices are introduced and manipulated as needed for various combinations of measurements? If the effect of such interventions is negligible, then resulting violations of Bell-type inequalities are non-local. Yet, as shown in the text, this expectation is incompatible with the basic principles of quantum theory.

To sum up, the concept of non-locality appears to contradict the principle of non-realism in quantum mechanics. As shown elsewhere, it also has falsifiable experimental implications [15]. Yet, like any new idea, this development feels out of sorts. How does it fit with the known evidence, and even with the basic principles of quantum theory? Previously, it has been suggested that Bell violations require *either* a failure of "realism", *or* a failure of "locality", but not both [10, 16, 17], yet the implications of this distinction did not quite make it into the textbooks. Recently, this list was expanded to include failures of "free will" in discussions about super-realism in quantum mechanics [18, 19]. Though, somehow these discussions have advanced with a narrow focus on Bell violations, without a rigorous discussion about the fitness of each concept in the wider context of orthodox quantum theory. Granted, there are many ways to violate Bell's inequality, but how many of these scenarios are compatible with the basic principles of quantum theory? Do they entail equally valid explanatory mechanisms? Historically speaking, the concept of non-realism was seen as fundamental for quantum theory from the beginning (even if poorly defined), while the concept of non-locality was a late addition. The goal of this essay was to accept the basic principles of quantum theory as they are, and to investigate their relationship to the concept of non-locality, one by one. Is it possible to reconcile them together? As it turns out, the answer is negative. The principles of quantum theory and the concept of non-locality are fundamentally incompatible with each other.

2. Non-locality *vs* the Correspondence Principle

The main reason to endorse the hypothesis of non-locality is that quantum theory predicts violations of Bell inequalities. This is a direct consequence of Bell's Theorem [20], which appears to suggest that quantum theory cannot be local. The structure of this argument is as follows:

1. It is a mathematical fact that local theories cannot predict phenomenon P in context C.
2. It is also a mathematical fact that quantum theory predicts phenomenon P in context C.
3. Therefore, quantum theory is not local.

If we take this argument at face value, it seems unquestionable. There is no doubt that statement 1 is correct. There seems to be no doubt that statement 2 is also correct. So, the conclusion should be true. If quantum theory can do what local theories cannot, then it is non-local. And yet, surprisingly, this conclusion entails a paradox. Any prediction of quantum theory should be an expression of the principles that govern its formalism. Yet, if quantum theory is non-local, as suggested above, then the same basic principles cannot be true. How is it possible for a quantum prediction, formulated in accordance with a set of principles, to contradict the same principles? The most likely explanation is that something is missing in the derivation of Bell's theorem. We need to take a closer look.

Four quantum principles will be shown to be relevant for this analysis. They are the Correspondence Principle, the Superposition Principle, the Complementarity Principle, and the Local Signaling Principle. Let us begin with the first one. From the start, quantum theory was explicitly developed in a manner that can explain the emergence of classical macroscopic phenomena. A major source of motivation for studying quantum behavior was to figure out how it explains the emergence of large-scale observations. Accordingly, the Correspondence Principle states that any prediction of quantum mechanics must approximate a corresponding prediction from classical mechanics, as the number of detected events increases [21]. In other words, individual quantum behavior may be classical, or it may be non-classical, but any prediction for large N observables can only be classical in nature. Consequently, it is not possible for quantum theory to contradict classical theories when it comes to large-scale statistical predictions.

Looking back at the first premise of Bell's Theorem, as outlined above, we read that "local theories cannot predict phenomenon P in context C". Yet, what is phenomenon P? It is not just any kind of observation. It concerns the ability of a statistical system to violate a Bell-type inequality. And what is Bell's inequality? It is an algebraic expression, containing several coefficients of correlation between pairs of variables. For example, the CHSH inequality [22] is a Bell-type inequality that requires four coefficients of correlation:

$$C_Q(a,b)+C_Q(a,b')+C_Q(a',b)-C_Q(a',b')\leq 2 \tag{1}$$

The only way to verify it is by collecting very large numbers of discrete events, several times over, in order to determine the values of those coefficients. Therefore, Bell inequalities are "large N" predictions, which cannot contradict corresponding classical predictions. As long as we are talking about quantum theory, the correspondence principle must apply. Hence, Bell's Theorem cannot possibly be correct. Either phenomenon P is not the same in both cases, or the context C is not the same. The bottom line is that Bell's theorem is not an "apples to apples" comparison.

