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The Geometrized Vacuum Physics based on the Algebra of Signature

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Abstract: The aim of the article is to develop geometrized physics of a vacuum on the basis of two basic postulates: 1) the constancy of the speed of light (more precisely, the speed of propagation of electromagnetic waves) in the vacuum; 2) the ‘vacuum balance condition’ associated with the statement that only mutually opposite formations are born from the vacuum, so that, on average, they completely compensate of the manifestations of each other. The Algebra of signatures is proposed as a mathematical basis for geometrized physics of a vacuum.

Key words: vacuum, physics of a vacuum, fully geometrized physics, vacuum balance, signature, algebra of signature.

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07.30.Hd (Vacuum testing methods)
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1 INTRODUCTION

The object of research in this article is vacuum. In the modern physics there are: a technical vacuum (the rarefied gas); a physical vacuum (the lowest energy state of a set of scalar, vector, tensor and spinor quantum fields); Einstein's vacuum (in the general case, a curved 4-dimensional space-time continuum surrounding neutral or charged physical bodies); a perfect vacuum (3-dimensional space in which there are no curvatures and particles at all).

At the beginning of this article, the main attention is paid to the perfect vacuum, with the aim of creating a mathematical apparatus of the "Algebra of signature", suitable for study of the vacuum phenomena and the development of the "zero" technologies. Then the possibilities of describing the curved region of the vacuum are considered.

This work is based on three experimentally confirmed facts:

1) electromagnetic waves propagate in a perfect vacuum with the speed of light c = 299 792 458 m/s;

2) all averaged characteristics in the mean of a flat area of the perfect vacuum (momentum, angular momentum, spin, etc.) are equal to zero.

3) If something is born from a perfect vacuum, it must be in a mutually opposite form (a particle and an antiparticle, a convexity and a concavity, a wave and an anti-wave, etc.). This property of the perfect vacuum in this article is called the "vacuum balance condition".

The foundations of the Algebra of signatures, developed in this work, are proposed as a universal mathematical apparatus suitable not only for studying the properties of vacuum, but also for any other liquid, solid and gaseous continuous media in which wave disturbances propagate at a constant speed.

The Definition numbers of the new terms introduced in this article are presented in Table 12.1.

2 THE PERFECT VACUUM

2.1 Longitudinal stratification of a perfect vacuum into $\lambda_{m,n}$-vacuums

Consider a 3-dimensional volume of a perfect vacuum ("vacuum"), in which there are no particles, curvatures and vacuum flows.

Definition 2.1.1 *The perfect vacuum for brevity we will call "vacuum".*

Let's probe the volume of a "vacuum" by laser beams that sent from three mutually perpendicular directions, so that they form a 3-dimensional cubic lattice (see Fig. 2.1*a,b*).

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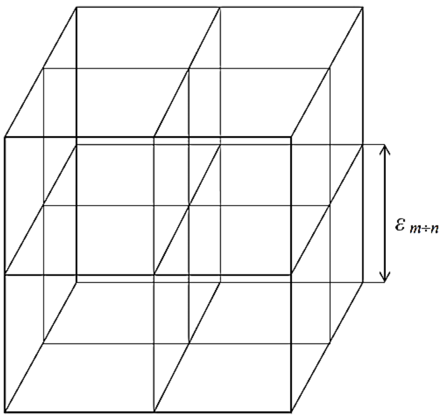

a) *b*)

Fig. 2.1 *a*) The 3-dimensional lattice in a "vacuum", which consists of the mutually perpendicular monochromatic light beams with a wavelength of $\lambda_{m,n}$; an edge length of a cubic cell of this lattice is $\varepsilon_{mn} \sim 10^2\lambda_{mn}$; *b*) Laser light beams in a vacuum, visualized with a finely dispersed sol

The light beams in a perfect vacuum are not visible, but they can be visualized using a finely dispersed sol with a low density (i.e., using small particles with a size of several microns, evenly distributed throughout the entire investigated volume of the "vacuum", so that the distance between the particles much larger than their size).

Of course, a "vacuum" filled with a transparent sol is not a perfect vacuum. Nevertheless, the rays propagate in the “vacuum” itself (i.e., between the particles of a low-density sol), while the influence of the sol on the metric-dynamic properties of the macroscopic volume of the “vacuum” in this case can be neglected.

A laser light beam is a narrowly directed propagation of mono-chromatic electromagnetic waves with a wavelength of $\lambda_{-4,-5}$, taken from the range of lengths $\Delta\lambda = 10^{-4} \div 10^{-5}$ cm. Therefore, a 3-dimensional lattice consisting of the mutually intersecting laser beams with an edge length of one cubic cell $\varepsilon_{-4,-5} \sim 100{\cdot}\lambda_{-4,-5}$ (see Fig. 2.1) will be called $\lambda_{-4,-5}$-vacuum (or $3D_{-4,-5}$-landscape).

Let’s divide the entire range of lengths of an electromagnetic (light) waves into a set of sub-ranges $\Delta\lambda = 10^m \div 10^n$ cm, where $n = m + 1$ (*m* and *n* are integers).

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Definition 2.1.2 *In this article, the eikonal of an electromagnetic wave with any wavelength λ is called "a ray of light with λ". In this case, the eikonal means the shortest distance from the point of emission of a light (electromagnetic) signal to the point of its reception. The diameter of an eikonal (i.e., a beam of light) depends on the wavelength of electromagnetic radiation λ, and is determined by the distance from the center of the eikonal (beam) to an obstacle that can take away at least 1% of the energy of the electromagnetic (light) signal transmitted from the emitter to the receiver antenna aperture.*

Similar to how it is shown in Fig. 2.1, we probe the investigated volume of the "vacuum" with other monochromatic light beams with wavelengths $\lambda_{m,n}$ from all subranges $\Delta\lambda = 10^m - 10^n$ cm. As a result, we get an almost infinite number of nested $\lambda_{m,n}$-vacuums (i.e., $3D_{m,n}$-landscapes) (see Fig. 2.2) with edge lengths of the one cubic cell $\varepsilon_{m,n} \sim 100 \cdot \lambda_{m,n}$.

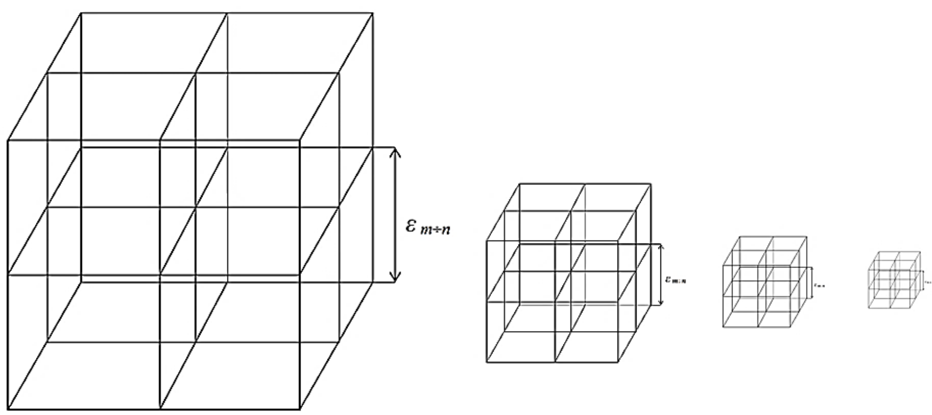

Fig. 2.2 Discrete set of $\lambda_{m,n}$-vacuums ($3D_{m,n}$-landscapes) of the same 3-dimensional area of the "vacuum", where $\lambda_{m,n} > \lambda_{(m+1)(n+1)} > \lambda_{(m+2)(n+2)} > \lambda_{(m+3)(n+3)} > \lambda_{(m+4)(n+4)}$...

The size of the edge of the cubic cell of the each $\lambda_{m,n}$-vacuum approximately equal (see Fig. 2.2)

$$\varepsilon_{m,n} \sim 100 \cdot \lambda_{m,n} \qquad (2.1.1)$$

follows from the condition of applicability of the geometric optics $\lambda_{m,n} \to 0$, i.e. when the thickness of the light beam is much less than the value of the corresponding cubic cell, and it can be neglected.

Definition 2.1.3 *$\lambda_{m,n}$-vacuum is a $3D_{m,n}$-landscape in vacuum, the geodetic lines of which are monochromatic beams of light with a wavelength $\lambda_{m,n}$ (see Figures 2.1, 2.2). In this case, the thickness of the light rays can be neglected in comparison with the dimensions of one cell of the $3D_{m,n}$-landscape. That is, the condition of applicability of geometric optics is satisfied.*

Definition 2.1.4 *Longitudinal stratification of the "vacuum" is a representation of a 3-dimensional volume of the "vacuum" in the form of an infinite discrete sequence $\lambda_{m,n}$-vacuums (see Fig. 2.2), nested into each other like nesting dolls.*

The question remains open: – Are there physical limitations on the frequency ω or wavelength λ of the electromagnetic wave, both in the direction of their increase and in the direction of their decreasing? If the critical values $\omega_{max} = 2\pi c/\lambda_{ma}$ and $\omega_{min} = 2\pi c/\lambda_{min}$ exist, then these will be very important characteristics of the «vacuum». As of today, as far as the author knows, the frequency range of the observed electromagnetic waves extends from 2 Hz to 10^{20} Hz, while restrictions on the expansion of this range have not been experimentally found.

2.2 The geodetic lines of the curved region of a $\lambda_{m,n}$-vacuum

Long-term experimental data show that monochromatic light rays in the entire observed wavelength range $\Delta\lambda$ propagate in a "vacuum" with the same speed of light c and according to the same laws of electrodynamics. Therefore, if the studied area of the vacuum is not curved, then all $\lambda_{m,n}$-vacuums (i.e., $3D_{m,n}$-landscapes) will be represented as ideal cubic lattices (Fig. 2.1, 2.2), since the geodesic lines of all these non-curved $\lambda_{m,n}$-vacuums are direct rays of light. In this case, $\lambda_{m,n}$-vacuums will differ from each other only in the length of the edge of the cubic cell $\varepsilon_{m,n} \sim 10^2 \cdot \lambda_{m,n}$ (see Fig. 2.2).

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However, if the investigated area of the vacuum turns out to be curved, then all $\lambda_{m,n}$-vacuums will slightly differ from each other due to the fact that light rays with different wavelengths have different thicknesses. This circumstance is theoretically substantiated in the sections of geometrical optics related to the resolving power of optical devices [17,18], and is confirmed by experimental data (see Fig. 2.3).

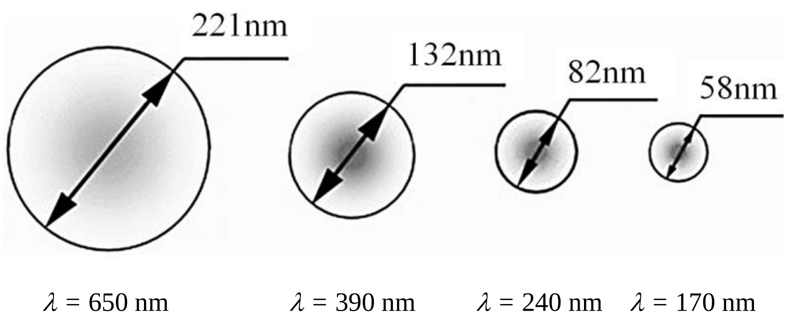

Fig. 2.3 Experimental data on the thickness of the laser beam as a function of the wavelength λ of the corresponding monochromatic radiation (see URL https://tech.onliner.by/2006/03/29/blu_ray_about)

In this case, each a $\lambda_{m,n}$-vacuum (i.e., a $3D_{m,n}$-landscape) will be unique (see Fig. 2.4), since the vacuum irregularities by averaged within the thickness of the probing light beam.

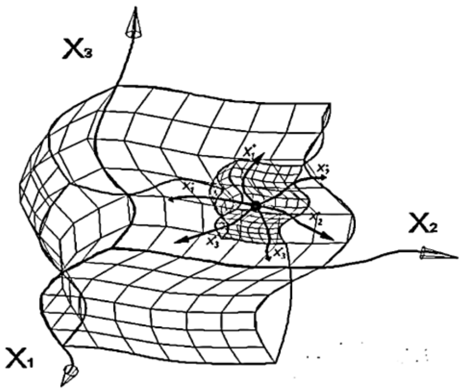

Fig. 2.4 The curved $\lambda_{m,n}$-vacuum is nested in the curved $\lambda_{f,d}$-vacuum (in case $\lambda_{f,d} > \lambda_{m,n}$)

Therefore, one $\lambda_{m,n}$-vacuum is only one 3-dimensional "cut" of the curved vacuum area. For a more complete description of the curved area of the vacuum,

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it is necessary to have an infinite set of the curved $\lambda_{m,n}$-vacuums nested into each other.

Thus, the investigated local volume of the vacuum is an infinitely complex system. The situation, however, is simplified by the fact that in the entire studied range of electromagnetic wavelengths from 10m to 10^{-19}m, all $\lambda_{m,n}$-vacuums obey the same physical laws. Therefore, the knowledge obtained in the study of one $\lambda_{k,r}$-vacuum automatically extends to all other $\lambda_{m,n}$-vacuums.

Below, the mathematical apparatus of the Algebra of signatures is developed, intended for the study of a local volume of only one $\lambda_{m,n}$-vacuum. But this apparatus is suitable for investigating not only all $\lambda_{m,n}$-vacuums, but also any other continuous media in which wave disturbances propagate at a constant speed.

2.3 The sixteen rotating 4-bases

Let's return to the consideration of the non-curved volume of one of the $\lambda_{m,n}$-vacuums (see Fig. 1.2) and investigate the "vacuum" area in the vicinity of the point O (Fig. 2.5).

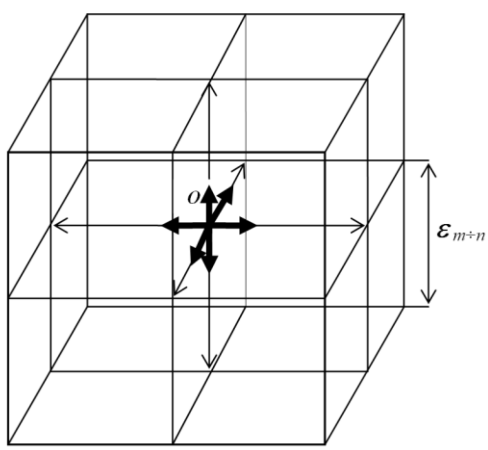

Fig. 2.5 The undistorted 3D-lattice of the $\lambda_{m,n}$-vacuum, revealed from the "vacuum" by means of mutually perpendicular monochromatic light beams with wavelength $\lambda_{m,n}$. The cells of such a lattice are cubes with an edge length ε_{mn} of approximately $10^2 \cdot \lambda_{m,n}$

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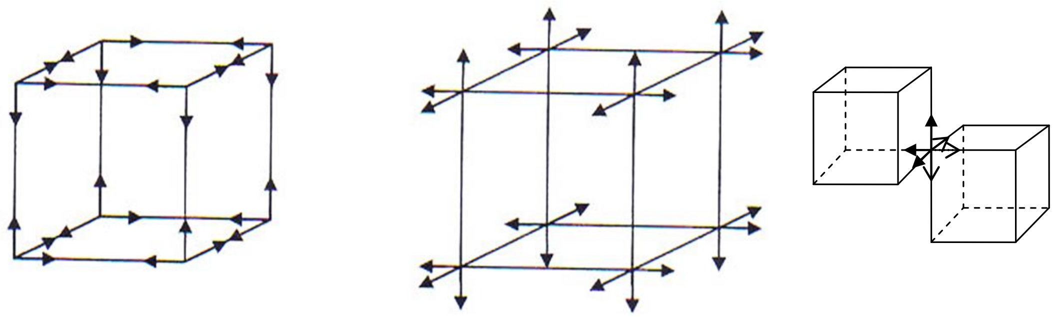
Let's calculate how many orthogonal 3-bases originate at the central point *O* (see Fig. 2.3).

If we extract 3-bases from point *O* in the different directions, then it turns out that there are 16 of them (see Fig. 2.6 *a*,*b*).



a) 8 internal 3-bases *b*) 8 external 3-antibases *c*) the adjacent cubic cells

Fig. 2.6 The sixteen 3-bases at the central point *O* of the studied volume of the $\lambda_{m,n}$-vacuum

The eight 3-bases belong to the cubic cell itself (see Fig. 2.6*a*), and the eight opposite 3-antibasises belong to adjacent cubic cells (see Fig. 2.6 *b*,*c*).

According to the "vacuum balance condition", any movement in a vacuum must be accompanied by a similar anti-movement. Therefore, if one 3-basis (together with a cubic cell) rotates clockwise, then this is possible only if an adjacent cubic cell (together with a 3-antibasis) rotates counterclockwise in the same way, since there is no fulcrum in vacuum.

In connection with the above, it is convenient for the eight 3-bases (see Fig. 2.6*a*) to add a fourth time axis *t*, and to the eight 3-antibases (see Fig.2.6*b*) add a fourth anti-axis (i.e., the opposite axis) of time – *t*.

Definition 2.3.1 *The time axis t is determined by the angular frequency of rotation of the 3-basis (i.e., the number of revolutions per unit of time). The rotation of a 3-basis with a constant angular velocity is described by the Expression* $d\varphi/dt = \omega$ *(where* φ *and* ω *are the phase and angular frequency of rotation of the 3-basis). Integrating this Expression, we get the time axis* $t = \varphi/\omega$*. The rotation of the 3-antibasis in the opposite direction similarly forms the anti-time axis* $-t = \varphi/\omega$*.*

Thus, at the considered point *O* of the $\lambda_{m,n}$-vacuum (see Fig. 2.5) there are 8 + 8 = 16 orthogonal 4-bases shown in Fig. 2.7.

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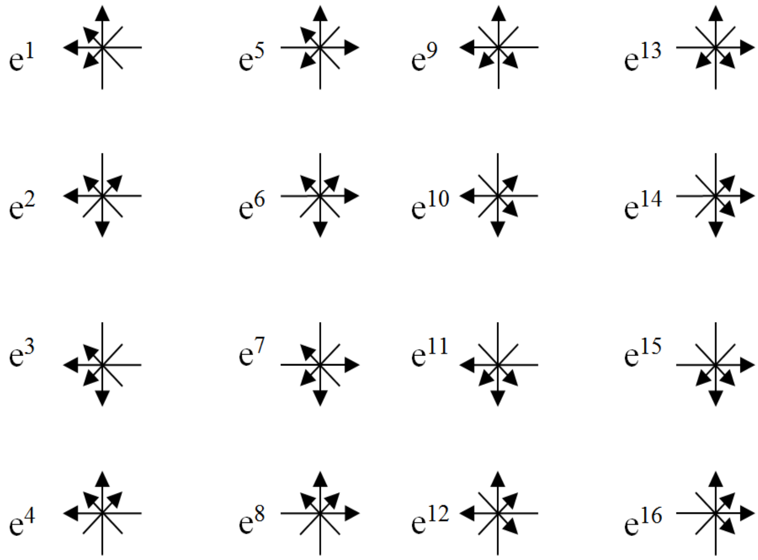
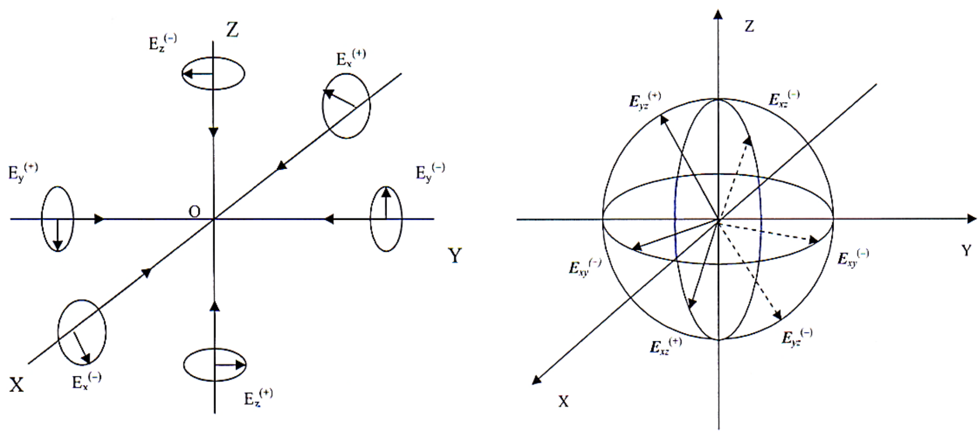


Fig. 2.7 Sixteen 4-bases starting at point O obtained by additions to the eight 3-bases of the fourth time axis *t* and to the eight 3-antibases of the fourth anti-time axis – *t*

The sixteen 4-bases (see Fig. 2.7) can be obtained within the framework of the theory of propagation of the electromagnetic waves.

Let six monochromatic rays of a light with circular polarization come to point *O*: two opposing rays of a light from each of three mutually perpendicular directions (see Fig. 2.8).



a) *b*)

Fig. 2.8 *a*) The rays and anti-rays (i.e., the counter rays) of a light with circular polarization, arriving at point *O* from three mutually perpendicular directions; *b*) Two 3-bases consisting of the electric field vectors $\mathbf{E}_x^{(+)}$, $\mathbf{E}_y^{(+)}$, $\mathbf{E}_z^{(+)}$ and $\mathbf{E}_x^{(-)}$, $\mathbf{E}_y^{(-)}$, $\mathbf{E}_z^{(-)}$ rotating at the point *O* in mutually opposite directions

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From the six rotating electric field vectors $\mathbf{E}_x^{(+)},\mathbf{E}_y^{(+)},\mathbf{E}_z^{(+)}$ and $\mathbf{E}_x^{(-)},\mathbf{E}_y^{(-)},\mathbf{E}_z^{(-)}$, shown in Fig. 2.6, can be formed the 16 rotating 3-bases. Of these: eight 3-bases rotate clockwise, and eight other 3-bases rotate counterclockwise.

3 THE ALGEBRA OF STIGNATUR

3.1 The stignature of an affine 4-dimensional space

Each of the sixteen 4-bases shown in Fig. 2.7 sets the direction of the axes of the 4-dimensional affine space.

In order to introduce the characteristic "stignature" of these spaces, we first define the concept of "base".

Let’s choose from the sixteen 4-bases, shown in Fig. 2.7, 4-basis $\mathbf{e}_i^{(5)}(\mathbf{e}_0^{(5)},\mathbf{e}_1^{(5)},\mathbf{e}_2^{(5)},\mathbf{e}_3^{(5)})$ and call it “base”.

We will conventionally assume that the directions of all unit vectors of the "base" are positive (see Fig. 3.1)

$$\mathbf{e}_i^{(5)}(\mathbf{e}_0^{(5)},\mathbf{e}_1^{(5)},\mathbf{e}_2^{(5)},\mathbf{e}_3^{(5)}) = (+1, +1,+ 1, +1) \rightarrow \{+ + + +\}. \qquad (3.1)$$

Here we have introduced an abbreviated notation {+ + + +}, which we will call the “stignature” of the affine space defined by the 4-basis $\mathbf{e}^{(5)}$ (i.e., the “base”).

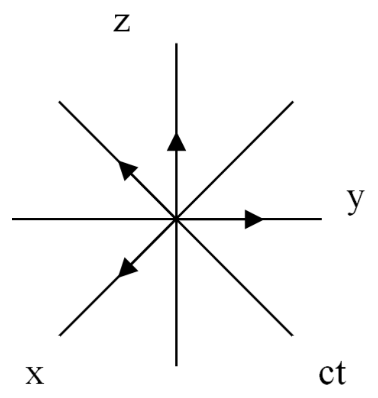

Fig. 3.1 The affine space, the directions of the axes of which are given by the 4-basis $\mathbf{e}_i^{(5)}(\mathbf{e}_0^{(5)},\mathbf{e}_1^{(5)},\mathbf{e}_2^{(5)},\mathbf{e}_3^{(5)})$ with the stignature {+ + + +}

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Definition 3.1.1 *The "stignature" of a 4-basis is a set of signs corresponding to the directions of the unit vectors in relation to the directions of the corresponding unit vectors of the "base".*

With respect to the directions of the unit vectors of the "base" (i.e., the 4-basis $\mathbf{e}^{(5)}$), the unit vectors of all the other 4-bases shown in Fig. 2.7, have the following signs and the corresponding stignatures:

Table 3.1

4-basis	Stignature	4-basis	Stignature
$\mathbf{e}_i^{(1)}(\mathbf{e}_0^{(1)}, \mathbf{e}_1^{(1)}, \mathbf{e}_2^{(1)}, \mathbf{e}_3^{(1)}) = (1,\ 1,\ -1,\ 1) \rightarrow$	$\{+ + - +\}$	$\mathbf{e}_i^{(9)}(\mathbf{e}_0^{(9)}, \mathbf{e}_1^{(9)}, \mathbf{e}_2^{(9)}, \mathbf{e}_3^{(9)}) = (-1,\ 1, -1,\ 1) \rightarrow$	$\{- + - +\}$
$\mathbf{e}_i^{(2)}(\mathbf{e}_0^{(2)}, \mathbf{e}_1^{(2)}, \mathbf{e}_2^{(2)}, \mathbf{e}_3^{(2)}) = (1,\ -1,\ -1,\ -1) \rightarrow$	$\{+ - - -\}$	$\mathbf{e}_i^{(10)}(\mathbf{e}_0^{(10)}, \mathbf{e}_1^{(10)}, \mathbf{e}_2^{(10)}, \mathbf{e}_3^{(10)}) = (-1,\ 1,\ -1,\ -1) \rightarrow$	$\{- - - -\}$
$\mathbf{e}_i^{(3)}(\mathbf{e}_0^{(3)}, \mathbf{e}_1^{(3)}, \mathbf{e}_2^{(3)}, \mathbf{e}_3^{(3)}) = (1,\ 1,\ -1,\ -1) \rightarrow$	$\{+ + - -\}$	$\mathbf{e}_i^{(11)}(\mathbf{e}_0^{(11)}, \mathbf{e}_1^{(11)}, \mathbf{e}_2^{(11)}, \mathbf{e}_3^{(11)}) = (-1,\ 1,\ -1,\ -1) \rightarrow$	$\{- + - -\}$
$\mathbf{e}_i^{(4)}(\mathbf{e}_0^{(4)}, \mathbf{e}_1^{(4)}, \mathbf{e}_2^{(4)}, \mathbf{e}_3^{(4)}) = (1, -1, -1,\ 1) \rightarrow$	$\{+ - - +\}$	$\mathbf{e}_i^{(12)}(\mathbf{e}_0^{(12)}, \mathbf{e}_1^{(12)}, \mathbf{e}_2^{(12)}, \mathbf{e}_3^{(12)}) = (-1, -1, -1,\ 1) \rightarrow$	$\{- - - +\}$
$\mathbf{e}_i^{(5)}(e_0^{(5)}, \mathbf{e}_1^{(5)}, \mathbf{e}_2^{(5)}, \mathbf{e}_3^{(5)}) = (1,\ 1,\ 1,\ 1) \rightarrow$	$\{+ + + +\}$	$\mathbf{e}_i^{(13)}(\mathbf{e}_0^{(13)}, \mathbf{e}_1^{(13)}, \mathbf{e}_2^{(13)}, \mathbf{e}_3^{(13)}) = (-1,\ 1,\ 1,\ 1) \rightarrow$	$\{- + + +\}$
$\mathbf{e}_i^{(6)}(\mathbf{e}_0^{(6)}, \mathbf{e}_1^{(6)}, \mathbf{e}_2^{(6)}, \mathbf{e}_3^{(6)}) = (1, -1,\ 1, -1) \rightarrow$	$\{+ - + -\}$	$\mathbf{e}_i^{(14)}(\mathbf{e}_0^{(14)}, \mathbf{e}_1^{(14)}, \mathbf{e}_2^{(14)}, \mathbf{e}_3^{(14)}) = (-1, -1,\ 1, -1) \rightarrow$	$\{- - + -\}$
$\mathbf{e}_i^{(7)}(\mathbf{e}_0^{(7)}, \mathbf{e}_1^{(7)}, \mathbf{e}_2^{(7)}, \mathbf{e}_3^{(7)}) = (1,\ 1,\ 1, -1) \rightarrow$	$\{+ + + -\}$	$\mathbf{e}_i^{(15)}(\mathbf{e}_0^{(15)}, \mathbf{e}_1^{(15)}, \mathbf{e}_2^{(15)}, \mathbf{e}_3^{(15)}) = (-1, 1, 1 -1) \rightarrow$	$\{- + + -\}$
$\mathbf{e}_i^{(8)}(\mathbf{e}_0^{(8)}, \mathbf{e}_1^{(8)}, \mathbf{e}_2^{(8)}, \mathbf{e}_3^{(8)}) = (1, -1,\ 1,\ 1) \rightarrow$	$\{+ - + +\}$	$\mathbf{e}_i^{(16)}(\mathbf{e}_0^{(16)}, \mathbf{e}_1^{(16)}, \mathbf{e}_2^{(16)}, \mathbf{e}_3^{(16)}) = (-1, -1,\ 1,\ 1) \rightarrow$	$\{- - + +\}$

3.2 The stignature matrix

The stignatures given in Table 3.1, are combined into a 16-component antisymmetric matrix:

$$stign\left(e_i^{(a)}\right)=\begin{pmatrix} \{++++\}^{00} & \{+++-\}^{10} & \{-++-\}^{20} & \{++-+\}^{30} \\ \{---+\}^{01} & \{-+++\}^{11} & \{--++\}^{21} & \{-+-+\}^{31} \\ \{+--+\}^{02} & \{++--\}^{12} & \{+---\}^{22} & \{+-++\}^{32} \\ \{--+-\}^{03} & \{+-+-\}^{13} & \{-+--\}^{23} & \{----\}^{33} \end{pmatrix}. \qquad (3.2)$$

Any other 4-basis out of the sixteen 4-bases shown in Fig. 2.7 could be chosen as a "base". In this case, the combinations of signs in the stignatures of the affine spaces would change, but the physical essence of the investigated non-curved volume of the $\lambda_{m,n}$-vacuum does not change. Nevertheless, it should be remembered that the "Algebra of signatures" developed here initially has the 16-fold degeneracy. This degeneracy under certain circumstances (in particular, with some types of curvature of the $\lambda_{m,n}$-vacuum) can lead to the splitting of the investigated volume of "vacuum" into the 16 different quantum states.

