Review

Quantum universe: a particle moving or a wave propagating?

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Abstract: The wave-particle description of quantum mechanics is briefly reviewed to set an analogy between the quantum mechanics of particles and fields and quantum cosmology. It is shown that quantum cosmology can be seen as a quantum field theory developed in the space of three metrics (the metrics of the spatial sections of the spacetime), where the classical evolution of the universe corresponds to a definite trajectory and the quantum mechanics of matter fields emerges as the first order quantum correction. Like in a quantum field theory developed in an isotropic background, the universes are created in universe-antiuniverse pairs, which might provide us with an explanation for the matter-antimatter asymmetry observed in our universe without going beyond the standard model. Moreover, the states of the matter and the antimatter fields of the pair of universes are entangled and that makes that their initial states are in a non-vacuum state (presumably in a thermal state). That might open the possibility of looking for observable imprints of the creation of universes in pairs is the astronomical data.

Keywords: quantum cosmology; multiverse; superspace; shape space; universe-antiuniverse pair;

1. Introduction

What is quantum mechanics all about? Is not in essence about the wave-particle duality? If so, how can we manage to apply this rule to the spacetime and particularly to the universe as a whole? What does it mean that the universe is either a particle or a wave, and more intriguing, what does it mean that it is a *wave-particle*?

The application of quantum mechanics to the universe as a whole demands almost a blind faith in this theory. After all, we are never going to see the universe as a whole, even less the quantum universe. However, this requirement is not unusual, we cannot see either the spacetime if it is not for the matter fields that live and propagate therein and even though we have a theory that explain how the spacetime evolves. Actually, what it explains is how the evolution of the (invisible) spacetime affects the (observable) matter fields we see. That is also what we can (and must) demand from a quantum theory of the whole universe, the explanation of effects that we see in the cosmological matter fields which are otherwise unexplained.

But, going back to the initial question, if the universe is a particle, where does it move (if the universe itself describes the spacetime)? if it is a wave, where does it propagate? In the case of a particle and a wave the equations of motion and the wave equation tell us where they move and propagate, they do it in the spacetime. We can follow a similar analysis to find where the universe moves or propagates. Even more, we can follow this analogy to study the relation between the wave and particle descriptions of the universe and to analyse how the quantum formalism relates them in both cases the matter fields and the universe as a whole. It turns out that, following that analogy, quantum cosmology can clearly be seen as a quantum field theory developed in a more general space called the superspace.

A definite trajectory in the superspace describes the behaviour of a classical universe, where the spacetime and the matter fields behaves purely classical (their equations are those given by the classical theories), and a wave propagating in the superspace is related to the quantum effects in both the spacetime and the matter fields. As it happens in a quantum field theory developed in the spacetime, one of these effects is that in some plausible cases the particles (i.e. universes) should be created in entangled pairs with their CP properties inversely related, that is, they have to be created in entangled universe-antiuniverse pairs, which might provide us with an explanation for the matter-antimatter asymmetry observed in our universe without going beyond the standard model. Moreover, the entanglement between the states of the two universes makes the initial state of the matter fields in the two universes depart from the vacuum state. That should have effects in for example the CMB distribution of a universe like ours. The aim of this review is to set the theoretical arena to make further developments and look for observable effects of the quantum state of the universe.

The outline is the following. In Sec. 2 we review the process(es) of quantisation of the trajectory of a particle. We start from the classical trajectory of the particle and end up in the development of a quantum field theory. In Sec. 3, we perform a similar analysis in the cosmological case. In Sec. 3.1 we analyse the geometrical properties of the space of three metrics, which can be seen as (part of) a $5+1$ Minkowski space. In Sec. 3.2 we show that quantum cosmology can be seen as a quantum field theory developed in the superspace. In Sec. 3.3 we review and analyse the semiclassical state of the universe and show why the universes are created in universe-antiuniverse pairs. In Sec. 4 we summarise and (briefly) explore some plausible observable effects.

2. Wave-particle description in quantum mechanics

The equations of the geodesics of the spacetime geometrically characterised by the metric tensor $g_{\mu\nu}$ can be obtained by extremizing the action [1,2],

$$S[x^\mu(\tau)] = \int L(x,\dot{x})d\tau = \frac{1}{2}\int\left(\frac{1}{N^2}g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu - m^2\right)Nd\tau, \tag{1}$$

where, $\dot{x}^\mu \equiv \frac{dx^\mu}{d\tau}$, $\tau \in [\tau_0, \tau_1]$ is the parameter that parametrises the geodesic, m^2 is a real constant and, $N = N(x)$, encodes the choice of the different parametrisations and makes the action (1) invariant under reparametrisations, $Nd\tau = \tilde{N}d\tilde{\tau}$ [2]. Variation of (1) with respect to the coordinates gives the equation of geodesics,

$$\ddot{x}^\mu + \Gamma^\mu_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta = 0, \tag{2}$$

with

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2}g^{\mu\nu}\left(\frac{\partial g_{\alpha\nu}}{\partial x^\beta} + \frac{\partial g_{\beta\nu}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\nu}\right), \tag{3}$$

and variation of (1) with respect to N also yields the momentum constraint

$$p_\mu p^\mu + m^2 = 0, \tag{4}$$

which under canonical quantisation and an appropriate choice of factor ordering turns out to be the Klein-Gordon equation,

$$\left(-\hbar^2\Box_x + m^2\right)\phi(x) = 0. \tag{5}$$

This first (very briefly explained) process quantisation can be seen as a *particle-to-wave quantisation*. As it is well known, the solutions of the Klein-Gordon equation (5) can be written in terms of normal modes $u_k(x)$ as (see, for example, Refs. [3,4])

