Numbers Based On Edges

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Abstract

Number is based in special edges, is introduced. The kind of natural extension from edges toward vertex and the set of vertices in the way that, the final notion is number, is studied. The result is obtained which is about the study on the classes of graphs in the matter of new notions. There is the extended notion about having edge amid two vertices toward having some edges in the word of neighbor and in another stage going into the atmosphere of having consecutive edges in the terminology of path and in the upper vision going on the notion about path with jargon and buzzword of distance as if the minimal vertices has concluded the new notions with the word, number.

Keywords: Special Edges, Neighbors, Distances, Numbers.
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1 Preliminary On The Concept

I'm going to refer to some books which are cited to the necessary and sufficient material which are covering the introduction and the preliminary of this outlet so look [**Ref.** [1], **Ref.** [2], **Ref.** [3], **Ref.** [4]] where **Ref.** [1] is about the textbook, **Ref.** [2] is common, **Ref.** [3] has good ideas and **Ref.** [4] is kind of disciplinary approaches in the good ways. Further references could be referred and could be addressed in **Refs.** [5–11].

2 Definition And Its Clarification

Definition 2.1. Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. Then

- an edge $e = xy$ is renamed to be
  - **EDGE-TYPE1** if $\mathcal{N}(x) = \mathcal{N}(y)$;
  - **EDGE-TYPE2** if $\mathcal{N}(x) \subseteq \mathcal{N}(y)$;
  - **EDGE-TYPE3** if $\mathcal{N}(y) \subseteq \mathcal{N}(x)$;
  - **EDGE-TYPE4** if $\forall z \in \mathcal{V} : \ \mathcal{D}(x, z) = \mathcal{D}(y, z)$;
  - **EDGE-TYPE5** if $\forall z \in \mathcal{V} : \ \mathcal{D}(x, z) \geq \mathcal{D}(y, z)$;
  - **EDGE-TYPE6** if $\forall z \in \mathcal{V} : \ \mathcal{D}(x, z) \leq \mathcal{D}(y, z)$;
- a vertex x is renamed to be

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- **VERTEX-TYPE1** if there's the vertex y such that xy is EDGE-TYPE1.
- **VERTEX-TYPE2** if there's the vertex y such that xy is EDGE-TYPE2.
- **VERTEX-TYPE3** if there's the vertex y such that xy is EDGE-TYPE3.
- **VERTEX-TYPE4** if there's the vertex y such that xy is EDGE-TYPE4.
- **VERTEX-TYPE5** if there's the vertex y such that xy is EDGE-TYPE5.
- **VERTEX-TYPE6** if there's the vertex y such that xy is EDGE-TYPE6.

- a set $\mathcal{B}$ is renamed to be
  - **SET-TYPE1** if for every vertex $y \in \mathcal{V}$, there's VERTEX-TYPE1 in $\mathcal{B}$;
  - **SET-TYPE2** if for every vertex $y \in \mathcal{V}$, there's VERTEX-TYPE2 in $\mathcal{B}$;
  - **SET-TYPE3** if for every vertex $y \in \mathcal{V}$, there's VERTEX-TYPE3 in $\mathcal{B}$;
  - **SET-TYPE4** if for every vertex $y \in \mathcal{V}$, there's VERTEX-TYPE4 in $\mathcal{B}$;
  - **SET-TYPE5** if for every vertex $y \in \mathcal{V}$, there's VERTEX-TYPE5 in $\mathcal{B}$;
  - **SET-TYPE6** if for every vertex $y \in \mathcal{V}$, there's VERTEX-TYPE6 in $\mathcal{B}$;
- a number $\mathcal{N}$ is renamed to be
  - **NUMBER-TYPE1** if there's SET-TYPE1 with minimum cardinality;
  - **NUMBER-TYPE2** if there's SET-TYPE2 with minimum cardinality;
  - **NUMBER-TYPE3** if there's SET-TYPE3 with minimum cardinality;
  - **NUMBER-TYPE4** if there's SET-TYPE4 with minimum cardinality;
  - **NUMBER-TYPE5** if there's SET-TYPE5 with minimum cardinality;
  - **NUMBER-TYPE6** if there's SET-TYPE6 with minimum cardinality;