The matrix (3.2) will be called the matrix of stignatures. This matrix is a separate mathematical object that has a number of properties. Let's list some of them:

1]. The sum of all 16 signatures (3.2) is equal to zero stignature

$$\begin{aligned} &\{++-+\} + \{+---\} + \{++--\} + \{+--+\} + \\ &+ \{++++\} + \{+-+-\} + \{+++-\} + \{+-++\} + \\ &+ \{-+-+\} + \{----\} + \{-+--\} + \{---+\} + \\ &+ \{-+++\} + \{--+-\} + \{-++-\} + \{--++\} = \{0000\}. \end{aligned} \qquad (3.3)$$

This Expression can also be represented as (3.3*a*)

$$\begin{array}{ccccc} \{+\ +\ +\ +\} & + & \{-\ -\ -\ -\} & = 0 \\ \{-\ -\ -\ +\} & + & \{+\ +\ +\ -\} & = 0 \\ \{+\ -\ -\ +\} & + & \{-\ +\ +\ -\} & = 0 \\ \{-\ -\ +\ -\} & + & \{+\ +\ -\ +\} & = 0 \\ \{+\ +\ -\ -\} & + & \{-\ -\ +\ +\} & = 0 \\ \{-\ +\ -\ -\} & + & \{+\ -\ +\ +\} & = 0 \\ \{+\ -\ +\ -\} & + & \{-\ +\ -\ +\} & = 0 \\ \underline{\{+\ -\ -\ -\}} & + & \underline{\{-\ +\ +\ +)} & = 0\,. \\ \{0\ 0\ 0\ 0\} & & \{0\ 0\ 0\ 0\} & \end{array}$$

where the summation of the signs «+» and «–» is performed in each row and column according to the rules:

$$«+» + «+» = 2«+», \quad «+» + «–» = «–» + «+» = 0, \quad «–» + «–» = 2«–». \qquad (3.3b)$$

2]. The sum of all 64 signs included in the matrix (3.2) is equal to zero

$$32«+» + 32«–» = 0. \qquad (3.3c)$$

3]. Four binary combinations of signs are possible:

$$H' \leftrightarrow \begin{pmatrix} + \\ - \end{pmatrix} \quad V \leftrightarrow \begin{pmatrix} - \\ + \end{pmatrix} \quad H \leftrightarrow \begin{pmatrix} + \\ + \end{pmatrix} \quad I \leftrightarrow \begin{pmatrix} - \\ - \end{pmatrix}, \qquad (3.4)$$

or in transposed form

$$H'^{+} \leftrightarrow (+\,-) \quad V^{+} \leftrightarrow (-\,+) \quad H^{+} \leftrightarrow (+\,+) \quad I^{+} \leftrightarrow (-\,-) \qquad (3.5)$$

4]. The combinations of these binary signs form are the 16 variants of stignature:

$$II = \begin{pmatrix} - & - \\ - & - \end{pmatrix} \equiv \{- - - -\}; \quad HI = \begin{pmatrix} + & - \\ + & - \end{pmatrix} \equiv \{+ + - -\}; \quad VI = \begin{pmatrix} - & - \\ + & - \end{pmatrix} \equiv \{- + - -\}; \quad H'I = \begin{pmatrix} + & - \\ - & - \end{pmatrix} \equiv \{+ - - -\};$$

$$IH = \begin{pmatrix} - & + \\ - & + \end{pmatrix} \equiv \{- - + +\}; \quad HH = \begin{pmatrix} + & + \\ + & + \end{pmatrix} \equiv \{+ + + +\}; \quad VH = \begin{pmatrix} - & + \\ + & + \end{pmatrix} \equiv \{- - + +\}; \quad H'H = \begin{pmatrix} + & + \\ - & + \end{pmatrix} \equiv \{- - + +\};$$

$$IV = \begin{pmatrix} - & - \\ - & + \end{pmatrix} \equiv \{- - - +\}; \quad HV = \begin{pmatrix} + & - \\ + & + \end{pmatrix} \equiv \{+ + - +\}; \quad VV = \begin{pmatrix} - & - \\ + & + \end{pmatrix} \equiv \{- + - +\}; \quad H'V = \begin{pmatrix} + & - \\ - & + \end{pmatrix} \equiv \{- - - +\};$$

$$IH' = \begin{pmatrix} - & + \\ - & - \end{pmatrix} \equiv \{- - + -\}; \quad HH' = \begin{pmatrix} + & + \\ + & - \end{pmatrix} \equiv \{+ + + -\}; \quad VH' = \begin{pmatrix} - & + \\ + & - \end{pmatrix} \equiv \{- + + -\}; \quad H'H' = \begin{pmatrix} + & + \\ - & - \end{pmatrix} \equiv \{- + + -\}.$$

(3.6)

5]. The Kronecker square of the two-row matrix of the binary signs (3.5) forms a matrix consisting of the 16 stignatures present in the matrix (3.2):

$$\begin{pmatrix} \{++\} & \{+-\} \\ \{-+\} & \{--\} \end{pmatrix}^{\otimes 2} = \begin{pmatrix} \{++++\} & \{+++-\} & \{+-++\} & \{+-+-\} \\ \{++-+\} & \{++--\} & \{+--+\} & \{+---\} \\ \{-+++\} & \{-++-\} & \{--++\} & \{--+-\} \\ \{-+-+\} & \{-+--\} & \{---+\} & \{----\} \end{pmatrix} \qquad (3.7)$$

where $\otimes$ is the symbol for Kronecker multiplication.

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6]. The stignature matrix (3.2) can be represented as the sum of the diagonal and antisymmetric matrices

$$\begin{pmatrix} \{++++\} & 0 & 0 & 0 \\ 0 & \{-+++\} & 0 & 0 \\ 0 & 0 & \{+---\} & 0 \\ 0 & 0 & 0 & \{----\} \end{pmatrix} + \begin{pmatrix} 0 & \{+++-\} & \{-++-\} & \{++-+\} \\ \{---+\} & 0 & \{--++\} & \{-+-+\} \\ \{+--+\} & \{++--\} & 0 & \{+-++\} \\ \{--+-\} & \{+-+-\} & \{-+--\} & 0 \end{pmatrix}$$

. (3.8)

3.3 The two-row stignatures and the Hadamard matrices

If we return the original units to the two-row stignatures (3.6), then we obtain the two-row matrices

$$\begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix} \quad \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \quad \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix} \quad \begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix} \qquad (3.9)$$

$$\begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix}; \quad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \quad \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \qquad (3.10)$$

$$\begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}.$$

Of these, eight matrices:

$$\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad (3.11)$$

$$\begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix} \quad \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix} \quad \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix} \quad \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}$$

are the Hadamard matrices, since they satisfy the condition

$$H(2) \otimes H^T(2) = 2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \qquad (3.12)$$

When raising to Kronecker powers of any of the matrices (3.11), the Hadamard matrices $H\,(n)$ are again obtained, satisfying the condition:

$$H(n) \otimes H^T(n) = nI, \tag{3.13}$$

where I is a diagonal unit matrix of dimension $n \times n$:

$$I = \begin{pmatrix} 1 & 0 & ... & 0 \\ 0 & 1 & ... & 0 \\ ... & ... & ... & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{3.14}$$

For example, (3.15)

$$H(2)^{\otimes 2} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{\otimes 2} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} & 1\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \\ 1\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} & -1\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix},$$

$$H(2)^{\otimes 3} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{\otimes 3} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{pmatrix}$$

(3.16)

and so on according to the algorithm

$$H(2)^{\otimes k} = H(2^k) = H(2) \otimes H(2)^{\otimes k\text{-}1} = H(2) \otimes H(2^{k\text{-}1}), \tag{3.17}$$

Recall that Hadamard matrices are used to construct the noise-proof protected error-correcting codes. In particular, it is believed that DNA molecules are built on the basis of the Hadamard matrices [10, 11].

If in the matrix (3.15) we again use the signs {+} and {–} instead of 1 and –1, then we obtain the rule for raising to the Kronecker power of the two-row stignatures. For example,

$$\begin{pmatrix} + & + \\ + & - \end{pmatrix}^{\otimes 2} = \begin{pmatrix} + & + \\ + & - \end{pmatrix} \otimes \begin{pmatrix} + & + \\ + & - \end{pmatrix} = \begin{pmatrix} +\begin{pmatrix} + & + \\ + & - \end{pmatrix} & +\begin{pmatrix} + & + \\ + & - \end{pmatrix} \\ +\begin{pmatrix} + & + \\ + & - \end{pmatrix} & -\begin{pmatrix} + & + \\ + & - \end{pmatrix} \end{pmatrix} = \begin{pmatrix} + & + & + & + \\ + & - & + & - \\ + & + & - & - \\ + & - & - & + \end{pmatrix} \tag{3.18}$$

$$\begin{pmatrix} + & + \\ + & - \end{pmatrix}^{\otimes 3} = \begin{pmatrix} + & + \\ + & - \end{pmatrix} \otimes \begin{pmatrix} + & + & + & + \\ + & - & + & - \\ + & + & - & - \\ + & - & - & + \end{pmatrix} = \begin{pmatrix} + & + & + & + & + & + & + & + \\ + & - & + & - & + & - & + & - \\ + & + & - & - & + & + & - & - \\ + & - & - & + & + & - & - & + \\ + & + & + & + & - & - & - & - \\ + & - & + & - & - & + & - & + \\ + & + & - & - & - & - & + & + \\ + & - & - & + & - & + & + & - \end{pmatrix}$$

The two-row stignatures corresponding to matrices (3.11)

$$\begin{matrix} \begin{pmatrix} - & + \\ + & + \end{pmatrix} & \begin{pmatrix} + & + \\ - & + \end{pmatrix} & \begin{pmatrix} + & - \\ + & + \end{pmatrix} & \begin{pmatrix} + & + \\ + & - \end{pmatrix} \\ \begin{pmatrix} + & - \\ - & - \end{pmatrix} & \begin{pmatrix} - & - \\ + & - \end{pmatrix} & \begin{pmatrix} - & + \\ - & - \end{pmatrix} & \begin{pmatrix} - & - \\ - & + \end{pmatrix} \end{matrix} \qquad (3.19)$$

will be called the two-row Hadamard stignatures.

3.4 The colored quaternions

The sixteen stignatures (3.2) correspond to the 16 types of "colored" quaternions

(3.20)

$z_1 = x_0 + ix_1 + jx_2 + kx_3$	$\{+ + + +\}$	$\{- - - -\}$	$z_9 = -x_0 - ix_1 - jx_2 - kx_3$
$z_2 = -x_0 - ix_1 - jx_2 + kx_3$	$\{- - - +\}$	$\{+ + + -\}$	$z_{10} = x_0 + ix_1 + jx_2 - kx_3$
$z_3 = x_0 - ix_1 - jx_2 + kx_3$	$\{+ - - +\}$	$\{- + + -\}$	$z_{11} = -x_0 + ix_1 + jx_2 - kx_3$
$z_4 = -x_0 - ix_1 + jx_2 - kx_3$	$\{- - + -\}$	$\{+ + - +\}$	$z_{12} = x_0 + ix_1 - jx_2 + kx_3$
$z_5 = x_0 + ix_1 - jx_2 - kx_3$	$\{+ + - -\}$	$\{- - + +\}$	$z_{13} = -x_0 - ix_1 + jx_2 + kx_3$
$z_6 = -x_0 + ix_1 - jx_2 - kx_3$	$\{- + - -\}$	$\{+ - + +\}$	$z_{14} = x_0 - ix_1 + jx_2 + kx_3$
$z_7 = x_0 - ix_1 + jx_2 - kx_3$	$\{+ - + -\}$	$\{- + - +\}$	$z_{15} = -x_0 + ix_1 - jx_2 + kx_3$
$z_8 = -x_0 + ix_1 + jx_2 + kdx_3$	$\{- + + +\}$	$\{+ - - -\}$	$z_{16} = x_0 - ix_1 - jx_2 - kx_3$

It was shown in [14, 15] that the “colors” of quaternions correspond to the “colors” of the vacuum chromodynamics.

It is easy to verify by direct calculation that the sum of all 16 types of "colored" quaternions (3.20) is equal to zero

$$\sum_{k=1}^{16} z_k = 0, \qquad (3.21)$$

that is, the superposition (i.e., the sum) of all types of "colored" quaternions is balanced with respect to the zero and satisfies the "vacuum balance condition".

3.5 The spectral-stignature analysis

Let’s point out a possible application of the Algebra of stignatures to expand the possibilities of the Fourier spectral analysis.

We recall the procedure known in quantum physics for the transition from a coordinate representation to a momentum representation.

Let there be some function of the space and time $\rho(ct,x,y,z)$. This function is represented as a product of two amplitudes:

$$\rho(ct,x,y,z) = \varphi(ct,x,y,z)\ \varphi(ct,x,y,z). \qquad (3.22)$$

Next, two Fourier transforms are performed

$$\psi(p_{ct}, p_x, p_y, p_z) = \int_{-\infty}^{\infty} \varphi(ct,x,y,z) \exp\{i\frac{p}{\eta}(ct - x - y - z)\} d\Omega, \qquad (3.23)$$

$$\psi^*(p_{ct}, p_x, p_y, p_z) = \int_{-\infty}^{\infty} \varphi(ct,x,y,z) \exp\{i\frac{p}{\eta}(-ct + x + y + z)\} d\Omega, \qquad (3.24)$$

where $p = 2\pi\eta/\lambda$ is the generalized frequency; λ is the wavelength; η - coefficient of proportionality (in quantum mechanics $\eta = \hbar$ is reduced Planck's constant); $d\Omega = dctdxdydz$ is elementary 4-dimensional volume.

The momentum (spectral) representation of the function $\rho(ct,x,y,z)$ is obtained as a result of the product of two amplitudes (3.23) and (3.24)

$$G(p_{ct}, p_x, p_y, p_z) = \psi(p_{ct}, p_x, p_y, p_z) \cdot \psi^*(p_{ct}, p_x, p_y, p_z). \qquad (3.25)$$

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By analogy with the procedure (3.24) – (3.25), we formulate the foundations of spectral-stignature (i.e., "color") Fourier analysis.

We represent the function $\rho(ct,x,y,z)$ as a product of the 8 "amplitudes"

$$\rho(ct,x,y,z)=\varphi_1(ct,x,y,z)\ \varphi_2(ct,x,y,z)\ \varphi_3(ct,x,y,z)\times\ldots\times\varphi_8(ct,x,y,z) = \prod_{k=1}^{8}\varphi_k(ct,x,y,z)\,. \tag{3.26}$$

Instead of the imaginary unit i, in integrals (3.23) and (3.24), we introduce the eight objects ζ_r (where r = 1,2,3,...,8), which satisfy the anticommutative relations of the Clifford algebra:

$$\zeta_m\zeta_k + \zeta_k\zeta_m = 0 \ \text{ for } \ m\neq k\,, \quad \zeta_m\zeta_m = 1, \tag{3.27}$$

$$\text{or} \quad \zeta_m\zeta_k + \zeta_k\zeta_m = 2\delta_{km}\,, \tag{3.28}$$

where δ_{km} is the Kronecker symbol (δ_{km}= 0 for $m\neq k$ and δ_{km} = 1 for $m = k$).

These requirements are satisfied, for example, by a set of the 8×8 matrices of type (3.29)

$$\zeta_1=\begin{pmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{pmatrix}\quad
\zeta_2=\begin{pmatrix}0&1&0&0&0&0&0&0\\-1&0&0&0&0&0&0&0\\0&0&0&-1&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&-1&0\end{pmatrix}\quad
\zeta_3=\begin{pmatrix}0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\-1&0&0&0&0&0&0&0\\0&-1&0&0&0&0&0&0\\0&0&0&0&0&0&-1&0\\0&0&0&0&0&0&0&-1\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\end{pmatrix}$$

$$\zeta_4=\begin{pmatrix}0&0&0&1&0&0&0&0\\0&0&-1&0&0&0&0&0\\0&1&0&0&0&0&0&0\\-1&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&-1\\0&0&0&0&0&0&1&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&1&0&0&0\end{pmatrix}\quad
\zeta_5=\begin{pmatrix}0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\\-1&0&0&0&0&0&0&0\\0&-1&0&0&0&0&0&0\\0&0&-1&0&0&0&0&0\\0&0&0&-1&0&0&0&0\end{pmatrix}\quad
\zeta_6=\begin{pmatrix}0&0&0&0&0&1&0&0\\0&0&0&0&-1&0&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&-1&0\\0&1&0&0&0&0&0&0\\-1&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&-1&0&0&0&0&0\end{pmatrix}$$

$$\zeta_7=\begin{pmatrix}0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&-1\\0&0&0&0&-1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&1&0&0&0&0&0\\0&0&0&-1&0&0&0&0\\-1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\end{pmatrix}\quad
\zeta_8=\begin{pmatrix}0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&-1&0&0&0\\0&0&0&1&0&0&0&0\\0&0&1&0&0&0&0&0\\0&-1&0&0&0&0&0&0\\-1&0&0&0&0&0&0&0\end{pmatrix}\quad
\delta_{km}=\begin{pmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{pmatrix}$$

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Let's perform the eight "color" Fourier transforms:

$$\psi_1(p_{ct},p_x,p_y,p_z)=\int_{-\infty}^{\infty}\varphi_1(ct,x,y,z)\exp\{\zeta_1\frac{p}{\eta}(ct+x+y+z)\}d\Omega, \qquad (3.30)$$
$$\psi_2(p_{ct},p_x,p_y,p_z)=\int_{-\infty}^{\infty}\varphi_2(ct,x,y,z)\exp\{\zeta_2\frac{p}{\eta}(-ct-x-y+z)\}d\Omega,$$
$$\psi_3(p_{ct},p_x,p_y,p_z)=\int_{-\infty}^{\infty}\varphi_3(ct,x,y,z)\exp\{\zeta_3\frac{p}{\eta}(ct-x-y+z)\}d\Omega,$$
$$\psi_4(p_{ct},p_x,p_y,p_z)=\int_{-\infty}^{\infty}\varphi_4(ct,x,y,z)\exp\{\zeta_4\frac{p}{\eta}(-ct-x+y-z)\}d\Omega,$$
$$\psi_5(p_{ct},p_x,p_y,p_z)=\int_{-\infty}^{\infty}\varphi_5(ct,x,y,z)\exp\{\zeta_5\frac{p}{\eta}(ct+x-y-z)\}d\Omega,$$
$$\psi_6(p_{ct},p_x,p_y,p_z)=\int_{-\infty}^{\infty}\varphi_6(ct,x,y,z)\exp\{\zeta_6\frac{p}{\eta}(-ct+x-y-z)\}d\Omega,$$
$$\psi_7(p_{ct},p_x,p_y,p_z)=\int_{-\infty}^{\infty}\varphi_7(ct,x,y,z)\exp\{\zeta_7\frac{p}{\eta}(ct-x+y-z)\}d\Omega,$$
$$\psi_8(p_{ct},p_x,p_y,p_z)=\int_{-\infty}^{\infty}\varphi_8(ct,x,y,z)\exp\{\zeta_8\frac{p}{\eta}(-ct+x+y+z)\}d\Omega,$$

where the objects ζ_m (3.29) perform the function of Clifford imaginary units.

We also find the eight complex conjugate "color" Fourier transforms:

$$\psi_1{}^*(p_{ct},p_x,p_y,p_z)=\int_{-\infty}^{\infty}\varphi_1(ct,x,y,z)\exp\{\zeta_1\frac{p}{\eta}(-ct-x-y-z)\}d\Omega, \qquad (3.31)$$
$$\psi_2{}^*(p_{ct},p_x,p_y,p_z)=\int_{-\infty}^{\infty}\varphi_2(ct,x,y,z)\exp\{\zeta_2\frac{p}{\eta}(c+x+y-z)\}d\Omega,$$
$$\psi_3{}^*(p_{ct},p_x,p_y,p_z)=\int_{-\infty}^{\infty}\varphi_3(ct,x,y,z)\exp\{\zeta_3\frac{p}{\eta}(-ct+x+y-z)\}d\Omega,$$
$$\psi_4{}^*(p_{ct},p_x,p_y,p_z)=\int_{-\infty}^{\infty}\varphi_4(ct,x,y,z)\exp\{\zeta_4\frac{p}{\eta}(ct+x-y+z)\}d\Omega,$$
$$\psi_5{}^*(p_{ct},p_x,p_y,p_z)=\int_{-\infty}^{\infty}\varphi_5(ct,x,y,z)\exp\{\zeta_5\frac{p}{\eta}(-ct-x+y+z)\}d\Omega,$$

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$$\psi_6{}^*(p_{ct},p_x,p_y,p_z)=\int_{-\infty}^{\infty}\varphi_6(ct,x,y,z)\exp\{\zeta_6\frac{p}{\eta}(ct-x+y+z)\}d\Omega,$$

$$\psi_7{}^*(p_{ct},p_x,p_y,p_z)=\int_{-\infty}^{\infty}\varphi_7(ct,x,y,z)\exp\{\zeta_7\frac{p}{\eta}(-ct+x-y+z)\}d\Omega,$$

$$\psi_8{}^*(p_{ct},p_x,p_y,p_z)=\int_{-\infty}^{\infty}\varphi_8(ct,x,y,z)\exp\{\zeta_8\frac{p}{\eta}(ct-x-y-z)\}d\Omega.$$

The integrals of the "color" Fourier transform (3.30) and (3.31) include the 16 linear forms with stignatures (3.2).

The spectral-stignature representation of the function $\rho(ct,x,y,z)$ is obtained as a result of the product of eight corresponding pairs of the "color" amplitudes (3.30) and their complex conjugate "color" amplitudes (3.31)

$$\Re(p_{ct},p_x,p_y,p_z)=\prod_{k=1}^{8}\psi_k(p_{ct},p_x,p_y,p_z)\psi_k{}^*(p_{ct},p_x,p_y,p_z). \qquad (3.32)$$

In this case, there are the 16 types of "colored" spirals with the corresponding stignatures

$\exp\{\zeta_1 2\pi/\lambda(\ ct+x+y+z)\}$	(3.33)	$\{+\ +\ +\ +\}$	(3.34)
$\exp\{\zeta_2 2\pi/\lambda(-ct-x-y+z)\}$		$\{-\ -\ -\ +\}$	
$\exp\{\zeta_3 2\pi/\lambda(\ ct-x-y+z)\}$		$\{+\ -\ -\ +\}$	
$\exp\{\zeta_4 2\pi/\lambda(-ct-x+y-z)\}$		$\{-\ -\ +\ -\}$	
$\exp\{\zeta_5 2\pi/\lambda(\ ct+x-y-z)\}$		$\{+\ +\ -\ -\}$	
$\exp\{\zeta_6 2\pi/\lambda(-ct+x-y-z)\}$		$\{-\ +\ -\ -\}$	
$\exp\{\zeta_7 2\pi/\lambda(\ ct-x+y-z)\}$		$\{+\ -\ +\ -\}$	
$\exp\{\zeta_8 2\pi/\lambda(-ct+x+y+z)\}$		$\{-\ +\ +\ +\}$	
$\exp\{\zeta_1 2\pi/\lambda(-ct-x-y-z)\}$		$\{-\ -\ -\ -\}$	
$\exp\{\zeta_2 2\pi/\lambda(\ ct+x+y-z)\}$		$\{+\ +\ +\ -\}$	
$\exp\{\zeta_3 2\pi/\lambda(-ct+x+y-z)\}$		$\{-\ +\ +\ -\}$	
$\exp\{\zeta_4 2\pi/\lambda(\ ct+x-y+z)\}$		$\{+\ +\ -\ +\}$	
$\exp(\zeta_5 2\pi/\lambda(-ct-x+y+z)\}$		$\{-\ -\ +\ +\}$	
$\exp\{\zeta_6 2\pi/\lambda(\ ct-x+y+z)\}$		$\{+\ -\ +\ +\}$	
$\exp\{\zeta_7 2\pi/\lambda(-ct+x-y+z)\}$		$\{-\ +\ -\ +\}$	
$\exp\{\zeta_8 2\pi/\lambda(\ ct-x-y-z)\}$		$\{+\ -\ -\ -\}$	
		$\{0\ \ 0\ \ 0\ \ 0\}_+$	

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Definition 3.5.1 *"Stignature" is an ordered set of signs in front of the corresponding terms of a linear form.*

The Expression (3.34) will be called a "rank", since in its numerator, actions on the signs (+) and (–) are performed by columns and/or by rows.

The result of adding signs in one column is written to the denominator under this column, and the result of adding signs in one line is written to the side of the rank {see the Expression (3.36)}.

The actions on the signs in the numerator and denominator of the rank are performed according to the arithmetic rules of addition (or subtraction):

$$\begin{array}{ll|ll} \{+\} + \{+\} = 2\{+\}; & \{-\} + \{+\} = \{0\}; & \{+\} - \{+\} = \{0\}; & \{-\} - \{+\} = 2\{-\}; \\ \{+\} + \{-\} = \{0\}; & \{-\} + \{-\} = 2\{-\}, & \{+\} - \{-\} = \{2+\}; & \{-\} - \{-\} = \{0\}, \end{array} \tag{3.35}$$

The type of the operation (addition or subtraction) on the signs in the numerator of the rank is shown as an index of its denominator $\{\ldots\}_{+}$ or $\{\ldots\}_{-}$.

The rank (3.34) can be represented as the sum of two ranks

$$\begin{array}{lclc} \{+\ \ +\ \ +\ +\} & + & \{-\ \ -\ \ -\ -\} & = 0 \\ \{-\ \ -\ \ -\ \ +\} & + & \{+\ \ +\ \ +\ \ -\} & = 0 \\ \{+\ \ -\ \ -\ \ +\} & + & \{-\ \ +\ \ +\ \ -\} & = 0 \\ \{-\ \ -\ \ +\ \ -\} & + & \{+\ \ +\ \ -\ \ +\} & = 0 \\ \{+\ \ +\ \ -\ \ -\} & + & \{-\ \ -\ \ +\ \ +\} & = 0 \\ \{-\ \ +\ \ -\ \ -\} & + & \{+\ \ -\ \ +\ \ +\} & = 0 \\ \{+\ \ -\ \ +\ \ -\} & + & \{-\ \ +\ \ -\ \ +\} & = 0 \\ \{\underline{-\ \ +\ \ +\ \ +}\} & + & \{\underline{+\ \ -\ \ -\ \ -}\} & = 0 \\ \{0\ \ 0\ \ 0\ \ 0)_{+} & & \{0\ \ 0\ \ 0\ \ 0)_{+} & = 0\,. \end{array} \tag{3.36}$$

where the signs are summed up both in the columns and in the rows.

The ranked Expression (3.36) is called “the splitting of the affine zero”, and it is of interest for the light-geometry of the vacuum, since it reflects the initial structure of "vacuum balance condition".

The Expressions (3.34) and (3.36) show that the "color" (i.e., spectral-stignature) Fourier analysis is balanced with respect to zero, and can be applied in the physics of the "vacuum".

In particular, color (or spectral-stignature) Fourier analysis can be useful for the development of "zero" (i.e., a vacuum) technologies, such as the compression of the vacuum communication channels.

4 THE ALGEBRA OF SIGNATURES

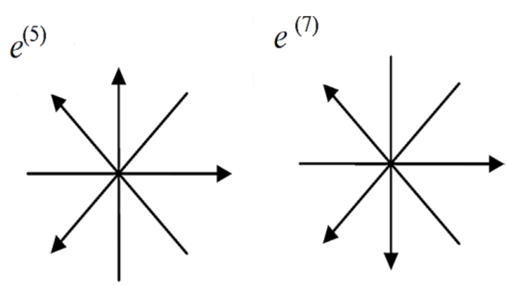
4.1 The metric spaces with different signatures

Let's pass from affine geometries to metric ones.

We consider an affine space with a 4-basis $\mathbf{e}_i^{(7)}(\mathbf{e}_0^{(7)},\mathbf{e}_1^{(7)},\mathbf{e}_2^{(7)},\mathbf{e}_3^{(7)})$ (see Fig. 2.7 and Fig. 4.1a) with the stignature {+ + + –}. Let's define in this space the 4-vector

$$d\mathbf{s}^{(7)} = \mathbf{e}_i^{(7)}dx_i^{(7)} = \mathbf{e}_0^{(7)}dx_0^{(7)} + \mathbf{e}_1^{(7)}dx_1^{(7)} + \mathbf{e}_2^{(7)}dx_2^{(7)} + \mathbf{e}_3^{(7)}dx_3^{(7)}, \qquad (4.1)$$

where $dx_i^{(7)}$ is the i-th projection of the 4-vector $d\mathbf{s}^{(7)}$ onto the axis $x_i^{(7)}$, the direction of which is determined by the basis vector $\mathbf{e}_i^{(7)}$.



a) {+ + + +} б) {+ + + –}

Fig. 4.1 Two 4-bases with different stignatures

Let's define a second 4-vector in an affine space with a 4-basis $\mathbf{e}_i^{(5)}$ $(\mathbf{e}_0^{(5)}, \mathbf{e}_1^{(5)}, \mathbf{e}_2^{(5)}, \mathbf{e}_3^{(5)})$ (see Fig. 2.7 and Fig. 4.1b), with the stignature {+ + + +}

$$d\mathbf{s}^{(5)} = \mathbf{e}_i^{(5)}dx_i^{(5)} = \mathbf{e}_0^{(5)}dx_0^{(5)} + \mathbf{e}_1^{(5)}dx_1^{(5)} + \mathbf{e}_2^{(5)}dx_2^{(5)} + \mathbf{e}_3^{(5)}dx_3^{(5)}. \qquad (4.2)$$

We find the scalar product of the 4-vectors (4.1) and (4.2)

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$$ds^{(5,7)\,2} = d\mathbf{s}^{(5)}d\mathbf{s}^{(7)} = \mathbf{e}_i^{(5)}\mathbf{e}_j^{(7)}dx^i\,dx^j = \tag{4.3}$$

$$= \mathbf{e}_0^{(5)}\mathbf{e}_0^{(7)}dx_0dx_0 + \mathbf{e}_1^{(5)}\mathbf{e}_0^{(7)}dx_1dx_0 + \mathbf{e}_2^{(5)}\mathbf{e}_0^{(7)}dx_2dx_0 + \mathbf{e}_3^{(5)}\mathbf{e}_0^{(7)}dx_3dx_0 +$$

$$+ \mathbf{e}_0^{(5)}\mathbf{e}_1^{(7)}dx_0dx_1 + \mathbf{e}_1^{(5)}\mathbf{e}_1^{(7)}dx_1dx_1 + \mathbf{e}_2^{(5)}\mathbf{e}_1^{(7)}dx_2dx_1 + \mathbf{e}_3^{(5)}\mathbf{e}_1^{(7)}dx_3dx_1 +$$

$$+ \mathbf{e}_0^{(5)}\mathbf{e}_2^{(7)}dx_0dx_2 + \mathbf{e}_1^{(5)}\mathbf{e}_2^{(7)}dx_1dx_2 + \mathbf{e}_2^{(5)}\mathbf{e}_2^{(7)}dx_2dx_2 + \mathbf{e}_3^{(5)}\mathbf{e}_2^{(7)}dx_3dx_2 +$$

$$+ \mathbf{e}_0^{(5)}\mathbf{e}_3^{(7)}dx_0dx_3 + \mathbf{e}_1^{(5)}\mathbf{e}_3^{(7)}dx_1dx_3 + \mathbf{e}_2^{(5)}\mathbf{e}_3^{(7)}dx_2dx_3 + \mathbf{e}_3^{(5)}\mathbf{e}_3^{(7)}dx_3dx_3.$$

For the case under consideration, the scalar products of the basis vectors $\mathbf{e}_i^{(5)}\mathbf{e}_j^{(7)}$ are equal:

$$\text{for } i = j \quad \mathbf{e}_0^{(5)}\mathbf{e}_0^{(7)} = 1,\ \ \mathbf{e}_1^{(5)}\mathbf{e}_1^{(7)} = 1,\ \ \mathbf{e}_2^{(5)}\mathbf{e}_2^{(7)} = 1,\ \ \mathbf{e}_3^{(5)}\mathbf{e}_3^{(7)} = -1, \tag{4.4}$$

$$\text{for } i \neq j \ \text{ all } \ \mathbf{e}_i^{(5)}\mathbf{e}_j^{(7)} = 0.$$

In this case, Expression (4.3) becomes a quadratic form (i.e., a 4-interval)

$$ds^{(5,7)2} = dx_0dx_0 + dx_1dx_1 + dx_2dx_2 - dx_3dx_3 = dx_0^2 + dx_1^2 + dx_2^2 - dx_3^2 \tag{4.5}$$

with signature (+ + + –).