$$\phi(x) = \sum_k a_k u_k(x) + a_k^* u_k^*(x), \tag{6}$$

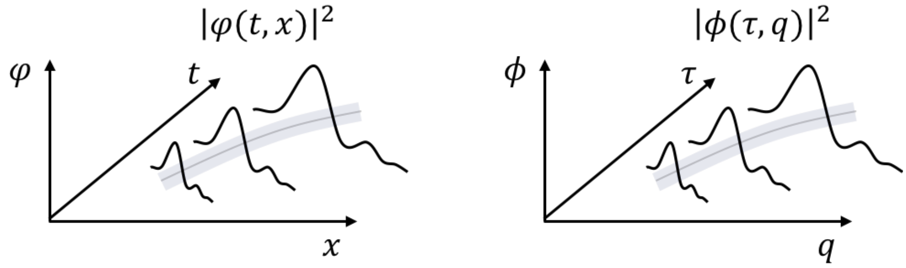

Figure 1. Left: in a quantum field theory the field is described in terms of particles that follow with the highest probability the classical trajectories given by the geodesics with however some uncertainties in their positions. Right: the wave function that describes the quantum state of the spacetime and the matter fields, all together, can be seen as a another field, say a super-field, that propagates in the superspace [5].

which is the starting point of another process of quantisation that turns the constants a_k and a_k^* into non commuting quantum operators, $\hat{a}_k$ and $\hat{a}_k^\dagger$, that satisfy the relations

$$[\hat{a}_k, \hat{a}_l^\dagger] = \delta_{kl}, \;\; [\hat{a}_k, \hat{a}_l] = [\hat{a}_k^\dagger, \hat{a}_l^\dagger] = 0. \tag{7}$$

Then, one defines the vacuum state and constructs the Fock space for the many particle states in terms of which it is described the field (or wave), $\phi(x)$. This (even more briefly explained) second quantisation can then be seen as a *wave-to-particle quantisation*.

We have not entered into the details, which are in the standard textbooks of the subject. We just reviewed it in the way presented above to set the base of an analogy that will allows us to interpret is a similar fashion the state of the quantum universe. The development presented above is one of the customary developments to obtain the Klein-Gordon equation and to develop the quantisation of a scalar field, except for one detail (in some cases). When the Klein-Gordon is constructed as the wave equation of a classical field the Planck constant does not appear in the wave equation. However, the appearance of $\hbar$ in (5) shows that in the present development, $\phi(x) \equiv \phi(x;\hbar)$, is not a classical field [2] and this is crucial in the forthcoming development.

Now, how does the wave function $\phi(x)$ represent the classical trajectory of a particle moving in the spacetime? Well, from the point of view of the wave function $\phi(x)$ the trajectory of the particle in the spacetime is nothing but the classical limit or the $\hbar^0$ order solution of the Klein-Gordon equation (5) (see, Fig. 1, left). For a WKB wave function $\phi(x) \propto e^{\pm \frac{i}{\hbar} S(x)}$, it yields the Hamilton-Jacobi equation [2]

$$g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu} - m^2 = 0, \tag{8}$$

which is the momentum constraint if we choose the the affine parameter, τ, as

$$\frac{\partial}{\partial \tau} = \pm g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial}{\partial x^\nu}. \tag{9}$$

In that case, it is obtained

$$\dot{x}^\mu \equiv \frac{\partial x^\mu}{\partial \tau} = \pm g^{\mu\nu} \frac{\partial S}{\partial x^\nu} = g^{\mu\nu} p_\nu, \tag{10}$$

which, together with (8), yields the equation of the geodesics (2) [2].

At order $\hbar^1$ one would expect to obtain the fluctuations or uncertainties in the position of the particle [6]. In particular, in the non relativistic limit, for which the wave function can be written as, $\phi(x) \propto e^{\pm \frac{i}{\hbar} mt} \psi(t, \vec{x})$, one obtains the Schrödinger equation for the trajectories followed by particles and antiparticles [5] (see, also, Ref. [7]),

$$(\pm) i\hbar \frac{\partial \psi}{\partial t} = \left(-\frac{\hbar^2}{2m} \nabla^2 + m \right) \psi(t, \vec{x}), \tag{11}$$

where ∇^2 is the Laplacian defined in the spatial section of the spacetime [5]. We shall now see that a similar development can be carried out in the case of the evolution of the universe. The particle description will describe the classical evolution of the spacetime and the wave description will entail the quantum corrections.

3. Wave-particle description in quantum cosmology

3.1. Superspace (or shape space)

A parallel development to that made in the preceding section can also be followed in the cosmological case. In order to see *where* the universe moves or propagates we can look at the equation of motion and the wave equation of the universe. In the case of the universe, the *coordinates* that describe the state of the universe are the components of the metric tensor[1], $g_{\mu\nu}$, which describe the geometry of the spacetime. However, in order to study the evolution of the universe it is convenient to determine a time like variable, t, and foliate the spacetime into orthogonal spatial sections, i.e. to split the spacetime into space and time [8] (see also, Ref. [9] for more details). The 4-dimensional metric tensor, $g_{\mu\nu}$, can then be expressed as [9]

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \left(h_{ij} N^i N^j - N^2 \right) dt^2 + h_{ij} \left(N^i dx^j + N^j dx^i \right) dt + h_{ij} dx^i dx^j, \tag{12}$$

where, N and N^i are the lapse function and the shift vector, respectively, and h_{ij} are the components of the 3-dimensional metric tensor induced in the spatial sections. The lapse function and the shift vector parametrise the foliation of the spacetime into space and time. Dynamically, they act like Lagrange multipliers so the only dynamical variables turn out to be the components of the metric tensor, h_{ij}. Thus, the evolution of the universe can be seen as the time evolution of the spatial sections, which is completely described by the six time-dependent variables, $h_{ij}(t)$. The 6-dimensional[2] space M of symmetric 3-dimensional metrics is the space where the universe move and propagates[3].

