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Article

Differential evolution with shadowed and general type-2 fuzzy systems for dynamic parameter adaptation in optimal design of fuzzy controllers

Patricia Ochoa[1], Oscar Castillo[2,*], Patricia Melin[3] and José Soria [4]

[1] Tijuana Institute of Technology; martha.ochoa18@tectijuana.edu.mx
[2] Tijuana Institute of Technology; ocastillo@tectijuana.mx
[3] Tijuana Institute of Technology; pmelin@tectijuana.mx
[4] Tijuana Institute of Technology; jsoria57@gmail.com
* Correspondence: ocastillo@tectijuana.mx

Abstract: This work is mainly focused on improving the differential evolution algorithm with the utilization of shadowed and general type 2 fuzzy systems to dynamically adapt one of the parameters of the evolutionary method. In this case, the mutation parameter is dynamically moved during the evolution process by using a shadowed and general type-2 fuzzy systems. The main idea of this work is to make a performance comparison between using shadowed and general type 2 fuzzy systems as controllers of the mutation parameter in differential evolution. The performance is compared with the problem of optimizing fuzzy controllers for a D.C. Motor. Simulation results show that general type-2 fuzzy systems are better when higher levels of noise are considered in the controller.

Keywords: Shadowed Type-2 Fuzzy Sets, Generalized Type-2 Fuzzy Systems and Differential Evolution algorithm.

1. Introduction

The utilization of new strategies to improve the functioning of certain processes is something very common today, and under this concept we have the Differential Evolution (DE) algorithm, which is used in multiple disciplines to perform optimization. The main approach for this work is the adaptation of a parameter of the DE algorithm using two variants of fuzzy logic, which are Shadowed and General Type 2 fuzzy systems.

Previously, a study was carried out using the differential evolution algorithm and the concept of Shadowed Type 2 fuzzy systems applied to benchmark functions and a control problem [1]. In this work, now we are aiming at comparing the two concepts of shadowed and general type 2 fuzzy systems in order to find out which method is better in improving the performance of the DE algorithm in the process of optimizing fuzzy controllers.

Today the utilization of shadowed type 2 fuzzy systems has become more common in the literature, and below we mention some of these recent works in different disciplines. For example, a shadowed set-based method and its application to large-scale group decision making was proposed in [2], a more comprehensible perspective for interval shadowed sets obtained from fuzzy sets was put forward in [3], an interval data driven construction of shadowed sets with application to linguistic word modelling was outlined in [4], and a shadowed set approximation of fuzzy sets based on nearest quota of fuzziness was described in [5]. In addition, an approach for parameterized shadowed type-2 fuzzy membership functions applied in control applications was outlined in [6], a two-threshold model for shadowed set with gradual representation of cardinality is presented in [7], and a hybrid design of shadowed type-2 fuzzy inference systems applied in diagnosis problems was put forward in [8], just to mention some related papers. In a similar fashion, the use of general type 2 fuzzy systems has become more common in different application areas, but mainly in the control area, and this work is mainly focused on this area. Some related works can be mentioned as follows: the optimal design of a general type-2 fuzzy classifier for the pulse level and its hardware implementation was presented in [9], a hybridized forecasting method based on weight adjustment of neural network using generalized type-2 fuzzy set was outlined in [10], parameter adaptation in the imperialist competitive algorithm using generalized type-2 fuzzy logic was described in [11], the optimization of fuzzy controller design using a differential evolution algorithm with dynamic parameter adaptation based on type-1 and interval type-2 fuzzy systems was put forward in [12], and a comprehensive review on type 2 fuzzy logic applications was outlined in [13].

In general, the most relevant contribution of the article is the comparison of the performance of shadowed type-2 and general type-2 fuzzy systems in achieving dynamic parameter adaptation in DE. This was achieved by making a comparison regarding the performance of DE in optimizing a fuzzy controller applied to nonlinear plant. A statistical comparison was used to verify which of the two types of fuzzy systems is better for parameter adjustment in a dynamical way for the DE algorithm. It can be mentioned that this has not been previously done in the current literature.

The article contains the following sections: Section 2 summarizes the basic constructs of Shadowed Type-2 Fuzzy Systems theory, Section 3 outlines the General Type-2 Fuzzy Systems theory, Section 4 explains in detail the differential evolution algorithm, Section 5 explains the method for dynamic parameter adjustment in differential evolution, Section 6 shows the experimentation done with the control problem and lastly in Section 7 the conclusions are offered, as well as some possible lines of future fruitful research work.

2. Type-2 Fuzzy Systems and Shadowed Sets

In the literature, the term of fuzzy set appears for the first time in 1965, which mainly tells us that as a system complexity increases, the preciseness of its perception and our ability to express its behavior decreases, and from this idea is that fuzzy systems emerge. However, today the fuzzy systems that we are now dealing with have evolved to be General Type-2 fuzzy systems, which can help to solve more complex systems or with higher uncertainty. The mathematical formulation of general type-2 fuzzy sets is expressed in Eq. 1:

$$\tilde{\tilde{A}} = \{((x,u), \mu_{\tilde{A}}(x)) | \forall x \in X, \forall u \in J_x^u \subseteq [0,1]\} \quad (1)$$

The general type-2 fuzzy set (GT2 FS) is currently used in different real-world applications, and there are some options to model or approximate a GT2 FS, one of them can be the vertical slices or z-slices representation [15], [16], [17]. The main part of this work focuses on the continuation of the previous work on fuzzy systems for dynamic parameter adaptation in harmony search and differential evolution. We continue taking into account the main idea of the previous work, which focuses on the representation of α planes, which mainly tells us that we can discretize the secondary axis of GT2 FS in several horizontal sections, which are called α planes, these α planes are expressed by Equation 2 and can be calculated as an interval type-2 fuzzy system (IT2FIS) [18]. Equation 3 expresses the modeling of a general fuzzy inference system (GT2FIS) as the union of the IT2FIS.

$$\tilde{A}_\alpha = \{((x,u), \alpha) | \forall x \in X, \forall u \in J_x \subseteq [0,1]\} \quad (2)$$

$$\tilde{\tilde{A}} = \bigcup \tilde{A}_\alpha \quad (3)$$

The purpose of the Shadowed Type-2 FIS [19] consists of reducing the computational cost represented by the use of α-planes, and the main characteristic of this proposal is to model the GT2FIS with only two optimal α-planes, eliminating the excessive precision, all that aforementioned knowledge is based on the concepts proposed by Pedrycz, who tells us about the theory of shadowed sets in [20] [21][22].

Equation 4 expresses the explanation of the shadowed set concept, which consists on performing two α-cuts on a fuzzy set, with α and β values, which are based on these α-cuts.

$$S_{\mu_A}(x) = \begin{cases} 1, if\ \mu_A(x) \geq \alpha \\ 0, if\ \mu_A(x) \leq \beta \\ [0,1], if\ \alpha \leq \mu_A(x) \geq \beta \end{cases} \tag{4}$$

There are 3 regions, which can have the following interpretation:

- The elevated region for the membership degrees with a value of 1.
- The reduced region for the membership degrees with a value of 0.
- The shaded region with degree of membership in [0, 1].

Using these regions as a reference, Pedrycz proposes that for finding the optimal α and β values, that they can be calculated using Eq. 5, which expresses the calculation to obtain the shadowed area.

$$elevated\ area_{(\alpha,\beta)}(\mu_A) + reduced\ area_{(\alpha,\beta)}(\mu_A) = shadowed\ area_{(\alpha,\beta)}(\mu_A) \tag{5}$$

The aforementioned can be represented graphically with Figure 1.