Definition 4.1.1 *The "signature" is an ordered set of signs in front of the corresponding terms of the quadratic form ("signature" is the term of the General Relativity).*

To determine the signature of a metric space with metric (4.5), instead of performing the operation of the scalar product of the vectors (4.3), we can multiply the signs of the stignatures of the 4-bases shown in Fig. 4.1:

$$\begin{array}{c} \{+\ +\ +\ +\} \\ \underline{\{+\ +\ +\ -\}} \\ (+\ +\ +\ -)_\times \end{array} \tag{4.6}$$

In the numerator of the rank (4.6), the signs in each column are multiplied according to the rules

$$\{+\} \times \{+\} = \{+\}; \qquad \{-\} \times \{+\} = \{+\} \times \{-\} = \{-\}; \tag{4.7}$$

the result of such a multiplication is written in the denominator (under the line) of the same column. Performing actions according to these rules will be called rank multiplication.

Definition 4.1.2 *The "rank" is an Expression that determines the arithmetic operation with signs of the stignatures of affine (linear) forms or with signs of the* *signatures of the metric (quadratic) forms. The sign after the parenthesis in the denominator of the rank shows what operation is performed with the signs in the columns and/or rows of the ranks numerator: $(...)_+$ is a rank addition, $(...)_-$ is a rank subtraction, $(...)_:$ is a rank division, $(...)_\times$ is a rank multiplication.*

Similarly to how it was done with the vectors $\boldsymbol{ds}^{(5)}$ and $\boldsymbol{ds}^{(7)}$ {see Expressions (4.3) – (4.5)}, in pairs, scalar multiply with each other vectors from all 16 affine spaces with 4-bases shown in Fig. 2.7. As a result, we get 16 × 16 = 256 metric 4-spaces with 4-intervals of the form

$$ds^{(ab)2} = \mathbf{e}_i^{(a)}\mathbf{e}_j^{(b)}\, dx^{i(a)}dx^{j(b)}, \qquad (4.8)$$

where $a = 1, 2, 3,\ldots,16$; $b = 1, 2, 3, \ldots ,16$.

The signatures of these 16 × 16 = 256 metric 4-spaces can be determined, similarly to (4.8), by rank multiplications of the signs of signatures of the corresponding affine spaces:

$$\begin{array}{cccc}
\{+-++\} & \{++++\} & \{-+++\} & \{++++\} \\
\underline{\{+++-\}} & \underline{\{+-+-\}} & \underline{\{+++-\}} & \underline{\{-++-\}} \\
(+-+-)_\times & (+-+-)_\times & (-++-)_\times & (-++-)_\times \\
\\
\{+--+\} & \{++-+\} & \{-+++\} & \{+-+-\} \\
\underline{\{+++-\}} & \underline{\{-++-\}} & \underline{\{-++-\}} & \underline{\{+-+-\}} \\
(+---)_\times & (-+--)_\times & (+++-)_\times & (++++)_\times \\
\\
\{+---\} & \{++-+\} & \{-+-+\} & \{+-++\} \\
\underline{\{+++-\}} & \underline{\{-+--\}} & \underline{\{--+-\}} & \underline{\{+-+-\}} \\
(+--+)_\times & (-++-)_\times & (+---)_\times & (+++-)_\times \\
\cdots & \cdots & \cdots & \cdots \\
\{+++-\} & \{-+--\} & \{-++-\} & \{+--+\} \\
\underline{\{--+-\}} & \underline{\{+-+-\}} & \underline{\{+-+-\}} & \underline{\{-++-\}} \\
(--++)_\times & (---+)_\times & (--+-)_\times & (----)_\times
\end{array} \qquad (4.9)$$

The point O (see Figure 2.5) is the intersection of all 256 metric 4-spaces with intervals (i.e., metric) (4.8) and the corresponding signature (4.9).

The sum of all 256 metric 4-spaces intersecting at point *O* is zero

$$\sum_{k=1}^{256} ds^{(k)} = \sum_{a=1}^{16}\sum_{b=1}^{16} e_i^{(a)} e_j^{(b)} dx^{i(a)} dx^{i(b)} = 0, \qquad (4.10)$$

this is easy to verify, since among 256 × 4 = 1024 signs of all 256 signatures there are 512 {+} and 512 {–}. Thus, Expression (4.10) satisfies the "vacuum balance condition".

A set of 256 metric 4-spaces (4-maps) form a single 256-page "atlas" with a binding at point *O*, with a total number of mathematical dimensions 256×4= 1024.

The Algebra of signature approach largely coincides with the local-reference (tetrad) formalism, which was developed by E. Cartan, R. Weizenbek, T. Levi-Civita, G. Shipov [5] and was often used by A. Einstein in the framework of differential geometry with absolute parallelism.

The difference between the Algebra of signature and the tetrad method in General Relativity is as follows. In geometry with absolute parallelism, at each point of a 4-manifold there are two 4-frames (i.e., two tetrads), which define one metric with an interval $ds^{(ab)2} = \mathbf{e}_i^{(a)}\mathbf{e}_j^{(b)}dx^{i(a)}dx^{j(b)}$ and signature (+ – – –), while in the Algebra of signature at each point of a 3-manifold (i.e., $\lambda_{m,n}$-vacuum) there are sixteen 4-bases (or 4-frames, or tetrad) (see Fig. 2.7), the scalar products of which form 256 metrics (4.8) having the corresponding signature from the collection of signatures (4.9).

4.2 Four kinds of the rank multiplication and the rank division rules

Within the framework of the Algebra of signature, the multiplication and division of signs in the numerators of the ranks can be performed according to the following four types of the arithmetic rules:

I - the rules for a commutative $\lambda_{m,n}$-vacuum:

$$\{+\}\times\{+\}=\{+\} \qquad \{-\}\times\{+\}=\{-\}$$
$$\{+\}\times\{-\}=\{-\} \qquad \{-\}\times\{-\}=\{+\} \tag{4.11}$$

$$\{+\}:\{+\}=\{+\} \qquad \{-\}:\{+\}=\{-\}$$
$$\{+\}:\{-\}=\{-\} \qquad \{-\}:\{-\}=\{+\}; \tag{4.12}$$

H - the rules for a non-commutative $\lambda_{m,n}$-vacuum:

$$\{+\}\times\{+\}=\{+\} \qquad \{-\}\times\{+\}=\{-\}$$
$$\{+\}\times\{-\}=\{+\} \qquad \{-\}\times\{-\}=\{-\} \tag{4.13}$$

$$\{+\}:\{+\}=\{+\} \qquad \{-\}:\{+\}=\{-\}$$
$$\{+\}:\{-\}=\{+\} \qquad \{-\}:\{-\}=\{-\}; \tag{4.14}$$

V - the rules for a non-commutative $\lambda_{m,n}$-antivacuum:

$$\{+\}\times\{+\}=\{-\} \qquad \{-\}\times\{+\}=\{-\}$$
$$\{+\}\times\{-\}=\{+\} \qquad \{-\}\times\{-\}=\{+\} \tag{4.15}$$

$$\{+\}:\{+\}=\{-\} \qquad \{-\}:\{+\}=\{-\}$$
$$\{+\}:\{-\}=\{+\} \qquad \{-\}:\{-\}=\{+\}; \tag{4.16}$$

H' - the rules for a commutative $\lambda_{m,n}$-antivacuum:

$$\{+\}\times\{+\}=\{-\} \qquad \{-\}\times\{+\}=\{+\}$$
$$\{+\}\times\{-\}=\{-\} \qquad \{-\}\times\{-\}=\{+\} \tag{4.17}$$

$$\{+\}:\{+\}=\{-\} \qquad \{-\}:\{+\}=\{+\}$$
$$\{+\}:\{-\}=\{-\} \qquad \{-\}:\{-\}=\{+\}; \tag{4.18}$$

As an example, we write down the rank (4.6) for the four types of the $\lambda_{m,n}$-vacuums (4.11) – (4.18)

$$\begin{array}{cccc} \{+\,+\,+\,+\} & \{+\,+\,+\,+\} & \{+\,+\,+\,+\} & \{+\,+\,+\,+\} \\ \underline{\{+\,+\,+\,-\}} & \underline{\{+\,+\,+\,-\}} & \underline{\{+\,+\,+\,-\}} & \underline{\{+\,+\,+\,-\}} \\ (+\,+\,+\,-)_{I\times} & (+\,+\,+\,+)_{H\times} & (-\,-\,-\,+)_{V\times} & (-\,-\,-\,-)_{H'\times} \end{array} \tag{4.19}$$

The sum of the signs in the denominators of these ranks is zero

$$(+\,+\,+\,-)+(+\,+\,+\,+)+(-\,-\,-\,+)+(-\,-\,-\,-)=0, \tag{4.19a}$$

or zero signature

$$(+\,+\,+\,-)+(+\,+\,+\,+)+(-\,-\,-\,+)+(-\,-\,-\,-)=(0\;0\;0\;0) \tag{4.19b}$$

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In this paper, we will only use the rule of rank multiplication and rank division of the signs (4.11) for the commutative $\lambda_{m,n}$-vacuum.

However, it should be borne in mind that in a more consistent theory, all four types of the $\lambda_{m,n}$-vacuums with the rules of multiplication and division (4.11) – (4.18) and four corresponding zero factorials should be present: $0_I! = 1$, $0_H! = -1$, $0_V! = 0_V{}^0 = i$, $0_{H'}! = -i$. These $\lambda_{m,n}$-vacuums are "supports" for each other and provide stability and complete balancing of the vacuum of type (4.19a) and/or type (4.19b).

A set of the 16 stignatures (3.2):

$$\begin{matrix} \{++++\} & \{+++-\} & \{-++-\} & \{++-+\} \\ \{---+\} & \{-+++\} & \{--++\} & \{-+-+\} \\ \{+--+\} & \{++--\} & \{+---\} & \{+-++\} \\ \{--+-\} & \{+-+-\} & \{-+--\} & \{----\} \end{matrix} \qquad (4.20)$$

forms various an Abelian groups: by the operations of the rank multiplication and rank division by the rules (4.11) – (4.18). This indicates that the foundations of the Algebra of signature contain hidden symmetries.

4.3 The first stage of compactification of the extra dimensions

One of the main tasks of any multidimensional theory is to determine the method of compactification (i.e., folding) of the additional mathematical dimensions to the observable three spatial dimensions and one time dimension.

A similar problem is faced by the Algebra of signature. However, we note in advance that the compactification of extra dimensions in the Algebra of signature leads to a nontrivial (i.e., to an unexpected) result.

Note that, for example, the 16 types of scalar products of the 4-bases shown in Fig. 4.2, lead to sixteen quadratic forms (metrics) of the form (4.8) $ds^{(ab)2} = \mathbf{e}_i{}^{(a)}\mathbf{e}_j{}^{(b)}dx^{i(a)}dx^{j(b)}$ with the same signature (– + – +). Therefore, these metrics can be averaged.

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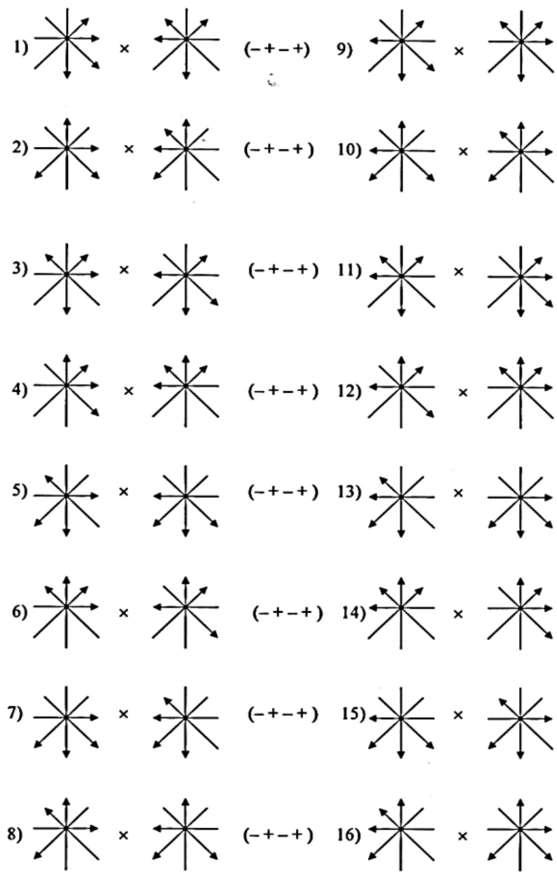

Fig. 4.2. The sixteen scalar products of the 4-bases, leading to the metrics with the same signature (– + – +)

Thus, it is possible to distinguish only 256/16 = 16 types of the metric 4-spaces with intervals (i.e., metrics)

$$
\begin{array}{ll}
ds^{(++++)2} = dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 = 0 & ds^{(----)2} = -dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2 = 0 \\
ds^{(---+)2} = -dx_0^2 - dx_1^2 - dx_2^2 + dx_3^2 = 0 & ds^{(+++-)2} = dx_0^2 + dx_1^2 + dx_2^2 - dx_3^2 = 0 \\
ds^{(+--+)2} = dx_0^2 - dx_1^2 - dx_2^2 + dx_3^2 = 0 & ds^{(-++-)2} = -dx_0^2 + dx_1^2 + dx_2^2 - dx_3^2 = 0 \\
ds^{(+---)2} = dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2 = 0 & ds^{(-+++)2} = -dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 = 0 \\
ds^{(--+-)2} = -dx_0^2 - dx_1^2 + dx_2^2 - dx_3^2 = 0 & ds^{(++-+)2} = dx_0^2 + dx_1^2 - dx_2^2 + dx_3^2 = 0 \\
ds^{(-+--)2} = -dx_0^2 + dx_1^2 - dx_2^2 - dx_3^2 = 0 & ds^{(+-++)2} = dx_0^2 - dx_1^2 + dx_2^2 + dx_3^2 = 0 \\
ds^{(+-+-)2} = dx_0^2 - dx_1^2 + dx_2^2 - dx_3^2 = 0 & ds^{(-+-+)2} = -dx_0^2 + dx_1^2 - dx_2^2 + dx_3^2 = 0 \\
ds^{(++--)2} = dx_0^2 + dx_1^2 - dx_2^2 - dx_3^2 = 0 & ds^{(--++)2} = -dx_0^2 - dx_1^2 + dx_2^2 + dx_3^2 = 0
\end{array}
\tag{4.21}
$$

with appropriate signatures

$$\begin{matrix} (++++) & (+++-) & (-++-) & (++-+) \\ (---+) & (-+++) & (--++) & (-+-+) \\ (+--+) & (++--) & (+---) & (+-++) \\ (--+-) & (+-+-) & (-+--) & (----) \end{matrix} \quad (4.22)$$

As a result of this averaging of metric 4-spaces with 16 types of the signatures, only 16 × 4 = 64 mathematical dimensions remain at the first stage of compactification.

4.4 The relationship between a signature and a 4-space topology

According to the classification of Felix Klein [2], metric spaces with intervals (4.21) can be divided into three topological classes:

1st class: is a 4-spaces, the signatures of which consist of four identical signs [2]:

$$\begin{matrix} x_0^2 + x_1^2 + x_2^2 + x_3^2 = 0 & (++++) \\ -x_0^2 - x_1^2 - x_2^2 - x_3^2 = 0 & (----) \end{matrix} \quad (4.23)$$

are zero metric 4-spaces. These "spaces" have only one valid point, located at the beginning of the light cone. All other points of these 4-spaces are imaginary. In fact, the first of the Expressions (4.23) describes not the space, but a single point (or the "white" point), and the second one is a single antipoint (or the "black" point).

2nd class: is a 4-spaces, the signatures of which consist of two positive and two negative signs [2]:

$$\begin{matrix} x_0^2 - x_1^2 - x_2^2 + x_3^2 = 0 & (+--+) \\ x_0^2 + x_1^2 - x_2^2 - x_3^2 = 0 & (++--) \\ x_0^2 - x_1^2 + x_2^2 - x_3^2 = 0 & (+-+-) \\ -x_0^2 + x_1^2 + x_2^2 - x_3^2 = 0 & (-++-) \\ -x_0^2 - x_1^2 + x_2^2 + x_3^2 = 0 & (--++) \\ -x_0^2 + x_1^2 - x_2^2 + x_3^2 = 0 & (-+-+) \end{matrix} \quad (4.24)$$

are various options for 4-dimensional tori.

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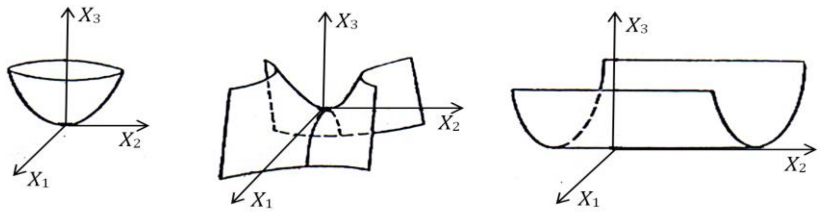
3rd class: is a 4-spaces, the signatures of which consist of three identical signs and one opposite [2]:

$$
\begin{array}{ll}
-x_0^2 - x_1^2 - x_2^2 + x_3^2 = 0 & (- - - +) \\
-x_0^2 - x_1^2 + x_2^2 - x_3^2 = 0 & (- - + -) \\
-x_0^2 + x_1^2 - x_2^2 - x_3^2 = 0 & (- + - -) \\
x_0^2 - x_1^2 - x_2^2 - x_3^2 = 0 & (+ - - -) \\
x_0^2 + x_1^2 + x_2^2 - x_3^2 = 0 & (+ + + -) \\
x_0^2 + x_1^2 - x_2^2 + x_3^2 = 0 & (+ + - +) \\
x_0^2 - x_1^2 + x_2^2 + x_3^2 = 0 & (+ - + +) \\
-x_0^2 + x_1^2 + x_2^2 + x_3^2 = 0 & (- + + +)
\end{array}
\qquad (4.25)
$$

are oval 4-surfaces: ellipsoids, elliptical paraboloids, two-sheet hyperboloids.

A simplified illustration of the relationship between the signature of a 2-dimensional space and its topology is shown in Fig. 4.3. It can be seen from this Figure that the signature of the quadratic form is uniquely related to the topology of 2-dimensional space.



a) signature (+ +) $x_3 = x_1^2 + x_2^2$ the parabolic surface

b) signature (– +); $x_3 = x_2^2 - x_1^2$ the saddle surface

c) signature (+ 0) $x_3 = x_1^2$ the U-shaped surface

Fig. 4.3. An illustration of the relationship between the signature of a 2-dimensional space and its topology [12]

The sixteen types of signatures (4.22), corresponding to the 16 types of topologies metric spaces, form the matrix

$$
sign\left(ds^{(ab)}\right) = \begin{pmatrix}
(++++)^{00} & (+++-)^{10} & (-++-)^{20} & (++-+)^{30} \\
(---+)^{01} & (-+++)^{11} & (--++)^{21} & (-+-+)^{31} \\
(+--+)^{02} & (++--)^{12} & (+---)^{22} & (+-++)^{32} \\
(--+-)^{03} & (+-+-)^{13} & (-+--)^{23} & (----)^{33}
\end{pmatrix}, \qquad (4.26)
$$

The properties of the signature matrix (4.26) partly coincide with the properties of the stignature matrix (3.2).

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4.5 The second stage of compactification of the extra dimensions. The "$\lambda_{m,n}$-vacuum balance condition"

At the second stage of compactification of the extra dimensions, we define the additive superposition of all 16 metrics (i.e., intervals) (4.21)

$$\begin{aligned} ds_\Sigma^2 = {} & ds^{(+---)2} + ds^{(++++)2} + ds^{(---+)2} + ds^{(+--+)2} + \\ & + ds^{(--+-)2} + ds^{(++--)2} + ds^{(-+--)2} + ds^{(+-+-)2} + \\ & + ds^{(-+++)2} + ds^{(----)2} + ds^{(+++-)2} + ds^{(-++-)2} + \\ & + ds^{(++-+)2} + ds^{(--++)2} + ds^{(+-++)2} + ds^{(-+-+)2} = 0. \end{aligned} \quad (4.26)$$

Indeed, adding the metrics (4.21), we obtain (4.27)

$$\begin{aligned} ds_\Sigma^2 = {} & (dx_0dx_0 - dx_1dx_1 - dx_2dx_2 - dx_3dx_3) + (dx_0dx_0 + dx_1dx_1 + dx_2dx_2 + dx_3dx_3) + \\ & + (-dx_0dx_0 - dx_1dx_1 + dx_2dx_2 - dx_3dx_3) + (dx_0dx_0 - dx_1dx_1 - dx_2dx_2 + dx_3dx_3) + \\ & + (-dx_0dx_0 - dx_1dx_1 + dx_2dx_2 - dx_3dx_3) + (dx_0dx_0 + dx_1dx_1 - dx_2dx_2 - dx_3dx_3) + \\ & + (-dx_0dx_0 + dx_1dx_1 - dx_2dx_2 - dx_3dx_3) + (dx_0dx_0 - dx_1dx_1 + dx_2dx_2 - dx_3dx_3) + \\ & + (-dx_0dx_0 + dx_1dx_1 + dx_2dx_2 + dx_3dx_3) + (-dx_0dx_0 - dx_1dx_1 - dx_2dx_2 - dx_3dx_3) + \\ & + (dx_0dx_0 + dx_1dx_1 + dx_2dx_2 - dx_3dx_3) + (-dx_0dx_0 + dx_1dx_1 + dx_2dx_2 - dx_3dx_3) + \\ & + (dx_0dx_0 + dx_1dx_1 - dx_2dx_2 + dx_3dx_3) + (-dx_0dx_0 - dx_1dx_1 + dx_2dx_2 + dx_3dx_3) + \\ & + (dx_0dx_0 - dx_1dx_1 + dx_2dx_2 + dx_3dx_3) + (-dx_0dx_0 + dx_1dx_1 - dx_2dx_2 + dx_3dx_3) = 0. \end{aligned}$$

Instead of summing homogeneous terms in Expression (4.27), only the signs in front of these terms can be summed. Therefore, Expression (4.21) can be represented in the ranked form

(4.28)

0 =	(0 0 0 0)	+	(0 0 0 0)	= 0
0 =	(+ + + +)	+	(– – – –)	= 0
0 =	(– – – +)	+	(+ + + –)	= 0
0 =	(+ – – +)	+	(– + + –)	= 0
0 =	(– – + –)	+	(+ + – +)	= 0
0 =	(+ + – –)	+	(– – + +)	= 0
0 =	(– + – –)	+	(+ – + +)	= 0
0 =	(+ – + –)	+	(– + – +)	= 0
0 =	(– + + +)	+	(+ – – –)	= 0
0 =	(0 0 0 0) +	+	(0 0 0 0) +	= 0

where the addition (or subtraction) of the signs is performed according to the following rules:

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$$\begin{array}{ll|ll} (+) + (+) = 2(+) & (–) + (+) = (0) & (+) – (+) = (0) & (–) – (+) = 2(–) \\ (+) + (–) = (0) & (–) + (–) = 2(–), & (+) – (–) = 2(+) & (–) – (–) = (0). \end{array} \quad (4.29)$$

The sum of epy signs, both in the columns of the ranked Expression (4.28) and in their rows between the ranks, is equal to zero. Therefore, this ranked Expression will be called "the splitting of the metric zero".

The additive imposition of the 16 metric 4-spaces with the intervals (4.21) and with the corresponding signatures (i.e., topologies) (4.22) at each point of the $\lambda_{m,n}$-vacuum leads to the formation of a zero Ricci-flat space. This space is very similar to the 6-dimensional Calabi-Yau manifold (see Fig. 4.4),

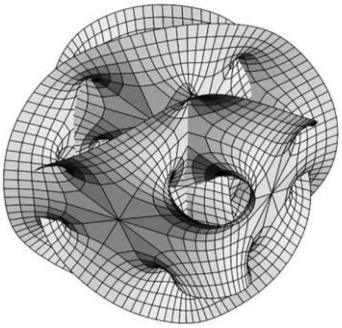

Fig. 4.4. One of the realizations of a two-dimensional projection of three-dimensional visualization local section of a 6-dimensional Calabi-Yau manifold

The second stage of compactification of the additional (mathematical) dimensions led to their complete reduction. At the same time, the ranking Expression (4.28) is a mathematical formulation of the "$\lambda_{m,n}$-vacuum balance condition".

Definition 4.5.1 *"The $\lambda_{m,n}$-vacuum balance condition" is a statement that any manifestations in the $\lambda_{m,n}$-vacuum should be mutually opposite: wave - anti-wave, convexity - concavity, motion - antimotion, compression - stretching, etc., so that on average they are equal to zero. The local $\lambda_{m,n}$-vacuum manifestations and anti-manifestations can be shifted and rotated relative to each other, but, on average, over the entire $\lambda_{m,n}$-vacuum region, they completely compensate for each other's manifestations, restoring the "$\lambda_{m,n}$-vacuum balance".*

4.6 Operations with metric ranks

Ranked Expression (4.28) allows to perform some operations in the vicinity of the investigated point O (see Fig. 2.5) without violating the "$\lambda_{m,n}$-vacuum balance condition". Such operations include, for example, a symmetric transfer of the first columns to the other side of the equality with the inverted signs:

$$
\begin{array}{lccccc}
0 = & \underline{(0\ \ 0\ \ 0)} & + & \underline{(0\ \ 0\ \ 0)} & = 0 \\
- = & (+\ \ +\ \ +) & + & (-\ \ -\ \ -) & = + \\
+ = & (-\ \ -\ \ +) & + & (+\ \ +\ \ -) & = - \\
- = & (-\ \ -\ \ +) & + & (+\ \ +\ \ -) & = + \\
+ = & (-\ \ +\ \ -) & + & (+\ \ -\ \ +) & = - \\
- = & (+\ \ -\ \ -) & + & (-\ \ +\ \ +) & = + \\
+ = & (+\ \ -\ \ -) & + & (-\ \ +\ \ +) & = - \\
- = & (-\ \ +\ \ -) & + & (+\ \ -\ \ +) & = + \\
+ = & \underline{(+\ \ +\ \ +)} & + & \underline{(-\ \ -\ \ -)} & = - \\
0 = & (0\ \ 0\ \ 0)_{+} & + & (0\ \ 0\ \ 0)_{+} & = 0
\end{array}
\qquad (4.30)
$$

or the transfer of any of the lines from the numerators of the ranks (4.28) to their the denominators, also with the inversion of a signs, for example:

$$
\begin{array}{lccccc}
0 = & \underline{(0\ \ 0\ \ 0\ \ 0)} & + & \underline{(0\ \ 0\ \ 0\ \ 0)} & = 0 \\
0 = & (+\ \ +\ \ +\ \ +) & + & (-\ \ -\ \ -\ \ -) & = 0 \\
0 = & (-\ \ -\ \ -\ \ +) & + & (+\ \ +\ \ +\ \ -) & = 0 \\
0 = & (+\ \ -\ \ -\ \ +) & + & (-\ \ +\ \ +\ \ -) & = 0 \\
0 = & (+\ \ +\ \ -\ \ -) & + & (-\ \ -\ \ +\ \ +) & = 0 \\
0 = & (-\ \ +\ \ -\ \ -) & + & (+\ \ -\ \ +\ \ +) & = 0 \\
0 = & (+\ \ -\ \ +\ \ -) & + & (-\ \ +\ \ -\ \ +) & = 0 \\
0 = & \underline{(-\ \ +\ \ +\ \ +)} & + & \underline{(+\ \ -\ \ -\ \ -)} & = 0 \\
0 = & (+\ \ +\ \ -\ \ +)_{+} & + & (-\ \ -\ \ +\ \ -)_{+} & = 0
\end{array}
\qquad (4.31)
$$

Such ranked operations correspond to certain symmetric vacuum manifestations, which will be considered below and investigated in [14, 15].

4.7 The double-sided $\lambda_{m,n}$-vacuum

As it was shown in the previous paragraph the $\lambda_{m,n}$-vacuum balance is not violated if at the ranks (4.28) transfer one line of a signs (i.e., the signature) from the numerator to the denominator with the change of a signs to opposite ones according to the rules of arithmetic.

For example, we transfer the signatures (– + + +) and (+ – – –) from the numerators of the ranks (4.28) to their the denominators

$$\begin{array}{cccc}
(+\ +\ +\ +) & + & (-\ -\ -\ -) & =0 \\
(-\ -\ -\ +) & + & (+\ +\ +\ -) & =0 \\
(+\ -\ -\ +) & + & (-\ +\ +\ -) & =0 \\
(-\ -\ +\ -) & + & (+\ +\ -\ +) & =0 \\
(+\ +\ -\ -) & + & (-\ -\ +\ +) & =0 \\
(-\ +\ -\ -) & + & (+\ -\ +\ +) & =0 \\
\underline{(+\ -\ +\ -)} & + & \underline{(-\ +\ -\ +)} & =0 \\
(+\ -\ -\ -)_+ & + & (-\ +\ +\ +)_+ & =0\ .
\end{array} \qquad (4.32)$$

In this case, in the denominator of the left-hand rank (4.32), we get the signature of the Minkowski 4-space (+ – – –), and in the denominator of the right-hand rank (4.31), we get the signature of the Minkowski 4-antispace (– + + +).

The ranked Expression (4.32) is equivalent to the fact that the addition (i.e., additive overlay) of the 7 metric spaces with signatures (topologies) indicated in the numerator of the left-hand rank (4.32) form a metric Minkowski 4-space with the interval

$$ds^{(+---)2} = c^2dt^2 - dx^2 - dy^2 - dz^2 = dx_0{}^2 - dx_1{}^2 - dx_2{}^2 - dx_3{}^2\ , \qquad (4.33)$$

where

$$ds^{(+---)2} = ds^{(++++)2} + ds^{(---+)2} + ds^{(+--+)2} + ds^{(--+-)2} + \qquad (4.34)$$
$$+\ ds^{(++--)2} + ds^{(-+--)2} + ds^{(+-+-)2},$$

this 4-space will be conventionally called the outer side of the $\lambda_{m,n}$-vacuum (or a «*subcont*» is an abbreviation from the conventional name "substantial continuum").

In the same way, the additive superposition of the 7 metric spaces with signatures indicated in the numerator of the right-hand rank (4.32) forms a metric Minkowski 4-antispace with the interval

$$ds^{(-+++)2} = -c^2dt^2 + dx^2 + dy^2 + dz^2 = -dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2\ , \qquad (4.35)$$

where

$$ds^{(-+++)2} = ds^{(----)2} + ds^{(+++-)2} + ds^{(-++-)2} + ds^{(++-+)2} + \qquad (4.36)$$
$$+\, ds^{(--++)2} + ds^{(+-++)2} + ds^{(-+-+)2}.$$

This 4-space will be conventionally called the inner side of the $\lambda_{m,n}$-vacuum (or an «*antisubcont*» is an abbreviation from the conventional name "antisubstantial continuum").