With the help of the spacetime foliation (12) the Einstein-Hilbert action, whose variation yields the Einstein's equations and therefore determine the evolution of the spacetime, can be written as[4] [9]

$$S_{EH} = \frac{1}{16\pi G} \int_{\mathcal{M}} dt d^3x N \left(\frac{1}{4N^2} G^{abcd} \dot{h}_{ab} \dot{h}_{cd} - \sqrt{h} \left(2\Lambda - {}^{(3)} R \right) \right), \tag{13}$$

where, G is the Newton's constant, Λ is the cosmological constant, ${}^{(3)}R$ is the scalar curvature of the spatial sections, $\dot{h}_{ab} = \frac{dh_{ab}}{dt}$, and G^{abcd} is given by

$$G^{abcd} = \frac{1}{2} h^{\frac{1}{2}} \left(h^{ac} h^{bd} + h^{ad} h^{bc} - 2 h^{ab} h^{cd} \right), \tag{14}$$

1 Plus the degrees of freedom of the matter fields, which are not considering here.

2 h_{ij} is a 3-dimensional symmetric tensor.

3 The *trajectory* of the universe is customary described in the so-called superspace, which is the quotient space $\mathcal{S}(\Sigma)$ of the class of 3-dimensional metrics on the spatial sections Σ that are related by diffeomorphisms (change of spatial coordinates), i.e. $\mathcal{S}(\Sigma) \equiv M/Diff\Sigma$, see Ref. [9]. However, it is not clear that the diffeomorphism constraint should be imposed before quantisation so we shall consider the space of the three metrics, M, the space where to develop the quantum theory of the universe.

4 We are assuming the value, $N^i = 0$.

where, $h = \det h_{ab}$, and, $h_{ac}h^{cb} = \delta_a^b$. Comparing the structure of the Einstein-Hilbert action (13) with the action (1), one can see that the tensor G^{abcd} provides the space M with a metric structure, in which the *line element* is defined as[5] [10]

$$ds^2 = G^{abcd} dh_{ab} dh_{cd}. \tag{16}$$

From this point of view, the evolution of the universe is nothing but the extremal trajectory in M, given by

$$\ddot{h}_{ab} + \Gamma^{cdef}_{ab} \dot{h}_{cd} \dot{h}_{ef} = -G_{abcd} \frac{\partial V}{\partial h_{cd}}, \tag{17}$$

where, $V = V(h_{ab})$, is the potential term of the action (13), i.e.

$$V(h_{ab}) = N\sqrt{h}\left(2\Lambda -^{(3)} R\right). \tag{18}$$

The trajectory given by (17) is not a geodesic because the existence of the potential (18). Apart from that, which is not very relevant[6], the classical evolution of the universe can formally be studied in a similar way as it has been done in the preceding section for the trajectory of a particle moving in a spacetime with geometry determined by the metric tensor, $g_{\mu\nu}$, and the quantisation procedures, both the *particle-wave* and the *wave-particle* procedures of quantisation, can also be applied in a similar way.

Even more, DeWitt showed [10] that the 6-dimensional space M is indeed a $5+1$ dimensional space with a *time-like* component τ given by [9,10,12]

$$\tau = \left(\frac{32}{3}\right)^{\frac{1}{2}} h^{1/4}, \tag{19}$$

which essentially represents the volume of the spatial sections of the spacetime. The hypersurfaces of constant τ are the *space-like* sections of M, labelled by $\bar{M}$ [10]. In terms of the variables, $q^\mu = \{\tau, \bar{q}^A\}$, where $\bar{q}^A$, with $A = 1, \ldots, 5$, are the five coordinates in $\bar{M}$, the line element (16) becomes

$$ds^2 = -d\tau + h_0^2 \tau^2 d\bar{s}^2 = -d\tau^2 + h_0^2 \tau^2 \bar{G}_{AB} d\bar{q}^A d\bar{q}^B, \tag{20}$$

where $d\bar{s}$ is the line element in $\bar{M}$, with

$$\bar{G}_{AB} = \mathrm{tr}\left(h^{-1} h_{,A} h^{-1} h_{,B}\right) \equiv h^{ij} \frac{\partial h_{jk}}{\partial \bar{q}^A} h^{kl} \frac{\partial h_{li}}{\partial \bar{q}^B}, \tag{21}$$

and, $h_0^2 = 3/32$. The metric (20) reveals the space M as *a set of "nested" 5-dimensional submanifolds, all having the same intrinsic shape* [10]. From that point of view, the 5-dimensional submanifold $\bar{M}$ can be seen as a proper realisation of what is called the 'shape space' [13] because any trajectory in $\bar{M}$ does not entail a change in the volume of the spatial sections of the spacetime but only a change in their shape (see Fig. 2).

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5 Let us notice that only six of the nine h_{ab} components are independent so it is sometimes simpler to work with the variables, $\{q^\mu\}$, $\mu = 1, \ldots, 6$, given by [10]

$$q^1 = h_{11}\,,\; q^2 = h_{22}\,,\; q^3 = h_{33}\,,\; q^4 = \sqrt{2}h_{23}\,,\; q^5 = \sqrt{2}h_{31}\,,\; q^6 = \sqrt{2}h_{12}, \tag{15}$$

in terms of which the line element (16) of the space M becomes, $ds^2 = G_{AB} dq^A dq^B$.