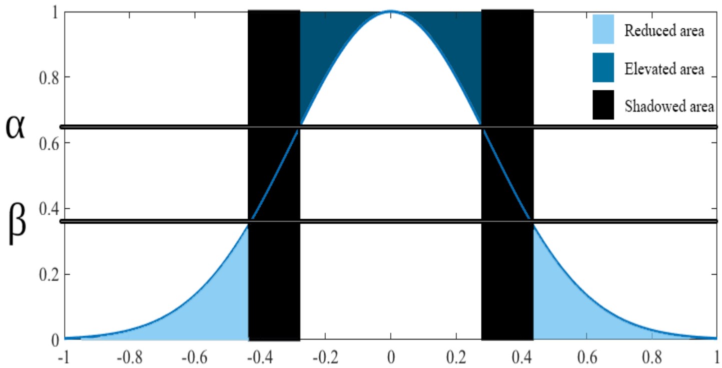

Figure 1. Graphical representation of a shadowed set.

The optimal α and β values can then be obtained by optimizing the $V(\alpha,\beta)$ function described by Eq. 6

$$V(\alpha,\beta) = \left| \int_{x\in A_r} \mu_A(x)dx + \int_{x\in A_e} (1-\mu_A(x))dx - \int_{x\in S} dx \right| \tag{6}$$

This is how we can take advantage of shadowed type-2 fuzzy sets to combine them into the structure of the differential evolution algorithm for dynamic parameter adaptation.

A fuzzy system can be built with a Trapezoidal Shadowed Type-2 fuzzy set membership function (TrapG ST2 MF) introduced in [23] and that is based on a Trapezoidal general type-2 (GT2) membership function with a Gaussian membership function as a secondary membership function. The mathematical knowledge of the membership functions is formulated in Eq. 7 and we can appreciate its graphical form in Figure 2.

$$\text{TrapG ST2 MF} = \begin{cases} \propto_o \begin{cases} \overline{\mu}_O = \dfrac{\overline{\mu}_t(x) + \underline{\mu}_t(x)}{2} - 1.449 \left| \dfrac{\overline{\mu}_t(x) - \underline{\mu}_t(x)}{10} \right| \\ \underline{\mu}_O = \dfrac{\overline{\mu}_t(x) + \underline{\mu}_t(x)}{2} + 1.449 \left| \dfrac{\overline{\mu}_t(x) - \underline{\mu}_t(x)}{10} \right| \end{cases} \\ \propto_I \begin{cases} \overline{\mu}_I = \dfrac{\overline{\mu}_t(x) + \underline{\mu}_t(x)}{2} - 0.9282 \left| \dfrac{\overline{\mu}_t(x) - \underline{\mu}_t(x)}{10} \right| \\ \underline{\mu}_I = \dfrac{\overline{\mu}_t(x) + \underline{\mu}_t(x)}{2} + 0.9282 \left| \dfrac{\overline{\mu}_t(x) - \underline{\mu}_t(x)}{10} \right| \end{cases} \end{cases} \quad (7)$$

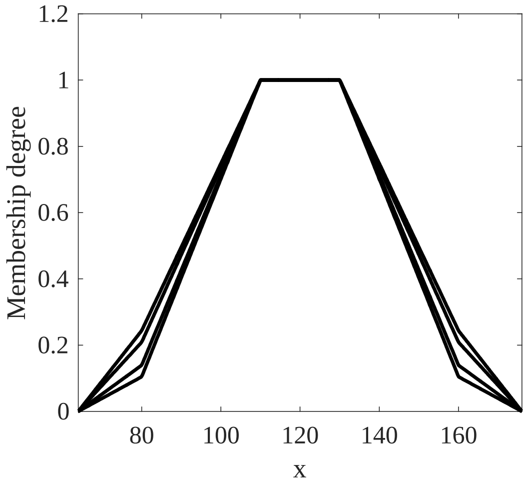

Figure 2. Graphical representation of a Trapezoidal ST2 MF.

3. General Type-2 Fuzzy Systems

Another important part of our work is the utilization of Generalized Type-2 fuzzy logic, which works under the same concept as Type-1 and interval Type-2 fuzzy logic systems, except that their mathematical functions contemplate different concepts since GT2FSs are well known for handling a higher level of uncertainty. There are different definitions about the mathematical functions used in a Generalized Type-2 fuzzy logic system, and for this work we are going to use the notation represented on [24-27]. The formulation of General Type-2 Fuzzy Sets is presented in Eq. 8.

$$\tilde{\tilde{A}} = \{((x,u), \mu_{\tilde{A}}(x,u)) \mid \forall_x \in X,\ \forall_u \in J_x \subseteq [0,1]\} \quad (8)$$

Where $J_x \subseteq [0, 1]$, x represents a primary membership function partition, and u represents a secondary membership function partition.

The graphical representation of a type-2 membership function is illustrated in Figure 3. On the other hand, we can notice the concept of footprint uncertainty (FOU) in Figure 4, which is shown in the third dimension and enables a clearer visualization of the real-world uncertainty modeling.

There is a difference in the nomenclature of each of the fuzzy systems:

The notation $\mu(x)$ is used for Type-1 and Interval Type-2 fuzzy systems.

The notation $f_x(u)$ is used for General Type-2 fuzzy logic systems.

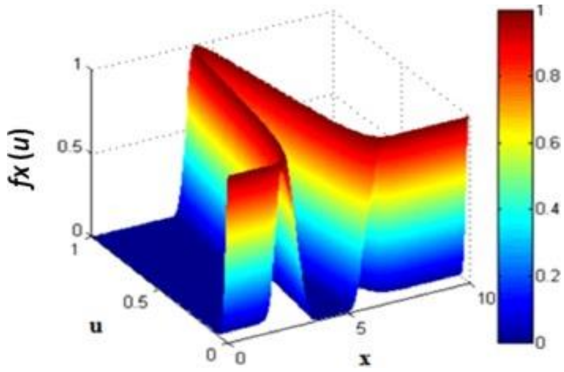

Figure 1. Visual representation of the GT2FS membership function.

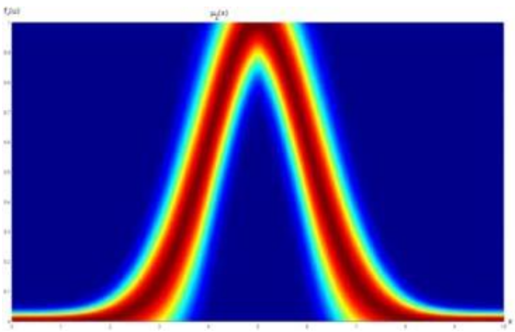

Figure 2. Visual representation of the FOU of the GT2FS membership function.