Definition 4.7.1 *The concepts of a «subcont» and an «antisubcont» are mental constructions, which are intended only to create the illusion of "visibility" of two adjacent mutually opposite sides of one $\lambda_{m,n}$-vacuum. These concepts are introduced only to facilitate the visualization of intra-vacuum processes, but they have nothing to do with reality. However, in terms of these mental concepts, real vacuum effects can be inspired.*

In expanded form, the ranks (4.32) have the following form (4.37)

$$\begin{array}{ll}
ds^{(++++)2} = dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 & ds^{(----)2} = -dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2 \\
ds^{(---+)2} = -dx_0^2 - dx_1^2 - dx_2^2 + dx_3^2 & ds^{(+++-)2} = dx_0^2 + dx_1^2 + dx_2^2 - dx_3^2 \\
ds^{(+--+)2} = dx_0^2 - dx_1^2 - dx_2^2 + dx_3^2 & ds^{(-++-)2} = -dx_0^2 + dx_1^2 + dx_2^2 - dx_3^2 \\
ds^{(--+-)2} = -dx_0^2 - dx_1^2 + dx_2^2 - dx_3^2 & ds^{(++-+)2} = dx_0^2 + dx_1^2 - dx_2^2 + dx_3^2 \\
ds^{(-+--)2} = -dx_0^2 + dx_1^2 - dx_2^2 - dx_3^2 & ds^{(+-++)2} = dx_0^2 - dx_1^2 + dx_2^2 + dx_3^2 \\
ds^{(+-+-)2} = dx_0^2 - dx_1^2 + dx_2^2 - dx_3^2 & ds^{(-+-+)2} = -dx_0^2 + dx_1^2 - dx_2^2 + dx_3^2 \\
\underline{ds^{(++--)2} = dx_0^2 + dx_1^2 - dx_2^2 - dx_3^2} & \underline{ds^{(--++)2} = -dx_0^2 - dx_1^2 + dx_2^2 + dx_3^2} \\
ds^{(+---)2} = dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2 & ds^{(-+++)2} = -dx_0^2 + d\,x_1^2 + dx_2^2 + dx_3^2
\end{array}$$

The operation described by the ranked Expression (4.32) makes it possible to "reveal" the two-sided of the $\lambda_{m,n}$-vacuum with the number of the mathematical dimensions 4 + 4 = 8 = 2^3. Therefore, we propose to call such a two-sided 8 - dimensional space a 2^3-$\lambda_{m,n}$-vacuum, under preservation of the 2^3-$\lambda_{m,n}$-vacuum balance condition

$$ds^{(+---)2} + ds^{(-+++)2} = 0, \qquad (4.38)$$

with a ranked equivalent

$$(+\,-\,-\,-) + (-\,+\,+\,+) = (0\ 0\ 0\ 0) \qquad (4.39)$$

or in transposed form

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$$\begin{array}{l} (+\,-\,-\,-) \\ \underline{(-\,+\,+\,+)} \\ (0\;0\;\;0\;0)_+ \end{array} \qquad (4.40)$$

In this terminology, the ranked Expression (4.28) is equivalent to the 2^6-$\lambda_{m,n}$-vacuum balance condition with the sixteen 4-dimensional sides (or faces), since the number of mathematical dimensions of a such 16-sided space is $4 \times 16 = 64 = 2^6$.

Let's recall that in the general theory of relativity (GR) of A. Einstein there is only one metric 4-space with one signature, for example, (+ – – –). Whereas in the light-geometry of vacuum developed here, the any $\lambda_{m,n}$-vacuum can have at least two sides (i.e., mutually opposite metric 4-spaces): the *outer side* (i.e., subcont) and the *inner side* (i.e., antisubcont), with the corresponding mutually opposite signatures (+ – – –) and (– + + +).

4.8 The binary triads

Note that not only the ranked Expression (4.32) leads to the antipode dyad: the Minkowski 4-space with signatures (+ – – –) and the Minkowski 4-antispace with signatures (– + + +).

The ranked binary triads presented below lead to this dyad too

$$\begin{array}{l} (-\,-\,-\,+) + (+\,+\,+\,-) = 0 \\ (+\,-\,+\,-) + (-\,+\,-\,+) = 0 \\ \underline{(+\,+\,-\,-)} + \underline{(-\,-\,+\,+)} = 0 \\ (+\,-\,-\,-)_+ + (-\,+\,+\,+)_+ = 0 \end{array} \qquad (4.41)$$

$$\begin{array}{l} (-\,-\,+\,-) + (+\,+\,-\,+) = 0 \\ (+\,+\,-\,-) + (-\,-\,+\,+) = 0 \\ \underline{(+\,-\,-\,+)} + \underline{(-\,+\,+\,-)} = 0 \\ (+\,-\,-\,-)_+ + (-\,+\,+\,+)_+ = 0 \end{array} \qquad (4.42)$$

$$\begin{array}{l} (-\,-\,+\,-) + (+\,+\,-\,+) = 0 \\ (+\,+\,-\,-) + (-\,-\,+\,+) = 0 \\ \underline{(+\,-\,-\,+)} + \underline{(-\,+\,+\,-)} = 0 \\ (+\,-\,-\,-)_+ + (-\,+\,+\,+)_+ = 0 \end{array} \qquad (4.43)$$

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These ranked Expressions (which we will call binary triads) also satisfy the $\lambda_{m,n}$-vacuum balance condition, and play an important role in vacuum chromodynamics developed in [14,15].

4.9 The transverse stratification of the «vacuum»

Like the ranked Expression (4.31), any pair of a metric 4-spaces with mutually opposite signatures can be represented as a sum of the seven metric 4-spaces with other signatures.

For example, the conjugate pair of intervals $ds^{(-\,+\,+\,-)2}$ and $ds^{(+\,-\,-\,+)2}$ with mutually opposite signatures (– + + –) and (+ – – +) can be expressed by an additive superposition of the seven metric 4-spaces with signatures

$$\begin{array}{lcll}
-(+\ +\ +\ +) & + & (-\ -\ -\ -) & = 0 \\
(-\ -\ -\ +) & + & (+\ +\ +\ -) & = 0 \\
(-\ -\ +\ -) & + & (+\ +\ -\ +) & = 0 \\
(+\ +\ -\ -) & + & (-\ -\ +\ +) & = 0 \\
(-\ +\ -\ -) & + & (+\ -\ +\ +) & = 0 \\
(+\ -\ +\ -) & + & (-\ +\ -\ +) & = 0 \\
\underline{(-\ +\ +\ +)} & + & \underline{(+\ -\ -\ -)} & = 0 \\
(-\ +\ +\ -)_+ & + & (+\ -\ -\ +)_+ & = 0
\end{array} \qquad (4.44)$$

Similarly, out of 256 metrics with signatures (4.9), 128 conjugate pairs of metrics can be distinguished, each of which can be expressed in terms of a superposition of a 7 + 7 = 14 metric 4-spaces. As a result of mathematical dimensions, it become 128 × 14 × 4 = 3584.

In turn, the conjugate pairs of a 4-spaces can be similarly decomposed into the sums of a 7 + 7 = 14 subspaces, and this can continue indefinitely.

The result is a vacuum light-geometry balanced with respect to the “split zero”, in which the “vacuum” is first represented in the form of an infinite number of $\lambda_{m,n}$-vacuums nested into each other (see § 2.1). This representation is called the longitudinal stratification "vacuum" (Definition № 2.1.5).

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Then each $\lambda_{m,n}$-vacuum splits into an infinite number of metric 4-subspaces with 16 types of signatures. At the same time, since all longitudinal layers (i.e., $\lambda_{m,n}$-vacuums) split in the same way, the entire "vacuum" is split into 4-subspaces with 16 types of signatures. Such a global splitting will be called a transverse stratification of the «vacuum».

Definition 4.9.1 *The transverse stratification of a "vacuum" is its representation as a global additive superposition of an infinite number of the metric 4-subspaces with 16 types of the signatures (topologies).*

Definition 4.9.2 *The transverse stratification of a $\lambda_{m,n}$-vacuum is its representation as an additive superposition of an infinite number of the metric 4-subspaces with 16 types of the signatures (topologies).*

5 THE SPINOR LIGHT-GEOMETRY

5.1 The spin-tensor representation of metrics with different signatures

Let's go back to considering the interval

$$ds^{(+---)2} = dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2 \text{ with signature } (+---). \quad (5.1)$$

For brevity, we omit the differentials in this Expression and write the quadratic form (5.1) in the form

$$s^2 = x_0^2 - x_1^2 - x_2^2 - x_3^2 \,. \quad (5.2)$$

As is known, the quadratic form (5.2) is the determinant of the Hermitian 2×2-matrix

$$\begin{pmatrix} x_0 + x_3 & x_1 + ix_2 \\ x_1 - ix_2 & x_0 - x_3 \end{pmatrix}_{\det} = \begin{vmatrix} x_0 + x_3 & x_1 + ix_2 \\ x_1 - ix_2 & x_0 - x_3 \end{vmatrix} = x_0^2 - x_1^2 - x_2^2 - x_3^2 = 0, \quad sign(+---). \quad (5.3)$$

The fact that this matrix is Hermitian can be easily verified by direct calculation

$$\begin{pmatrix} x_0 + x_3 & x_1 + ix_2 \\ x_1 - ix_2 & x_0 - x_3 \end{pmatrix}^{+} = \begin{pmatrix} x_0 + x_3 & x_1 + ix_2 \\ x_1 - ix_2 & x_0 - x_3 \end{pmatrix}. \quad (5.4)$$

In the spinor theory, matrices of the form (5.4) are called the mixed Hermitian spin-tensors of the second rank [9].

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We represent the 2×2-matrix (5.4) in the expanded form

$$A_4 = \begin{pmatrix} x_0 + x_3 & x_1 + ix_2 \\ x_1 - ix_2 & x_0 - x_3 \end{pmatrix} = x_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - x_1 \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} - x_2 \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - x_3 \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad (5.5)$$

where

$$\sigma_0^{(+---)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \quad \sigma_1^{(+---)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}; \quad \sigma_2^{(+---)} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \quad \sigma_3^{(+---)} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

is a set of Pauli matrices.

In the spinor theory an A_4-matrices of the form (5.5) are uniquely associated with quaternions of the type

$$q = x_0 + \vec{e}_1 x_1 + \vec{e}_2 x_2 + \vec{e}_3 x_3 , \qquad (5.6)$$

under the isomorphism

$$\vec{e}_1 \to \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}; \quad \vec{e}_2 \to \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \quad \vec{e}_3 \to \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \qquad (5.7)$$

Similarly, each quadratic form with the corresponding signature:

: (5.8)

$$s^{(++++)2} = x_0^2 + x_1^2 + x_2^2 + x_3^2 \qquad s^{(----)2} = -x_0^2 - x_1^2 - x_2^2 - x_3^2$$

$$s^{(---+)2} = -x_0^2 - x_1^2 - x_2^2 + x_3^2 \qquad s^{(+++-)2} = x_0^2 + x_1^2 + x_2^2 - x_3^2$$

$$s^{(+--+)2} = x_0^2 - x_1^2 - x_2^2 + x_3^2 \qquad s^{(-++-)2} = -x_0^2 + x_1^2 + x_2^2 - x_3^2$$

$$s^{(+---)2} = x_0^2 - x_1^2 - x_2^2 - x_3^2 \qquad s^{(-+++)2} = -x_0^2 + x_1^2 + x_2^2 + x_3^2$$

$$s^{(--+-)2} = -x_0^2 - x_1^2 + x_2^2 - x_3^2 \qquad s^{(++-+)2} = x_0^2 + x_1^2 - x_2^2 + x_3^2$$

$$s^{(-+--)2} = -x_0^2 + x_1^2 - x_2^2 - x_3^2 \qquad s^{(+-++)2} = x_0^2 - x_1^2 + x_2^2 + x_3^2$$

$$s^{(+-+-)2} = x_0^2 - x_1^2 + x_2^2 - x_3^2 \qquad s^{(-+-+)2} = -x_0^2 + x_1^2 - x_2^2 + x_3^2$$

$$s^{(++--)2} = x_0^2 + x_1^2 - x_2^2 - x_3^2 \qquad s^{(--++)2} = -x_0^2 - x_1^2 + x_2^2 + x_3^2$$

can be represented as a spin-tensors or as an A_4-matrix, which are shown in the Table 5.1:

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Table 5.1

1	$\begin{pmatrix} x_0+ix_3 & ix_1+x_2 \\ ix_1-x_2 & x_0-ix_3 \end{pmatrix}_{\det} = x_0^2+x_1^2+x_2^2+x_3^2=0; \qquad sign(++++)$ $\begin{pmatrix} x_0+ix_3 & ix_1+x_2 \\ ix_1-x_2 & x_0-ix_3 \end{pmatrix} = x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + x_3\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix};$ *where* $\sigma_0^{(++++)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \quad \sigma_1^{(++++)} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}; \quad \sigma_2^{(++++)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; \quad \sigma_3^{(++++)} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}.$
2	$\begin{pmatrix} x_0+x_3 & ix_1+x_2 \\ ix_1-x_2 & x_0-x_3 \end{pmatrix}_{\det} = x_0^2+x_1^2+x_2^2-x_3^2=0; \qquad sign(+++-)$ $\begin{pmatrix} x_0+x_3 & ix_1+x_2 \\ ix_1-x_2 & x_0-x_3 \end{pmatrix} = x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - x_3\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix};$ *where* $\sigma_0^{(+++-)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \quad \sigma_1^{(+++-)} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}; \quad \sigma_2^{(+++-)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; \quad \sigma_3^{(+++-)} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}.$
3	$\begin{pmatrix} x_0+ix_3 & ix_1+x_2 \\ ix_1-x_2 & -x_0+ix_3 \end{pmatrix}_{\det} = -x_0^2+x_1^2+x_2^2-x_3^2=0; \qquad sign(-++-)$ $\begin{pmatrix} x_0+ix_3 & ix_1+x_2 \\ ix_1-x_2 & -x_0+ix_3 \end{pmatrix} = -x_0\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - x_3\begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix};$ *where* $\sigma_0^{(-++-)} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}; \quad \sigma_1^{(-++-)} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}; \quad \sigma_2^{(-++-)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; \quad \sigma_3^{(-++-)} = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}.$
4	$\begin{pmatrix} x_0+ix_3 & x_1+x_2 \\ -x_1+x_2 & x_0-ix_3 \end{pmatrix}_{\det} = x_0^2+x_1^2-x_2^2+x_3^2=0; \qquad sign(++-+)$ $\begin{pmatrix} x_0+ix_3 & x_1+x_2 \\ -x_1+x_2 & x_0-ix_3 \end{pmatrix} = x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} + x_3\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix};$ *where* $\sigma_0^{(++-+)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \quad \sigma_1^{(++-+)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; \quad \sigma_2^{(++-+)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}; \quad \sigma_3^{(++-+)} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix};$

5	$\begin{pmatrix} x_0+x_3 & ix_1+x_2 \\ -ix_1+x_2 & -x_0+x_3 \end{pmatrix}_{\det} = -x_0^2 - x_1^2 - x_2^2 + x_3^2 = 0; \quad sign(---+)$ $\begin{pmatrix} x_0+x_3 & ix_1+x_2 \\ -ix_1+x_2 & -x_0+x_3 \end{pmatrix} = -x_0\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} + x_3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix};$ *where* $\sigma_0^{(---+)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1^{(---+)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \sigma_2^{(---+)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \sigma_3^{(---+)} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}.$
6	$\begin{pmatrix} x_0+x_3 & ix_1+x_2 \\ ix_1-x_2 & -x_0+x_3 \end{pmatrix}_{\det} = -x_0^2 + x_1^2 + x_2^2 + x_3^2 = 0; \quad sign(-+++)$ $\begin{pmatrix} x_0+x_3 & ix_1+x_2 \\ ix_1-x_2 & -x_0+x_3 \end{pmatrix} = -x_0\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + x_3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix};$ *where* $\sigma_0^{(-+++)} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1^{(-+++)} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \quad \sigma_2^{(-+++)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \sigma_3^{(-+++)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$
7	$\begin{pmatrix} x_0+x_3 & x_1+x_2 \\ x_1-x_2 & -x_0+x_3 \end{pmatrix}_{\det} = -x_0^2 - x_1^2 + x_2^2 + x_3^2 = 0; \quad sign(--++)$ $\begin{pmatrix} x_0+x_3 & x_1+x_2 \\ x_1-x_2 & -x_0+x_3 \end{pmatrix} = -x_0\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + x_3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix};$ *where* $\sigma_0^{(--++)} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}; \quad \sigma_1^{(--++)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}; \quad \sigma_2^{(--++)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; \quad \sigma_3^{(--++)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix};$
8	$\begin{pmatrix} x_0+x_3 & -x_1+x_2 \\ x_1+x_2 & -x_0+x_3 \end{pmatrix}_{\det} = -x_0^2 + x_1^2 - x_2^2 + x_3^2 = 0; \quad sign(-+-+)$ $\begin{pmatrix} x_0+x_3 & -x_1+x_2 \\ x_1+x_2 & -x_0+x_3 \end{pmatrix} = -x_0\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} + x_3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix};$ *where* $\sigma_0^{(-+-+)} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1^{(-+-+)} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2^{(-+-+)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \sigma_3^{(-+-+)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$

9	$\begin{pmatrix} x_0 - ix_3 & x_1 - ix_2 \\ x_1 + ix_2 & x_0 + ix_3 \end{pmatrix}_{\det} = x_0^2 - x_1^2 - x_2^2 + x_3^2 = 0; \quad sign(+--+)$ $\begin{pmatrix} x_0 - ix_3 & x_1 - ix_2 \\ x_1 + ix_2 & x_0 + ix_3 \end{pmatrix} = x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} + x_3\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix};$ *where* $\sigma_0^{(+--+)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1^{(+--+)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \sigma_2^{(+--+)} = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}; \quad \sigma_3^{(+--+)} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}.$
10	$\begin{pmatrix} x_0 - x_3 & x_1 + x_2 \\ -x_1 + x_2 & x_0 + x_3 \end{pmatrix}_{\det} = x_0^2 + x_1^2 - x_2^2 - x_3^2 = 0; \quad sign(++--)$ $\begin{pmatrix} x_0 - x_3 & x_1 + x_2 \\ -x_1 + x_2 & x_0 + x_3 \end{pmatrix} = x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + x_1\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} - x_3\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix};$ *where* $\sigma_0^{(++--)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1^{(++--)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \sigma_2^{(++--)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \sigma_3^{(++--)} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$
11	$\begin{pmatrix} x_0 + x_3 & x_1 + ix_2 \\ x_1 - ix_2 & x_0 - x_3 \end{pmatrix}_{\det} = \begin{vmatrix} x_0 + x_3 & x_1 + ix_2 \\ x_1 - ix_2 & x_0 - x_3 \end{vmatrix} = x_0^2 - x_1^2 - x_2^2 - x_3^2 = 0; \quad sign(+---)$ $\begin{pmatrix} x_0 + x_3 & x_1 + ix_2 \\ x_1 - ix_2 & x_0 - x_3 \end{pmatrix} = x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - x_3\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix};$ *where* $\sigma_0^{(+---)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1^{(+---)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \sigma_2^{(+---)} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3^{(+---)} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}.$
12	$\begin{pmatrix} x_0 + ix_3 & x_1 + x_2 \\ x_1 - x_2 & x_0 - ix_3 \end{pmatrix}_{\det} = x_0^2 - x_1^2 + x_2^2 + x_3^2 = 0; \quad sign(+-++)$ $\begin{pmatrix} x_0 + ix_3 & x_1 + x_2 \\ x_1 - x_2 & x_0 - ix_3 \end{pmatrix} = x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + x_3\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix};$ *where* $\sigma_0^{(+-++)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1^{(+-++)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \sigma_2^{(+-++)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \sigma_3^{(+-++)} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}.$

13	$\begin{pmatrix} -x_0+ix_3 & x_1+x_2 \\ x_1-x_2 & x_0+ix_3 \end{pmatrix}_{\det} = -x_0^2-x_1^2+x_2^2-x_3^2=0; \quad sign(--+-)$ $\begin{pmatrix} -x_0+ix_3 & x_1+x_2 \\ x_1-x_2 & x_0+ix_3 \end{pmatrix} = -x_0\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - x_3\begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix};$ *where* $\sigma_0^{(--+-)} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \ \sigma_1^{(--+-)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \ \sigma_2^{(--+-)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \ \sigma_3^{(--+-)} = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}.$
14	$\begin{pmatrix} x_0-x_3 & x_1+x_2 \\ x_1-x_2 & x_0+x_3 \end{pmatrix}_{\det} = x_0^2-x_1^2+x_2^2-x_3^2=0; \quad sign(+-+-)$ $\begin{pmatrix} x_0-x_3 & x_1+x_2 \\ x_1-x_2 & x_0+x_3 \end{pmatrix} = x_0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - x_3\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix};$ *where* $\sigma_0^{(+-+-)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ \sigma_1^{(+-+-)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \ \sigma_2^{(+-+-)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \ \sigma_3^{(+-+-)} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$
15	$\begin{pmatrix} -x_0+ix_3 & x_1+x_2 \\ -x_1+x_2 & x_0+ix_3 \end{pmatrix}_{\det} = -x_0^2+x_1^2-x_2^2-x_3^2=0; \quad sign(-+--)$ $\begin{pmatrix} -x_0+ix_3 & x_1+x_2 \\ -x_1+x_2 & x_0+ix_3 \end{pmatrix} = -x_0\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + x_1\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} - x_3\begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix};$ *where* $\sigma_0^{(-+--)} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \ \sigma_1^{(-+--)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \ \sigma_2^{(-+--)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \ \sigma_3^{(-+--)} = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}.$
16	$\begin{pmatrix} -x_0+ix_3 & x_1+ix_2 \\ x_1-ix_2 & x_0+ix_3 \end{pmatrix}_{\det} = -x_0^2-x_1^2-x_2^2-x_3^2=0; \quad sign(----)$ $\begin{pmatrix} -x_0+ix_3 & x_1+ix_2 \\ x_1-ix_2 & x_0+ix_3 \end{pmatrix} = -x_0\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - x_1\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} - x_2\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - x_3\begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix};$ *where* $\sigma_0^{(----)} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \ \sigma_1^{(----)} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \ \sigma_2^{(----)} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_3^{(----)} = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}.$

Each an A_4-matrix from the Table 5.1 is assigned a "colored" quaternion with the corresponding stignature (3.20), where the objects represented below are used as the imaginary units

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$$\vec{e}_1 \to \sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \vec{e}_2 \to \sigma_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \vec{e}_3 \to \sigma_3 = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix} \quad \vec{e}_4 \to \sigma_4 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

$$\vec{e}_5 \to \sigma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad \vec{e}_6 \to \sigma_6 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \vec{e}_7 \to \sigma_7 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \quad \vec{e}_8 \to \sigma_8 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\vec{e}_9 \to \sigma_9 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \vec{e}_{10} \to \sigma_{10} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \vec{e}_{11} \to \sigma_{11} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \quad \vec{e}_{12} \to \sigma_{12} = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

$$\vec{e}_{13} \to \sigma_{13} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad \vec{e}_{14} \to \sigma_{14} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \quad \vec{e}_{15} \to \sigma_{15} = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix} \quad \vec{e}_{16} \to \sigma_{16} = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}, \tag{5.9}$$

where σ_{ij} are the Pauli-Cayley spin-matrices, which are generators of the Clifford algebra satisfying the conditions

$$\sigma_i^{(....)}\sigma_j^{(....)} + \sigma_j^{(....)}\sigma_i^{(....)} = \begin{cases} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \; npu \; i \neq j; \\ 2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \; npu \; i = j, \end{cases} \tag{5.10}$$

Table 5.1 shows only a special cases of the spin-tensor representations of quadratic forms. For example, the determinants of all thirty-five 2×2 matrices (Hermitian spin-tensors): (5.11)

$$\begin{pmatrix} x_0+x_3 & x_1+ix_2 \\ x_1-ix_2 & x_0-x_3 \end{pmatrix} \begin{pmatrix} x_0-x_3 & x_1-ix_2 \\ x_1+ix_2 & x_0+x_3 \end{pmatrix} \begin{pmatrix} x_0+x_3 & x_1-ix_2 \\ x_1+ix_2 & x_0-x_3 \end{pmatrix} \begin{pmatrix} x_0-x_3 & x_1+ix_2 \\ x_1-ix_2 & x_0+x_3 \end{pmatrix} \begin{pmatrix} ix_1-x_2 & -x_0+x_3 \\ x_0+x_3 & ix_1+x_2 \end{pmatrix}$$

$$\begin{pmatrix} x_0+x_1 & x_3+ix_2 \\ x_3-ix_2 & x_0-x_1 \end{pmatrix} \begin{pmatrix} x_0-x_1 & x_3-ix_2 \\ x_3+ix_2 & x_0+x_1 \end{pmatrix} \begin{pmatrix} x_0+x_1 & x_3-ix_2 \\ x_3+ix_2 & x_0-x_1 \end{pmatrix} \begin{pmatrix} x_0-x_1 & x_3+ix_2 \\ x_3-ix_2 & x_0+x_1 \end{pmatrix} \begin{pmatrix} ix_2-x_1 & -x_0+x_3 \\ x_0+x_3 & ix_2+x_1 \end{pmatrix}$$

$$\begin{pmatrix} x_0+x_2 & x_1+ix_3 \\ x_1-ix_3 & x_0-x_2 \end{pmatrix} \begin{pmatrix} x_0-x_2 & x_1-ix_3 \\ x_1+ix_3 & x_0+x_2 \end{pmatrix} \begin{pmatrix} x_0+x_2 & x_1-ix_3 \\ x_1+ix_3 & x_0-x_2 \end{pmatrix} \begin{pmatrix} x_0-x_2 & x_1+ix_3 \\ x_1-ix_3 & x_0+x_2 \end{pmatrix} \begin{pmatrix} ix_1-x_3 & -x_0+x_2 \\ x_0+x_2 & ix_1+x_3 \end{pmatrix}$$

$$\begin{pmatrix} x_0+x_3 & x_2+ix_1 \\ x_2-ix_1 & x_0-x_3 \end{pmatrix} \begin{pmatrix} x_0-x_3 & x_2-ix_1 \\ x_2+ix_1 & x_0+x_3 \end{pmatrix} \begin{pmatrix} x_0+x_3 & x_2-ix_1 \\ x_2+ix_1 & x_0-x_3 \end{pmatrix} \begin{pmatrix} x_0-x_3 & x_2+ix_1 \\ x_2-ix_1 & x_0+x_3 \end{pmatrix} \begin{pmatrix} ix_3-x_2 & -x_0+x_1 \\ x_0+x_1 & ix_3+x_2 \end{pmatrix}$$

$$\begin{pmatrix} x_0+x_1 & x_2+ix_3 \\ x_2-ix_3 & x_0-x_1 \end{pmatrix} \begin{pmatrix} x_0-x_1 & x_2-ix_3 \\ x_2+ix_3 & x_0+x_1 \end{pmatrix} \begin{pmatrix} x_0+x_1 & x_2-ix_3 \\ x_2+ix_3 & x_0-x_1 \end{pmatrix} \begin{pmatrix} x_0-x_1 & x_2+ix_3 \\ x_2-ix_3 & x_0+x_1 \end{pmatrix} \begin{pmatrix} ix_2-x_3 & -x_0+x_1 \\ x_0+x_1 & ix_2+x_3 \end{pmatrix}$$

$$\begin{pmatrix} x_0+x_2 & x_3+ix_1 \\ x_3-ix_1 & x_0-x_2 \end{pmatrix} \begin{pmatrix} x_0-x_2 & x_3-ix_1 \\ x_3+ix_1 & x_0+x_2 \end{pmatrix} \begin{pmatrix} x_0+x_2 & x_3-ix_1 \\ x_3+ix_1 & x_0-x_2 \end{pmatrix} \begin{pmatrix} x_0-x_2 & x_3+ix_1 \\ x_3-ix_1 & x_0+x_2 \end{pmatrix} \begin{pmatrix} ix_3-x_1 & -x_0+x_2 \\ x_0+x_2 & ix_3+x_1 \end{pmatrix}$$

$$\begin{pmatrix} ix_2-x_1 & -x_0+x_3 \\ x_0+x_3 & ix_2+x_1 \end{pmatrix} \begin{pmatrix} ix_2-x_1 & x_0+x_3 \\ -x_0+x_3 & ix_2+x_1 \end{pmatrix} \begin{pmatrix} ix_1-x_3 & x_0+x_2 \\ -x_0+x_2 & ix_1+x_3 \end{pmatrix} \begin{pmatrix} ix_2-x_3 & x_0+x_1 \\ -x_0+x_1 & ix_2+x_3 \end{pmatrix} \begin{pmatrix} ix_3-x_1 & x_0+x_2 \\ -x_0+x_2 & ix_3+x_1 \end{pmatrix}$$

equal to the same quadratic form $s^{(+---)2} = x_0^2 - x_1^2 - x_2^2 - x_3^2$.

In a number of cases, the discrete degeneracy (i.e., latent multivaluedness) of the initial ideal state of the $\lambda_{m,n}$-vacuum when deviating from ideality, can lead to splitting (quantization) into a discrete set of dissimilar states of its transverse layers.

Sixteen types of A_4-matrices are equivalent to 16 "color" quaternions (3.20). For clarity, all types of "colored" A_4-matrices are summarized in Table 5.2.