6 In fact, one can use a generalisation of the Maupertuis principle [2,11] to construct a new metric in which the evolution of the universe follows a geodesic. Let us consider the reparametrisation given by, $d\tilde{t} = V(t)dt$ and $G^{abcd} \rightarrow \tilde{G}^{abcd} = V(t) G^{abcd}$. In that case, the action (13) turns out to be

$$S_{EH} = \frac{1}{16\pi G} \int_M d\tilde{t} d^3x N \left(\frac{1}{4N^2} \tilde{G}^{abcd} h'_{ab} h'_{cd} - 1\right),$$

where, $h'_{ab} = \frac{dh_{ab}}{d\tilde{t}}$. In the superspace determined by the supermetric $\tilde{G}^{abcd}$ the evolution of the universe turns out to be a geodesic.

On the other hand, the structure of the line element (20) has a clear similarity with a Friedmann-Robertson-Walker spacetime. In fact, it has the same formal structure of the Milne spacetime (for the Milne spacetime, see Ref. [14]). As it is well-known the Milne spacetime is a 'merely unconventional coordination of flat spacetime' [3]. Something similar occurs in M. Let us notice that with the introduction of the variables [10]

$$y^0 = \tau \cosh h_0 \bar{s} \ , \ \ y^1 = \tau \sinh h_0 \bar{s}, \tag{22}$$

the line element (20) turns out to be

$$ds^2 = -(dy^0)^2 + (dy^1)^2, \tag{23}$$

with, $0 < y^0 < \infty$ and $-\infty < y^1 < \infty$ (see Fig. 2). If τ represents the volume of the spatial sections of the universe $\bar{s}$ represents their *shape*. Universes scaling their volume conserving the shape are represented by the lines of constant $\bar{s}$. On the other hand, the points of the curves of constant τ represent different shapes of the spatial sections of the universe with a fixed given volume.

3.2. QFT in the superspace

As we have already said, we can follow a similar approach to that done in Sec. 2 to perform the two procedures of quantisation, the particle-to-wave and the wave-to-particle procedures, in the cosmological case too. The variation of the action (13) with respect to the lapse function N yields the classical Hamiltonian constraint, which in terms of the variables, $q^\mu = \{\tau, \bar{q}^A\}$ (see, Eq. (20)), can be written as

$$\mathcal{H} = G^{AB} p_A p_B + m^2(q, \varphi) = 0, \tag{24}$$

where $m^2(q, \varphi)$ contains all the potential terms of the gravitational field plus the Hamiltonian of the matter fields (which for simplicity have not been considered so far) that can generically be encapsulated in a variable φ,

$$m^2(q, \varphi) = m_g^2(q) + 2\mathcal{H}_m(q, \varphi), \tag{25}$$

where

$$m_g^2(q) = \frac{h_0^2 \tau^2}{(16\pi G)^2} \Big(2\Lambda - {}^3R(q) \Big), \tag{26}$$

and

$$\mathcal{H}_m = \frac{1}{2 h_0^2 \tau^2} p_\varphi^2 + \ldots + h_0^2 \tau^2 V(\varphi), \tag{27}$$

where the dots indicate terms that contain spatial derivatives of the matter fields, which for simplicity we shall consider negligible.

Under canonical quantisation of the momenta in the Hamiltonian constraint (24) one obtains the Wheeler-DeWitt equation, which with an appropriate choice of factor ordering can also be written as,

$$\Big(-\hbar^2 \Box_q + m^2(q, \varphi) \Big) \phi(q, \varphi) = 0, \tag{28}$$

where, $\phi(q, \varphi)$, is the so-called wave function of the universe [15], and

$$\Box_q = \nabla \vec{\nabla} = \frac{1}{\sqrt{-G}} \frac{\partial}{\partial q^A} \left(\sqrt{-G}\, G^{AB} \frac{\partial}{\partial q^B} \right), \tag{29}$$

with, $G = \det G_{AB}$. Let us first notice the formal similarity of (28) with Eq. (5). Thus, (28) can be seen as a Klein-Gordon equation for the wave function, $\phi(q, \varphi)$, which can now be interpreted as a field that *propagates* through the space M. On the other hand, we have seen that M has the geometrical structure of a $5+1$-dimensional Friedmann-

Robertson-Walker universe, where we perfectly know how to develop the quantisation of a field that propagates therein.

A technical problem however is that the 'mass' of the field (see 25-26) is not a constant. In the cases where it only depends on the volume variable τ, the quantisation can be performed in a parallel way as it is done in a homogeneous and isotropic spacetime, where the mass term may depend on the time variable. However, in general the ${}^3R$ curvature depends on all the components of the spatial metric and thus the space M turns out to be a dispersive medium for the wave function of the universe. It does not invalidate the formalism but it becomes more complicated from a technical point of view. For that reason we shall mainly focus on the case for which, ${}^3R \ll 2\Lambda$, which on the other hand is a very plausible condition for the initial state of the universe[7]. Then, we can assume the value

$$m_g^2(q) \approx \frac{2\Lambda h_0^2}{(16\pi G)^2}\tau^2 \equiv m_0^2\tau^2, \tag{30}$$

in the Wheeler-DeWitt equation (28), and perform the (wave-to-particle) quantisation in the customary way.