3 Relationships And Its illustrations

Theorem 3.1. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. $\mathcal{G}$ is complete graph if and only if any of edge/vertex/set/number is renamed*

- *TYPE1;*
- *TYPE4.*

Theorem 3.2. *Let $\mathcal{K}_{m,n}$ be a complete graph. Then all edges in same parts are renamed to*

- *EDGE-TYPE1 and EDGE-TYPE4 if $m = n$;*
- *either EDGE-TYPE2 and EDGE-TYPE5 or EDGE-TYPE3 and EDGE-TYPE6 if $m \geq n$ or $m \leq n$.*

Theorem 3.3. *Let $\mathcal{S}_{1,n}$ be a star graph. Then all edges are renamed to*

- *either EDGE-TYPE2 and EDGE-TYPE5 or EDGE-TYPE3 and EDGE-TYPE6 if $m \geq n$ or $m \leq n$.*

Theorem 3.4. *Let $\mathcal{W}_{1,n}$ be a wheel graph. Then all edges are renamed to*

- *EDGE-TYPE1 and EDGE-TYPE4 if edges aren't incident to center;*
- *either EDGE-TYPE2 and EDGE-TYPE5 or EDGE-TYPE3 and EDGE-TYPE6 if edges are incident to center;*

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Theorem 3.5. *Let $\mathcal{K}_{m,n}$ be a complete graph. Then all vertices in same parts are renamed to*

- *VERTEX-TYPE1 and VERTEX-TYPE4 if $m = n$;*
- *either VERTEX-TYPE2 and VERTEX-TYPE5 or VERTEX-TYPE3 and VERTEX-TYPE6 if $m \geq n$ or $m \leq n$.*

Theorem 3.6. *Let $\mathcal{S}_{1,n}$ be a star graph. Then all vertices are renamed to*

- *either VERTEX-TYPE2 and VERTEX-TYPE5 or VERTEX-TYPE3 and VERTEX-TYPE6.*

Theorem 3.7. *Let $\mathcal{W}_{1,n}$ be a wheel graph. Then all vertices are renamed to*

- *VERTEX-TYPE1 and VERTEX-TYPE4 if edges aren't incident to center;*
- *either VERTEX-TYPE2 and VERTEX-TYPE5 or VERTEX-TYPE3 and VERTEX-TYPE6 if edges are incident to center;*

Theorem 3.8. *Let $\mathcal{K}_{m,n}$ be a complete graph. Then all $\mathcal{B}$s have two members from each part and their extensions where $\mathcal{B}$s are renamed to in same parts are renamed to*

- *SET-TYPE1 and SET-TYPE4 if $m = n$;*
- *either SET-TYPE2 and SET-TYPE5 or SET-TYPE3 and SET-TYPE6 if $m \geq n$ or $m \leq n$.*

Theorem 3.9. *Let $\mathcal{S}_{1,n}$ be a star graph. Then all $\mathcal{B}$s are singleton and their extensions where $\mathcal{B}$s are renamed to*

- *either SET-TYPE2 and SET-TYPE5 or SET-TYPE3 and SET-TYPE6.*

Theorem 3.10. *Let $\mathcal{W}_{1,n}$ be a wheel graph. Then all $\mathcal{B}$s are singleton and their extensions where $\mathcal{B}$s are renamed to*

- *SET-TYPE1 and SET-TYPE4 if edges aren't incident to center;*
- *either SET-TYPE2 and SET-TYPE5 or SET-TYPE3 and SET-TYPE6 if edges are incident to center;*

4 Results And Its Beyond

Theorem 4.1. *$\mathcal{G}$ is a complete graph or a star graph if and only if one is renamed to*

- *NUMBER-TYPE1;*
- *NUMBER-TYPE4.*

Theorem 4.2. *$\mathcal{G}$ is a star graph if and only if one is renamed to either*

- *NUMBER-TYPE2 and NUMBER-TYPE5;*
- *or NUMBER-TYPE3, NUMBER-TYPE6.*

Theorem 4.3. *$\mathcal{K}_{m,n}$ is a complete bipartite graph if and only if two is renamed to either*

- *NUMBER-TYPE1 and NUMBER-TYPE4 if $m = n$;*

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- *or NUMBER-TYPE2, NUMBER-TYPE3, NUMBER-TYPE5 and NUMBER-TYPE6 if* $m \geq n$ *or* $m \leq n$.