Table 5.2

Quadratic form	A_4-matrix	Stignature
$x_0^2+x_1^2+x_2^2+x_3^2$	$x_0\begin{pmatrix}1&0\\0&1\end{pmatrix}+x_1\begin{pmatrix}0&i\\i&0\end{pmatrix}+x_2\begin{pmatrix}0&1\\-1&0\end{pmatrix}+x_3\begin{pmatrix}i&0\\0&-i\end{pmatrix}$	$\{++++\}$
$x_0^2-x_1^2-x_2^2+x_3^2$	$x_0\begin{pmatrix}1&0\\0&1\end{pmatrix}-x_1\begin{pmatrix}0&-1\\-1&0\end{pmatrix}-x_2\begin{pmatrix}0&i\\-i&0\end{pmatrix}+x_3\begin{pmatrix}-i&0\\0&i\end{pmatrix}$	$\{+--+\}$
$x_0^2+x_1^2+x_2^2-x_3^2$	$x_0\begin{pmatrix}1&0\\0&1\end{pmatrix}+x_1\begin{pmatrix}0&i\\i&0\end{pmatrix}+x_2\begin{pmatrix}0&1\\-1&0\end{pmatrix}-x_3\begin{pmatrix}-1&0\\0&1\end{pmatrix}$	$\{+++-\}$
$x_0^2+x_1^2-x_2^2-x_3^2$	$x_0\begin{pmatrix}1&0\\0&1\end{pmatrix}+x_1\begin{pmatrix}0&1\\-1&0\end{pmatrix}-x_2\begin{pmatrix}0&-1\\-1&0\end{pmatrix}-x_3\begin{pmatrix}1&0\\0&-1\end{pmatrix}$	$\{++--\}$
$-x_0^2+x_1^2+x_2^2-x_3^2$	$-x_0\begin{pmatrix}-1&0\\0&1\end{pmatrix}+x_1\begin{pmatrix}0&i\\i&0\end{pmatrix}+x_2\begin{pmatrix}0&1\\-1&0\end{pmatrix}-x_3\begin{pmatrix}-i&0\\0&-i\end{pmatrix}$	$\{-++-\}$
$x_0^2-x_1^2-x_2^2-x_3^2$	$x_0\begin{pmatrix}1&0\\0&1\end{pmatrix}-x_1\begin{pmatrix}0&-1\\-1&0\end{pmatrix}-x_2\begin{pmatrix}0&-i\\i&0\end{pmatrix}-x_3\begin{pmatrix}1&0\\0&-1\end{pmatrix}$	$\{+---\}$
$x_0^2+x_1^2-x_2^2+x_3^2$	$x_0\begin{pmatrix}1&0\\0&1\end{pmatrix}+x_1\begin{pmatrix}0&1\\-1&0\end{pmatrix}-x_2\begin{pmatrix}0&-1\\-1&0\end{pmatrix}+x_3\begin{pmatrix}i&0\\0&-i\end{pmatrix}$	$\{++-+\}$
$x_0^2-x_1^2+x_2^2+x_3^2$	$x_0\begin{pmatrix}1&0\\0&1\end{pmatrix}-x_1\begin{pmatrix}0&-1\\-1&0\end{pmatrix}+x_2\begin{pmatrix}0&1\\-1&0\end{pmatrix}+x_3\begin{pmatrix}i&0\\0&-i\end{pmatrix}$	$\{+-++\}$
$-x_0^2-x_1^2-x_2^2+x_3^2$	$-x_0\begin{pmatrix}-1&0\\0&1\end{pmatrix}-x_1\begin{pmatrix}0&-i\\i&0\end{pmatrix}-x_2\begin{pmatrix}0&-1\\-1&0\end{pmatrix}+x_3\begin{pmatrix}1&0\\0&1\end{pmatrix}$	$\{---+\}$
$-x_0^2-x_1^2+x_2^2-x_3^2$	$-x_0\begin{pmatrix}1&0\\0&-1\end{pmatrix}-x_1\begin{pmatrix}0&-1\\-1&0\end{pmatrix}+x_2\begin{pmatrix}0&1\\-1&0\end{pmatrix}-x_3\begin{pmatrix}-i&0\\0&-i\end{pmatrix}$	$\{--+-\}$
$-x_0^2+x_1^2+x_2^2+x_3^2$	$-x_0\begin{pmatrix}-1&0\\0&1\end{pmatrix}+x_1\begin{pmatrix}0&i\\i&0\end{pmatrix}+x_2\begin{pmatrix}0&1\\-1&0\end{pmatrix}+x_3\begin{pmatrix}1&0\\0&1\end{pmatrix}$	$\{-+++\}$

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$x_0^2 - x_1^2 + x_2^2 - x_3^2$	$x_0\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - x_1\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix} + x_2\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix} - x_3\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$	$\{+ - + -\}$
$x_0^2 + x_1^2 - x_2^2 + x_3^2$	$-x_0\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix} - x_1\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix} + x_2\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix} + x_3\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$	$\{- - + +\}$
$x_0^2 - x_1^2 + x_2^2 + x_3^2$	$-x_0\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} + x_1\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix} - x_2\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix} - x_3\begin{pmatrix}-i & 0\\ 0 & -i\end{pmatrix}$	$\{- + - +\}$
$-x_0^2 + x_1^2 - x_2^2 + x_3^2$	$-x_0\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix} + x_1\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix} - x_2\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix} + x_3\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$	$\{- + - +\}$
$-x_0^2 - x_1^2 - x_2^2 - x_3^2$	$-x_0\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} - x_1\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix} - x_2\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} - x_3\begin{pmatrix}-i & 0\\ 0 & -i\end{pmatrix}$	$\{- - - -\}$

The Algebra of signature associates a zero-balanced superposition of the affine spaces with the 16 possible signatures:

$$
\begin{aligned}
ds_\Sigma = {} & (-dx_0 - dx_1 - dx_2 - dx_3) + (dx_0 + dx_1 + dx_2 + dx_3) + \\
& + (dx_0 + dx_1 + dx_2 - dx_3) + (-dx_0 - dx_1 - dx_2 + dx_3) + \\
& + (-dx_0 + dx_1 + dx_2 - dx_3) + (dx_0 - dx_1 - dx_2 + dx_3) + \\
& + (dx_0 + dx_1 - dx_2 + dx_3) + (-dx_0 - dx_1 + dx_2 - dx_3) + \\
& + (-dx_0 - dx_1 + dx_2 + dx_3) + (dx_0 + dx_1 - dx_2 - dx_3) + \\
& + (dx_0 - dx_1 + dx_2 + dx_3) + (-dx_0 + dx_1 - dx_2 - dx_3) + \\
& + (-dx_0 + dx_1 - dx_2 + dx_3) + (dx_0 - dx_1 + dx_2 - dx_3) + \\
& + (dx_0 - dx_1 - dx_2 - dx_3) + (-dx_0 + dx_1 + dx_2 + dx_3) = 0,
\end{aligned}
\qquad (5.12)
$$

with one of the variants of an additive superposition of a 16-and A_4-matrices:

$$
\begin{aligned}
& x_0\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} + x_1\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} + x_2\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix} + x_3\begin{pmatrix}i & 0\\ 0 & -i\end{pmatrix} + \\
+ \; & x_0\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix} + x_1\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix} + x_2\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} + x_3\begin{pmatrix}-i & 0\\ 0 & i\end{pmatrix} + \\
+ \; & x_0\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} + x_1\begin{pmatrix}0 & i\\ -i & 0\end{pmatrix} + x_2\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix} + x_3\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} +
\end{aligned}
$$

$$
\begin{gathered}
+x_0\begin{pmatrix}-1&0\\0&1\end{pmatrix}+x_1\begin{pmatrix}0&1\\-1&0\end{pmatrix}+x_2\begin{pmatrix}0&-1\\1&0\end{pmatrix}+x_3\begin{pmatrix}-1&0\\0&1\end{pmatrix}+\\
+x_0\begin{pmatrix}1&0\\0&-1\end{pmatrix}+x_1\begin{pmatrix}0&-i\\i&0\end{pmatrix}+x_2\begin{pmatrix}0&1\\-1&0\end{pmatrix}+x_3\begin{pmatrix}i&0\\0&i\end{pmatrix}+\\
+x_0\begin{pmatrix}-1&0\\0&1\end{pmatrix}+x_1\begin{pmatrix}0&-1\\1&0\end{pmatrix}+x_2\begin{pmatrix}0&i\\-i&0\end{pmatrix}+x_3\begin{pmatrix}-1&0\\0&1\end{pmatrix}+\\
+x_0\begin{pmatrix}1&0\\0&-1\end{pmatrix}+x_1\begin{pmatrix}0&1\\-1&0\end{pmatrix}+x_2\begin{pmatrix}0&-1\\1&0\end{pmatrix}+x_3\begin{pmatrix}-i&0\\0&-i\end{pmatrix}+\\
+x_0\begin{pmatrix}-1&0\\0&1\end{pmatrix}+x_1\begin{pmatrix}0&-1\\1&0\end{pmatrix}+x_2\begin{pmatrix}0&1\\-1&0\end{pmatrix}+x_3\begin{pmatrix}i&0\\0&-i\end{pmatrix}+\\
+x_0\begin{pmatrix}1&0\\0&-1\end{pmatrix}+x_1\begin{pmatrix}0&i\\-i&0\end{pmatrix}+x_2\begin{pmatrix}0&-1\\1&0\end{pmatrix}+x_3\begin{pmatrix}1&0\\0&-1\end{pmatrix}+\\
+x_0\begin{pmatrix}-1&0\\0&1\end{pmatrix}+x_1\begin{pmatrix}0&1\\-1&0\end{pmatrix}+x_2\begin{pmatrix}0&1\\-1&0\end{pmatrix}+x_3\begin{pmatrix}-i&0\\0&i\end{pmatrix}+\\
+x_0\begin{pmatrix}1&0\\0&-1\end{pmatrix}+x_1\begin{pmatrix}0&-i\\i&0\end{pmatrix}+x_2\begin{pmatrix}0&-1\\1&0\end{pmatrix}+x_3\begin{pmatrix}1&0\\0&1\end{pmatrix}+\\
+x_0\begin{pmatrix}-1&0\\0&1\end{pmatrix}+x_1\begin{pmatrix}0&-1\\1&0\end{pmatrix}+x_2\begin{pmatrix}0&1\\-1&0\end{pmatrix}+x_3\begin{pmatrix}-1&0\\0&-1\end{pmatrix}+\\
+x_0\begin{pmatrix}1&0\\0&-1\end{pmatrix}+x_1\begin{pmatrix}0&i\\-i&0\end{pmatrix}+x_2\begin{pmatrix}0&-i\\-i&0\end{pmatrix}+x_3\begin{pmatrix}-1&0\\0&1\end{pmatrix}+\\
+x_0\begin{pmatrix}-1&0\\0&1\end{pmatrix}+x_1\begin{pmatrix}0&1\\-1&0\end{pmatrix}+x_2\begin{pmatrix}0&-1\\1&0\end{pmatrix}+x_3\begin{pmatrix}-i&0\\0&-i\end{pmatrix}+\\
+x_0\begin{pmatrix}1&0\\0&-1\end{pmatrix}+x_1\begin{pmatrix}0&-1\\1&0\end{pmatrix}+x_2\begin{pmatrix}0&1\\-1&0\end{pmatrix}+x_3\begin{pmatrix}1&0\\0&-1\end{pmatrix}+\\
+x_0\begin{pmatrix}-1&0\\0&1\end{pmatrix}+x_1\begin{pmatrix}0&1\\-1&0\end{pmatrix}+x_2\begin{pmatrix}0&i\\i&0\end{pmatrix}+x_3\begin{pmatrix}i&0\\0&i\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}.
\end{gathered}
\tag{5.13}
$$

The Expression (5.13) is equal to the zero 2×2-matrix corresponding to the $\lambda_{m,n}$-vacuum balance condition.

The mathematical apparatus presented here is convenient for solving a number of a problems associated with the multilayer intravacuum rotational processes.

5.2 Using a spin-tensors with different stignatures

Let's look at two examples using spin-tensors.

Example 1 Suppose are given a matrix-column, and her Hermitian conjugate matrix-line

$$\begin{pmatrix} s_1 \\ s_2 \end{pmatrix}, \quad \left(s_1^*, s_2^*\right), \tag{5.14}$$

that describe the state of the spinor.

The spin projection on the coordinate axis for the case when the metric 4-space has the signature $(+---)$ can be determined using the spin-tensors (5.4)

$$\left(s_1^*, s_2^*\right)\begin{pmatrix} x_0 + x_3 & x_1 + ix_2 \\ x_1 - ix_2 & x_0 - x_3 \end{pmatrix}\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} =$$

$$= x_0\left(s_1^*, s_2^*\right)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} - x_1\left(s_1^*, s_2^*\right)\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} - \tag{5.15}$$

$$- x_2\left(s_1^*, s_2^*\right)\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} - x_3\left(s_1^*, s_2^*\right)\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} =$$

$$= \left(s_1^* s_1 + s_2^* s_2\right)x_0 - \left(-s_2^* s_1 - s_2^* s_1\right)x_1 - \left(is_2^* s_1 - is_1^* s_2\right)x_2 - \left(-s_1^* s_1 + s_2^* s_2\right)x_3,$$

Example 2 Let a forward wave be described by

$$\tilde{\vec{E}}^{(+)} = \bar{a}_+ e^{-i\frac{2\pi}{\lambda}(ct-r)}, \tag{5.16}$$

and its reverse wave

$$\tilde{\vec{E}}^{(-)} = \bar{a}\_e^{i\frac{2\pi}{\lambda}(ct-r)}, \tag{5.17}$$

where a_+ and a_- are the forward and reverse wave amplitudes. In general, the complex numbers:

$$\bar{a}_+ = a_+ e^{i\varphi_+}, \qquad \bar{a}\_ = a\_e^{-i\varphi_-}, \qquad \bar{a}_+^* = a_+ e^{-i\varphi_+}, \qquad \bar{a}\_^* = a\_e^{i\varphi_-},$$

contain an information about the phases of the waves φ_+ and φ_-.

Mutually opposing waves (5.16) and (5.17) can be represented as a two-component spinor:

$$\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = |\psi\rangle = \begin{pmatrix} \bar{a}_+ e^{-i\frac{2\pi}{\lambda}(ct-r)} \\ \bar{a}_- e^{i\frac{2\pi}{\lambda}(ct-r)} \end{pmatrix}. \quad (5.18)$$

and an Hermitian conjugated spinor to him

$$(s_1^*, s_2^*) = |\psi\rangle^+ = \langle\psi| = \begin{pmatrix} \bar{a}_+^* e^{i\frac{2\pi}{\lambda}(ct-r)} & \bar{a}_-^* e^{-i\frac{2\pi}{\lambda}(ct-r)} \end{pmatrix} \quad (5.19)$$

The conditions of the normalization in this case are expressed by the equation

$$(s_1^*, s_2^*)\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \langle\psi|\psi\rangle = \begin{pmatrix} \bar{a}_+^* e^{i\frac{2\pi}{\lambda}(ct-r)} & \bar{a}_-^* e^{-i\frac{2\pi}{\lambda}(ct-r)} \end{pmatrix}\begin{pmatrix} \bar{a}_+ e^{-i\frac{2\pi}{\lambda}(ct-r)} \\ \bar{a}_- e^{i\frac{2\pi}{\lambda}(ct-r)} \end{pmatrix} = |\bar{a}_+|^2 + |\bar{a}_-|^2. \quad (5.20)$$

To find the spin projections (i.e., circular polarization) of the light beam on the coordinate axes, we use spin-tensors [8]

$$A_3 = \begin{pmatrix} x_3 & x_1 + ix_2 \\ x_1 - ix_2 & -x_3 \end{pmatrix} = x_1\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + x_2\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} + x_3\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (5.21)$$

which is associated with the 3-dimensional metric

$$\det(A_3) = \begin{pmatrix} x_3 & x_1 - ix_2 \\ x_1 - ix_2 & -x_3 \end{pmatrix}_{\det} = \begin{vmatrix} x_3 & x_1 - ix_2 \\ x_1 + ix_2 & -x_3 \end{vmatrix} = -(x_1^2 + x_2^2 + x_3^2). \quad (5.22)$$

with signature (– – –).

Putting $x_1 = x_2 = x_3 = 1$ into the Expression (5.21), we consider the projection of the spin on the coordinate axes

$$(s_1^*, s_2^*)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + (s_1^*, s_2^*)\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + (s_1^*, s_2^*)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} =$$

$$= (s_2^* s_1 + s_2^* s_1) + (-is_2^* s_1 + is_1^* s_2) + (s_1^* s_1 - s_2^* s_2). \quad (5.23)$$

Substituting to this Expression the spinors (5.19) and (5.20), we obtain the following three spin projection on the corresponding coordinate axis $x_1 = x$, $x_2 = y$, $x_3 = z$:

$$\langle s_x\rangle = \langle\psi|-\sigma_1|\psi\rangle = (s_1^*, s_2^*)\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}s_1\\ s_2\end{pmatrix} =$$

$$= \left(\bar{a}_+^* e^{i\frac{2\pi}{\lambda}(ct-r)},\quad \bar{a}_-^* e^{-i\frac{2\pi}{\lambda}(ct-r)}\right)\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\bar{a}_+ e^{-i\frac{2\pi}{\lambda}(ct-r)}\\ \bar{a}_- e^{i\frac{2\pi}{\lambda}(ct-r)}\end{pmatrix} = \bar{a}_-^*\bar{a}_+ e^{-i\frac{4\pi}{\lambda}(ct-r)} + \bar{a}_+^*\bar{a}_- e^{i\frac{4\pi}{\lambda}(ct-r)};$$

(5.24)

$$\langle s_y\rangle = \langle\psi|-\sigma_2|\psi\rangle = (s_1^*, s_2^*)\begin{pmatrix}0 & i\\ -i & 0\end{pmatrix}\begin{pmatrix}s_1\\ s_2\end{pmatrix} =$$

$$= \left(\bar{a}_+^* e^{i\frac{2\pi}{\lambda}(ct-r)},\quad \bar{a}_-^* e^{-i\frac{2\pi}{\lambda}(ct-r)}\right)\begin{pmatrix}0 & i\\ -i & 0\end{pmatrix}\begin{pmatrix}\bar{a}_+ e^{-i\frac{2\pi}{\lambda}(ct-r)}\\ \bar{a}_- e^{i\frac{2\pi}{\lambda}(ct-r)}\end{pmatrix} =$$

$$= \bar{a}_-^*\bar{a}_+ e^{-i\frac{4\pi}{\lambda}(ct-r)} + \bar{a}_+^*\bar{a}_- e^{i\frac{4\pi}{\lambda}(ct-r)} = i\left[\bar{a}_+^*\bar{a}_- e^{i\frac{4\pi}{\lambda}(ct-r)} - \bar{a}_-^*\bar{a}_+ e^{-i\frac{4\pi}{\lambda}(ct-r)}\right];$$

(5.25)

$$\langle s_z\rangle = \langle\psi|-\sigma_3|\psi\rangle = (s_1^*, s_2^*)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}s_1\\ s_2\end{pmatrix} =$$

$$\left(\bar{a}_+^* e^{i\frac{2\pi}{\lambda}(ct-r)},\quad \bar{a}_-^* e^{-i\frac{2\pi}{\lambda}(ct-r)}\right)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\bar{a}_+ e^{-i\frac{2\pi}{\lambda}(ct-r)}\\ \bar{a}_- e^{i\frac{2\pi}{\lambda}(ct-r)}\end{pmatrix} = |\bar{a}_+|^2 - |\bar{a}_-|^2. \qquad (5.26)$$

In the case $\varphi_+ = \varphi_- = 0$, Expressions (5.24) – (5.26) take the following simplified form:

$$\langle s_x\rangle = 2a_+a_- \cos\left[\frac{4\pi}{\lambda}(ct-r)\right] = 2a_+a_- \cos[2(t\omega - kr)], \qquad (5.27)$$

$$\langle s_y\rangle = 2a_+a_- \sin\left[\frac{4\pi}{\lambda}(ct-r)\right] = 2a_+a_- \sin[2(t\omega - kr)],$$

$$\langle s_z\rangle = |a_+|^2 - |a_-|^2.$$

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In the case of equality of the amplitudes of the forward and reverse waves $a_+ = a_-$, instead of the Expressions (5.27) we obtain the following averaged spin projections

$$\langle s_z \rangle = 0, \tag{5.29}$$

$$\langle s_x \rangle = 2a_+^2 \cos[2(\omega t - kr)],$$

$$\langle s_y \rangle = 2a_+^2 \sin[2(\omega t - kr)].$$

The projection of the spin (i.e., the rotating vector of the electric field) on the direction of propagation of the light beam Z is unchanged and equal to zero. Moreover, its projection onto the XY plane, perpendicular to the direction of propagation of this ray, rotates around the Z axis with an angular velocity $\omega = 4\pi c/\lambda$.

Thus, the spinor concept of the propagation of a conjugate pair of waves leads to a description of the circular polarization without invoking an additional hypotheses.

Similarly can be performed the analysis of wave propagation in a 3 - dimensional metric spaces with a signatures: (– – –), (+ – –), (– + –), (– – +), (+ + +), (– + +), (+ – +), (+ + –).

6 DIRAC'S "VACUUM"

6.1 The Dirac stratification of a $\lambda_{m,n}$-vacuum

Consider the Dirac's stratification of a quadratic form using for example the metric (6.1)

$$ds^2 = c^2dt^2 + dx^2 + dy^2 + dz^2 = dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2$$ with signature (+ + + +).

Let's represent this metric as a product of the two affine (linear) forms

$$ds^2 = ds'ds'' = (\gamma_0 dx_0' + \gamma_1 dx_1' + \gamma_2 dx_2' + \gamma_3 dx_3')\cdot(\gamma_0 dx_0'' + \gamma_1 dx_1'' + \gamma_2 dx_2'' + \gamma_3 dx_3'') \tag{6.2}$$

Expanding the parentheses in this Expression, we get

$$ds'ds'' = \sum_{\mu=0}^{3}\sum_{\eta=0}^{3}\gamma_\mu\gamma_\eta dx^\mu dx^\eta = \frac{1}{2}\sum_{\mu=0}^{3}\sum_{\eta=0}^{3}(\gamma_\mu\gamma_\eta + \gamma_\eta\gamma_\mu)dx^\mu dx^\eta. \quad (6.3)$$

There are at least two options for determining the quantities γ_μ that satisfy the equality condition of the Expressions (6.1) and (6.3):

1) the method of the Clifford aggregates (for example, quaternions);

2) Dirac's method.

In the first case, the linear forms included in the Expression (6.2) are represented as a pair of the affine aggregates:

$$ds' = \gamma_0 cdt' + \gamma_1 dx' + \gamma_2 dy' + \gamma_3 dz' \quad (6.4)$$

$$ds'' = \gamma_0 cdt'' + \gamma_1 dx'' + \gamma_2 dy'' + \gamma_3 dz'' \quad (6.5)$$

with the stignature {+ + + +},

where γ_μ are an objects satisfying the anticommutative condition of the Clifford algebra

$$\gamma_\mu\gamma_\eta + \gamma_\eta\gamma_\mu = 2\delta_{\mu\eta}, \quad (6.6)$$

where

$$\delta_{\mu\eta} = \begin{cases} 1 \ for \ \ \mu = \eta, \\ 0 \ for \ \ \mu \neq \eta. \end{cases} \text{ – the Kronecker symbols.} \quad (6.7)$$

In the second case, the Dirac method assumes instead of the Crohneker symbols (6.7) to use the unit matrix

$$\delta_{\mu\eta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (6.8)$$

then condition (6.6) is satisfied, for example, by the following set of the 4×4-Dirac matrices:

$$\gamma_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \gamma_1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad \gamma_2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \quad \gamma_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}. \tag{6.9}$$

These matrices can be regarded as generators of the corresponding Clifford algebra.

In this case, Expression (6.3) takes the matrix form

$$\left(ds_{ii}^2\right) = \sum_{\mu=0}^{3}\sum_{\eta=0}^{3} \gamma_\mu \gamma_\eta dx^\mu dx^\eta = \frac{1}{2}\sum_{\mu=0}^{3}\sum_{\eta=0}^{3} (\gamma_\mu \gamma_\eta + \gamma_\eta \gamma_\mu) dx^\mu dx^\eta, \tag{6.10}$$

where

$$\left(ds_{ii}^2\right) = \begin{pmatrix} ds_{00}^2 & 0 & 0 & 0 \\ 0 & ds_{11}^2 & 0 & 0 \\ 0 & 0 & ds_{22}^2 & 0 \\ 0 & 0 & 0 & ds_{33}^2 \end{pmatrix}. \tag{6.11}$$

The Expression (6.10), taking into account (6.8), can be represented as

$$\left(ds_{ii}^2\right) = \sum_{\mu=0}^{3}\sum_{\eta=0}^{3} \gamma_\mu \gamma_\eta dx^\mu dx^\eta = c^2 dt^2 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + dx^2 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} +$$

$$+ dy^2 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + dz^2 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{6.12}$$

Let's return to the quadratic form (6.1) and its Dirac stratification (6.10)

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$$\left(ds_{ii}^2\right)=\sum_{\mu=0}^{3}\sum_{\eta=0}^{3}\gamma_\mu\gamma_\eta dx^\mu dx^\eta=\sum_{\mu=0}^{3}\sum_{\eta=0}^{3}b_{\mu\eta}dx^\mu dx^\eta, \quad (6.13)$$

where
$$\gamma_\mu\gamma_\eta=b_{\mu\eta}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}. \quad (6.14)$$

Consider all possible ways to write the Expression (6.13).

We use the following basis of sixteen all possible $\gamma_\mu^{(\rho)}$-matrices of Dirac:

$$\gamma_0^{(0)}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\quad \gamma_1^{(0)}=\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\quad \gamma_2^{(0)}=\begin{pmatrix}i&0&0&0\\0&i&0&0\\0&0&i&0\\0&0&0&i\end{pmatrix}\quad \gamma_3^{(0)}=\begin{pmatrix}0&i&0&0\\i&0&0&0\\0&0&0&i\\0&0&i&0\end{pmatrix}$$

$$\gamma_0^{(1)}=\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{pmatrix}\quad \gamma_1^{(1)}=\begin{pmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{pmatrix}\quad \gamma_2^{(1)}=\begin{pmatrix}0&0&0&i\\0&0&i&0\\0&i&0&0\\i&0&0&0\end{pmatrix}\quad \gamma_3^{(1)}=\begin{pmatrix}0&0&i&0\\0&0&0&i\\i&0&0&0\\0&i&0&0\end{pmatrix}$$

$$\gamma_0^{(2)}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\quad \gamma_1^{(2)}=\begin{pmatrix}0&0&0&1\\0&0&1&0\\1&0&0&0\\0&1&0&0\end{pmatrix}\quad \gamma_2^{(2)}=\begin{pmatrix}i&0&0&0\\0&i&0&0\\0&0&0&i\\0&0&i&0\end{pmatrix}\quad \gamma_3^{(2)}=\begin{pmatrix}0&0&0&i\\0&0&i&0\\i&0&0&0\\0&i&0&0\end{pmatrix}$$

$$\gamma_0^{(3)}=\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\quad \gamma_1^{(3)}=\begin{pmatrix}0&0&1&0\\0&0&0&1\\0&1&0&0\\1&0&0&0\end{pmatrix}\quad \gamma_2^{(3)}=\begin{pmatrix}0&i&0&0\\i&0&0&0\\0&0&i&0\\0&0&0&i\end{pmatrix}\quad \gamma_3^{(3)}=\begin{pmatrix}0&0&i&0\\0&0&0&i\\0&i&0&0\\i&0&0&0\end{pmatrix}.$$

(6.15)

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The Dirac method, in contrast to the method of the affine aggregates, allows one to simultaneously "stratify" metric 4-spaces with four metrics that are components of the matrix (6.11).

In the Algebra of signatures, we consider quadratic forms (5.8) with a sixteen possible signatures:

(6.16)

$$ds^{(++++)2} = dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 \qquad ds^{(----)2} = -dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2$$

$$ds^{(---+)2} = -dx_0^2 - dx_1^2 - dx_2^2 + dx_3^2 \qquad ds^{(+++-)2} = dx_0^2 + dx_1^2 + dx_2^2 - dx_3^2$$

$$ds^{(+--+)2} = dx_0^2 - dx_1^2 - dx_2^2 + dx_3^2 \qquad ds^{(-++-)2} = -dx_0^2 + dx_1^2 + dx_2^2 - dx_3^2$$

$$ds^{(+---)2} = dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2 \qquad ds^{(-+++)2} = -dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2$$

$$ds^{(--+-)2} = -dx_0^2 - dx_1^2 + dx_2^2 - dx_3^2 \qquad ds^{(++-+)2} = dx_0^2 + dx_1^2 - dx_2^2 + dx_3^2$$

$$ds^{(-+--)2} = -dx_0^2 + dx_1^2 - dx_2^2 - dx_3^2 \qquad ds^{(+-++)2} = dx_0^2 - dx_1^2 + dx_2^2 + dx_3^2$$

$$ds^{(+-+-)2} = dx_0^2 - dx_1^2 + dx_2^2 - dx_3^2 \qquad ds^{(-+-+)2} = -dx_0^2 + dx_1^2 - dx_2^2 + dx_3^2$$

$$ds^{(++--)2} = dx_0^2 + dx_1^2 - dx_2^2 - dx_3^2 \qquad ds^{(--++)2} = -dx_0^2 - dx_1^2 + dx_2^2 + dx_3^2$$

Each of them can also be "stratified" by the Dirac method

$$\left(ds_{ii}^{(a,b)2}\right) = \sum_{\mu=0}^{3}\sum_{\eta=0}^{3} \gamma_{\mu}^{(a)}\gamma_{\eta}^{(b)} dx^{\mu} dx^{\eta} , \qquad (6.17)$$

where

$$\gamma_{\mu}^{(a)}\gamma_{\eta}^{(b)} = b_{\mu\eta}^{(ab)} , \qquad (6.17a)$$

but in this case each a $b_{\mu\eta}^{(ab)}$-matrix has the corresponding signature:

$$b_{\mu\eta}^{00}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\quad b_{\mu\eta}^{10}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix}\quad b_{\mu\eta}^{20}=\begin{pmatrix}-1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix}\quad b_{\mu\eta}^{30}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&1\end{pmatrix}$$

$$b_{\mu\eta}^{01}=\begin{pmatrix}-1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&1\end{pmatrix}\quad b_{\mu\eta}^{11}=\begin{pmatrix}-1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\quad b_{\mu\eta}^{21}=\begin{pmatrix}-1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\quad b_{\mu\eta}^{31}=\begin{pmatrix}-1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&1\end{pmatrix}$$

$$b_{\mu\eta}^{02}=\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&1\end{pmatrix}\quad b_{\mu\eta}^{12}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\quad b_{\mu\eta}^{22}=\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\quad b_{\mu\eta}^{32}=\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

$$b_{\mu\eta}^{03}=\begin{pmatrix}-1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix}\quad b_{\mu\eta}^{13}=\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix}\quad b_{\mu\eta}^{23}=\begin{pmatrix}-1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\quad b_{\mu\eta}^{33}=\begin{pmatrix}-1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}. \tag{6.18}$$

The signs in front of the ones in the diagonal $b_{\mu\eta}{}^{(ab)}$-matrices correspond to the sets of signs in the components of the signature matrix (4.26)

$$\begin{matrix}(++++)^{00} & (+++-)^{10} & (-++-)^{20} & (++-+)^{30}\\ (---+)^{01} & (-+++)^{11} & (--++)^{21} & (-+-+)^{31}\\ (+--+)^{02} & (++--)^{12} & (+---)^{22} & (+-++)^{32}\\ (--+-)^{03} & (+-+-)^{13} & (-+--)^{23} & (----)^{33}.\end{matrix}$$

In this paragraph, for brevity, we will temporarily omit the superscripts and instead of the "$b_{\mu\eta}{}^{(ab)}$-matrix" we will write the "$b_{\mu\eta}$-matrix".

Let's return to the Dirac stratification of the quadratic form (6.10)

$$\left(ds_{ii}^{2}\right)=\sum_{\mu=0}^{3}\sum_{\eta=0}^{3}\gamma_{\mu}\gamma_{\eta}dx^{\mu}dx^{\eta}=\sum_{\mu=0}^{3}\sum_{\eta=0}^{3}b_{\mu\eta}dx^{\mu}dx^{\eta}, \tag{6.19}$$

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where
$$\gamma_\mu \gamma_\eta = b_{\mu\eta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad (6.20)$$

and consider all possible options for its disclosure.

For each of the sixteen $\gamma_\mu{}^{(\rho)}$-matrices (6.15), we can choose a second $\gamma_\chi{}^{(\tau)}$-matrix from the same set, such that their product is equal to the $b_{\mu\eta}$-matrix (6.20). For example:

$$\begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \end{pmatrix} \begin{pmatrix} 0 & -i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \qquad (6.21)$$

Each a $\gamma_\mu{}^{(\rho)}$-matrix (6.15) can have one of 16 possible stignatures. For example: (6.22)

$$\gamma_{11}^{00} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad \gamma_{11}^{10} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad \gamma_{11}^{20} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad \gamma_{11}^{30} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\gamma_{11}^{01} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \quad \gamma_{11}^{11} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad \gamma_{11}^{21} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \quad \gamma_{11}^{31} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\gamma_{11}^{02} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \quad \gamma_{11}^{12} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad \gamma_{11}^{22} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \quad \gamma_{11}^{32} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

$$\gamma_{11}^{03} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \quad \gamma_{11}^{13} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \quad \gamma_{11}^{23} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad \gamma_{11}^{33} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}.$$

For each of these $\gamma_{\mu\rho}{}^{ij}$-matrices we can also choose a second $\gamma_{\chi\tau}{}^{nj}$-matrix, the product with which leads to the $b_{\mu\eta}$-matrix (6.20). Thus, taking into account 16

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signatures, the 16 × 16 = 256 $\gamma_{\mu\rho}$-matrices are obtained from 16 $\gamma_{\mu\rho}{}^{ij}$-matrices (6.15).