Let us first consider the 'conformal volume variable' λ, given by $|\tau| = h_0^{-1}e^{h_0\lambda}$, with $h_0^2 = 3/32$ (see, Eq. (21)), in terms of which the supermetric G_{AB} can be written as,

$$G_{AB} = \mathcal{A}^2(\lambda)\begin{pmatrix} -1 & 0 \\ 0 & \bar{G}_{AB} \end{pmatrix}, \tag{31}$$

with, $\mathcal{A}(\lambda) = e^{h_0\lambda}$, and therefore, $\sqrt{-G} = \mathcal{A}^6(\lambda)\sqrt{\bar{G}}$, where, $\bar{G} = \det \bar{G}_{AB}$. In that case, following the standard formalism of a quantum field theory (see, for instance, Refs. [3,4]), we can decompose the wave function of the universe $\phi(q)$ in normal modes as

$$\phi(q) = \int d^5\mathbf{k}\, [a_\mathbf{k} u_\mathbf{k}(q) + a_\mathbf{k}^\dagger u_\mathbf{k}^*(q)], \tag{32}$$

with,

$$u_\mathbf{k}(q) = \frac{1}{\mathcal{A}^2(\lambda)}\mathcal{Y}_\mathbf{k}(\bar{q})\chi_\mathbf{k}(\lambda), \tag{33}$$

where $\mathcal{Y}_\mathbf{k}(\bar{q})$ are the orthonormal eigenfunctions of the 5-dimensional Laplacian[8]

$$\Box_{\bar{q}}\mathcal{Y}_\mathbf{k}(\bar{q}) = -(k^2+1)\mathcal{Y}_\mathbf{k}(\bar{q}), \tag{34}$$

where $\Box_{\bar{q}}$ is given by (29) with the 5-dimensional metric $\bar{G}_{AB}$ instead of G_{AB} (and without the minus sign in the square roots) and, $k = |\mathbf{k}|$. The amplitude $\chi_\mathbf{k}(\lambda)$ must then satisfy the equation

$$\frac{d^2\chi_\mathbf{k}}{d\lambda^2} + \omega_k^2(\lambda)\chi_\mathbf{k} = 0, \tag{35}$$

with

$$\omega_k^2(\lambda) = k^2 + 1 + \hbar^{-2}\mathcal{A}^2 m^2(\lambda) - 2\left(\frac{\dot{\mathcal{A}}^2}{\mathcal{A}^2} + \frac{\ddot{\mathcal{A}}}{\mathcal{A}}\right) \tag{36}$$

$$= \tilde{k}^2 + \tilde{m}_0^2 e^{4h_0\lambda}, \tag{37}$$

where we have used,

$$\frac{\dot{\mathcal{A}}^2}{\mathcal{A}^2} = \frac{\ddot{\mathcal{A}}}{\mathcal{A}} = h_0^2, \tag{38}$$

[7] Let us notice that the condition, ${}^3R \ll 2\Lambda$, does not assume that the universe is homogeneous.

[8] The scalar curvature is negative [10].

and,

$$\tilde{k}^2 = k^2 + 1 - 4h_0^2 = k^2 + \frac{9}{10} \approx k^2 \ , \ \tilde{m}_0^2 = \frac{m_0^2}{\hbar^2 h_0^2}. \tag{39}$$

The normalisation condition reads

$$\chi_{\mathbf{k}} \partial_\lambda \chi_{\mathbf{k}}^* - \chi_{\mathbf{k}}^* \partial_\lambda \chi_{\mathbf{k}} = i. \tag{40}$$

The wave equation (35) is then readily solvable in terms of Bessel or Hankel functions. We have two set of orthonormal modes given by [3]

$$\bar{\chi}_{\mathbf{k}}(\tau) = \left[(4h_0/\pi)\sinh(\pi\tilde{k}/2h_0)\right]^{-\frac{1}{2}} \mathcal{J}_{-i\tilde{k}/2h_0}(\tilde{m}\tau), \tag{41}$$

$$\chi_{\mathbf{k}}(\tau) = \frac{1}{2}(\pi/2h_0)^{\frac{1}{2}} e^{\frac{\pi\tilde{k}}{4h_0}} \mathcal{H}^{(2)}_{i\tilde{k}/2h_0}(\tilde{m}\tau). \tag{42}$$

Now, we have to impose some boundary condition on the state of the wave function $\chi_{\mathbf{k}}(\tau)$. For this it is worth noticing that the modes $\bar{\chi}_{\mathbf{k}}$ are regular in the limit of zero volume, i.e., $\bar{\chi}_{\mathbf{k}} \rightarrow e^{-\frac{ik}{\hbar}|\lambda|}$. It seems therefore a good boundary condition to impose the regularity of the wave function at the singular frontier[9], $\tau = 0$. In that case, the solution of the field is given by (41). On the other hand, in the limit of large volumes, $\tau \rightarrow \infty$, the modes $\chi_{\mathbf{k}}(\tau)$ and $\chi_{\mathbf{k}}^*(\tau)$ can be approximated by exponential wave functions, $e^{\pm\frac{i\tilde{m}}{\hbar}\tau}$. We shall see in the next section that this kind of wave functions represent (semi) classical universes with their time variable being reversely related, i.e. they represent universes and antiuniverses. It turns then out that the vacuum state of the field $\bar{\chi}_{\mathbf{k}}$ is full of universe-antiuniverse pairs. This can be seen by noticing that the two set of modes are related by a Bogolyubov transformation,

$$\bar{\chi}_{\mathbf{k}} = \alpha_k \chi_{\mathbf{k}} + \beta_k \chi_{\mathbf{k}}^*, \tag{43}$$

where [3]