We can summarize this as follows:

1. Classical mechanics is a local theory.
2. Quantum mechanics cannot contradict classical mechanics for large N.

3. Bell's inequality is a large N prediction.
4. Therefore, quantum mechanics can only make local predictions for Bell measurements.
5. Ergo, Bell's Theorem cannot be correct.

It is important to note that this is an exercise in qualitative analysis. This argument does not show in any way that the mathematical aspects of Bell's Theorem are questionable. Without a doubt, quantum theory predicts violations of Bell inequalities, and numerous experiments have confirmed them. The relevant implication here is that the interpretation is wrong. Namely, quantum Bell violations are valid, but they cannot possibly be non-local. They must be structurally similar to corresponding classical phenomena. The bottom line is that it does not seem possible for quantum non-locality to be true and for the correspondence principle to be valid at the same time.

3. Non-locality *vs* the Superposition Principle

Though, if Bell's theorem is wrong, where is the mistake? As suggested above, either phenomenon P or the context C are out of synch in the two premises. On closer inspection, phenomenon P is strictly about Bell's inequality, and it is the same in both steps of the argument. Therefore, the mistake should be found in the description of the context C. The most likely solution is that Bell's theorem is comparing one type of contexts for local theories in general, and a different type of contexts for quantum theory. Indeed, there is a remarkable difference between these two kinds of contexts, because it is both subtle and strikingly obvious at the same time. On the one hand, quantum theory has an identity of its own, which is why it cannot be confused with any other theory. More importantly, quantum mechanics is radically different from classical mechanics. On the other hand, when we compare the statistical implications of several theories, we expect them to be different in special cases, but we need to be explicit: do they have the same scope? As it turns out, quantum predictions cannot have the same scope as the predictions that Bell described as local.

The reasons for this conclusion can be summarized as follows. Many predictions of quantum theory are derived by finding solutions to relevant wave-functions. Yet, these predictions cannot violate the principles of quantum mechanics. In this case, the relevant concept is the Superposition Principle. When two different measurement outcomes are predicted in quantum mechanics, the formal aspect is that two different solutions of Schrodinger's equation are being derived. Yet, according to the superposition principle, if two valid solutions of a quantum wave-function are possible, then the linear superposition (or vector sum) of these two solutions is also a valid solution [23]. What is the physical significance of this? The relevant implication is that we can describe all the possible measurement outcomes of a quantum system as if they exist together, in superposition, prior to measurement. Yet, what does it mean to say that many outcomes are in superposition? It means that all of them can act at the same time, like different spectral components of a complex wave. Consequently, if all of them can act at the same time, then none of them can express the properties of the system, as a whole. By definition, *to be in superposition is to be a part of something.*

Let us review why this is important. Any valid derivation of Bell's inequality presupposes that one and the same population is measured for every coefficient of correlation. If it was admissible to make such measurements in different populations, then Bell violations would be a trivial side-effect. The logical mechanism behind Bell inequalities is that well-defined properties cannot display coefficients of correlation that contradict each other. Yet, quantum theory predicts violations for non-identical projections that can be superposed to recreate the input profile. Therefore, it is not possible to reduce all the coefficients of a Bell test to a single sub-ensemble of quanta. (Adding the same component to itself several times cannot reproduce the full system.) Instead, every joint measurement must target a different sub-ensemble. This means that Bell's theorem compared classical properties that apply to full populations with quantum properties that apply to partial incompatible slices of such populations. Accordingly, the result of Cetto et al. [12] should not be surprising.

As shown in the previous section, quantum theory predicts Bell violations for large N, but it cannot violate the correspondence principle. At first, that seemed a little arbitrary. Though, as it turns out, quantum mechanics must also obey the superposition principle, and therefore its predictions can only apply to partial incompatible measurements. Therefore, the correct argument is:

1. Local theories cannot predict phenomenon P for complete measurements.
2. Quantum mechanics predicts phenomenon P for partial measurements.
3. Local theories also allow for P when measurements are partial.
4. Ergo, Bell's theorem doesn't work.