Each a $\gamma_{\mu\rho}{}^{ij}$-matrix (6.22) can be transformed into one of 16 mixed matrices. Let's clarify this statement using the example of the $\gamma_{11}{}^{13}$-matrix:

$$ {}_{00}\gamma_{11}^{13}=\begin{pmatrix}0&0&1&0\\0&0&0&-1\\1&0&0&0\\0&-1&0&0\end{pmatrix}\quad {}_{10}\gamma_{11}^{13}=\begin{pmatrix}0&0&i&0\\0&0&0&-i\\i&0&0&0\\0&-1&0&0\end{pmatrix}\quad {}_{20}\gamma_{11}^{13}=\begin{pmatrix}0&0&1&0\\0&0&0&-i\\i&0&0&0\\0&-1&0&0\end{pmatrix}\quad {}_{30}\gamma_{11}^{13}=\begin{pmatrix}0&0&i&0\\0&0&0&-i\\1&0&0&0\\0&-i&0&0\end{pmatrix} $$

$$ {}_{01}\gamma_{11}^{13}=\begin{pmatrix}0&0&i&0\\0&0&0&-i\\i&0&0&0\\0&-1&0&0\end{pmatrix}\quad {}_{11}\gamma_{11}^{13}=\begin{pmatrix}0&0&i&0\\0&0&0&-1\\1&0&0&0\\0&-1&0&0\end{pmatrix}\quad {}_{21}\gamma_{11}^{13}=\begin{pmatrix}0&0&i&0\\0&0&0&-i\\1&0&0&0\\0&-1&0&0\end{pmatrix}\quad {}_{31}\gamma_{11}^{13}=\begin{pmatrix}0&0&i&0\\0&0&0&-1\\i&0&0&0\\0&-1&0&0\end{pmatrix} $$

$$ {}_{02}\gamma_{11}^{13}=\begin{pmatrix}0&0&1&0\\0&0&0&-i\\i&0&0&0\\0&-1&0&0\end{pmatrix}\quad {}_{12}\gamma_{11}^{13}=\begin{pmatrix}0&0&1&0\\0&0&0&-1\\i&0&0&0\\0&-i&0&0\end{pmatrix}\quad {}_{22}\gamma_{11}^{13}=\begin{pmatrix}0&0&1&0\\0&0&0&-i\\i&0&0&0\\0&-i&0&0\end{pmatrix}\quad {}_{32}\gamma_{11}^{13}=\begin{pmatrix}0&0&1&0\\0&0&0&-i\\1&0&0&0\\0&-1&0&0\end{pmatrix} $$

$$ {}_{03}\gamma_{11}^{13}=\begin{pmatrix}0&0&i&0\\0&0&0&-i\\1&0&0&0\\0&-i&0&0\end{pmatrix}\quad {}_{13}\gamma_{11}^{13}=\begin{pmatrix}0&0&1&0\\0&0&0&-i\\1&0&0&0\\0&-i&0&0\end{pmatrix}\quad {}_{23}\gamma_{11}^{13}=\begin{pmatrix}0&0&i&0\\0&0&0&-1\\i&0&0&0\\0&-i&0&0\end{pmatrix}\quad {}_{33}\gamma_{11}^{13}=\begin{pmatrix}0&0&i&0\\0&0&0&-i\\i&0&0&0\\0&-i&0&0\end{pmatrix} \tag{6.23}$$

When all 256 $\gamma_{\mu\rho}{}^{ij}$-matrices (6.23) are mixed in this way, a basis of 16^3=256×16=4096 ${}_{nk}\gamma_{\mu\rho}{}^{ij}$-matrices is obtained. Therefore, in this case, the $b_{\mu\eta}$-matrix (6.20) can be given by one of 4096 products of pairs of a ${}_{nk}\gamma_{\mu\rho}{}^{ij}$-matrices.

In turn, all sixteen $b_{\mu\eta}$-matrices (6.18) can be given by 16^4 = 65536 different versions of the pair products of the ${}^{vc}{}_{nk}\gamma_{lm}{}^{ij}$-matrices.

Similarly, we can continue to build up the basis of generalized Dirac γ-matrices to infinity.

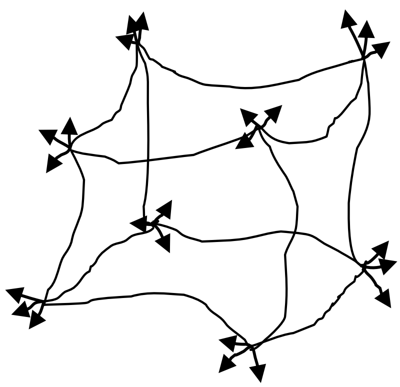
Above, the Dirac stratification of only one quadratic form (6.1) was considered. All other metrics (6.16) are "foliated" in the same way.

The entire collection of ${}^{vc}{}_{nk}\,\gamma\,{}_{lm}{}^{ij}$-matrices will be called generalized Dirac matrices, and the $\lambda_{m,n}$-vacuum prepared by means of these matrices, will be called the Dirac stratification of the $\lambda_{m,n}$-vacuum.

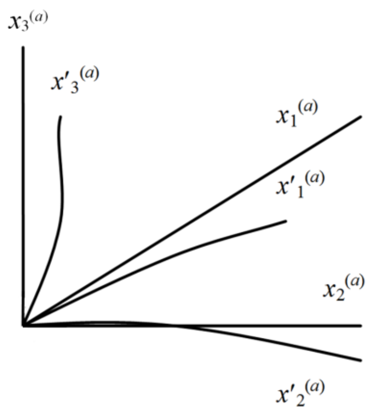
7 THE CURVED AREA OF A VACUUM

7.1 The curved area of a $\lambda_{m,n}$-vacuum

Let's consider a curved 3D area of a vacuum. If the wavelength $\lambda_{m,n}$ of the test monochromatic light beams is much less than the dimensions of the vacuum irregularities, then in this area the cubic cell of the $\lambda_{m,n}$-vacuum (i.e., the cubic cell of $3D_{m,n}$-landscape, limited by these rays) will be curved (see Fig. 7.1).



a)



b)

Fig. 7.1 *a*) The deformed cubic cell of the $\lambda_{m,n}$-vacuum; *b*) One of a corners of the curved cube

We consider one of the eight vertices of the curved cube of a $\lambda_{m,n}$-vacuum (see Fig. 7.1 *a*). Let's replace the distorted edges emerging from this vertex with the distorted axes of the curvilinear coordinate system $x'^{0(a)}$, $x'^{1(a)}$, $x'^{2(a)}$, $x^{3\prime(a)}$ (see Fig. 7.1 *b*). The same edges of the original, ideal cube will be denoted by the pseudo-Cartesian coordinate system $x^{0(a)}$, $x^{1(a)}$, $x^{2(a)}$, $x^{3(a)}$.

The distortions of the angle of the considered cube of a $\lambda_{m,n}$-vacuum can be decomposed into two components:

1) the change in the lengths (compression or expansion) of the axes $x'^{0(a)}$, $x'^{1(a)}$, $x'^{2(a)}$, $x^{3\prime(a)}$ while maintaining right angles between these axes;

2) the deviations of the angles between the axes $x'^{0(a)}$, $x'^{1(a)}$, $x'^{2(a)}$, $x'^{3(a)}$ from straight lines while maintaining their lengths.

Let's consider these affine distortions separately.

1) Suppose that only the lengths of the axes $x'^{0(a)}$, $x'^{1(a)}$, $x'^{2(a)}$, $x^{3\prime(a)}$ have changed during the curvature. Then these axes can be expressed in terms of the axes of the original ideal cube $x^{0(a)}$, $x^{1(a)}$, $x^{2(a)}$, $x^{3(a)}$ using the corresponding transformations of coordinates:

$$x'^{0(a)} = \alpha_{00}{}^{(a)}x^{0(a)} + \alpha_{01}{}^{(a)}x^{1(a)} + \alpha_{02}{}^{(a)}x^{2(a)} + \alpha_{03}{}^{(a)}x^{3(a)};$$
$$x'^{1(a)} = \alpha_{10}{}^{(a)}x^{0(a)} + \alpha_{11}{}^{(a)}x^{1(a)} + \alpha_{12}{}^{(a)}x^{2(a)} + \alpha_{13}{}^{(a)}x^{3(a)}; \qquad (7.1)$$
$$x'^{2(a)} = \alpha_{20}{}^{(a)}x^{0(a)} + \alpha_{21}{}^{(a)}x^{1(a)} + \alpha_{22}{}^{(a)}x^{2(a)} + \alpha_{23}{}^{(a)}x^{3(a)};$$
$$x'^{3\,(a)} = \alpha_{30}{}^{(a)}x^{0(a)} + \alpha_{31}{}^{(a)}x^{1(a)} + \alpha_{32}{}^{(a)}x^{2(a)} + \alpha_{33}{}^{(a)}x^{3(a)},$$

where

$$\alpha_{ij}{}^{(a)} = dx'^{i(a)}/dx^{j(a)} \qquad (7.2)$$

is the Jacobian of the transformation, or the components of the elongation tensor.

2) Let now only the angles between the axes of the coordinate system $x'^{0(a)}$, $x'^{1(a)}$, $x'^{2(a)}$, $x^{3\prime(a)}$ be subject to change, and the lengths of these axes remain unchanged. In this case, it is sufficient to consider only the change in the angles between the basis vectors $\mathbf{e}'_0{}^{(a)}$, $\mathbf{e}'_1{}^{(a)}$, $\mathbf{e}'_2{}^{(a)}$, $\mathbf{e}'_3{}^{(a)}$ of the distorted frame of reference.

It is known from vector analysis that the basis vectors of the distorted 4-basis $\mathbf{e}'_0{}^{(a)}$, $\mathbf{e}'_1{}^{(a)}$, $\mathbf{e}'_2{}^{(a)}$, $\mathbf{e}'_3{}^{(a)}$ can be expressed in terms of the original basis vectors $\mathbf{e}_0{}^{(a)}$, $\mathbf{e}_1{}^{(a)}$, $\mathbf{e}_2{}^{(a)}$, $\mathbf{e}_3{}^{(a)}$ of the orthogonal 4-basis by means of the following system of linear equations:

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$$\mathbf{e}'_0{}^{(a)} = \beta^{00(a)}\,\mathbf{e}_0{}^{(a)} + \beta^{01(a)}\,\mathbf{e}_1{}^{(a)} + \beta^{02(a)}\,\mathbf{e}_2{}^{(a)} + \beta^{03(a)}\,\mathbf{e}_3{}^{(a)};$$
$$\mathbf{e}'_1{}^{(a)} = \beta^{10(a)}\,\mathbf{e}_0{}^{(a)} + \beta^{11(a)}\,\mathbf{e}_1{}^{(a)} + \beta^{12(a)}\,\mathbf{e}_2{}^{(a)} + \beta^{13(a)}\,\mathbf{e}_3{}^{(a)}; \qquad (7.3)$$
$$\mathbf{e}'_2{}^{(a)} = \beta^{20(a)}\,\mathbf{e}_0{}^{(a)} + \beta^{21(a)}\,\mathbf{e}_1{}^{(a)} + \beta^{22(a)}\,\mathbf{e}_2{}^{(a)} + \beta^{23(a)}\,\mathbf{e}_3{}^{(a)};$$
$$\mathbf{e}'_3{}^{(a)} = \beta^{30(a)}\,\mathbf{e}_0{}^{(a)} + \beta^{31(a)}\,\mathbf{e}_1{}^{(a)} + \beta^{32(a)}\,\mathbf{e}_2{}^{(a)} + \beta^{03(a)}\,\mathbf{e}_3{}^{(a)},$$

where

$$\beta^{\,pm(a)} = (\mathbf{e}'_p{}^{(a)} \cdot \mathbf{e}_m{}^{(a)}) = \cos\,(\mathbf{e}'_p{}^{(a)\,\wedge}\mathbf{e}_m{}^{(a)}) \qquad (7.4)$$

is direction cosines.

The systems of the Equations (7.1) and (7.3) can be represented in a compact form:

$$x'^{i\,(a)} = \alpha_{ij}{}^{(a)}\, x^{\,j(a)}\,, \qquad (7.5)$$

$$\mathbf{e}'_p{}^{(a)} = \beta^{\,pm(a)}\,\mathbf{e}_m{}^{(a)}\,. \qquad (7.6)$$

The distortions of the remaining 7 angles of the curved cube of a $\lambda_{m,n}$-vacuum (see Fig. 7.1) (more precisely, the fifteen remaining 4-bases, see Fig. 2.7) are described in a similar way.

Let's write, for example, vector (4.1) in a distorted 4-basis

$$d\mathbf{s}'^{(7)} = \mathbf{e}'_i{}^{(7)} dx'^{\,i\,(7)}. \qquad (7.7)$$

Taking into account (7.5) and (7.6), the vector (7.7) can be represented as

$$d\mathbf{s}'^{(7)} = \beta^{\,pm(7)}\,\mathbf{e}_m{}^{(7)}\alpha_{pj}{}^{(7)} dx^{j(7)}. \qquad (7.8)$$

Similarly, all vertices of the distorted cube of a $\lambda_{m,n}$-vacuum can be represented by the vectors

$$d\mathbf{s}'^{(a)} = \beta^{\,pm(a)}\,\mathbf{e}_m{}^{(a)}\,\alpha_{pj}{}^{(a)} dx^{j(a)}, \qquad (7.9)$$

where a = 1,2, ...,16.

7.2 The curved metric 4-spaces

Let's consider two the vectors (4.1) and (4.2) given in the 5th and 7th curved affine spaces

$$d\mathbf{s}'^{(5)} = \beta^{ln(5)}\mathbf{e}_n{}^{(5)}\alpha_{lj}{}^{(5)} dx^j, \qquad (7.10)$$

$$d\mathbf{s}'^{(7)} = \beta^{pm(7)}\mathbf{e}_m{}^{(7)}\alpha_{pi}{}^{(7)}dx^i. \quad (7.11)$$

We find the scalar product of these vectors

$$ds'^{(7,5)2} = d\mathbf{s}'^{(7)}d\mathbf{s}'^{(5)} = \beta^{pm(7)}\mathbf{e}_m{}^{(7)}\alpha_{pi}{}^{(7)}\beta^{ln(5)}\mathbf{e}_n{}^{(5)}\alpha_{lj}{}^{(5)}dx^i dx^j = c_{ij}{}^{(7,5)}dx^i dx^j, \quad (7.12)$$

where

$$c_{ij}{}^{(7,5)} = \beta^{pm(7)}\mathbf{e}_m{}^{(7)}\alpha_{pi}{}^{(7)}\beta^{ln(5)}\mathbf{e}_n{}^{(5)}\alpha_{lj}{}^{(5)} \quad (7.13)$$

are the components of the metric tensor of the (7,5)-th metric 4-space.

Thus, we have obtained the interval of the (7,5)-th metric 4-space

$$ds'^{(7,5)2} = c_{ij}{}^{(7,5)}dx^i dx^j \quad (7.14)$$

with signature (4.6) (+ + + –) and metric tensor

$$c_{ij}^{(7,5)} = \begin{pmatrix} c_{00}^{(7,5)} & c_{10}^{(7,5)} & c_{20}^{(7,5)} & c_{30}^{(7,5)} \\ c_{01}^{(7,5)} & c_{11}^{(7,5)} & c_{21}^{(7,5)} & c_{31}^{(7,5)} \\ c_{02}^{(7,5)} & c_{12}^{(7,5)} & c_{22}^{(7,5)} & c_{32}^{(7,5)} \\ c_{03}^{(7,5)} & c_{13}^{(7,5)} & c_{23}^{(7,5)} & c_{33}^{(7,5)} \end{pmatrix}. \quad (7.15)$$

Similarly, the scalar product of any two of the 16 vectors (7.9)

$$d\mathbf{s}'^{(a)} = \beta^{pm(a)}\mathbf{e}_m{}^{(a)}\alpha_{pi}{}^{(a)}dx^i, \quad (7.16)$$

$$d\mathbf{s}'^{(b)} = \beta^{ln(b)}\mathbf{e}_n{}^{(b)}\alpha_{lj}{}^{(b)}dx^j \quad (7.17)$$

leads to the formation of an «atlas» consisting of 16×16=256 all possible curved 4-sheets (i.e. metric 4-spaces) with the metrics

$$ds'^{(a,b)2} = c_{ij}{}^{(a,b)}dx^i dx^j, \quad (7.18)$$

where a = 1,2,3, ..., 16; b = 1, 2, 3, ..., 16, with the corresponding signatures (4.9) and the metric tensors

$$c_{ij}^{(a,b)} = \begin{pmatrix} c_{00}^{(a,b)} & c_{10}^{(a,b)} & c_{20}^{(a,b)} & c_{30}^{(a,b)} \\ c_{01}^{(a,b)} & c_{11}^{(a,b)} & c_{21}^{(a,b)} & c_{31}^{(a,b)} \\ c_{02}^{(a,b)} & c_{12}^{(a,b)} & c_{22}^{(a,b)} & c_{32}^{(a,b)} \\ c_{03}^{(a,b)} & c_{13}^{(a,b)} & c_{23}^{(a,b)} & c_{33}^{(a,b)} \end{pmatrix}, \quad (7.19)$$

where

$$c_{ij}{}^{(a,b)} = \beta^{pm(a)}\mathbf{e}_m{}^{(a)}\alpha_{pi}{}^{(a)}\beta^{ln(b)}\mathbf{e}_n{}^{(b)}\alpha_{lj}{}^{(b)} \quad (7.20)$$

are the components of the metric tensor of the (a,b)-th curved metric 4-space.

7.3 The 4-strain tensor

In the classical theory of elasticity and in the general theory of relativity, the actual state of the local volume of an elastic-plastic medium (in particular, the Einstein vacuum) is described by only one "frozen in" reference system with the corresponding 4-basis. This results in the analysis of the only one quadratic form

$$ds'^2 = g_{ij}\, dx^j dx^j \quad \text{with signature } (+\,-\,-\,-), \qquad (7.21)$$

where g_{ij} are the components of the metric tensor of the local area of the curved metric space (there are 16 of these components, but due to the symmetry $g_{ji} = g_{ij}$, only 10 are significant).

The quadratic form (7.21) is compared with the quadratic form of the initial ideal state of the same local area of the elastic-plastic medium [6]

$$ds_0^2 = g_{ij}^0 dx^i dx^j \quad \text{with the same signature } (+\,-\,-\,-). \qquad (7.22)$$

Subtracting the metric of the initial state (7.22) from the metric of the actual state (7.21), we obtain [6]

$$ds'^2 - ds_0^2 = (g_{ij} - g_{ij}^0) dx^i dx^j = 2\varepsilon_{ij} dx^i dx^j, \qquad (7.23)$$

where

$$\varepsilon_{ij} = \frac{1}{2}(g_{ij} - g_{ij}^0), \qquad (7.24)$$

is tensor of the 4-strains, which is the subject of the classical theory of elasticity.

The light-geometry of the $\lambda_{m,n}$-vacuum developed here, based on the Algebra of signatures, differs from the classical theory of an elasticity in that the investigated local volume of an elastic-plastic medium (in this case the $\lambda_{m,n}$-vacuum) is described by more than one 4-basis associated with one of the eight corners of the investigated cube (see Fig. 7.1*a*,*b*), but with the all sixteen curved 4-bases (see the same Fig. 7.1 *a*), the beginning of which is at the investigated point *O* (see Fig. 2.5 and 2.6).

This circumstance leads to the fact that instead of one metric of the type (7.21) in the light-geometry of a $\lambda_{m,n}$-vacuum, there are 256 metrics (7.18)

$$ds^{(a,b)2} = c_{ij}^{(a,b)}dx^i dx^j \qquad (7.25)$$

with the corresponding signatures (4.9) or (4.20), which describe the same volume of the studied space (in particular, a $\lambda_{m,n}$-vacuum) from different sides.

In this case, the metric-dynamic state of the investigated volume of the elastic-plastic medium (in particular, a $\lambda_{m,n}$-vacuum) is described not by 16 numbers (i.e., by the components of the metric tensor g_{ji}), but by the 256×16 = 4096 components of the 256 tensors $c_{ji}^{(a,b)}$ (7.19).

This not only achieves a much more accurate description of the curved volume of an elastic-plastic medium (in particular, a $\lambda_{m,n}$-vacuum) in the vicinity of point O (see Fig. 2.5), but also prepares of the logical basis for identifying a number of vacuum effects that were previously not considered due to the lack of a proper mathematical apparatus.

We note once again that the mathematical apparatus of the light-geometry of a $\lambda_{m,n}$-vacuum based on the Algebra of signatures developed here is suitable for studying the properties of not only objective and/or subjective emptiness, but also any other 3-dimensional continuous media in which wave disturbances (light, sound, phonons) propagate at a constant speed.

7.4 The first stage of the compactification of a curved dimensions

Just as it was done in § 4.3, at the first stage of compactification of the additional curved mathematical dimensions in the Algebra of signature, the averaging of metric 4-spaces with the same signature is performed.

For example, for the metrics with signature (– + – +) (see Fig. 4.2), we have the following averaged metric tensor

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$$c_{ij}^{(p)} = \begin{pmatrix} c_{00}^{(p)} & c_{10}^{(p)} & c_{20}^{(p)} & c_{30}^{(p)} \\ c_{01}^{(p)} & c_{11}^{(p)} & c_{21}^{(p)} & c_{31}^{(p)} \\ c_{02}^{(p)} & c_{12}^{(p)} & c_{22}^{(p)} & c_{32}^{(p)} \\ c_{03}^{(p)} & c_{13}^{(p)} & c_{23}^{(p)} & c_{33}^{(p)} \end{pmatrix} = \frac{1}{16}\left\{ \begin{array}{l} \begin{pmatrix} c_{00}^{(14,2)} & c_{10}^{(14,2)} & c_{20}^{(14,2)} & c_{30}^{(14,2)} \\ c_{01}^{(14,2)} & c_{11}^{(14,2)} & c_{21}^{(14,2)} & c_{31}^{(14,2)} \\ c_{02}^{(14,2)} & c_{12}^{(14,2)} & c_{22}^{(14,2)} & c_{32}^{(14,2)} \\ c_{03}^{(14,2)} & c_{13}^{(14,2)} & c_{23}^{(14,2)} & c_{33}^{(14,2)} \end{pmatrix} + \\ + \begin{pmatrix} c_{00}^{(13,1)} & c_{10}^{(13,1)} & c_{20}^{(13,1)} & c_{30}^{(13,1)} \\ c_{01}^{(13,1)} & c_{11}^{(13,1)} & c_{21}^{(13,1)} & c_{31}^{(13,1)} \\ c_{02}^{(13,1)} & c_{12}^{(13,1)} & c_{22}^{(13,1)} & c_{32}^{(13,1)} \\ c_{03}^{(13,1)} & c_{13}^{(13,1)} & c_{23}^{(13,1)} & c_{33}^{(13,1)} \end{pmatrix} + \ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ + \begin{pmatrix} c_{00}^{(1,13)} & c_{10}^{(1,13)} & c_{20}^{(1,13)} & c_{30}^{(1,13)} \\ c_{01}^{(1,13)} & c_{11}^{(1,13)} & c_{21}^{(1,13)} & c_{31}^{(1,13)} \\ c_{02}^{(1,13)} & c_{12}^{(1,13)} & c_{22}^{(1,13)} & c_{32}^{(1,13)} \\ c_{03}^{(1,13)} & c_{13}^{(1,13)} & c_{23}^{(1,13)} & c_{33}^{(1,13)} \end{pmatrix} \end{array} \right\}, \quad (7.26)$$

where p corresponds to the 14-th signature (– + – +), according to the following conditional numbering of signatures:

$$sign\left(c_{ij}^{(p)}\right) = \begin{matrix} (++++)^1 & (+++-)^5 & (-++-)^9 & (++-+)^{13} \\ (---+)^2 & (-+++)^6 & (--++)^{10} & (-+-+)^{14} \\ (+--+)^3 & (++--)^7 & (+---)^{11} & (+-++)^{15} \\ (--+-)^4 & (+-+-)^8 & (-+--)^{12} & (----)^{16} \end{matrix} \qquad (7.27)$$

and the average interval (i.e., metric)

$$<ds^{(-+-+)\,2}> = c_{ij}^{(14)} dx^i\, dx^j. \qquad (7.28)$$

Similarly, because of the 16-fold degeneracy of the 256 intervals (7.18) of the curved metric 4-spaces, we can obtain 256/16 = 16 averaged intervals (i.e., metric) with 16-th possible signatures

$$\begin{matrix} <ds^{(+---)2}> & <ds^{(++++)2}> & <ds^{(---+)2}> & <ds^{(+--+)2}> \\ <ds^{(--+-)2}> & <ds^{(++--)2}> & <ds^{(-+--)2}> & <ds^{(+-+-)2}> \\ <ds^{(-+++)2}> & <ds^{(----)2}> & <ds^{(+++-)2}> & <ds^{(-++-)2}> \\ <ds^{(++-+)2}> & <ds^{(--++)2}> & <ds^{(+-++)2}> & <ds^{(-+-+)2}>, \end{matrix} \qquad (7.29)$$

where ‹·› - means averaging.

If the additive superposition of all these 16 averaged intervals (7.29) is equal to zero

$$ds_\Sigma^2=\sum_{p=1}^{16}c_{ij}^{(p)}dx^idx^j= c_{ij}^{(1)}dx^idx^j+ c_{ij}^{(2)}dx^idx^j+ c_{ij}^{(3)}dx^idx^j+ c_{ij}^{(4)}dx^idx^j+$$
$$+c_{ij}^{(5)}dx^idx^j+ c_{ij}^{(6)}dx^idx^j+ c_{ij}^{(7)}dx^idx^j+ c_{ij}^{(8)}dx^idx^j+ \qquad (7.30)$$
$$+c_{ij}^{(9)}dx^idx^j+ c_{ij}^{(10)}dx^idx^j+ c_{ij}^{(11)}dx^idx^j+ c_{ij}^{(12)}dx^idx^j+$$
$$+c_{ij}^{(13)}dx^idx^j+ c_{ij}^{(14)}dx^idx^j+ c_{ij}^{(15)}dx^idx^j+ c_{ij}^{(16)}dx^idx^j=0.,$$

then this Expression can be used in stochastic light-geometry for an averaged flat $\lambda_{m,n}$-vacuum, since it is a condition for observing of the $\lambda_{m,n}$-vacuum balance.

In this case, the all 16×16 = 256 components of the 16 averaged metric tensors $c_{ij}^{(p)}$ can be random functions of time. But, according to the $\lambda_{m,n}$-vacuum condition, these metric-dynamic fluctuations should overflow into each other in such a way that the total metric (7.30), on average, remains equal to the zero.

On the basis of the total interval (i.e., metric) (7.30) the $\lambda_{m,n}$-vacuum thermodynamics can be developed, considering the most complex, near-zero "overflows" of the local $\lambda_{m,n}$-vacuum curvatures. The concept of the $\lambda_{m,n}$-vacuum entropy and temperature (as the essence of the chaos and intensity of a local $\lambda_{m,n}$-vacuum fluctuations) can be introduced.

We can talk about: cooling of the $\lambda_{m,n}$-vacuum to "freezing": heating of the $\lambda_{m,n}$-vacuum to "evaporation"; and many other effects similar to the processes occurring in ordinary (atomistic) continuous media.

The features of the $\lambda_{m,n}$-vacuum thermodynamics are associated with processes when the gradients of the $\lambda_{m,n}$-vacuum fluctuations approach the speed of light ($dc_{ij}^{(p)}/dx_a \sim c$), or to zero ($dc_{ij}^{(p)}/dx_a \sim 0$).

More detailed consideration of the $\lambda_{m,n}$-vacuum thermodynamics is beyond the scope of this article. However, some aspects of this direction of the research are considered in [15].

7.5 The second stage of the compactification of a curved dimensions

Just as it was done in § 4.7, Expression (7.30) can be reduced to two terms

$$\langle ds^{(-)2}\rangle + \langle ds^{(+)2}\rangle = \langle g_{ij}{}^{(+)}\rangle dx^i dx^j + \langle g_{ij}{}^{(-)}\rangle dx^i dx^j = 0, \qquad (7.31)$$

where

$$\langle g_{ij}^{(-)}\rangle dx^i dx^j = \langle g_{ij}^{(+---)}\rangle dx^i dx^j = \frac{1}{7}\sum_{p=1}^{7} c_{ij}^{(p)} dx^i dx^j \qquad (7.32)$$

is the quadratic form (i.e., metric), which is the result of averaging of the seven metrics (7.29) with signatures included in the numerator of the left-hand rank (4.32) or (7.34)

$$\langle g_{ij}^{(+)}\rangle dx^i dx^j = \langle g_{ij}^{(-+++)}\rangle dx^i dx^j = \frac{1}{7}\sum_{p=8}^{14} c_{ij}^{(p)} dx^i dx^j \qquad (7.33)$$

is the quadratic form (i.e., metric), which is the result of averaging of the seven metrics (7.29) with signatures included in the numerator of the right-hand rank (4.32) or (7.34)

$$\begin{array}{ccccc}
(+\ +\ +\ +) & + & (-\ -\ -\ -) & = 0 \\
(-\ -\ -\ +) & + & (+\ +\ +\ -) & = 0 \\
(+\ -\ -\ +) & + & (-\ +\ +\ -) & = 0 \\
(-\ -\ +\ -) & + & (+\ +\ -\ +) & = 0 \\
(+\ +\ -\ -) & + & (-\ -\ +\ +) & = 0 \\
(-\ +\ -\ -) & + & (+\ -\ +\ +) & = 0 \\
\underline{(+\ -\ +\ -)} & + & \underline{(-\ +\ -\ +)} & = 0 \\
(+\ -\ -\ -)_+ & + & (-\ +\ +\ +)_+ & = 0
\end{array} \qquad (7.34)$$

Thus, from the entire set of the $\lambda_{m,n}$-vacuum fluctuations, we can distinguish:

- averaged "outer" side 2^3-$\lambda_{m,n}$-vacuum (i.e., averaged subcont) with the average metric

$$ds^{(+\,-\,-\,-)2} = ds^{(-)2} = g_{ij}{}^{(-)}dx^i dx^j \text{ with signature } (+\,-\,-\,-), \qquad (7.35)$$

where
$$g_{ij}^{(+)} = \begin{pmatrix} g_{00}^{(+)} & g_{10}^{(+)} & g_{20}^{(+)} & g_{30}^{(+)} \\ g_{01}^{(+)} & g_{11}^{(+)} & g_{21}^{(+)} & g_{31}^{(+)} \\ g_{02}^{(+)} & g_{12}^{(+)} & g_{22}^{(+)} & g_{31}^{(+)} \\ g_{03}^{(+)} & g_{13}^{(+)} & g_{23}^{(+)} & g_{33}^{(+)} \end{pmatrix}; \qquad (7.36)$$

- averaged "inner" side 2^3-$\lambda_{m,n}$-vacuum (i.e., averaged antisubcont) with the average metric

$$ds^{(-\,+\,+\,+)2} = ds^{(+)2} = g_{ij}{}^{(+)}dx^i dx^j \text{ with signature } (-\,+\,+\,+), \qquad (7.37)$$

where
$$g_{ij}^{(-)} = \begin{pmatrix} g_{00}^{(-)} & g_{10}^{(-)} & g_{20}^{(-)} & g_{30}^{(-)} \\ g_{01}^{(-)} & g_{11}^{(-)} & g_{21}^{(-)} & g_{31}^{(-)} \\ g_{02}^{(-)} & g_{12}^{(-)} & g_{22}^{(-)} & g_{31}^{(-)} \\ g_{03}^{(-)} & g_{13}^{(-)} & g_{23}^{(-)} & g_{33}^{(-)} \end{pmatrix}. \qquad (7.38)$$

To shorten the notation, the averaging signs in the metrics (7.35) – (7.38) are omitted.

In Figure 7.2 conditionally shows the averaged section of a two-sided 2^3-$\lambda_{m,n}$-vacuum, the outer side of which (i.e., subcont) is described by the metric $ds^{(+\,-\,-\,-)2}$ (7.35), and the inner side (i.e., antisubcont) is described by the metric $ds^{(-\,+\,+\,+)2}$ (7.37).