$$\alpha_k = \left[\frac{e^{\pi\tilde{k}/h_0}}{2\sinh(\pi\tilde{k}/h_0)}\right]^{\frac{1}{2}}, \beta_k = \left[\frac{e^{-\pi\tilde{k}/h_0}}{2\sinh(\pi\tilde{k}/h_0)}\right]^{\frac{1}{2}}, \tag{44}$$

with, $|\alpha_k|^2 - |\beta_k|^2 = 1$. It means that the vacuum state of the $\bar{\chi}_{\mathbf{k}}$ modes, $|\bar{0}_{\mathbf{k}}\bar{0}_{-\mathbf{k}}\rangle$ can be written as [4]

$$|\bar{0}_{\mathbf{k}}\bar{0}_{-\mathbf{k}}\rangle = \prod_{\mathbf{k}} \frac{1}{|\alpha_k|^{1/2}} \left(\sum_{n=0}^{\infty} \left(\frac{\beta_k}{\alpha_k}\right)^n |n_{\mathbf{k}} n_{-\mathbf{k}}\rangle \right), \tag{45}$$

with a number of universes given by

$$N_k = |\beta_k|^2 = \frac{1}{e^{\frac{2\pi\tilde{k}}{h_0}} - 1}, \tag{46}$$

which corresponds to a thermal distribution with temperature

$$T = \frac{h_0}{2\pi}. \tag{47}$$

The superspace turns out to be then full of universe-antiuniverse pairs. In the cosmological case we should pick out one of these pairs as representing our universe and its corresponding antiuniverse. However, the thermal distribution might be important in the case of quantum gravity because it might mean that the gravitational vacuum is formed by a thermal distribution of gravitons.

9 It would be more appropriate to consider the vacuum state of the invariant representation [16] as the boundary condition. However, the argument seems to be clearer (and the conclusion still valid) with the representations given by (41-42).

Even though this is not the most general model[10], one can obtain some general conclusions from it. In the QFT of matter fields in an expanding background, the isotropy of the space makes that the particles are created in pairs with opposite values of the field modes, $\mathbf{k}$ and $-\mathbf{k}$ (see (45)), and in the case of a complex field in particle-antiparticle pairs with opposite momenta. In the superspace, the space-like subspace $\bar{M}$ is homogeneous and isotropic. Therefore, if the potential of the Wheeler-DeWitt equation is also isotropic in $\bar{M}$ (or negligible) the universes must be created in pairs with opposite values of the 5-dimensional $\mathbf{k}$ ($\equiv \bar{\mathbf{k}}_{ab}$) modes. The value of each mode of the Fourier decomposition (32) is proportional to the momentum conjugated to the components of the scaled metric tensor[11], $\bar{h}_{ab} = h^{-1/3} h_{ab}$. It means that any change that is produced by the momentum associated to $+\bar{\mathbf{k}}_{ab}$ in the shape of the universe with metric $\bar{h}_{ab}$ is being also produced in the shape of the partner universe with opposite sign, $-\bar{\mathbf{k}}_{ab}$. The shape of the two universes is changing in a symmetric way as well therefore as their 'shape complexity'. Thus, the creation point is a Janus point [13]. In the next section we shall see that the physical time variables of the two universes must be reversely related. That means that the universes are created in symmetric pairs, one of them filled with matter and the other filled with antimatter, having these properties always a relative meaning. One can then conclude that, quite generally, the universes of the multiverse are created in symmetric universe-antiuniverse pairs whose properties are expected to be entangled [17–19].

3.3. Appearance of space and matter

It is well known [20–22] that the general solution of the Wheeler-DeWitt equation (28) can be written, in the semiclassical regime, as the symmetrically matched pair[12]

$$\phi(q,\varphi) = \phi^{+}(q,\varphi) + \phi^{-}(q,\varphi), \tag{48}$$

with

$$\phi^{\pm} = C(q) e^{\pm \frac{i}{\hbar} S(q)} \psi_{\pm}(q,\varphi). \tag{49}$$

It turns out that $S(q)$ must satisfy the $\hbar^0$ order equation

$$G^{AB} \frac{\partial S}{\partial q^A} \frac{\partial S}{\partial q^B} + m_g^2(q) = 0, \tag{50}$$

which is the Hamiltonian constraint of the background spacetime if we choose the WKB time variable $t_{\pm}^{w}$ as [20,24]

$$\frac{\partial}{\partial t_w} = \pm \nabla S \cdot \nabla. \tag{51}$$

In that case,

$$\dot{q}^A = \pm G^{AB} \frac{\partial S}{\partial q^B} \text{ , and } \frac{\partial S}{\partial q^A} = \pm G_{AB} \dot{q}^B = p_A, \tag{52}$$

so the Hamilton-Jacobi (50) becomes the Hamiltonian constraint for the background spacetime (see, Eq. (24)). Furthermore, like it happened in the case of the trajectory of a particle (see, Eqs. (8-10)), using (50-52) one can recover the classical trajectory of the universe in the superspace. Therefore, at the classical level, i.e. in the limit $\hbar \to 0$, one recovers from the semiclassical solutions (49) the classical trajectory of the universe in the superspace, i.e. one recovers the classical description of the background spacetime of the universe. In that sense, these solutions describe the classical spacetime of the universes they represent. It is worth noticing the freedom that we have to choose the sign of the time variable in (51), $+t_w$ or $-t_w$. The Hamiltonian constraint (24) is invariant under a reversal change in the time variable because the quadratic terms in the momenta.

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10 Recall that we have required, $^3R \ll \Lambda$.

11 The metric $\bar{h}_{ab}$ has unit determinant so it has only 5 independent components so they become an appropriate choice of coordinates for the space $\bar{M}$.