It is important to point out that quantum non-locality is often presented as a matter of fact when Bell violations are confirmed by experiment [1-10]. In fact, those claims only mean to suggest that quantum theory is proven to be valid. The "non-locality" tag is attached on the basis of Bell's theorem (not on the basis of direct demonstrations). Hence, it is not a contradiction to suggest that quantum theory is both correct and local, because the basis for belief in non-locality is missing.

4. Non-locality *vs* the Complementarity Principle

As shown above, it is not necessary to believe that quantum theory is non-local, but one could ask: is it necessary to believe that it is local? After all, quantum "non-realism" is a very subtle concept. It is not meant to suggest that nothing is real before the act of measurement, but rather that well-defined observable properties are not real outside of their context. Indeed, a more sensible interpretation, given current knowledge, is that all the possible quantum properties may be real for unmeasured individual particles. This is supported by numerous quantum experiments in which single quanta are likely to propagate alone, one at a time. Accordingly, it might be meaningful to suggest that quantum experiments are indeed sampling the full input population, even when they are designed to target incompatible wave-function components. If so, non-locality would still be a plausible interpretation.

For clarity, this objection can be explained with a concrete example. Let us consider a multimode optical beam, observable in the form of white light (Figure 2). According to quantum theory, every single photon in this projection expresses the full wave-function profile of its context. In plain language, it is white. The question is: what happens if the beam is guided to pass through a prism? The projection splits into the known colors of the rainbow, and different quantum detectors can record arrival events. If there is only one photon in the beam, only one detector will fire. For example, suppose that the photon is detected as blue. It will contribute to the "bliue" sub-ensemble of detected events, representing a partial measurement. Therefore, it satisfies the conditions explained above. Nonetheless, the photon cannot be described as "blue" before the prism. It should be described as "white". If so, is it not the case that partial quantum measurements sample complete input profiles? If so, shouldn't they be described as fair samples of input profiles, just like classical populations that cannot violate Bell inequalities?

This is a very interesting objection. Nonetheless, it is invalid because it contradicts another fundamental concept in quantum theory: the Complementarity Principle [24]. According to this postulate, incompatible quantum profiles cannot be real at the same time. For example, if two coherent projections intersect, individual beams cannot be described as real in the area of overlap. The only thing that is observable there is the interference profile. Conversely, when the beams separate, no trace of interference can exist. The only thing that can be real at that stage is a projection with two separated beams, as if they never overlapped with each other. In the same manner, when we have a multimode frequency profile, we have two incompatible realities. Before the prism, only white quanta exist. After the prism, only blue quanta exist in the adequate channel. It does not matter what history these quanta had before the measurement. Once detected, they can only express information about the context that was selected for observation. As a result, it cannot be true that we are sampling the full input profile, because every single quantum is blue and only blue at the detector. It is as if it was never white before it passed through the prism.

Another way to think about this is in terms of the measurement problem [25]. Before the prism, quantum behavior is governed by the full input wave-function. At the moment of observation, the properties of each quantum are governed by collapsed wave-functions. In other words, they express two incompatible realities. Yet, Bell experiments only deal with distributions that express output profiles. The only way to see if a system violates a Bell-type inequality is to put together a very large number of collapsed quanta, and to post-select the coincidences between their distributions. That is how we determine the relevant coefficients of correlation. Therefore, it is not physically meaningful to interpret a quantum Bell violation as a property of a complete input profile.