Theorem 4.4. *Let* $\mathcal{K}_{m,n}$ *be a complete graph. Then two are renamed to*

- *NUMBER-TYPE1 and NUMBER-TYPE4 if* $m = n$;
- *either NUMBER-TYPE2 and NUMBER-TYPE5 or NUMBER-TYPE3 and NUMBER-TYPE6 if* $m \geq n$ *or* $m \leq n$.

Theorem 4.5. *Let* $\mathcal{S}_{1,n}$ *be a star graph. Then one renamed to*

- *either NUMBER-TYPE2 and NUMBER-TYPE5 or NUMBER-TYPE3 and NUMBER-TYPE6.*

Theorem 4.6. *Let* $\mathcal{W}_{1,n}$ *be a wheel graph. Then one is renamed to*

- *NUMBER-TYPE1 and NUMBER-TYPE4 if edges aren't incident to center;*
- *either NUMBER-TYPE2 and NUMBER-TYPE5 or NUMBER-TYPE3 and NUMBER-TYPE6 if edges are incident to center;*

Theorem 4.7. *Let* $\mathcal{W}_{1,n}$ *be a wheel graph. Then one is renamed to*

- *NUMBER-TYPE1 and NUMBER-TYPE4 if edges aren't incident to center;*
- *either NUMBER-TYPE2 and NUMBER-TYPE5 or NUMBER-TYPE3 and NUMBER-TYPE6 if edges are incident to center;*

Theorem 4.8. *Let* $\mathcal{G}$ *be a path graph or cycle graph or acyclic graph or tree. Then any of edge/vertex/set/number is renamed to either TYPE5 or TYPE6 as if there isn't any of TYPE1, TYPE2, TYPE3, TYPE4.*

Theorem 4.9. *Let* $\mathcal{G}$ *be a graph. Then number which is renamed to any of TYPE1, TYPE2, TYPE3, TYPE4, TYPE5 and TYPE6, is* $\leq \mathcal{O}(\mathcal{G})$.

Theorem 4.10. *Consider a graph and its subgraph. Then graph's number which is renamed to any of TYPE1, TYPE2, TYPE3, TYPE4, TYPE5 and TYPE6, is greater than subgraph's corresponded number.*

References

1. R. Balakrishnan, K. Ranganathan, **A Textbook of Graph Theory**, New York, 2012.
2. Adrian Bondy, U.S.R Murty, **Graph Theory**, New York, 2008.
3. Michael Capobianco, and John C. Molluzzo, **Examples and counterexamples in graph theory**, New York, 1978.
4. Chris Godsil, and Gordon Royle, **Algebraic Graph Theory**, New York, 2001.
5. Aleksander Kelenc, Aoden Teo Masa Toshi, Riste Skrekovski, Ismael G. Yero, **On Metric Dimensions of Hypercubes**. Preprint available at https://arxiv.org/abs/2102.10916.
6. Martin Knor, Riste Skrekovski, Ismael G. Yero, **A note on the metric and edge metric dimensions of 2-connected graphs**. Preprint available at https://arxiv.org/abs/2103.00215.

7. Satvik Mashkaria, Gergely Odor, Patrick Thiran, **On the robustness of the metric dimension to adding a single edge**. Preprint available at https://arxiv.org/abs/2010.11023.

8. Milica Milivojevic Danas, **The difference between several metric dimension graph invariants**. Preprint available at https://arxiv.org/abs/2012.07471.

9. Talmeez Ur Rehman, Naila Mehreen, **Partition dimension and strong metric dimension of chain cycle**. Preprint available at https://arxiv.org/abs/2007.09499.

10. Jelena Sedlar, Riste Skrekovski, **Vertex and edge metric dimensions of unicyclic graphs**. Preprint available at https://arxiv.org/abs/2104.00577.

11. Jelena Sedlar, Riste Skrekovski, **Extremal mixed metric dimension with respect to the cyclomatic number**. Preprint available at https://arxiv.org/abs/2012.08590.