The α-plane for a General Type-2 fuzzy set, in this case $\tilde{A}$, is denoted by $\tilde{A}_{\alpha}$, and it is the union of all primary membership functions of Ã, which secondary membership degrees are higher or equal to α ($0 \leq \alpha \leq 1$) [28- 29]. The visual representation of an alpha plane can be found in Figure 5, in the same way the expression of the alpha plane is given by Eq. 9.

$$\tilde{A}_{\alpha} = \{(x,u), \mu_{\tilde{A}}(x,u) \geq \alpha | \forall_x \in X, \forall_u \in J_x \subseteq [0,1]\} \quad (9)$$

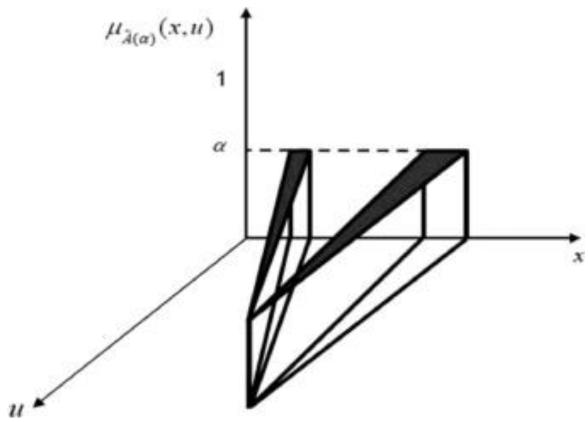

Figure 3. Representation of an alpha-plane corresponding to a type-2 fuzzy set.

4. Differential Evolution Algorithm

Differential evolution is a metaheuristic with which we have previously worked, and it has always provided good results in the different experiments that we have carried out. This is an algorithm that is mainly composed of the following operations:

Equations 10-15 define the initialization of the population structure, Eq. 16 represents the initialization of the algorithm, Eq. 17 represents the mutation performed by the algorithm, Eq. 18 shows the crossover process and finally Eq. 19 expresses the last step, which is the selection.

A more detailed explanation of the equations can be found in previous works [30-34].

Structure of the Population

$$P_{x,g} = (\boldsymbol{x}_{i,g}), i = 0,1,\dots,Np-1,\ g = 0,1,\dots,g_{max}, \quad (10)$$

$$\boldsymbol{x}_{i,g} = (x_{j,i,g}), \quad j = 0,1,\dots,D-1 \quad (11)$$

$$P_{v,g} = (\boldsymbol{v}_{i,g}), i = 0,1,\dots,Np-1,\ g = 0,1,\dots,g_{max}, \quad (12)$$

$$\boldsymbol{v}_{i,g} = (v_{j,i,g}), \quad j = 0,1,\dots,D-1 \quad (13)$$

$$P_{u,g} = (\boldsymbol{u}_{i,g}), i = 0,1,\dots,Np-1,\ g = 0,1,\dots,g_{max}, \quad (14)$$

$$\boldsymbol{u}_{i,g} = (u_{j,i,g}), \quad j = 0,1,\dots,D-1 \quad (15)$$

Initialization

$$x_{j,i,0} = rand_j(0,1) \cdot (b_{j,U} - b_{j,L}) + b_{j,L} \quad (16)$$

Mutation

$$\boldsymbol{v}_{i,g} = \boldsymbol{x}_{r_0,g} + \mathrm{F} \cdot (\boldsymbol{x}_{r_1,g} - \boldsymbol{x}_{r_2,g}) \quad (17)$$

Crossover

$$\boldsymbol{u}_{i,g} = u_{j,i,g} \begin{cases} v_{j,i,g} & if\ (rand_j(0,1) \le Cr\ or\ j = j_{rand}) \\ x_{j,i,g} & otherwise \end{cases} \quad (18)$$

Selection

$$\boldsymbol{x}_{i,g+1} = \begin{cases} \boldsymbol{u}_{i,g} & if\ f(\boldsymbol{u}_{i,g}) \le f(\boldsymbol{x}_{i,g}) \\ \boldsymbol{x}_{i,g} & otherwise \end{cases} \quad (19)$$

5. Differential Evolution algorithm with dynamic parameter adaptation

The structure of each of the fuzzy systems created for experimentation is explained in more detail below. We consider shadowed and generalized type-2 fuzzy systems, which contain one input and one output. As input variable we consider the "generations", which is represented in Eq. 20, the experiment refers to generations for the Fuzzy DE. In this case, the current experiment represents the current generation number, and the maximum of experiments represents the maximum number of generations. For the output parameter we are using the variable F representing the mutation of the differential evolution algorithm.

$$\boldsymbol{Generations} = \frac{\boldsymbol{Current\ generation}}{\boldsymbol{Maximun\ of\ generation}} \quad (20)$$

Equation 21 represents the mutation parameter, and this parameter is the output of the fuzzy system. In other words, F is the fuzzy parameter, which changes dynamically in DE.

$$F = \frac{\sum_{i=1}^{r_F} \mu_i^F\ (F_{1i})}{\sum_{i=1}^{r_F} \mu_i^F} \quad (21)$$

Where F, is the output and the mutation parameter; r_{hmr}, is the number of rules of the fuzzy systems corresponding to F; F_{1i}, is the output result for rule i; μ_i^F, is the membership function of rule i.

The inputs and outputs of both fuzzy systems are granulated into three membership functions, and they are called *low*, *medium* and *high.*

The rules that make both systems are based on previous experimentation experience, these rules can be observed in Table 1.

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Table 1. Rules of the ST2FDE fuzzy system.

Generation \ F	Low	Medium	High
Low	–	–	Low
Medium	–	Medium	–
High	High	–	–

➢ **Shadowed Type 2 fuzzy systems**

In the first instance we have a fuzzy system, which we called ST2FDE since it represents the fuzzy system using Shadowed Type-2 fuzzy sets. This fuzzy system is composed of an input called generations and an output called F that represents mutation in the differential evolution algorithm, another characteristic of the system is that it corresponds to a Mamdani type.

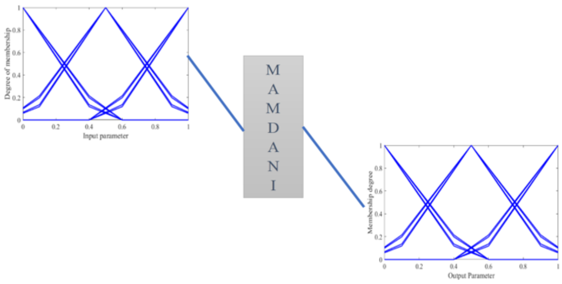

Figure 6. Shadowed Type-2 fuzzy system.

➢ **Generalized Type 2 fuzzy systems**

The second fuzzy system used in this paper is of Generalized Type-2 fuzzy form, just like our ST2FDE system contains an input called generations and an output called F that corresponds to the mutation. The type of membership functions that the system contains are triangular and their mathematical knowledge is expressed in Eq. 22, we called the Generalized Type-2 fuzzy system as GT2FDE.

$$\mu(x,u) = trigausstype2(x,u[a_1,b_1,c_1,a_2,b_2,c_2,\rho])$$

$$\mu(x,u) = exp\left[-\frac{1}{2}\left(\frac{u-P_X}{\sigma_u}\right)\right] where$$

$$\mu_1(x) = \max\left(\min\left(\frac{x-a_1}{b_1-a_1},\frac{c_1-x}{c_1-b_1}\right),0\right)\ and$$

$$\mu_2(x) = \max\left(\min\left(\frac{x-a_2}{b_2-a_2},\frac{c_2-x}{c_2-b_2}\right),0\right)$$

$$\overline{\mu}(x) = \begin{cases}\max(\mu_1(\mathrm{x}),\mu_2(\mathrm{x}))\ \forall_x \notin (b_1,b_2) \\ 1 \qquad \forall_x \in (b_1,b_2)\end{cases} \tag{22}$$

$$\underline{\mu}(x) = \min(\mu_1(\mathrm{x}),\mu_2(\mathrm{x}))$$

$$\rho_x = max\left(\min\left(\frac{x-a_x}{b_x-a_x}\right),\left(\frac{c_x-x}{c_x-b_x}\right),0\right), where$$

$$a_x = \frac{a_1+a_2}{2}, b_x = \frac{b_1+b_2}{2}, c_x = \frac{c_1+c_2}{2},$$

$$\delta = \overline{\mu}(x) - \underline{\mu}(x)$$

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$$\sigma_u = \frac{1+\rho}{2\sqrt{3}}\,\delta + \varepsilon$$

Where a_1, b_1 *and* c_1 are the upper membership function parameters and a_2, b_2 *and* c_2 are the lower membership function parameters, respectively. In addition, ρ is the fraction of uncertainty of the secondary membership function support.