12 In relation with this equation, see the very interesting analysis made in Ref. [23].

However, the value of these momenta in (52) is not invariant under the reversal change of the time variable. It means that we have two possible values of the momenta, $+p_A$ and $-p_A$, which are associated to the conjugated solutions (48) of the Wheeler-DeWitt equation. It means that, as it happens in particle physics, the universes are created in pairs with opposite values of their momenta so that the total momentum is conserved.

At the next order in $\hbar$ of the variables of the background spacetime the Wheeler-DeWitt equation (28) is fulfilled provided that the following two equations are satisfied

$$i\hbar\frac{\partial\psi_-}{\partial t_w} = \mathcal{H}_m\psi_- \;,\; -i\hbar\frac{\partial\psi_+}{\partial t_w} = \mathcal{H}_m\psi_+. \tag{53}$$

The 'wrong' sign in the second of the time dependent Schrödinger equations (TDSE) of (53) is not problematic [25]. It only means that it is the TDSE of the CP related fields to those of the first TDSE of (53), i.e. $\psi_+^* = \psi_-(t_w, \bar{\varphi})$. Thus the matched pair of wave functions in (48) represents a universe filled with matter and another universe filled with antimatter, i.e. it represents a universe-antiuniverse pair [19,26]. From the point of view of internal inhabitants the universes are both filled with matter[13] and the correct TDSE of the matter fields is recovered from the $\hbar^1$ order of the Wheeler-DeWitt equation in terms of the local time variable each single universe of the entangled pair. From a global point of view, however, the time asymmetry perceived by the internal observers is completely restored.

4. Conclusions (and expectable imprints)

We have shown the parallelism between the quantisation of the trajectory of a particle in the spacetime and the quantisation of the trajectory of the universe in the superspace. The particle version of the universe is the 'test particle' of the superspace. This is the $\hbar^0$ limit of the wave function of the universe, which can generally be seen as a wave that propagates in the superspace.

Following the second quantisation procedure of a quantum field theory the wave function of the whole spacetime manifold can be quantum mechanically described in the Fock space where the many particle states describe many universe states (or the space of spacetime quantum fluctuations in quantum gravity) whose classical evolution is given by the $\hbar \to 0$ limit of the Wheeler-DeWitt equation. Under quite plausible conditions (an isotropic or negligible potential term in the Wheeler-DeWitt equation), the universes must be created in entangled pairs. The next order in $\hbar$ of the wave description of the quantum universes yields the Schrödinger equation of the matter fields. The two universes of the entangled pair have time variables that are reversely related. That makes the matter content of the two universes be CP related with respect to each other so they form a relative universe-antiuniverse pair.

The question is what kind of observables imprints are now expected. In some exactly solvable models [27] one can see that the general quantum state of the universe is a quantum superposition of entangled states in which the modes of the spacetime and the modes of the matter fields are completely correlated (see, Eq. (61) of Ref. [27]). The principle of superposition of the matter fields alone is only satisfied in the semiclassical regime of the spacetime, i.e. for sufficiently large universes, where the modes of the matter fields decouple from those of the spacetime. Thus, violations of the superposition principle of the cosmic matter fields may unveil quantum effects of the spacetime in their evolution. High order corrections can also be expected [28,29].

On the other hand, the creation of universes in entangled universe-antiuniverse pairs make that the initial state of the matter fields in each universe of the entangled pair is a non-vacuum state, presumably a thermal like state with a time dependent temperature [18]. A non vacuum initial state induces some imprints in the cosmological data, for instance, in the spectrum of fluctuations of the mater fields [18,29]. Furthermore,

[13] It is always the partner universe the one that is filled with antimatter.

it might explain the matter-antimatter anisotropy observed in the universe without going beyond the standard model [30].

The formalism of quantum cosmology presented here opens the possibility of studying the quantum structure of the multiverse as a quantum field theory developed in the superspace. This can presumably be applied as well to the case of quantum gravity. That, together with the new expected observational data (gravitational waves, pre-inflationary data, ...) may open new lines of research for the quantum description of the spacetime.

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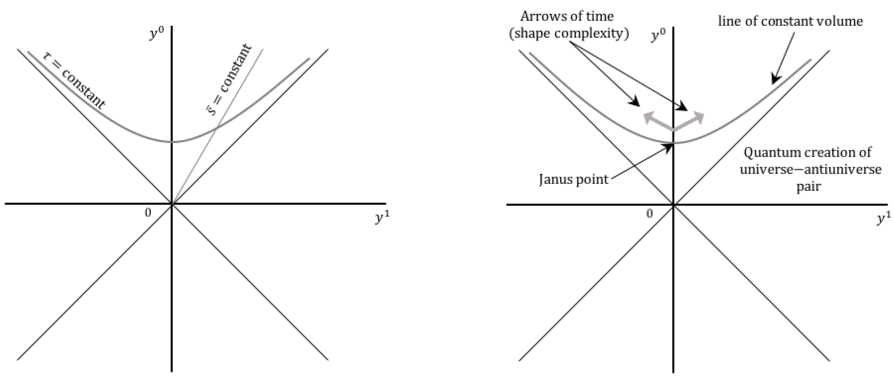

Figure 2. Left: the space of three metrics, M, turns out to be a particular coordination of (part) of the Minkowki superspace. Lines of constant τ are lines of constant volume of the spatial sections of the spacetime (with different shape). Lines of constant $\bar{s}$ correspond to different volumes of the same shape (a scaling universe). Right: assuming the universes are created with a given volume, shape complexity grows at both sides of the Janus point [13]. Quantum mechanically the time reversal invariance of the Einstein-Hilbert action (13) would induce the creation of universe-antiuniverse pairs.