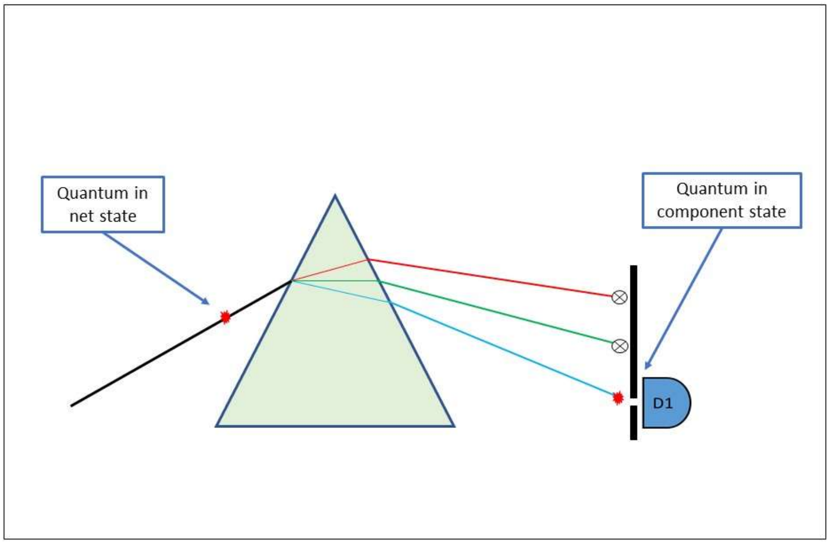

Figure 2. A single quantum with two realities. According to quantum theory, single quanta must express the full profile of their governing wave-function, even if propagating one at a time. In this case, a singe quantum belongs to a heavily attenuated beam of sunlight. Therefore, it has to be described as white. After the prism, the quantum can only produce a single event in one of the component states of the input spectrum. Doesn't this entail that partial quantum measurements sample complete input profiles? If so, isn't quantum behavior non-local? As shown in the text, this loophole is not allowed by quantum theory, because it contradicts the Complementarity Principle.

And so, we have come back to the same problem. The only reason to believe in non-locality is if quantum correlations and classical correlations are evaluated by sampling the same type of statistical contexts. In this case, both kinds of correlations must apply to complete input profiles. Yet, this can only be true if sharp quantum measurements violate the Complementarity Principle. Therefore, we see yet again that quantum behavior can only be non-local if quantum theory is incorrect.

5. Non-locality *vs* the Local Signaling Principle

As shown above, quantum mechanical violations of Bell's inequality cannot contradict classical observations, because of the Correspondence Principle. They only apply to partial sub-ensembles, obeying the Superposition Principle. Finally, they cannot be reduced to a common group of quanta indirectly, because of the Complementarity Principle. Yet, the principle of non-realism appears to leave open a loophole for non-locality. The goal of quantum observation is to extract information, but it sounds like any quantum should be able to express any kind of information. Therefore, it sounds possible that one and the same sub-ensemble could "choose" to collapse in various ways, such as to always become post-selected, no matter which measurement combination is triggered by the mechanism of observation. The relevant information would be attributed to incompatible slices of the input wave-function, but one and the same group of "actors" would always be there to express it, non-locally. Admittedly, this sounds a little bizarre, but the question is whether quantum theory is open to such a possibility or not? The apparent difficulty here is that we cannot know for sure what would have happened if the measurement was different for *the same exact pair* of entangled quanta, given that it is already detected in some way.

As it turns out, there is an answer, because quantum mechanics is governed by yet another important rule, known as the Local Signaling principle. According to this concept, quantum phenomena cannot display observable interactions faster than the speed of light. Even if we assume that quantum behavior is non-local, for the purpose of the argument, quantum information exchange can only be local. Usually, this is known as non-signaling non-locality [26. 27]. As a rule, it is invoked in discussions about quantum communication protocols, in which it is assumed that quantum collapse is non-local, but encoded messages require the exchange of quantum keys through local channels. The relevant question to ask is: why do we need quantum keys? The answer is that two observers need to apply coordinated measurement sequences, such as to post-select the correct sub-ensemble of message carriers. For example, it may be that Alice and Bob have an event log of 1 million detections each, but only 1 thousand of them are encoded. Quantum communication is secure precisely because it is very hard to guess which quantum is the one that carries the next bit of information (in addition to other factors). However, if we assume that it is always the same exact 1 thousand quanta that generate coincidences in every measurement setting, then we do not need a list to identify them, because they are always the same. Therefore, it is possible to devise clever algorithms with coincident measurements that extract quantum messages faster than light without an exchange of keys. For example, we could entangle more than two quanta at a time and use two of them to identify the other ones that carry encoded information. Thus, if this loophole was valid, then quantum correlations would be capable of detectable instantaneous signaling. If that was true, then the local signaling principle of quantum theory would be violated in a verifiable manner. Notably, this would also produce insurmountable contradictions in the predictions of quantum theory and violate many well-established laws of physics [15]. Though, if such contradictions were possible, they would have already been discovered in the study of quantum monogamy and other n-quantum entanglement phenomena [28].