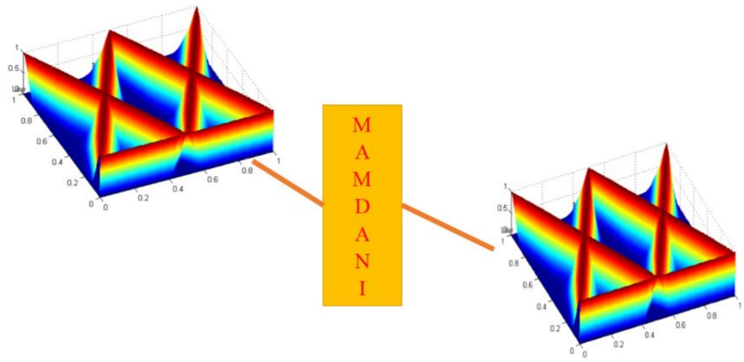

Figure 7. General Type-2 fuzzy logic system.

Table 2 shows the mathematical expression used by the GT2FDE fuzzy system. This table summarizes the parameterized knowledge of the fuzzy system.

Table 2. Parameters of the General Type-2 fuzzy sets.

	Generalized Type-2 fuzzy logic sets
Low	$\mu_1(x) = \max\left(\min\left(\frac{x-0.5}{-0.08+0.5}, \frac{0.4-x}{0.4+0.08}\right), 0\right)$ *and*
	$\mu_2(x) = \max\left(\min\left(\frac{x+0.4}{0.08+0.4}, \frac{0.5-x}{0.5-0.08}\right), 0\right)$
	$\overline{\mu}(x) = \begin{cases} \max(\mu_1(x), \mu_2(x)) & \forall_x \notin (-0.08, 0.08) \\ 1 & \forall_x \in (-0.08, 0.08) \end{cases}$
	$\underline{\mu}(x) = \min(\mu_1(x), \mu_2(x))$
	$\rho_x = max\left(\min\left(\frac{x-a_x}{b_x-a_x}\right), \left(\frac{c_x-x}{c_x-b_x}\right), 0\right), where$
	$a_x = \frac{-0.5-0.4}{2}, b_x = \frac{-0.8-0.08}{2}, c_x = \frac{-0.4-0.5}{2},$
	$\delta = \overline{\mu}(x) - \underline{\mu}(x)$
	$\sigma_u = \frac{1+\rho}{2\sqrt{3}}\,\delta + \varepsilon$
	$Where\ \rho = 0.5$
Medium	$\mu_1(x) = \max\left(\min\left(\frac{x+0.084}{0.4+0.084}, \frac{0.92-x}{0.92-0.4}\right), 0\right)$ *and*
	$\mu_2(x) = \max\left(\min\left(\frac{x-0.084}{0.5-0.084}, \frac{1.07-x}{1.07-0.5}\right), 0\right)$
	$\overline{\mu}(x) = \begin{cases} \max(\mu_1(x), \mu_2(x)) & \forall_x \notin (0.4, 0.5) \\ 1 & \forall_x \in (0.4, 0.5) \end{cases}$
	$\underline{\mu}(x) = \min(\mu_1(x), \mu_2(x))$
	$\rho_x = max\left(\min\left(\frac{x-a_x}{b_x-a_x}\right), \left(\frac{c_x-x}{c_x-b_x}\right), 0\right), where$

	$a_x = \frac{-0.084+0.084}{2}, b_x = \frac{0.4-0.5}{2}, c_x = \frac{0.92-1.09}{2},$ $\delta = \overline{\mu}(x) - \underline{\mu}(x)$ $\sigma_u = \frac{1+\rho}{2\sqrt{3}}\delta + \varepsilon$ $Where\ \rho = 0.5$
High	$\mu_1(x) = \max\left(\min\left(\frac{x-0.4}{0.92-0.4}, \frac{1.4-x}{1.4-0.92}\right), 0\right)\ and$ $\mu_2(x) = \max\left(\min\left(\frac{x-0.5}{1.07-0.5}, \frac{1.5-x}{1.5-1.07}\right), 0\right)$ $\overline{\mu}(x) = \begin{cases} \max(\mu_1(\mathrm{x}), \mu_2(\mathrm{x})) & \forall_x \notin (0.92, 1.07) \\ 1 & \forall_x \in (0.92, 1.07) \end{cases}$ $\underline{\mu}(x) = \min(\mu_1(\mathrm{x}), \mu_2(\mathrm{x}))$ $\rho_x = max\left(\min\left(\frac{x-a_x}{b_x-a_x}\right), \left(\frac{c_x-x}{c_x-b_x}\right), 0\right), where$ $a_x = \frac{0.4+0.5}{2}, b_x = \frac{0.92-1.07}{2}, c_x = \frac{1.4-1.5}{2},$ $\delta = \overline{\mu}(x) - \underline{\mu}(x)$ $\sigma_u = \frac{1+\rho}{2\sqrt{3}}\delta + \varepsilon$ $Where\ \rho = 0.5$

6. Experiments whit the D.C. Motor Speed Controller

For the experimentation, it was decided to use a reference control problem, which is used in real applications in the industry. We decided to use the direct current (D.C.) motor speed control problem, and the purpose of the experimentation is to improve the response capacity using the two proposals for fuzzy systems, namely GT2FDE and ST2FDE.

We illustrate in Figure 8 a schematic view showing the form of a D.C. Motor [35].

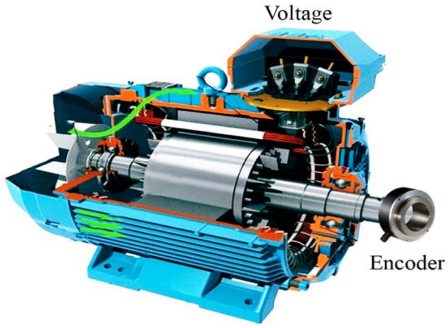

Figure 8. Control in a D.C. Motor.

The structure of the controller with respect to the fuzzy system is of two inputs, which are the error and the error change, and the output that corresponds to the voltage. The controller is of the Mamdani type, and the aforementioned can be appreciated in Figure 9. Another important aspect of the controller is its rules, which are represented in Table 3, and they form a rule base of 15 fuzzy rules.