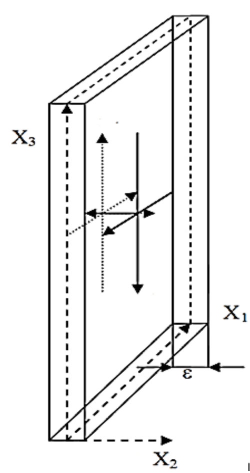

Fig. 7.2 The **s**implified illustration of a section of two-sided of a 2^3-$\lambda_{m\div n}$-vacuum, the outer side (i.e., subcont) of which is described by the metric $ds^{(+\,-\,-\,-)2}$, and the inner side (i.e., antisubcont) is described by the metric $ds^{(-\,+\,+\,+)2}$, for $\varepsilon \to 0$

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8 THE COMPONENTS OF THE METRIC TENSOR

8.1 The 4-strains tensor of a 2^3-$\lambda_{m,n}$-vacuum

Let the initial non-curved metric-dynamic state of the investigated area of the outer side of the 2^3-$\lambda_{m,n}$-vacuum (i.e., the averaged subcont) be characterized by the averaged metric

$$ds_0^{(-)2} = g_{ij0}^{(-)}dx^i\, dx^j \quad \text{with signature } (+\,-\,-\,-), \qquad (8.1)$$

and the curved state of the same area of the subcont is given by the averaged metric

$$ds^{(-)2} = g_{ij}^{(-)}dx^i dx^j \text{ with the same signature } (+\,-\,-\,-). \qquad (8.2)$$

The difference between the curved state of the investigated area of the subcont and its non-curved state is determined by the Expression (7.23)

$$ds^{(-)2} - ds_0^{(-)2} = (g_{ij}^{(-)} - g_{ij0}^{(-)})\, dx^i dx^j = 2\varepsilon_{ij}^{(-)}dx^i dx^j\,, \qquad (8.3)$$

where

$$\varepsilon_{ij}^{(-)} = ½\,(g_{ij}^{(-)} - g_{ij0}^{(-)}) \qquad (8.4)$$

is the 4-strains tensor of the local area of the subcont.

The relative elongation of the curved area of the subcont is [6]

$$l^{(-)} = \frac{ds^{(-)} - ds^{0(-)}}{ds^{0(-)}} = \frac{ds^{(-)}}{ds^{0(-)}} - 1. \qquad (8.5)$$

Where does it follow

$$ds^{(-)2} = (1 + l^{(-)})^2 ds_0^{(-)2}. \qquad (8.6)$$

Substituting (8.6) into (8.3) and taking into account (8.4), we have [6]

$$\varepsilon_{ij}^{(-)} = ½\,[(1 + l^{(-)})^2 - 1]\, g_{ij0}^{(-)}, \qquad (8.7)$$

or in the expanded form

$$\varepsilon_{ij}^{(-)} = ½\,[(1 + l_i^{(-)})(1 + l_j^{(-)}) \cos\beta_{ij}^{(-)} - \cos\beta_{ij0}^{(-)}]\, g_{ij0}^{(-)}, \qquad (8.8)$$

where

$\beta_{ij0}^{(-)}$ is the angle between the axes x_i and x_j of the frame of reference "frozen in" in the initial non-curved state of the investigated area of the subcont;

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$\beta_{ij}^{(-)}$ is the angle between the axes x_i' and x_j' of the distorted frame of reference "frozen in" into the curved state of the same area of the subcont.

For $\beta_{ij0}^{(-)} = \pi/2$, the Expression (8.8) takes the form

$$\varepsilon_{ij}^{(-)} = ½\,[(1 + l_i^{(-)})(1 + l_j^{(-)})\cos\beta_{ij}^{(-)} - 1]\, g_{ij0}^{(-)}. \qquad (8.9)$$

For the diagonal components of the 4-strain tensor $\varepsilon_{ii}^{(-)}$, the Expression (8.9) is simplified

$$\varepsilon_{ii}^{(-)} = ½\,[(1 + l_i^{(-)})^2 - 1]\, g_{ii0}^{(-)}, \qquad (8.10)$$

whence it follows [6]

$$l_i^{(-)} = \sqrt{1+\frac{2\varepsilon_{ii}^{(-)}}{g_{ii}^{0(-)}}} - 1 = \sqrt{1+\frac{g_{ii}^{(-)} - g_{ii}^{0(-)}}{g_{ii}^{0(-)}}} - 1 = \sqrt{\frac{g_{ii}^{(-)}}{g_{ii}^{0(-)}}} - 1. \qquad (8.11)$$

If the deformations $\varepsilon_{ij}^{(-)}$ are small, then, the expanding Expression (8.11) in a series, and limiting ourselves to the first term of this series, we obtain the relative elongation of the local area of the subcont

$$l_i^{(-)} \approx \frac{\varepsilon_{ii}^{(-)}}{g_{ii}^{0(-)}}. \qquad (8.12)$$

Similarly, the deformation of the local area of the inner side of the 2^3-$\lambda_{m,n}$-vacuum length (i.e., averaged antisubcont) is determined by the Expression

$$ds^{(+)2} - ds_0^{(+)2} = (g_{ij}^{(+)} - g_{ij0}^{(+)})dx^i dx^j = 2\varepsilon_{ij}^{(+)} dx^i dx^j, \qquad (8.13)$$

where

$$\varepsilon_{ij}^{(+)} = ½\,(g_{ij}^{(+)} - g_{ij0}^{(+)}) \qquad (8.14)$$

is the 4-strains tensor of the local area of the antisubcont;

$$ds_0^{(+)2} = g_{ij0}^{(+)} dx^i dx^j \quad \text{with the signature } (- + + +) \qquad (8.15)$$

is the metric of the non-curved state of the local area of the antisubcont;

$$ds^{(+)2} = g_{ij}^{(+)} dx^i dx^j \quad \text{with the same signature } (- + + +) \qquad (8.16)$$

is the metric of the curved state of the local area of the antisubcont.

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The relative elongation of the local area of the antisubcont is determined by the Expression

$$l^{(+)} = \frac{ds^{(+)} - ds^{0(+)}}{ds^{0(+)}} = \frac{ds^{(+)}}{ds^{0(+)}} - 1. \qquad (8.17)$$

Let's define the 4-strains tensor of the two-sided 2^3-$\lambda_{m,n}$-vacuum as the mean

$$\varepsilon_{ij}^{(\pm)} = ½\,(\varepsilon_{ij}^{(+)} + \varepsilon_{ij}^{(-)}) = ½\,(\varepsilon_{ij}^{(-+++)} + \varepsilon_{ij}^{(+---)}), \qquad (8.18)$$

or, taking into account (8.4) and (8.14), we obtain

$$\varepsilon_{ij}^{(\pm)} = ½\,(g_{ij}^{(+)} + g_{ij}^{(-)}) - ½\,(g_{ij0}^{(+)} + g_{ij0}^{(-)}) = ½\,(g_{ij}^{(+)} + g_{ij}^{(-)}), \qquad (8.19)$$

since according to the $\lambda_{m,n}$-vacuum balance condition:

$$g_{ij0}^{(+)} + g_{ij0}^{(-)} = g_{ij0}^{(-+++)} + g_{ij0}^{(+---)} = 0.$$

The relative lengthening of the local area of the two-sided 2^3-$\lambda_{m,n}$-vacuum $l_i^{(\pm)}$ in this case should be calculated using the formula [15]

$$l_i^{(\pm)} = ½\left(l_i^{(+)} + l_i^{(-)}\right), \qquad (8.20)$$

where

$$l_i^{(+)} = \sqrt{1 + \frac{2\varepsilon_{ii}^{(\pm)}}{g_{ii}^{0(+)}}} - 1 = \sqrt{1 + \frac{g_{ii}^{(+)} + g_{ii}^{(-)}}{g_{ii}^{0(+)}}} - 1,$$

$$l_i^{(-)} = \sqrt{1 + \frac{2\varepsilon_{ii}^{(\pm)}}{g_{ii}^{0(-)}}} - 1 = \sqrt{1 + \frac{g_{ii}^{(+)} + g_{ii}^{(-)}}{g_{ii}^{0(-)}}} - 1. \qquad (8.21)$$

Since in any case one of the components $g_{ij0}^{(-)}$ or $g_{ij0}^{(+)}$ is a negative number, the relative lengthening (8.20) may turn out to be a complex number.

In this regard, we note the following the important circumstance. If both sides of the Expression (8.19) are multiplied by $dx^i dx^j$, then we obtain the averaged quadratic form [15]

$$ds^{(\pm)2} = ½\,(ds^{(-)2} + ds^{(+)2}), \qquad (8.22)$$

which resembles the Pythagorean theorem $c^2 = a^2 + b^2$.

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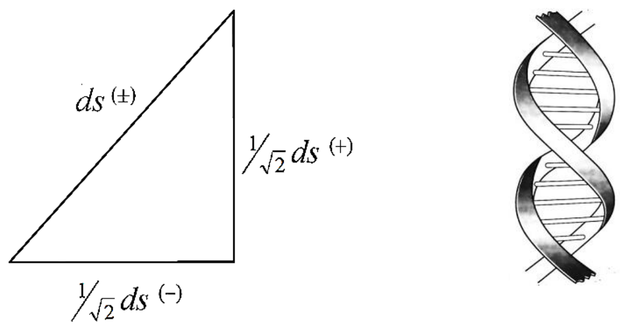
This means that the line segments $(\frac{1}{2})^{1/2}ds^{(-)}$ and $(\frac{1}{2})^{1/2}ds^{(+)}$ are always mutually perpendicular to each other, i.e. $ds^{(-)} \perp ds^{(+)}$ (see Fig.8.1*a*). In this case, two lines directed in the same direction can always be mutually perpendicular only if they form a double helix (see Fig. 8.1*b*).



a) *b*)

Fig. 8.1 *a*) The ratio of the segments $ds^{(-)}$ and $ds^{(+)}$; *b*) If project the double helix onto a plane, then at the intersection the its segments $ds^{(-)}$ and $ds^{(+)}$ are always mutually perpendicular

Thus, the averaged metric (8.22) corresponds to a segment of the "braid" (i.e., the double helix) consisting of two mutually perpendicular spirals $s^{(-)}$ and $s^{(+)}$. In this case, just like the averaged relative elongation (8.20), the segment of this "double helix" can be described by a complex number [15]

$$ds^{(\pm)} = \frac{1}{\sqrt{2}}(ds^{(-)} + i ds^{(+)}), \qquad (8.23)$$

the square of the modulus of this Expression is (8.22).

Definition 8.1.1 *A k-braid is the result of averaging the metrics with different signatures (where k is the number of averaged metrics, i.e. the number of "threads" in the "braid").*

In particular, the averaged metric (8.22) is called the 2-braid, since it is "twisted" from the 2 lines (i.e., "threads"):

$$ds^{(-)} = ds^{(+---)} \text{ and } ds^{(-)} = ds^{(-+++)}.$$

At the next deeper 16-sided level of consideration, the metric-dynamic properties of the local area of the 2^6-$\lambda_{m,n}$-vacuum are characterized by an additive

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superposition (or averaging) of sixteen metrics with the all 16 possible signatures, i.e. a 16-braid [15]:

$$ds_\Sigma^2 = 1/16\,(ds^{(+---)2} + ds^{(++++)2} + ds^{(---+)2} + ds^{(+--+)2} + $$
$$+ ds^{(--+-)2} + ds^{(++--)2} + ds^{(-+--)2} + ds^{(+-+-)2} + \qquad (8.24)$$
$$+ ds^{(-+++)2} + ds^{(----)2} + ds^{(+++-)2} + ds^{(-++-)2} + $$
$$+ ds^{(++-+)2} + ds^{(--++)2} + ds^{(+-++)2} + ds^{(-+-+)2}) = 0.$$

In this case, we have sixteen of the 4-strains tensors of all kinds of the 4-spaces

$$\varepsilon_{ij}^{(p)} = \begin{pmatrix} \varepsilon_{ij}^{(1)} & \varepsilon_{ij}^{(2)} & \varepsilon_{ij}^{(3)} & \varepsilon_{ij}^{(4)} \\ \varepsilon_{ij}^{(5)} & \varepsilon_{ij}^{(6)} & \varepsilon_{ij}^{(7)} & \varepsilon_{ij}^{(8)} \\ \varepsilon_{ij}^{(9)} & \varepsilon_{ij}^{(10)} & \varepsilon_{ij}^{(11)} & \varepsilon_{ij}^{(12)} \\ \varepsilon_{ij}^{(13)} & \varepsilon_{ij}^{(14)} & \varepsilon_{ij}^{(15)} & \varepsilon_{ij}^{(16)} \end{pmatrix}, \qquad (8.25)$$

where

$$\varepsilon_{ij}^{(p)} = ½\,(c_{ij}^{(p)} - c_{ij0}^{(p)}) \qquad (8.26)$$

is the 4-strains tensor of the *p*-th metric 4-space;

$c_{ij0}^{(p)}$ – the metric tensor of the uncurved local area of the *p*-th 4-space;

$c_{ij}^{(p)}$ – the metric tensor of the same, but a curved local area of the *p*-th 4-space.

At the 16-sided level of consideration, the total 4-strains tensor $\varepsilon_{ij(16)}$ of the local area of the 2^6-$\lambda_{m,n}$-vacuum is equal to

$$\varepsilon_{ij(16)} = 1/16\,(\varepsilon_{ij}^{(1)} + \varepsilon_{ij}^{(2)} + \varepsilon_{ij}^{(3)} + \varepsilon_{ij}^{(4)} + \varepsilon_{ij}^{(5)} + \varepsilon_{ij}^{(6)} + \varepsilon_{ij}^{(7)} + \varepsilon_{ij}^{(8)} + \varepsilon_{ij}^{(9)} +$$
$$+ \varepsilon_{ij}^{(10)} + \varepsilon_{ij}^{(11)} + \varepsilon_{ij}^{(12)} + \varepsilon_{ij}^{(13)} + \varepsilon_{ij}^{(14)} + \varepsilon_{ij}^{(15)} + \varepsilon_{ij}^{(16)}), \qquad (8.27)$$

and the relative elongation of the local area of the 2^6-$\lambda_{m,n}$-vacuum in this case can be calculated by the formula

$$l_{i\,(16)} = \eta_1\, l_i^{(1)} + \eta_2\, l_i^{(2)} + \eta_3\, l_i^{(3)} + \ldots + \eta_4\, l_i^{(16)},$$

where

$$l_i^{(p)} = \sqrt{1 + \frac{2\varepsilon_{ii}^{(p)}}{c_{ii}^{0(p)}}} - 1. \qquad (8.29)$$

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where η_m (where m = 1,2,3, ..., 16) is the orthonormal basis of objects satisfying the anticommutation relation of the Clifford algebra

$$\eta_m\eta_n + \eta_n\eta_m = 2\delta_{mn}\,, \tag{8.30}$$

where δ_{nm} is the unit 16×16-matrix.

In this case, the segment of the 16-braid consists of 16 "threads" [15]:

$$\begin{aligned} ds_{(16)} = {} & \eta_1\, ds^{(+---)} + \eta_2\, ds^{(++++)} + \eta_3\, ds^{(---+)} + \eta_4\, ds^{(+--+)} + \\ & + \eta_5\, ds^{(--+-)} + \eta_6\, ds^{(++--)} + \eta_7\, ds^{(-+--)} + \eta_8\, ds^{(+-+-)} + \\ & + \eta_9\, ds^{(-+++)} + \eta_{10}\, ds^{(----)} + \eta_{11}\, ds^{(+++-)} + \eta_{12}\, ds^{(-++-)} + \\ & + \eta_{13}\, ds^{(++-+)} + \eta_{14}\, ds^{(--++)} + \eta_{15}\, ds^{(+-++)} + \eta_{16}\, ds^{(-+-+)} = 0. \end{aligned} \tag{8.31}$$

If the all 16 linear forms $ds^{(+---)}$, $ds^{(++++)}$, … , $ds^{(-+-+)}$ can be represented in diagonal form, then in accordance with (5.12) and (5.13) the Expression (8.31) can be represented in the spin-tensor form

$$\begin{aligned} ds_{(16)} = {} & \sqrt{g_{00}^{(1)}}dx_0\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix} + \sqrt{g_{11}^{(1)}}dx_1\begin{pmatrix}0 & -i\\i & 0\end{pmatrix} + \sqrt{g_{22}^{(1)}}dx_2\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix} + \sqrt{g_{33}^{(1)}}dx_3\begin{pmatrix}i & 0\\0 & -i\end{pmatrix} + \\ & + \sqrt{g_{00}^{(2)}}dx_0\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix} + \sqrt{g_{11}^{(2)}}dx_1\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix} + \sqrt{g_{22}^{(2)}}dx_2\begin{pmatrix}0 & -i\\i & 0\end{pmatrix} + \sqrt{g_{33}^{(2)}}dx_3\begin{pmatrix}-i & 0\\0 & i\end{pmatrix} + \\ & + \sqrt{g_{00}^{(3)}}dx_0\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix} + \sqrt{g_{11}^{(3)}}dx_1\begin{pmatrix}0 & i\\-i & 0\end{pmatrix} + \sqrt{g_{22}^{(3)}}dx_2\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix} + \sqrt{g_{33}^{(3)}}dx_3\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix} + \\ & + \sqrt{g_{00}^{(4)}}dx_0\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix} + \sqrt{g_{11}^{(4)}}x_1\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix} + \sqrt{g_{22}^{(4)}}x_2\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix} + \sqrt{g_{33}^{(4)}}x_3\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix} + \\ & + \sqrt{g_{00}^{(5)}}dx_0\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix} + \sqrt{g_{11}^{(5)}}dx_1\begin{pmatrix}0 & -i\\i & 0\end{pmatrix} + \sqrt{g_{22}^{(5)}}dx_2\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix} + \sqrt{g_{33}^{(5)}}dx_3\begin{pmatrix}i & 0\\0 & i\end{pmatrix} + \\ & + \sqrt{g_{00}^{(6)}}dx_0\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix} + \sqrt{g_{11}^{(6)}}dx_1\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix} + \sqrt{g_{22}^{(6)}}dx_2\begin{pmatrix}0 & i\\-i & 0\end{pmatrix} + \sqrt{g_{33}^{(6)}}dx_3\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix} + \\ & + \sqrt{g_{00}^{(7)}}dx_0\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix} + \sqrt{g_{11}^{(7)}}dx_1\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix} + \sqrt{g_{22}^{(7)}}dx_2\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix} + \sqrt{g_{33}^{(7)}}dx_3\begin{pmatrix}-i & 0\\0 & -i\end{pmatrix} + \\ & + \sqrt{g_{00}^{(8)}}dx_0\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix} + \sqrt{g_{11}^{(8)}}dx_1\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix} + \sqrt{g_{22}^{(8)}}dx_2\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix} + \sqrt{g_{33}^{(8)}}dx_3\begin{pmatrix}i & 0\\0 & -i\end{pmatrix} + \end{aligned}$$

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$$
\begin{aligned}
&+\sqrt{g_{00}^{(9)}}dx_0\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}+\sqrt{g_{11}^{(9)}}dx_1\begin{pmatrix}0 & i\\ -i & 0\end{pmatrix}+\sqrt{g_{22}^{(9)}}dx_2\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}+\sqrt{g_{33}^{(9)}}dx_3\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}+\\
&+\sqrt{g_{00}^{(10)}}dx_0\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}+\sqrt{g_{11}^{(10)}}dx_1\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}+\sqrt{g_{22}^{(10)}}dx_2\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}+\sqrt{g_{33}^{(10)}}dx_3\begin{pmatrix}-i & 0\\ 0 & i\end{pmatrix}+\\
&+\sqrt{g_{00}^{(11)}}dx_0\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}+\sqrt{g_{11}^{(11)}}dx_1\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}+\sqrt{g_{22}^{(11)}}dx_2\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}+\sqrt{g_{33}^{(11)}}dx_3\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}+\\
&+\sqrt{g_{00}^{(12)}}dx_0\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}+\sqrt{g_{11}^{(12)}}dx_1\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}+\sqrt{g_{22}^{(12)}}dx_2\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}+\sqrt{g_{33}^{(12)}}dx_3\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}+\\
&+\sqrt{g_{00}^{(13)}}dx_0\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}+\sqrt{g_{11}^{(13)}}dx_1\begin{pmatrix}0 & i\\ -i & 0\end{pmatrix}+\sqrt{g_{22}^{(13)}}dx_2\begin{pmatrix}0 & -i\\ -i & 0\end{pmatrix}+\sqrt{g_{33}^{(13)}}dx_3\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}+\\
&+\sqrt{g_{00}^{(14)}}dx_0\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}+\sqrt{g_{11}^{(14)}}dx_1\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}+\sqrt{g_{22}^{(14)}}dx_2\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}+\sqrt{g_{33}^{(14)}}dx_3\begin{pmatrix}-i & 0\\ 0 & -i\end{pmatrix}+\\
&+\sqrt{g_{00}^{(15)}}dx_0\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}+\sqrt{g_{00}^{(15)}}dx_1\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}+\sqrt{g_{00}^{(15)}}dx_2\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}+\sqrt{g_{00}^{(15)}}dx_3\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}+\\
&+\sqrt{g_{00}^{(16)}}dx_0\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}+\sqrt{g_{11}^{(16)}}dx_1\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}+\sqrt{g_{22}^{(16)}}dx_2\begin{pmatrix}0 & i\\ i & 0\end{pmatrix}+\sqrt{g_{33}^{(16)}}dx_3\begin{pmatrix}i & 0\\ 0 & i\end{pmatrix}. \qquad (8.32)
\end{aligned}
$$

Within the framework of the Algebra of signature, it is possible much more deeper 2^n-sided levels of the consideration of the metric-dynamic properties of the curved region of a $\lambda_{m,n}$-vacuum, with an increase in the number of a components of the metric tensors to infinity (see Fig. 8.2).

9 THE KINEMATICS OF THE VACUUM LAYERS

9.1 The kinematics of a $\lambda_{m,n}$-vacuum layers

Under the kinematics of a vacuum layers is meant such a section of the light-geometry of the vacuum, in which the displacements (or movements) of the different sides of the $\lambda_{m,n}$-vacuum are considered independently of their deformations.

With a more consistent approach, performed in [15], it turns out that any displacements (or movements) of the local area of each layer of the $\lambda_{m,n}$-vacuum

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are inevitably accompanied by its deformation, and vice versa, the deformation of the local area of the any layer of the $\lambda_{m,n}$-vacuum necessarily accompanied by its displacement (or flow).

Interconnected flows and deformations (i.e., 4-deformations) of the local area of the $\lambda_{m,n}$-vacuum are considered in the Section "Dynamics of the $\lambda_{m,n}$ - vacuum layers" in [15].

In this article, we consider only the kinematic models of the behavior of the $\lambda_{m,n}$-vacuum layers. These models, despite the above disadvantages, allow theoretically predicting a number of the previously unknown vacuum effects that can be tested in practice.

9.2 The nonzero components of the metric tensor

Let the metric-dynamic states of the two 4-dimensional sides of the 2^3-$\lambda_{m,n}$-vacuum local area be given by the intervals (7.35) and (7.37) (see Fig. 7.2). Then the nonzero components of the metric tensors (7.36) and (7.38)

$$g_{\alpha\beta}^{(+)} = \begin{pmatrix} \dots & \dots & \dots & \dots \\ \dots & g_{11}^{(+)} & g_{21}^{(+)} & g_{31}^{(+)} \\ \dots & g_{12}^{(+)} & g_{22}^{(+)} & g_{31}^{(+)} \\ \dots & g_{13}^{(+)} & g_{23}^{(+)} & g_{33}^{(+)} \end{pmatrix}, \quad g_{\alpha\beta}^{(-)} = \begin{pmatrix} \dots & \dots & \dots & \dots \\ \dots & g_{11}^{(-)} & g_{21}^{(-)} & g_{31}^{(-)} \\ \dots & g_{12}^{(-)} & g_{22}^{(-)} & g_{31}^{(-)} \\ \dots & g_{13}^{(-)} & g_{23}^{(-)} & g_{33}^{(-)} \end{pmatrix} \qquad (9.1)$$

determine the local curvature of the 3-dimensional cell of the $\lambda_{m,n}$-vacuum. Here the indices of the Greek alphabet α, β correspond to 3-dimensional consideration (i.e., $\alpha, \beta = 1,2,3$).

The scalar curvature of a 3-dimensional $\lambda_{m,n}$-vacuum cell in a two-sided consideration within the framework of the Algebra of signatures is determined by the averaged Expression [15]

$$R^{(\pm)} = \tfrac{1}{2}\,(R^{(-)}+R^{(+)}) \,, \qquad (9.2)$$

where the scalar curvature of each of the two sides is determined in the same way as in general relativity

$$R^{(-)} = g^{(-)\alpha\beta} R_{\alpha\beta}{}^{(-)} \quad \text{и} \quad R^{(+)} = g^{(+)\alpha\beta} R_{\alpha\beta}{}^{(+)}, \tag{9.3}$$

where

$$R_{\alpha\beta} = \frac{\partial \Gamma^{l}_{\alpha\beta}}{\partial x^{l}} - \frac{\partial \Gamma^{l}_{\alpha l}}{\partial x^{\beta}} + \Gamma^{l}_{\alpha\beta}\Gamma^{m}_{lm} - \Gamma^{m}_{\alpha l}\Gamma^{l}_{m\beta} \tag{9.4}$$

is the Ricci tensor of the outer (–), or inner (+) "side" of the $\lambda_{m,n}$-vacuum local area;

$$\Gamma^{\lambda}_{\alpha\beta} = \frac{1}{2} g^{\lambda\mu}\left(\frac{\partial g_{\mu\beta}}{\partial x^{\alpha}} + \frac{\partial g_{\alpha\mu}}{\partial x^{\beta}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\mu}}\right) \tag{9.5}$$

is the Christoffel symbols of the outer (–), or inner (+) side of the $\lambda_{m,n}$-vacuum local area, where $g^{\alpha\beta}$ is, respectively, $g^{(-)\alpha\beta}$ or $g^{(+)\alpha\beta}$.

The 3-strain tensor of a 3-dimensional cell of the $\lambda_{m,n}$-vacuum in this case is given by the averaged Expression [15]

$$\varepsilon_{\alpha\beta}{}^{(\pm)} = ½ \left(\varepsilon_{\alpha\beta}{}^{(+)} + \varepsilon_{\alpha\beta}{}^{(-)}\right), \tag{9.6}$$

where

$$\varepsilon_{\alpha\beta}{}^{(-)} = ½ \left(g_{\alpha\beta}{}^{(-)} - g_{\alpha\beta 0}{}^{(-)}\right) \tag{9.7}$$

is 3-strains tensor of the outer side of the $\lambda_{m,n}$-vacuum cell;

$$\varepsilon_{\alpha\beta}{}^{(+)} = ½ \left(g_{\alpha\beta}{}^{(+)} - g_{\alpha\beta 0}{}^{(+)}\right) \tag{9.8}$$

is 3-strains tensor of the inner side of the same $\lambda_{m,n}$-vacuum cell.

The theory of the deformation of the $\lambda_{m,n}$-vacuum local 3-dimensional region can be developed by analogy with the theory of elasticity of the conventional (atomistic) continuous elastic-plastic media [6], taking into account the two-sided (or 2^n-side) consideration.

9.3 The zero components of the metric tensor

To clarify the physical meaning of the zero components of the metric tensors (7.36) and (7.38)

$$g_{0j}^{(-)} = \begin{pmatrix} g_{00}^{(-)} & g_{10}^{(-)} & g_{20}^{(-)} & g_{30}^{(-)} \\ g_{01}^{(-)} & \dots & \dots & \dots \\ g_{02}^{(-)} & \dots & \dots & \dots \\ g_{03}^{(-)} & \dots & \dots & \dots \end{pmatrix}, \quad g_{i0}^{(+)} = \begin{pmatrix} g_{00}^{(+)} & g_{10}^{(+)} & g_{20}^{(+)} & g_{30}^{(+)} \\ g_{01}^{(+)} & \dots & \dots & \dots \\ g_{02}^{(+)} & \dots & \dots & \dots \\ g_{03}^{(+)} & \dots & \dots & \dots \end{pmatrix} \quad (9.9)$$

let's use the kinematics of two-sided of a 2^3-$\lambda_{m,n}$-vacuum.

Let the initial (stationary and non-curved) state of the 2^3-$\lambda_{m,n}$-vacuum be given by a set of pseudo-Euclidean metrics (4.33) and (4.35)

$$\begin{cases} ds_0^{(-)2} = c^2dt^2 - dx^2 - dy^2 - dz^2 = ds^{(-)\prime} ds^{(-)\prime\prime} = cdt' cdt'' - dx' dx'' - dy' dy'' - dz' dz'', \\ ds_0^{(+)2} = -c^2dt^2 + dx^2 + dy^2 + dz^2 = ds^{(+)\prime} ds^{(+)\prime\prime} = -cdt' cdt'' + dx' dx'' + dy' dy'' + dz' dz'', \end{cases}$$

where (9.10)

$ds^{(-)\prime} = c\,dt' + idx' + jdy' + kdz'$ is a mask of the subcont; (9.11)

$ds^{(-)\prime\prime} = c\,dt'' + idx'' + jdy'' + kdz''$ is an interior of the subcont (9.12)

$ds^{(+)\prime} = -c\,dt' + idx' + jdy' + kdz'$ is a mask of the antisubcont; (9.13)

$ds^{(+)\prime\prime} = c\,dt'' - idx'' - jdy'' - kdz''$ is an interior of the antisubcont, (9.14)

this is affine aggregates, in particular, quaternions with the multiplication Table of imaginary units, for example,[9]

Table 9.1

	i	j	k
i	-1	k	$-j$
j	$-k$	-1	i
k	j	$-i$	-1

Let's consider the four kinematic cases:

1) Let in the first case the mask and the interior sides of the outer and inner sides of the 2^3-$\lambda_{m,n}$-vacuum move relative to the initial stationary state along the x-axis with the same velocity v_x, but in different directions. This is formally described by the transformation of a coordinates [3, 4]:

$t' = t, \quad x' = x + v_x t, \quad y' = y, \quad z' = z$ – for a mask; (9.15)

$t'' = t, \quad x'' = x - v_x t, \quad y'' = y, \quad z'' = z$ – for an interior. (9.16)

The equality of the moduli of the velocities v_x of the mask and the interior sides is due to the $\lambda_{m,n}$-vacuum balance condition, which requires that an adequate anti-motion corresponds to each movement in the $\lambda_{m,n}$-vacuum.

Differentiating (9.15) and (9.16), and substituting the results of differentiation into metrics (9.10), we obtain the set of the metrics

$$\begin{cases} ds^{(-)2} = (1+v_x^2/c^2)c^2dt^2 - dx^2 - dy^2 - dz^2; & (9.17) \\ ds^{(+)2} = -(1+v_x^2/c^2)c^2dt^2 + dx^2 + dy^2 + dz^2, & (9.18) \end{cases}$$

describing the kinematics of the joint movement of the outer side (i.e., subcont) and the inner side (i.e., antisubcont) of the 2^3-$\lambda_{m,n}$-vacuum, subject to the condition

$$ds^{(-)2} + ds^{(+)2} = 0.$$

The zero components of the metric tensors (9.9) in this case are equal to

$$g_{0j}^{(-)} = \begin{pmatrix} 1+v_x^2/c^2 & 0 & 0 & 0 \\ 0 & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots \end{pmatrix}, \quad g_{i0}^{(+)} = \begin{pmatrix} -1-v_x^2/c^2 & 0 & 0 & 0 \\ 0 & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots \end{pmatrix}. \qquad (9.19)$$

2) In the second case, let the mask and the interior sides of the subcont and antisubcont move relative to their initial stationary state in the same direction – along the x-axis with the same velocity v_x. This is formally described by transformations of the coordinates [3, 4]:

$$t' = t, \quad x' = x - v_x t, \quad y' = y, \quad z' = z \quad \text{– for a mask;} \qquad (9.20)$$

$$t'' = t, \quad x'' = x - v_x t, \quad y'' = y, \quad z'' = z \quad \text{– for an interior.} \qquad (9.21)$$

Differentiating (9.20) and (9.21) and substituting the results of the differentiation into the metrics (9.10), we obtain the set of metrics:

$$\begin{cases} ds^{(-)2} = (1-v_x^2/c^2)c^2dt^2 + v_x dxdt + v_x dtdx - dx^2 - dy^2 - dz^2, & (9.22) \\ ds^{(+)2} = -(1-v_x^2/c^2)c^2dt^2 - v_x dxdt - v_x dxdt + dx^2 + dy^2 + dz^2. & (9.23) \end{cases}$$

In this case, the $\lambda_{m,n}$-vacuum balance condition is also met, since

$$ds^{(-)2} + ds^{(+)2} = 0,$$

but additional cross terms $v_x dxdt$ appear.