In short, we failed yet again to reconcile the hypothesis of non-locality with the basic principles of quantum theory. Even if we allow this approach to bend all the rules of common sense, it still cannot replicate the known features of quantum behavior. Based on this, we can draw a more general conclusion, namely that quantum non-locality must always contradict at least one basic principle of quantum theory. The interesting nuance here is that quantum behavior has many unverifiable aspects. So, it seems that we can say anything we want about those aspects, without consequence. Yet, as it turns out, this is not true, because the principles of quantum mechanics are quite rigid in this regard. Non-locality is either directly in conflict with those principles, or it must be modified in very suspicious ways. Yet, if we do these necessary changes, then we end up with verifiable contradictions with quantum theory. The main source of these contradictions is the fact that local models and non-local models produce different predictions about conditional observables, even when they make identical predictions for unconditional observables.

As a corollary, the only way for quantum non-locality to be correct is if quantum theory is false. In other words, we need experimental evidence proving that quantum theory is wrong before we can entertain any hope that quantum non-locality is real. Yet, quantum theory has made numerous predictions that are already confirmed beyond reasonable doubt. To even hope that quantum non-locality is true, we need to find a reason to believe that many of those confirmations are wrong, and this is simply unreasonable. We can also speculate that a future theory might supersede quantum theory, but that future theory will have to incorporate the confirmed facts of the current theory, with the implicit patterns that follow from them. Consequently, even that future theory would have to be incompatible with quantum non-locality, at least with regard to Bell observables. By implication, every time we read a news report stating that quantum theory was confirmed yet again, the logical conclusion is that quantum non-locality was falsified yet again.

6. Discussion

Quantum non-locality is one of the most famous concepts in modern physics and possibly a major motivator for many people to be interested in science. Thus, it is a complex feeling to discover that it has a superior alternative, let alone that it was never compatible with quantum theory. Still, how was it possible for such a gross mistake to go unnoticed for so long? A possible answer, given our personal experience, is that the link between quantum theory and quantum non-locality was not perceived as worthy of discussion, at least for the recent generations of physicists. The relevant debates were often dominated by quantitative concerns. Is quantum theory accurate? If “yes”, then “non-locality”. Do quantum experiments agree with quantum predictions? If “yes”, then “non-locality”. It is safe to say that any other answer was unthinkable. Even the proponents of local ontologies have been trying to find loopholes and alternatives to quantum theory, instead of questioning its interpretation. A possible source of this tendency is the persistent retelling of the story that Einstein hated quantum theory (rather than its “spooky” interpretations). Somehow, this grew into the idea that quantum theory can only be correct if Einstein’s opinions were false. Thus, every time a new experiment was shown to confirm quantum predictions, the headlines across the world began with the words: “Sorry, Einstein…” (Presumably, if quantum theory is correct, then Einstein is automatically wrong, and therefore non-locality is true). In retrospect, Einstein and Bohr debated the nature of quantum reality, but did not spar over the accuracy of the formalism. Insofar as Bohr “won” those debates, he argued correctly that quantum observables are contextual (and therefore, not indicative of “real” properties outside of their context). The importance of dualism in quantum theory (and the inadequacy of “wave-only” or “particle only” interpretations) has stood the test of time. Nonetheless, we now begin to realize that Einstein was also correct to insist that “the Moon is there when no one is looking”. Just because some aspects of quantum behavior are unknowable, it does not follow that anything goes in their interpretation. As shown by the work of Cetto et al. [12], we still have a lot discover about quantum “non-realism”, before we can add fantastic qualities to it. Indeed, when quantum theory is confirmed by experiments, this is not the end of the discussion, but only the beginning.

Funding: This research received no external funding.

Conflicts of Interest: The author declares no conflict of interest.