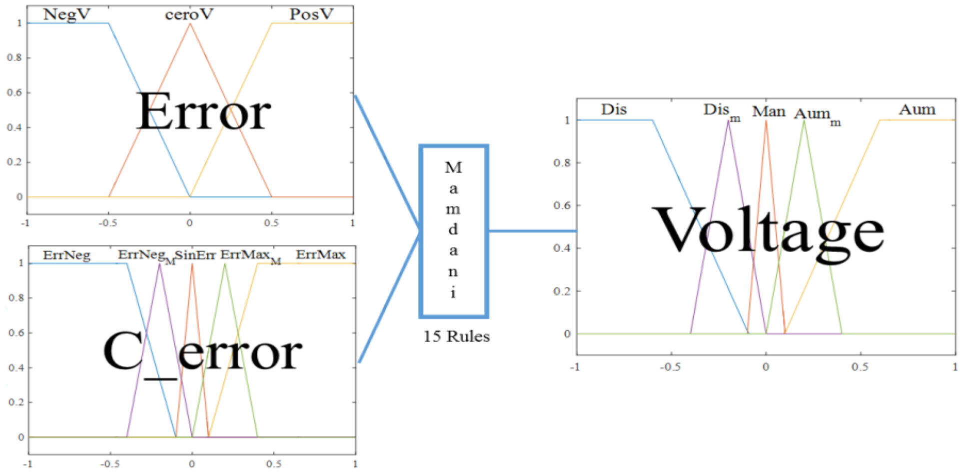

Figure 9. The structure of the fuzzy controller of the Motor.

Table 3. Fuzzy Rules for Motor Control.

No.	Inputs		Output
	Error	Change in Error	Voltage
1	NegV	ErrNeg	Dis
2	NegV	SinErr	Dis
3	NegV	ErrMax	Dis_m
4	CeroV	ErrNeg	Aum_m
5	CeroV	ErrMax	Dis_m
6	PosV	ErrNeg	Aum_m
7	PosV	SinErr	Aum
8	PosV	ErrMax	Aum
9	CeroV	SinErr	Man
10	NegV	ErrNeg_M	Dis
11	CeroV	ErrNeg_M	Aum_m
12	PosV	ErrNeg_M	Aum
13	PosV	ErrMax_M	Aum
14	CeroV	ErrMax_M	Dis_m
15	NegV	ErrMax_M	Dis

The main characteristic of the controller is to achieve moving from a resting state to a desired reference speed of 40m/s. Figure 10 illustrates the reference for the speed of the controller with respect to time.

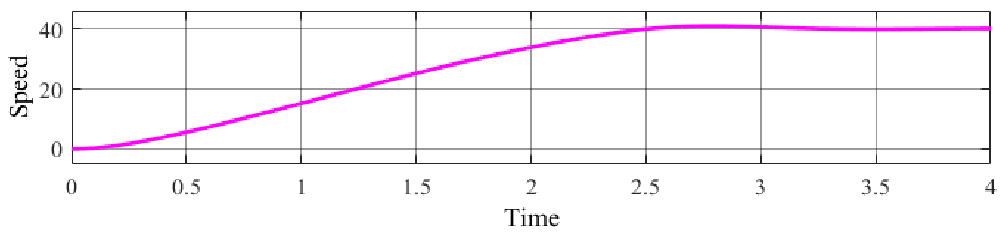

Figure 10. Speed response without optimization.

The experimentation of this work is mainly based on separately using the two fuzzy systems to optimize membership function parameters of the fuzzy system of the controller. The fuzzy controller is formed by 45 parameters that represent the sum of the points that make up each of the membership functions.

Figure 11 expresses the composition of the complete vector formed by all the fuzzy system parameters and based on these parameters the evolutionary algorithm combined with the fuzzy system searches for the best architecture for the fuzzy controller.

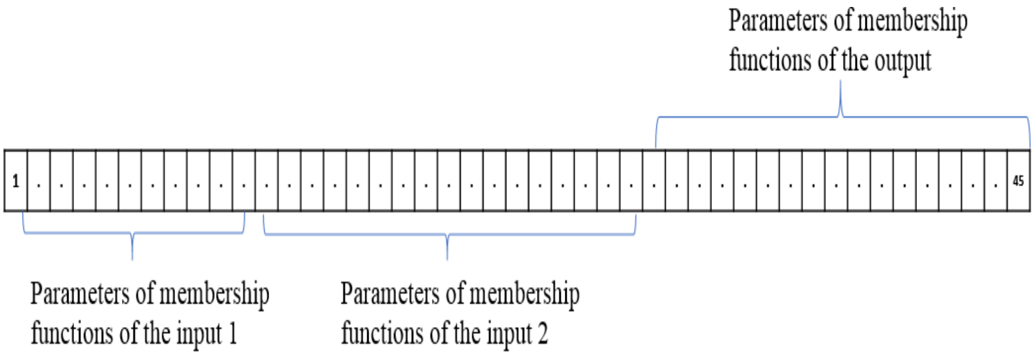

Figure 11. Chromosome for the fuzzy controller (membership functions parameters).

The experimentation was performed using the parameters shown in Table 4, and to validate which of the two proposed fuzzy systems has better performance, we decided to add a noise level to the controller. The different noise levels applied to this controller are: 0.5, 0.7 and 0.9 (Gaussian random number).

Table 4. Parameters of the algorithm.

Parameters	ST2FDE and GT2FDE
Population	50
Dimensions	45
Generations	30
Number of experiments	30
F	Dynamic
Cr	0.3

In this case, the objective function is defined by the root mean square error (RMSE) of the real values with respect to the reference speed for the motor, as it is illustrated in Eq. 23:

$$RMSE= \sqrt{\frac{1}{N}\sum_{t=1}^{N}(x_t - \hat{x}_t)^2} \tag{23}$$

The 30 experiments were carried out applying each of the fuzzy systems varying the level of noise (0.5, 0.7 and 0.9) and from which the best results, the worst results, averages and standard deviations were obtained.

Table 5 summarizes the aforementioned information from the experimentation using the Shadowed Type-2 fuzzy system (ST2FDE).

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Table 5. Comparison of results using ST2FDE.

ST2FDE				
Method	**ST2FDE without noise FLC**	**ST2FDE with noise 0.5 FLC**	**ST2FDE with noise 0.7 FLC**	**ST2FDE with noise 0.9 FLC**
Best	9.66E-01	9.41E-01	5.59E-01	4.52E-01
Worst	9.98E-01	9.96E-01	6.11E-01	6.56E-01
Average	9.84E-01	9.73E-01	5.86E-01	5.81E-01
Std.	8.45E-03	1.17E-02	1.40E-02	6.13E-02

The visual representation of the best results obtained by the performed experimentation with the fuzzy ST2FDE system is presented in Figs. 12, 13, 14 and 15. These figures show the controller simulation with the different variants that we used. In these figures, the x-axis is the time measured in seconds and the y-axis is the speed measured in radians per second.

Figure 12 represents the results obtained without using noise in the controller.