The zero components of the metric tensors (9.9) in the second case are

$$g_{0j}^{(-)} = \begin{pmatrix} 1-v_x^2/c^2 & v_x & 0 & 0 \\ v_x & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots \end{pmatrix}, \quad g_{i0}^{(+)} = \begin{pmatrix} -1+v_x^2/c^2 & -v_x & 0 & 0 \\ -v_x & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots \end{pmatrix}. \qquad (9.24)$$

3) Let the mask and the interior sides of the subcont and the antisubcont (i.e., the outer and inner sides of the 2^3-$\lambda_{m,n}$-vacuum) rotate around the z-axis in the same direction with an angular velocity Ω. This is described by changing variables:

$$t' = t, \quad x' = x\cos\Omega t - y\sin\Omega t, \quad z' = z, \quad y' = x\sin\Omega t + y\cos\Omega t, \qquad (9.25)$$

$$t'' = t, \quad x'' = x\cos\Omega t - y\sin\Omega t, \quad z'' = z, \quad y'' = x\sin\Omega t + y\cos\Omega t.$$

Differentiating (9.25) and substituting the results of the differentiation into the metrics (9.10), we obtain the set of the metrics:

$$\begin{cases} ds^{(-)2} = [1-(\Omega^2/c^2)(x^2+y^2)]c^2dt^2 + 2\Omega ydxdt - 2\Omega xdydt - dx^2 - dy^2 - dz^2, \quad (9.26) \\ ds^{(-)2} = -[1-(\Omega^2/c^2)(x^2+y^2)]c^2dt^2 - 2\Omega ydxdt + 2\Omega xdydt + dx^2 + dy^2 + dz^2. \end{cases}$$

In the cylindrical coordinates

$$\rho^2 = x^2 + y^2, \quad z = z, \quad t = t, \quad \varphi = \mathrm{arctg}(y/x) - \Omega t. \qquad (9.27)$$

the metrics (9.26) take the form (9.28)

$$\begin{cases} ds^{(-)2} = (1-\rho^2\Omega^2/c^2)\, c^2dt^2 - \rho^2\Omega/c\, d\varphi dt - \rho^2\Omega/c\, dtd\varphi - d\rho^2 - \rho^2 d\varphi^2 - dz^2, \\ ds^{(+)2} = -(1-\rho^2\Omega^2/c^2)\, c^2dt^2 + \rho^2\Omega/c d\varphi dt + \rho^2\Omega/c dt d\varphi + d\rho^2 + \rho^2 d\varphi^2 + dz^2. \end{cases}$$

In this case the $\lambda_{m,n}$-vacuum balance condition is observed $ds^{(-)2} + ds^{(+)2} = 0$, and the zero components of the metric tensors (9.9) are

$$g_{0j}^{(-)} = \begin{pmatrix} 1-\rho^2\Omega^2/c^2 & -\rho^2\Omega/c & 0 & 0 \\ -\rho^2\Omega/c & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots \end{pmatrix}, \quad g_{0i}^{(+)} = \begin{pmatrix} -1+\rho^2\Omega^2/c^2 & \rho^2\Omega/c & 0 & 0 \\ \rho^2\Omega/c & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots \end{pmatrix}. \qquad (9.29)$$

4) The case can also be considered when the mask and the interior sides of the subcont and the antisubcont rotate in mutually opposite directions with an angular velocity Ω. This is described by changing variables:

$$t' = t, \quad x' = x\cos\Omega t - y\sin\Omega t, \quad z' = z, \quad y' = x\sin\Omega t + y\cos\Omega, \quad (9.30)$$

$$t'' = t, \quad x'' = -x\cos\Omega t + y\sin\Omega t, \quad z'' = z, \quad y'' = -x\sin\Omega t - y\cos\Omega t. \quad (9.31)$$

and leads to similar results.

It can be seen from the considered kinematic examples that the zero components of the metric tensor (9.9) are associated with the translational and/or rotational motion of various 2^3-$\lambda_{m,n}$-vacuum layers.

The state of motion of the local 3-dimensional area of a 2^3-$\lambda_{m,n}$-vacuum is characterized by averaged zero components of the metric tensor

$$g_{i0}^{(\pm)} = \tfrac{1}{2}\left(g_{i0}^{(+)} + g_{i0}^{(-)}\right). \quad (9.32)$$

In all four considered cases, the averaged zero components of the metric tensor (9.32) are equal to zero $g_{i0}^{(\pm)} = \tfrac{1}{2}\left(g_{i0}^{(+)} + g_{i0}^{(-)}\right) = 0$. This means that inside the local 3-dimensional area $\lambda_{m,n}$-vacuum, mutually opposite intravacuum processes can occur, but, on the whole, this area remains motionless.

Nevertheless, there are cases when intravacuum processes due to phase shifts can compensate each other not locally, but globally. In this case, the local 3-dimensional area of the $\lambda_{m,n}$-vacuum can participate (as a whole) in the some intricate closed motion. Let's consider such a case with a specific example.

Let in some local area of the $\lambda_{m,n}$-vacuum the kinematics of intravacuum processes be such that

$t' = t, \quad x' = x + v_{1x}t, \quad y' = y, \quad z' = z$ – for a mask of the subcont; (9.33)

$t'' = t, \quad x'' = x - v_{2x}t, \quad y'' = y, \quad z'' = z$ – for an interior of the subcont; (9.34)

$t' = t, \quad x' = x + v_{3x}t, \quad y' = y, \quad z' = z$ – for a mask of the antisubcont; (9.35)

$t'' = t, \quad x'' = x - v_{4x}t, \quad y'' = y, \quad z'' = z$ – for an interior of the antisubcont, (9.36)

where $v_{1x} \neq v_{2x} \neq v_{3x} \neq v_{4x}$, but the overall balance of movement is observed

$$v_{1x} - v_{2x} + v_{3x} - v_{4x} = 0. \qquad (9.37)$$

In this case, the outer and the inner sides of the 2^3-$\lambda_{m,n}$-vacuum (i.e., the subcont and the antisubcont) are described by a set of metrics

$$\begin{cases} ds^{(-)2} = (1 + v_{1x} v_{2x}/c^2)c^2dt^2 - v_{1x}dtdx + v_{2x}dxdt - dx^2 - dy^2 - dz^2; & (9.38) \\ ds^{(+)2} = -(1 + v_{3x} v_{4x}/c^2)c^2dt^2 + v_{3x}dtdx - v_{4x}dxdt + dx^2 + dy^2 + dz^2. & (9.39) \end{cases}$$

in this case, the averaged zero components of the metric tensor (9.32) are

$$g_{00}^{(\pm)} = (v_{1x} v_{2x} - v_{3x} v_{4x})/2c^2, \quad g_{01}^{(\pm)} = (v_{3x} - v_{1x})/2, \quad g_{10}^{(\pm)} = (v_{2x} - v_{4x})/2 \qquad (9.40)$$

$$\text{for } (v_{1x} + v_{3x}) - (v_{2x} + v_{4x}) = 0. \qquad (9.41)$$

This means that the considered 3-dimensional local area of the $\lambda_{m,n}$-vacuum participates in an intricate motion along the x-axis with the formal observance of the $\lambda_{m,n}$-vacuum balance condition with respect to the total momentum (9.37).

The technical capabilities to create and control of the intra-vacuum flows are considered in Sections P14.6 – P14.7 in [15].

9.4 The limiting velocity of movement of the $\lambda_{m.n}$-vacuum layers

Let's consider the metric (9.22)

$$ds^{(-)2} = (1 - v_x^2/c^2)c^2dt^2 + 2v_xdxdt - dx^2 - dy^2 - dz^2. \qquad (9.42)$$

We select in Expression (9.42) a complete square

$$ds^{(-)2} = dt^2\left[c\sqrt{1-\frac{v_x^2}{c^2}} - \frac{v_x}{cdt}\frac{dx}{\sqrt{1-\frac{v_x^2}{c^2}}}\right]^2 - \frac{dx^2}{1-\frac{v_x^2}{c^2}} - dy^2 - dz^2, \qquad (9.43)$$

and introduce the notation

$$c' = c\sqrt{1-\frac{v_x^2}{c^2}} - \frac{v_x}{cdt}\frac{x}{\sqrt{1-\frac{v_x^2}{c^2}}},$$

$$t'=t,\quad x'=\frac{x}{\sqrt{1-\frac{v_x^2}{c^2}}},\quad y'=y,\quad z'=z. \tag{9.44}$$

In this notation, the metric (9.42) takes the form

$$ds^{(-)2}=c'^2dt'^2-dx'^2-dy'^2-dz'^2. \tag{9.45}$$

If an area of one of the 2^3-$\lambda_{m,n}$-vacuum sides moves as a whole with a velocity v_x [see Expressions (9.20) – (9.23)], then for an outside stationary observer, a direct ray of light will propagate with a speed

$$c'=c\sqrt{1-\frac{v_x^2}{c^2}}-\frac{v_x}{cdt}\frac{x}{\sqrt{1-\frac{v_x^2}{c^2}}}. \tag{9.46}$$

This is similar to how a stationary observer measures the speed of waves propagating along the moving surface of a river. Such an observer will find that the speed of propagation of the wave disturbances depends on the speed of the river flow, while relative to the water itself, the speed of the wave propagation remains unchanged, and depends only on the properties of the water itself (its density, temperature, impurities, etc.).

From Expressions (9.46) it follows that in the case of (9.20) – (9.23) the velocity of propagation of the outer side of the 2^3-$\lambda_{m,n}$-vacuum (i.e., subcont) cannot exceed the speed of light c. At low velocity (i.e., when $v_x << c$) for an outside observer, the speed c' turns out to be somewhat less than the speed of light

$$c'=c-\frac{v_x x}{cdt}.$$

Thus, for the case (9.20) – (9.23), the conclusions the Algebra of signature and special theory of relativity coincide, i.e., the main physical conclusions remain the same.

However, for the case (9.15) – (9.18), the situation is different. Let's consider this version of intra-vacuum processes using the example of the motion of a subcont described by the metric (9.17)

$$ds^{(-)2} = (1+ v_x^2/c^2)c^2dt^2 - dx^2 - dy^2 - dz^2. \quad (9.47)$$

In this case, the introduction of the notation

$$c' = c\sqrt{1+\frac{v_x^2}{c^2}}, \quad t'=t, \quad x'=x, \quad y'=y, \quad z'=z. \quad (9.48)$$

transform the metric (9.47) to the metric (9.45), but there are no restrictions on the oncoming velocities v_x of the mask and the interior sides of the subcont. This circumstance requires a separate detailed consideration, since it allows the possibility of organizing the superluminal intra-vacuum communication channels.

9.5 The Inert properties of layers of a $\lambda_{m,n}$-vacuum

Let's return to the consideration of metrics (9.10)

$$ds^{(+---)2} = ds^{(-)2} = c^2dt^2 - dx^2 - dy^2 - dz^2 , \quad (9.49)$$

$$ds^{(-+++)2} = ds^{(+)2} = -c^2dt^2 + dx^2 + dy^2 + dz^2 . \quad (9.50)$$

We take the value c^2dt^2 in the right parts of these metrics out of brackets

$$ds^{(-)2} = c^2dt^2(1-\frac{v^2}{c^2}), \quad (9.51)$$

$$ds^{(+)2} = -c^2dt^2(1-\frac{v^2}{c^2}), \quad (9.52)$$

where $v = (dx^2 + dy^2 + dz^2)^{1/2}/dt = dl/dt$ is 3-dimensional velocity.

Let's extract the root from the two sides of the resulting Expressions (9.51) and (9.52). As a result, according to the notation (9.11) – (9.14), we get

$$ds^{(-)}{}' = cdt\sqrt{1-\frac{v^2}{c^2}} \quad \text{– for a mask of the subcont;} \quad (9.53)$$

$$ds^{(-)}{}'' = -cdt\sqrt{1-\frac{v^2}{c^2}} \quad \text{– for an interior of the subcont;} \quad (9.54)$$

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$$ds^{(+)}{}' = icdt\sqrt{1-\frac{v^2}{c^2}}$$ – for a mask of the antisubcont; (9.55)

$$ds^{(+)}{}' = -icdt\sqrt{1-\frac{v^2}{c^2}}$$ – for an interior of the antisubcont. (9.56)

For example, consider the 4-dimensional velocity vector of the subcont mask

$$u_i^{(-)} = dx^i / ds^{(-)}{}'. \tag{9.57}$$

Substituting (9.53) in (9.57), we get the components of the 4-velocity [3]

$$u_i^{(-)} = \left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}},\ \frac{v_x}{c\sqrt{1-\frac{v^2}{c^2}}},\ \frac{v_y}{c\sqrt{1-\frac{v^2}{c^2}}},\ \frac{v_z}{c\sqrt{1-\frac{v^2}{c^2}}}\right]. \tag{9.58}$$

Let the mask sides of the subcont move only in the direction of the x-axis, then its 4-velocity has components

$$u_i^{(-)} = \left[\frac{1}{\sqrt{1-\frac{v_x^2}{c^2}}},\ \frac{v_x}{c\sqrt{1-\frac{v_x^2}{c^2}}},\ 0,\ 0\right]. \tag{9.59}$$

Let's now define the 4-acceleration of the subcont mask [3, 4]

$$\frac{du_i^{(-)}}{cdt} = \left[\frac{d}{cdt}\left(\frac{1}{\sqrt{1-\frac{v_x^2}{c^2}}}\right),\ \frac{d}{cdt}\left(\frac{v_x}{c\sqrt{1-\frac{v_x^2}{c^2}}}\right),\ 0,\ 0\right] \tag{9.60}$$

and consider only its x-component

$$\frac{du_x^{(-)}}{cdt} = \frac{d}{cdt}\left(\frac{v_x}{c\sqrt{1-\frac{v_x^2}{c^2}}}\right), \tag{9.61}$$

where the value

$$\frac{d}{dt}\left(\frac{v_x}{\sqrt{1-\frac{v_x^2}{c^2}}}\right) = a_x^{(-)} \qquad (9.62)$$

has the dimension of the *x*-component of the 3-dimensional acceleration.

On the left-hand side of the Expression (9.62), we perform the differentiation operation [4]

$$a_x^{(-)} = \left(\frac{1}{\sqrt{1-\frac{v_x^2}{c^2}}} + \frac{v_x^2}{c^2\left(1-\frac{v_x^2}{c^2}\right)^{\frac{3}{2}}}\right)\frac{dv_x}{dt} \qquad (9.63)$$

and introduce the notation

$$dv_x/dt = a_x^{(-)}{}'. \qquad (9.64)$$

In this case, the Expression (9.63) takes the form

$$a_x^{(-)} = \left(\frac{1}{\sqrt{1-\frac{v_x^2}{c^2}}} + \frac{v_x^2}{c^2\left(1-\frac{v_x^2}{c^2}\right)^{\frac{3}{2}}}\right)a_x^{(-)}{}', \qquad (9.65)$$

where $a_x^{(-)}$ is the actual acceleration of the area of the subcont mask, taking into account its inert properties;

$a_x^{(-)}{}'$ is ideal acceleration of the same area of the subcont mask.

We represent the Expression (9.65) in the following form

$$a_x^{(-)} = \mu_x^{(-)} a_x^{(-)}{}', \qquad (9.66)$$

where

$$\mu_x^{(-)} = \left(\frac{1}{\sqrt{1-\frac{v_x^2}{c^2}}} + \frac{v_x^2}{c^2\left(1-\frac{v_x^2}{c^2}\right)^{\frac{3}{2}}} \right) \qquad (9.67)$$

is the dimensionless coefficient of an inertia, linking the actual and ideal accelerations of the studied local area of the subcont mask, and showing how the inertia (i.e., resistance) of this area changes when the velocity of its movement changes.

From the Expression (9.67) it follows that if $v_x = 0$, then the inertia coefficient $\mu_x^{(-)} = 1$ and $a_x^{(-)} = a_x^{(-)}{'}$. This means that the area of the subcont mask does not resist the beginning of its movement.

When the velocity v_x approaches the speed of light c the inertia coefficient $\mu_x^{(-)}$ tends to infinity, while further acceleration of the subcont mask becomes impossible.

The Expression (9.66) is a massless analogue of Newton's second law

$$F_x = ma_x', \qquad (9.68)$$

where F_x is the component of the force vector; m is mass of the body; a_x' is a component of its ideal acceleration.

Comparing the Expressions (9.66) and (9.68), we find that in $\lambda_{m,n}$-vacuum dynamics, the massless coefficient of inertia $\mu_x^{(-)}$ of the local area of the subcont mask is analogous to the density of the inert mass of a continuous medium in the post-Newtonian physics.

By successive substitution of intervals (9.54) – (9.56) into Expression (9.57), we can get the inertia coefficients $\mu_x^{(-)}{''}$, $\mu_x^{(+)}{'}$, $\mu_x^{(+)}{''}$ for the other three affine layers of the 2^3-$\lambda_{m,n}$-vacuum.

The general coefficient of inertia of the local area of a 2^3-$\lambda_{m,n}$ -vacuum is a function of all four coefficients of inertia

$$\mu_x^{(\pm)} = f\,(\mu_x^{(-)}{'}, \mu_x^{(-)}{''}, \mu_x^{(+)}{'}, \mu_x^{(+)}{''}) \qquad (9.69)$$

An explicit form of this function can be obtained by describing the 2^3-$\lambda_{m,n}$-vacuum dynamics (see [15]).

9.6 The kinematics of rupture of the local area of the "vacuum"

The vacuum light-geometry opens up opportunities for the development of "zero" (i.e., vacuum) technologies. The mathematical apparatus of the Algebra of signatures makes it possible to predict a number of a vacuum effects that, in principle, cannot be predicted by modern "one-sided" physics.

It will be possible to talk about "zero" technologies in more detail after the presentation of a $\lambda_{m,n}$-vacuum dynamics (see [15]).

In this article, we will consider only the kinematic aspects of the possibility of "rupture" the local area of the $\lambda_{m,n}$-vacuum.

Let's integrate the Expression (9.62) [4]:

$$\frac{v_x}{\sqrt{1-\frac{v_x^2}{c^2}}} = a_x t + const\,. \qquad (9.70)$$

Integrating Expression (9.70) once again, and assuming $x_0 = 0$ at $t = 0$, we obtain the following change in the length of the subcont mask along the x-axis at its accelerated movement [4]:

$$x - x_0 = \Delta x = \frac{c^2}{a_x}\left(\sqrt{1+\frac{a_x^2 t^2}{c^2}} - 1\right).$$

Let the initial (i.e. stationary) state of the local area of the subcont be given by the metric (9.49)

$$ds^{(-)2} = c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2\,. \qquad (9.71)$$

The uniformly accelerated motion of a given area of the subcont along the x-axis is formally specified by the transformation of coordinates [4]:

$$t' = t, \quad x' = x + \Delta x = x + \frac{c^2}{a_x}\left(\sqrt{1 + \frac{a_x{}^2 t^2}{c^2}} - 1\right), \quad y' = y, \quad z' = z. \tag{9.72}$$

Differentiating coordinates (9.72), and substituting the results of differentiation in metric (9.71), we obtain the metric [4]

$$ds_a^{(-)2} = \frac{c^2 dt^2}{1 + \frac{a_x{}^2 t^2}{c^2}} - \frac{2a_x\, t\, dt\, dx}{\sqrt{1 + \frac{a_x{}^2 t^2}{c^2}}} - dx^2 - dy^2 - dz^2, \tag{9.73}$$

describing the uniformly accelerated motion of the local area of the subcont (i.e., the outer side of the 2^3-$\lambda_{m,n}$-vacuum) in the direction of the x-axis.

If an additional flow with negative acceleration created in the same area of the subcont

$$\frac{d}{dt}\left(\frac{v_x}{\sqrt{1 - \frac{v_x^2}{c^2}}}\right) = -a_x\,, \tag{9.74}$$

then, performing mathematical calculations similar to (9.70) – (9.73), we obtain the metric

$$ds_b^{(-)2} = \frac{c^2 dt^2}{1 - \frac{a_x^2 t^2}{c^2}} - \frac{2a_x\, t\, dt dx}{\sqrt{1 - \frac{a_x^2 t^2}{c^2}}} - dx^2 - dy^2 - dz^2\,. \tag{9.75}$$

In this case, the average metric-dynamic state of the subcont local area will be characterized by the averaged metric

$$< ds^{(-)} >^2 = \frac{1}{2}\left(ds_a^{(-)2} + ds_b^{(-)2}\right) =$$

$$= \frac{c^2 dt^2}{1 - \frac{a_x^4 t^4}{c^4}} - \frac{a_x t\left(\sqrt{1 - \frac{a_x^2 t^2}{c^2}} + \sqrt{1 + \frac{a_x^2 t^2}{c^2}}\right) dt dx}{\sqrt{1 - \frac{a_x^4 t^4}{c^4}}} - dx^2 - dy^2 - dz^2 \tag{9.76}$$

with signature (+ – – –).

Whence we see that for

$$\frac{a_x^4 t^4}{c^4} = 1, \quad \text{or} \quad |a_x|t = c, \quad \text{or} \quad |a_x| = c\,/\Delta t, \tag{9.77}$$

the first and second terms in the averaged metric (9.76) turn to infinity. This singularity can be interpreted as a "rupture" of the investigated area of the subcont (i.e., the outer side of the 2^3-$\lambda_{m,n}$-vacuum).

The "rupture" of a subcont is an incomplete action. For a complete "rupture" of the local area of the 2^3-$\lambda_{m,n}$-vacuum, it is necessary to "rupture" its inner side (i.e., antisubcont) described by the metric (9.50) with the signature (– + + +). To do this, it is necessary to create similar uniformly accelerated and equally slowed flows in the antisubcont of the same area of the 2^3-$\lambda_{m,n}$-vacuum, so that its average state is determined by the averaged metric [15]

$$< ds^{(+)} >^2 = \frac{1}{2}\left(ds_a^{(+)2} + ds_b^{(+)2}\right) =$$

$$= -\frac{c^2 dt^2}{1 - \frac{a_x^4 t^4}{c^4}} + \frac{a_x t\left(\sqrt{1 - \frac{a_x^2 t^2}{c^2}} + \sqrt{1 + \frac{a_x^2 t^2}{c^2}}\right) dt dx}{\sqrt{1 - \frac{a_x^4 t^4}{c^4}}} + dx^2 + dy^2 + dz^2, \tag{9.78}$$

with a signature (– + + +), which "rupture" under the same conditions

$$\frac{a_x^4 t^4}{c^4} = 1, \quad \text{or} \quad |a_x|t = c, \quad \text{or} \quad |a_x| = c\,/\Delta t. \tag{9.79}$$

The averaging metrics (9.76) and (9.78) leads to the fulfillment of the $\lambda_{m,n}$-vacuum balance condition

$$<< ds >>^2 = ½(< ds^{(+)} >^2 + < ds^{(-)} >^2) = 0, \tag{9.80}$$

which in this situation is equivalent to Newton's third law: "Action is equal to reaction"

$$F_x^{(+)} - F_x^{(-)} = m a_x^{(+)} - m a_x^{(-)} = a_x^{(+)} - a_x^{(-)}. \tag{9.81}$$

That is, the process of "rupture" of the $\lambda_{m,n}$-vacuum local area is similar to the rupture of an ordinary (atomistic) solid body, to which sufficiently large and equal forces, more precisely accelerations, are applied from both sides.

It is not excluded that the conditions described above for the "rupture" of the $\lambda_{m,n}$-vacuum are formed in the collision of the accelerated elementary particles. It is possible that a strong collision of particles leads to the emergence of a web of vacuum "cracks", while the closed cracks scatter in the form of a many new "particles" and "antiparticles".

10 CONCLUSION

The article proposes a method for studying the local volume of a perfect “vacuum” (i.e., an empty space in which there are no particles at all) by probing it with light rays directed from the three mutually perpendicular directions.

As a result of such probing, a 3-dimensional lattice (i.e., a 3D-landscape), consisting of the light rays, is formed in a “vacuum” (see Fig. 2.1). This 3- dimensional landscape is called $\lambda_{m,n}$-vacuum, where $\lambda_{m,n}$ is the wavelength of the probing rays taken from the sub-range $\Delta\lambda = 10^m \div 10^n$ cm.

There are an infinite number of such $\lambda_{m,n}$-vacuums in the investigated volume of the "vacuum" depending on the wavelength $\lambda_{m,n}$ of probing beams (see Fig. 2.2). However, all $\lambda_{m,n}$-vacuums obey the same laws, therefore only one of them is studied in detail in the article.

The analysis of the properties of the $\lambda_{m,n}$-vacuum led to the development of the Algebra of signatures. Let's list the main features and differences of this mathematical construction from the mathematical apparatus of known theories:

1. In the theory of elasticity, in the mechanics of continuous media, and in the general theory of relativity, only one metric is considered, for example,

$$ds^{(+---)2} = g_{ij}^{(-)}dx^i dx^j \quad \text{с сигнатурой} \quad (+---), \qquad (10.1)$$

which determines the metric-dynamic state of the local area of only one side of the continuous medium (in particular, the empty space, i.e., "vacuum"), which in a number of cases leads to paradoxes. Whereas the Algebra of signatures takes into account the totality of 16 different metrics (7.29)

$$\begin{array}{llll}
ds^{(+---)2} & ds^{(++++)2} & ds^{(---+)2} & ds^{(+--+)2} \\
ds^{(--+-)2} & ds^{(++--)2} & ds^{(-+--)2} & ds^{(+-+-)2} \\
ds^{(-+++)2} & ds^{(----)2} & ds^{(+++-)2} & ds^{(-++-)2} \\
ds^{(++-+)2} & ds^{(--++)2} & ds^{(+-++)2} & ds^{(-+-+)2},
\end{array} \qquad (10.2)$$

with 16 corresponding signatures (or topologies) (4.28) that satisfy of the $\lambda_{m,n}$-vacuum balance condition: (10.3)

$$\begin{array}{llclc}
0 = & \underline{(0\ \ 0\ \ 0\ \ 0)} & + & \underline{(0\ \ 0\ \ 0\ \ 0)} & = 0 \\
0 = & (+\ +\ +\ +) & + & (-\ -\ -\ -) & = 0 \\
0 = & (-\ -\ -\ +) & + & (+\ +\ +\ -) & = 0 \\
0 = & (+\ -\ -\ +) & + & (-\ +\ +\ -) & = 0 \\
0 = & (-\ -\ +\ -) & + & (+\ +\ -\ +) & = 0 \\
0 = & (+\ +\ -\ -) & + & (-\ -\ +\ +) & = 0 \\
0 = & (-\ +\ -\ -) & + & (+\ -\ +\ +) & = 0 \\
0 = & (+\ -\ +\ -) & + & (-\ +\ -\ +) & = 0 \\
0 = & \underline{(-\ +\ +\ +)} & + & \underline{(+\ -\ -\ -)} & = 0 \\
0 = & (0\ \ 0\ \ 0\ \ 0)_+ & + & (0\ \ 0\ \ 0\ \ 0)_+ & = 0
\end{array}$$

2. From the ranked Expression (10.3), subject to the requirements of the $\lambda_{m,n}$-vacuum balance condition, follows the ranked Expression (4.32)

$$\begin{array}{lclc}
(+\ +\ +\ +) & + & (-\ -\ -\ -) & =0 \\
(-\ -\ -\ +) & + & (+\ +\ +\ -) & =0 \\
(+\ -\ -\ +) & + & (-\ +\ +\ -) & =0 \\
(-\ -\ +\ -) & + & (+\ +\ -\ +) & =0 \\
(+\ +\ -\ -) & + & (-\ -\ +\ +) & =0 \\
(-\ +\ -\ -) & + & (+\ -\ +\ +) & =0 \\
\underline{(+\ -\ +\ -)} & + & \underline{(-\ +\ -\ +)} & =0 \\
(+\ -\ -\ -)_+ & + & (-\ +\ +\ +)_+ & =0\ .
\end{array}$$

which leads to put forward a hypothesis that the "vacuum" has at least two sides: the outer side with the signature of the Minkowski 4-space (+ – – –); and the inner side with the opposite signature of the Minkowski 4-antispace (– + + +).

3. Within the framework of the Algebra of signatures, there are two types of the $\lambda_{m,n}$-vacuums (commutative and anticommutative) and two similar types of the $\lambda_{m,n}$-antivacuums with the sign multiplication rules (4.11) – (4.18). It is assumed that the joint consideration of the $\lambda_{m,n}$-vacuums and $\lambda_{m,n}$-antivacuums will significantly expand the possibilities of the theory.

4. The multidimensional space of the Algebra of signatures is supersymmetric, since at each point of a given single manifold, both commutative and anticommutative operations on sets of numbers are specified. There is a clear analogy between the multidimensional, supersymmetric and Ricci planar spaces of the Algebra of signatures and the Calabi - Yau manifold.

Taking into account the full set of metrics (10.2) makes it possible to outline ways to solve a number of problems that previously could not be solved. For example, within the framework of geometrized physics of the “vacuum”, developed on the basis of the Algebra of signatures, it is possible to propose a theoretical basis for the development of "zero" (i.e. vacuum) technologies, such as: "transmission of the information through the superluminal communication channels", "compression of the communication channels based on spectral-signature (color) Fourier analysis” (see §3.5), “using intravacuum flows for movement in the space”, etc. [13,14,15].

The formal mathematical apparatus of the Algebra of signatures is a differential, multisignature, transversely and longitudinally stratified, supersymmetric and infinite-dimensional light-geometry, which become more and more complicated as one immerses in infinite deep of the vacuum. But initially, the Algebra of signatures provides algorithms for the compactification and/or convolution of the many extra dimensions and topological layers to describe the metric-dynamic properties of the 3-dimensional volume of the "vacuum".

Constant observance of the "$\lambda_{m,n}$-vacuum balance condition" allow the Algebra of signatures to avoid the paradoxes.

According to the author, the formal mathematical apparatus of the Algebra of signatures creates a logical platform for planning experiments related to the study of "vacuum" effects and performing actions on the "vacuum" as a real object of research.

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12 TERMS AND DEFINITIONS

New terms and numbers of their definitions are presented in the Table 12.1.

Table 12.1

The Term	Number of the Definition
Perfect vacuum	2.1.1
"Vacuum"	2.1.1
$\lambda_{m,n}$-vacuum	2.1.3
Longitudinal stratification of the "vacuum"	2.1.4
Time axis	2.3.1
Stignature	3.1.1, 3.5.1
Signature	4.1.1
Rank	4.1.2
$\lambda_{m,n}$-vacuum balance condition	4.5.1
Subcont	4.7.1
Antisubcont	4.7.1
Transverse stratification of a "vacuum"	4.9.1
Transverse stratification of a $\lambda_{m,n}$-vacuum	4.9.2
k-braid	8.1.1

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