References:

1. A. Aspect, P. Grangier, and G. Roger, “Experimental tests of realistic local theories via Bell’s theorem”, *Phys. Rev. Lett.* **47**, 460 (1981).
2. A. Aspect, P. Grangier, and G. Roger, “Experimental realization of Einstein-Podolsky-Rosen-Bohm gedankenexperiment: a new violation of Bell’s inequalities”, *Phys. Rev. Lett.* **49**, 91 (1982).
3. A. Aspect, J. Dalibard, and G. Roger, “Experimental test of Bell’s inequalities using time-varying analyzers”, *Phys. Rev. Lett.* **49**, 1804 (1982).
4. B. G. Christensen et al., “Detection-Loophole-Free Test of Quantum Nonlocality, and Applications”, *Phys. Rev. Lett.* **111**, 130406 (2013).
5. M. Giustina et al., “Bell violation using entangled photons without the fair-sampling assumption”, *Nature* **497**, 227 (2013).
6. M. Giustina et al., “Significant-loophole-free test of Bell's theorem with entangled photons”, *Phys. Rev. Lett.* **115**, 250401 (2015).
7. L. K. Shalm et al., “A strong loophole-free test of local realism”, *Phys. Rev. Lett.* **115**, 250402 (2015).
8. A. Zeilinger, “Experiment and the foundations of quantum physics”, *Rev. Mod. Phys.* **71**, S288 (1999).
9. Y. Shih, “Two-photon entanglement and quantum reality”, *Advances in Atomic, Molecular, and Optical Physics* **41**, 1 (1999).
10. R. A. Bertlman and A. Zeilinger (eds), *Quantum [Un]speakables: From Bell to Quantum Information* (Springer, 2002).
11. A. Fine, “Joint distributions, quantum correlations, and commuting observables”, *J. Math. Phys.* **23**, 1306 (1982).
12. A. M. Cetto, A. Valdés-Hernández and L. de la Peña, “On the spin projection operator and the probabilistic meaning of the bipartite correlation function”, *Found. Phys.* **50**, 27 (2020).
13. E. Santos, “Does quantum mechanics violate the Bell inequalities?”, *Phys. Rev. Lett.* **66**, 1388 (1991).
14. E. Santos, “Critical analysis of the empirical tests of local hidden-variable theories”, *Phys. Rev. A* **46**, 3646 (1992).

15. G. Mardari, “How to Erase Quantum Monogamy?”, *Quantum Rep.* **3**, 53 (2021).
16. W. M. de Muynck, W. De Baere, and H. Martens, “Interpretations of quantum mechanics, joint measurement of incompatible observables, and counterfactual definiteness”, *Found. Phys.* **24**, 1589 (1994).
17. R. D. Gill, “Statistics, Causality and Bell’s Theorem”, *Stat. Sci.* **29**, 512 (2014).
18. G. ‘t Hooft, *The Cellular Automaton Interpretation of Quantum Mechanics* (Springer, 2016).
19. L. Vervoort, “Probability Theory as a Physical Theory Points to Superdeterminism”, *Entropy* **21**, 848 (2019).
20. J. S. Bell, *Speakable and unspeakable in quantum mechanics* (Cambridge, 1987).
21. N. Bohr, *Collected Works, Vol. 3: The correspondence Principle (1918-1923),* (North-Holland, 1976).
22. J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, “Proposed Experiment to Test Local Hidden-Variable Theories”, *Phys. Rev. Lett*. **23**, 880 (1969).
23. P.A.M. Dirac, *The Principles of Quantum Mechanics* (Clarendon Press, 1947).
24. M. Jammer, *The Philosophy of Quantum Mechanics* (John Wiley and Sons, 1974).
25. J. A. Wheeler and W. H. Zurek (eds), *Quantum Theory and Measurement* (Princeton, 1983).
26. C. Simon, V. Buzek, and N. Gisin, “The no-signaling condition and quantum dynamics,” *Phys. Rev. Lett*. **87**, 170405 (2001).
27. N. Gisin et al., “Constraints on nonlocality in networks from no-signaling and independence”, *Nat. Commun.* **11**, 2378 (2020).
28. J. Pan, Z. Chen, C. Lu, H. Weinfurter, A. Zeilinger and M. Żukowski, “Multiphoton entanglement and interferometry”, *Rev. Mod. Phys*. **84**, 777 (2012).