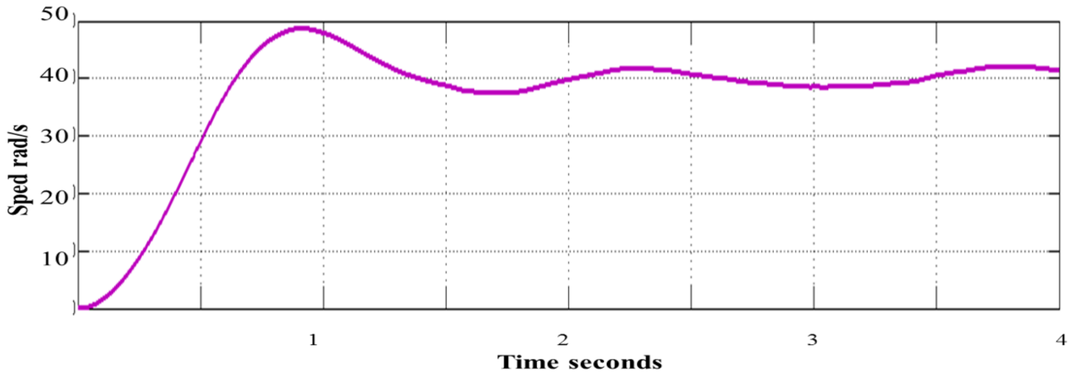

Figure 12. ST2FDE without noise FLC.

Figure 13 represents the results obtained with a level of noise of 0.5 in the controller.

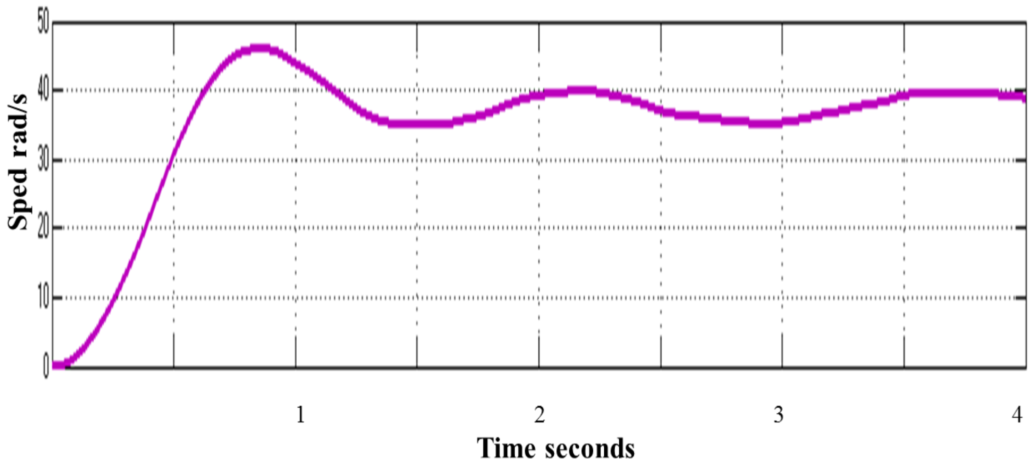

Figure 13. ST2FDE with noise 0.5 FLC.

Figure 14 represents the result obtained with a level of noise of 0.7.

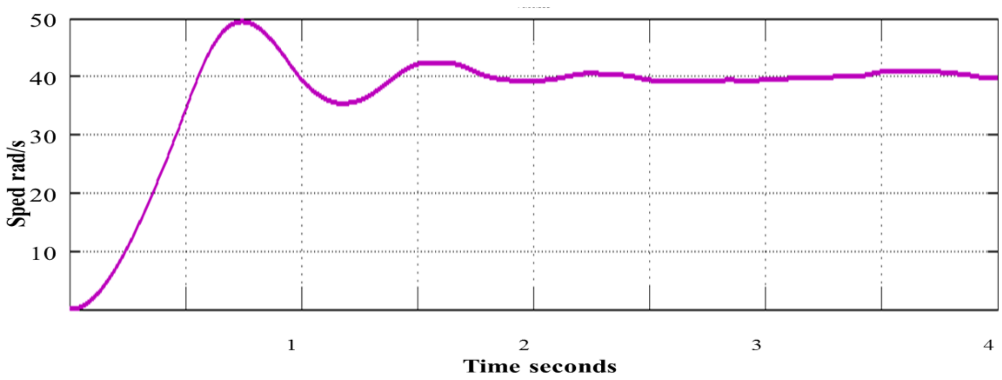

Figure 14. ST2FDE with noise 0.7 FLC.

Figure 15 represents the results obtained with a level of noise of 0.9.

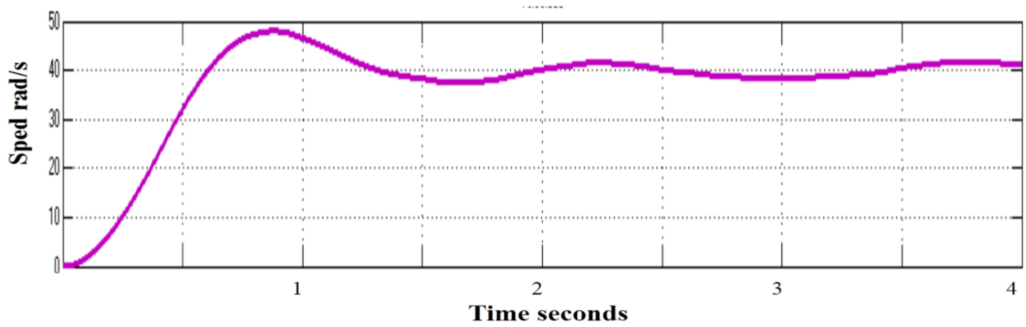

Figure 15. ST2FDE with noise 0.9 FLC.

Fig. 16 shows the convergence for each of the cases by utilizing the Shadowed Type-2 fuzzy alternative. This figure includes the experimentation of the controller without noise, with noise levels of 0.5, 0.7, and 0.9, and the figure clearly shows that when there is more noise the fuzzy system produces better the results.

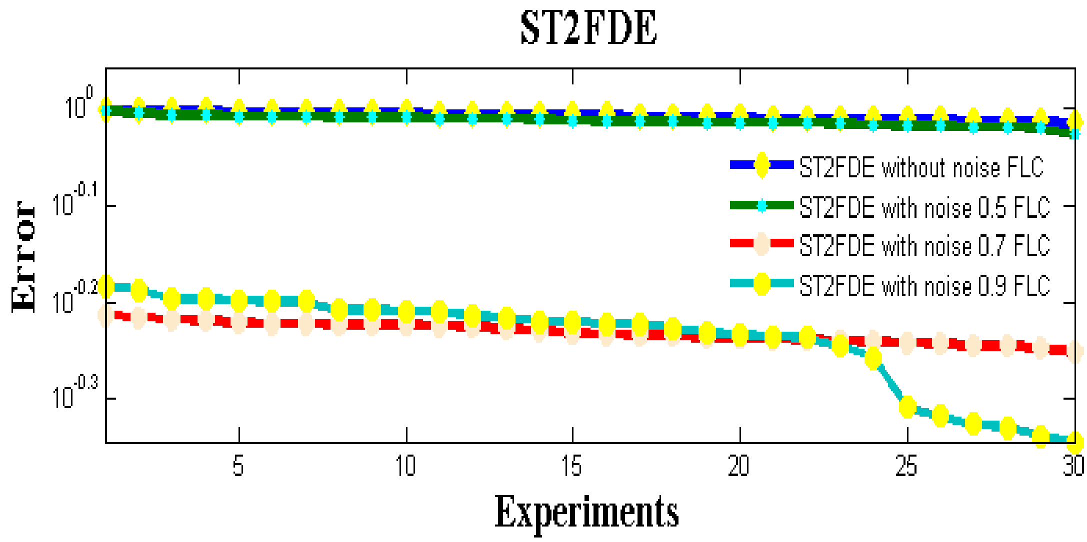

Figure 16. Graphical representation of the error using ST2FDE.