References

1. Hartle, J.B.; Hawking, S.W. Path-integral derivation of black-hole radiance. *Phys. Rev. D* **1976**, *13*, 2188–2203.
2. Garay, I.; Robles-Pérez, S. Classical geodesics from the canonical quantisation of spacetime coordinates. *submitted* **2018**, [arXiv:1901.05171].
3. Birrell, N.D.; Davies, P.C.W. *Quantum fields in curved space*; Cambridge University Press, Cambridge, UK, 1982.
4. Mukhanov, V.F.; Winitzki, S. *Quantum Effects in Gravity*; Cambridge University Press, Cambridge, UK, 2007.
5. Robles-Pérez, S.J. Quantum cosmology in the light of quantum mechanics. *Galaxies* **2019**, *7*, 50.
6. Garay, I.; Robles-Pérez, S. In preparation.
7. Kiefer, C.; Singh, T.P. Quantum gravitational corrections to the functional Schrödinger equation. *Phys. Rev. D* **1991**, *44*, 1067.
8. Arnowitt, R.; Deser, S.; Misner, C.W. The dynamics of general relativity. In *Gravitation: an introduction to current research*; Witten, L., Ed.; Wiley, New York, 1962; chapter 7, pp. 227–264, [arXiv:gr-qc/0405109].
9. Kiefer, C. *Quantum gravity*; Oxford University Press, Oxford, UK, 2007.
10. De Witt, B.S. Quantum Theory of Gravity. I. The Canonical Theory. *Phys. Rev.* **1967**, *160*, 1113–1148.
11. Biesiada, M.; Rugh, S. Maupertuis principle, Wheeler's superspace and an invariant criterion for local instability in general relativity, [arXiv:gr-qc/9408030].
12. Higuchi, A.; Wald, R.M. Applications of a new proposal for solving the problem of time to some simple quantum cosmological models. *Phys. Rev. D* **1995**, *51*, 544–561, [arXiv:gr-qc/9407038v1].
13. Barbour, J. Shape Dynamics. An introduction. Quantum field theory and gravity; Finster, F.; Müller, O.; Nardmann, M.; Tolksdorf, J.; Zeidler, E., Eds. Springer, Basel, 2012, pp. 257–297, [arXiv:1105.0183].
14. Griffiths, J.B.; Podolsky, J. *Exact space-times in Einstein's general relativity*; Cambridge monographs on mathematical physics, Cambridge University Press, Cambridge, UK, 2009.
15. Hartle, J.B.; Hawking, S.W. Wave function of the Universe. *Phys. Rev. D* **1983**, *28*, 2960.
16. Robles-Pérez, S. Invariant vacuum. *Phys. Lett. B* **2017**, *774*, 608–615, [arXiv:1706.05474].
17. Robles-Pérez, S.; González-Díaz, P.F. Quantum entanglement in the multiverse. *JETP* **2014**, *118*, 34, [arXiv:1111.4128].
18. Robles-Pérez, S.J. Cosmological perturbations in the entangled inflationary universe. *Phys. Rev. D* **2018**, *97*, 066018, [arXiv:1708.03860].
19. Robles-Pérez, S.J. Time reversal symmetry in cosmology and the creation of a universe-antiuniverse pair. *Universe* **2019**, *5*, 150, [arXiv:1901.03387].
20. Hartle, J.B. The quantum mechanics of cosmology. In *Quantum Cosmology and Baby Universes*; Coleman, S.; Hartle, J.B.; Piran, T.; Weinberg, S., Eds.; World Scientific, London, UK, 1990; Vol. 7.
21. Halliwell, J.J. Introductory lectures on quantum cosmology. In *Quantum Cosmology and Baby Universes*; Coleman, S.; Hartle, J.B.; Piran, T.; Weinberg, S., Eds.; World Scientific, London, UK, 1990; Vol. 7.
22. Hartle, J.B. Spacetime quantum mechanics and the quantum mechanics of spacetime. Gravitation and Quantization: Proceedings of the 1992 Les Houches Summer School; Julia, B.; Zinn-Justin, J., Eds. North Holland, Amsterdam, 1995, [gr-qc/9304006].
23. Barbour, J.B. Time and complex numbers in canonical quantum gravity. *Phys. Rev. D* **1993**, *47*, 5422–5429.
24. Kiefer, C. Wave packets in minisuperspace. *Phys. Rev. D* **1988**, *38*, 1761.
25. Rubakov, V.A. Quantum cosmology. *Lecture at NATO ASI 'Structure Formation in the Universe', Cambridge, 1999* **1999**, [arXiv:gr-qc/9910025].
26. Robles-Pérez, S.J. Restoration of matter-antimatter symmetry in the multiverse **2017**. [arXiv:1706.06304].
27. Robles-Pérez, S. Quantum cosmology of a conformal multiverse. *Phys. Rev. D* **2107**, *96*, 063511, [arXiv:1706.06023].
28. Kiefer, C.; Krämer, M. Quantum gravitational contributions to the CMB anisotropy spectrum. *Phys. Rev. Lett.* **2012**, *108*, 021301.
29. Brizuela, D.; Kiefer, C.; Krämer, M.; Robles-Pérez, S. Quantum-gravity effects for excited states of inflationary perturbations. *Phys. Rev. D* **2019**, [arXiv:1903.01234].
30. Robles-Pérez, S.J. Quantum creation of a universe-antiuniverse pair. Proceedings of the POTOR-6 Conference of the Polish Society on Relativity, 2020.