Table 6 shows the results obtained from the experimentation using the Generalized Type-2 fuzzy system, without noise in the controller, and with noise levels of 0.5, 0.7 and 0.9. This table shows the best, the worst, the mean and the standard deviation results for each case.

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Table 6. Comparison of results using GT2FDE.

	GT2FDE			
Method	**GT2FDE without noise FLC**	**GT2FDE with noise 0.5 FLC**	**GT2FDE with noise 0.7 FLC**	**GT2FDE with noise 0.9 FLC**
Best	9.73E-01	9.38E-01	5.48E-01	4.35E-02
Worst	9.95E-01	9.91E-01	6.08E-01	6.53E-01
Average	9.85E-01	9.75E-01	5.79E-01	5.51E-01
Std.	5.88E-03	1.25E-02	1.70E-02	7.46E-02

The visual representation of the best results obtained by experimentation with the fuzzy GT2FDE system is presented in Figs. 17, 18, 19 and 20, which show us the simulation of the controller with the different variants that we use. In this case, the x-axis is the time measured in seconds and the y-axis is the speed measured in radians per second, and this is for all the aforementioned figures.

Figure 17 represents the results obtained without using noise in the controller.

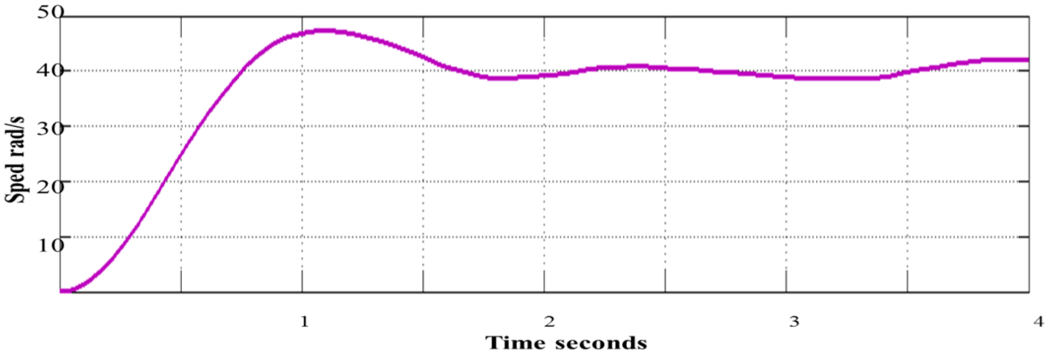

Figure 17. GT2FDE without noise in the FLC.

Figure 18 illustrates the results obtained with noise level of 0.5 in the controller.

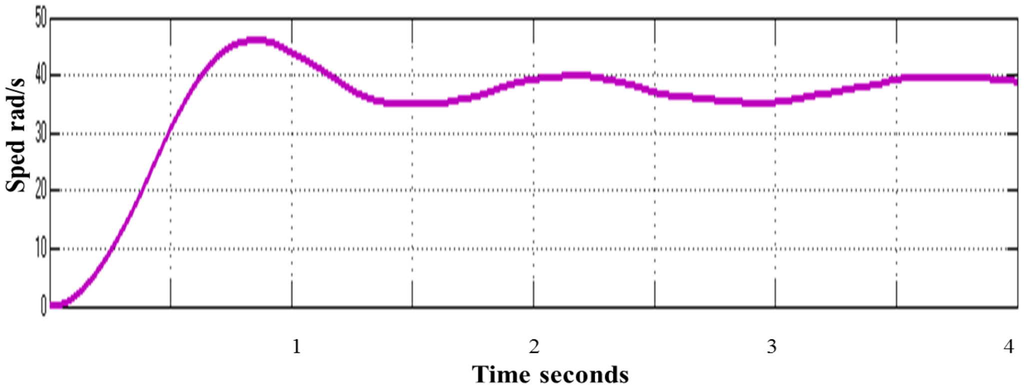

Figure 18. GT2FDE with noise of 0.5 in the FLC.

Figure 19 represents the results obtained with a level of noise of 0.7.

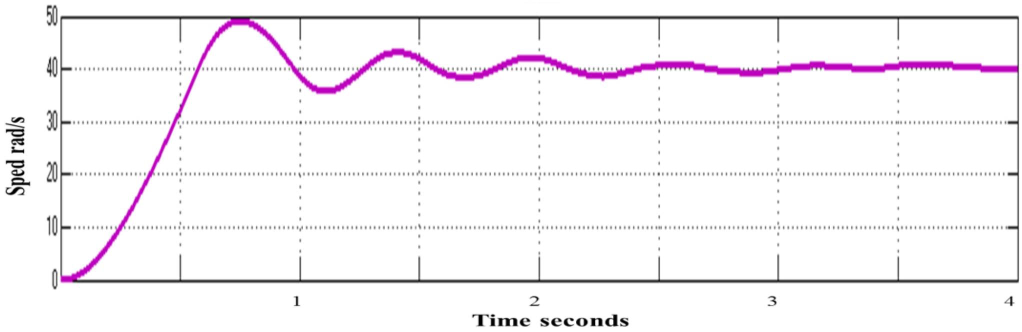

Figure 19. GT2FDE with noise of 0.7 in the FLC.

Figure 20 illustrates the results obtained with a noise level of 0.9.

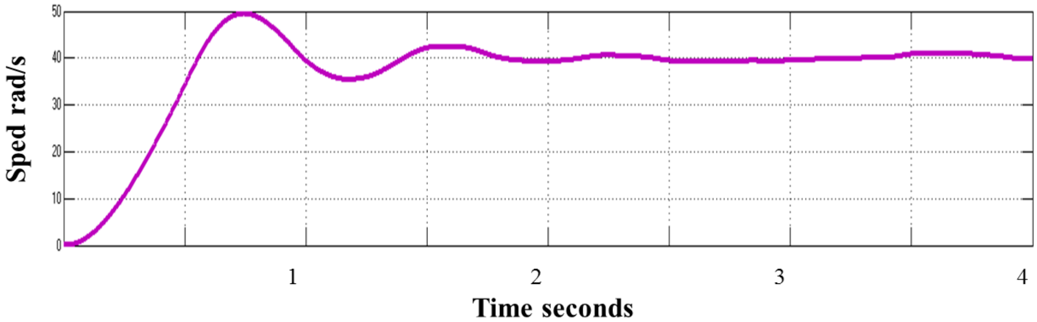

Figure 20. GT2FDE with a noise of 0.9 in the FLC.

Fig. 21 shows the convergence of each of the cases when using the General Type-2 fuzzy alternative. This figure shows the different variants used in the experimentation, and as in the experimentation with the ST2FDE we can appreciate that with a higher noise level in the controller we obtain better results.

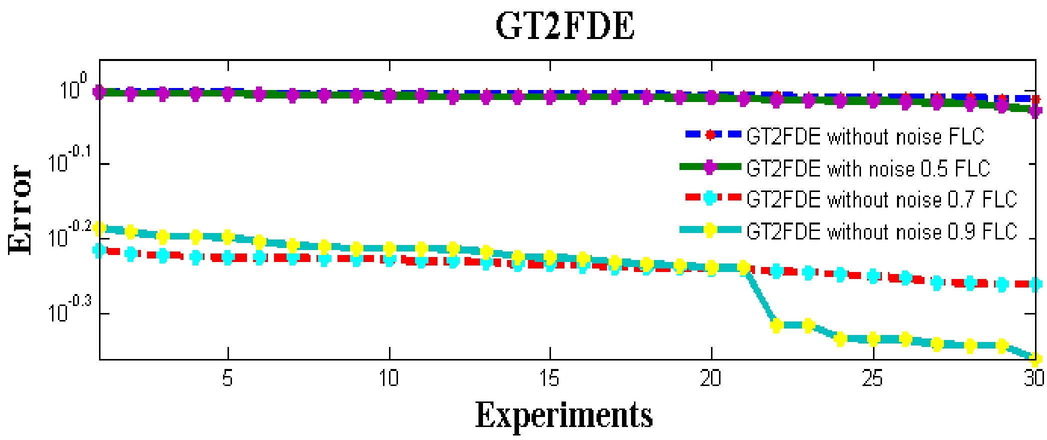

Figure 21. Graphical representation of the error using GT2FDE.

Fig. 22 shows a comparative of the best results achieved by each of the variants with noise and without noise used for two fuzzy systems ST2FDE and GT2FDE. We can appreciate that the GT2FDE fuzzy system is slightly better for most of the variants.

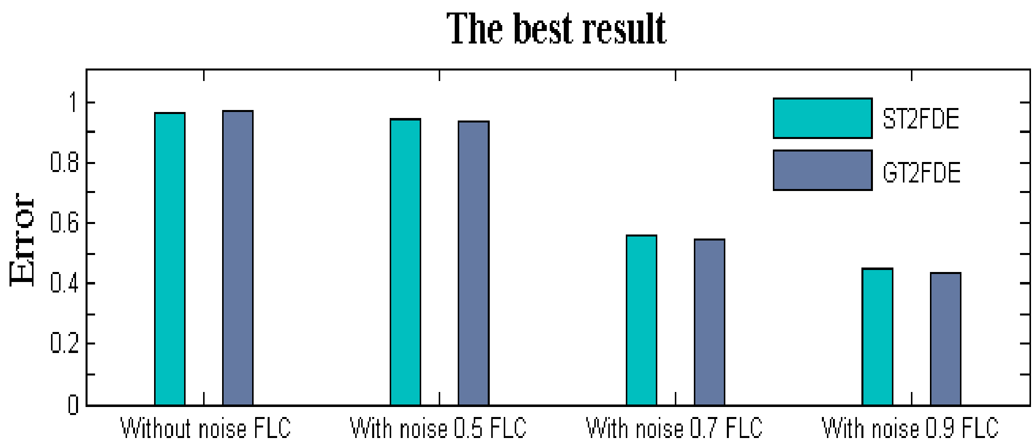

Figure 22. Comparison of the best results between ST2FDE and GT2FDE.

In order to make a decision on which of the used systems has the best result or, in other words, which of the two systems is better depending on the achieved error, we carry out a statistical test using the z test.

The parameters used to perform the statistical test are summarized in Table 7.

Table 7. Summary of parameters for the z-test.

Parameter	Value
Level of Confidence	95%
Alpha	0.05 %
H_a	$\mu_1 < \mu_2$
H_0	$\mu_1 \geq \mu_2$
Critical Value	-1.645

In this case, μ_1 represents the variants using GT2FDE and μ_2 represents the variants using ST2FDE.

The null and alternative hypotheses that we propose for the statistical test are the following:

H_0: The results of the GT2FDE methodology without noise and with noise are higher than the methodology ST2FDE without noise and with noise.

H_a: The results of the GT2FDE methodology without noise and with noise are lower than the methodology ST2FDE without noise and with noise.

Based on the values shown in Table 7, the rejection zone is for values lower than -1.64. Eq. 24 for calculating the z value of the z-test is presented below:

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sigma_{\bar{X}_1 - \bar{X}_2}} \quad (24)$$

Table 8 shows the z values obtained for the different statistical tests to compare the performance of the two fuzzy systems.

Table 8. Summary of results of the statistical z-tests.

	Statistical Tests			
Case study	μ_1	μ_2	Z value	Evidence
Speed control in a D.C. Motor	GT2FDE without FCL noise	ST2FDE without FCL noise	0.5321	Not Significant
	GT2FDE with FCL 0.5 noise	ST2FDE without FCL 0.5 noise	0.6398	Not Significant
	GT2FDE with FCL 0.7 noise	ST2FDE without FCL 0.7 noise	-1.7410	Significant
	GT2FDE with FCL 0.9 noise	ST2FDE without FCL 0.9 noise	-1.7018	Significant

Statistical tests show us that when there is a higher noise level, then the Generalized Type-2 fuzzy system obtains better results, and the Shadowed Type-2 fuzzy system is better for lower noise levels.

To verify the efficiency of the GT2FDE fuzzy system, which is statistically better than ST2FDE, we also performed a comparison with the best results obtained in [36]. This previous work used a structure of the fuzzy system that is similar to the one we use here, an input and an output, where we used the differential evolution algorithm and harmony search (HS).

Table 9 summarizes a comparison of the best results obtained using a high-speed interval Type-2 fuzzy system for parameter adjustment in the DE and HS algorithms [36] and the GT2FDE methodology proposed in this work.

It is relevant to mention that the comparison is only with the best results since the reference does not provide means and standard deviations to be able to perform a sound statistical test. However, based on the information summarized in Table 9 we can state the proposed method in this paper outperforms the methods presented in [36].

Table 9. Comparison between the GT2FDE and other methods.

		Method	**Best**
D.C. Motor Speed Controller	**RMSE**	Original DE	4.72E-01
		DEFIS 1	4.57E-01
		DEFIS 2	4.80E-01
		DEFIS 3	2.36E-01
		Original HS	4.72E-01
		HSFIS 1	4.57E-01
		HSFIS 2	4.80E-01
		HSFIS 3	2.36E-01
		GT2FDE with noise 0.9 FLC	4.35E-02

7. Conclusions

The conclusions for the work presented in this article are summarized below. First of all, we can highlight and affirm that the utilization of type-2 fuzzy logic is better for higher levels of uncertainty. The experimentation that was performed consisted of using two kinds of systems: Shadowed and General Type-2 fuzzy systems, which were used under the same conditions, and we can observe that each of the fuzzy systems by themselves improve when increasing the noise level. However, statistically we can say that GT2FDE is better at higher noise levels, while ST2FDE is statistically better without noise and with lower noise levels.

This shows us what the literature affirms in most of the works that generalized type-2 fuzzy systems are better whenever the noise levels or disturbances are higher, which is what actually occurs in real world problems.

Table 9 shows us that the generalized type 2 fuzzy systems have better results when compared to the work of a High-Speed Interval Type 2 Fuzzy Systems presented in [36]. It is important to mention that the comparison is only with the best result since the reference does not have means and standard deviations to be able to perform a statistical test.

In general, the work carried out shows good results when comparing the two kinds of systems and regarding the comparison with the High-Speed Interval Type 2 Fuzzy Systems combined with the DE and HS algorithms. We can appreciate that we achieved better results with our proposed GT2FDE methodology because the general type-2 fuzzy systems help the differential evolution algorithm a lot in terms of achieving a better performance.

As future work we envision that the proposed method could be also applied in other problems in areas such as, pattern recognition, time series prediction, medical diagnosis and others [37-42].

Compliance with Ethical Standards:

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Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors

Data Availability Statement: My manuscript has no associated data

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