

## Article

# Strain and grain size of CeO<sub>2</sub> and TiO<sub>2</sub> nanoparticles: Comparing structural and morphological methods

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**Abstract:** Various crystallite size estimation methods were used to analyze X-ray diffractograms of spherical cerium dioxide and donut-like titanium dioxide anatase nanoparticles aiming to evaluate their reliability and limitations. The microstructural parameters were estimated from Scherrer, Monshi, Williamson-Hall, and their variants: i) uniform deformation model, ii) uniform strain deformation model, and iii) uniform deformation energy density model, and also size-strain plot, and Halder-Wagner method. For that, an improved systematic Matlab code was developed to estimate the crystallite sizes and strain, and the linear regression analysis was used to compare all the models based on the coefficient of determination, where the Halder Wagner method gave the highest value (close to 1). Therefore, being the best candidate to fit the X-ray Diffraction data of metal-oxide nanoparticles. Advanced Rietveld was introduced for comparison purposes. Refined microstructural parameters were obtained from a nanostructured 40.5 nm Lanthanum hexaboride nanoparticles and correlated with the above estimation methods and transmission electron microscopy images. In addition, electron density modelling was also studied for final refined nanostructures, and  $\mu$ -Raman spectra were recorded for each material estimating the mean crystallite size and comparing by means of a phonon confinement model.

**Keywords:** X-ray diffraction; CeO<sub>2</sub>; TiO<sub>2</sub>; crystallite size; strain; TEM;  $\mu$ -Raman

## 1. Introduction

Nanotechnology is the branch of Science in which materials are obtained and studied on a nanometric scale, named as nanoparticles (NPs), where novel and outstanding physical and chemical effects are found between 1 and 100 nm [1]. NPs can be obtained through various physical, chemical or biological synthesis methods. The NPs manufacturing process is essential since it can affect the photocatalytic, adsorptive, thermal, and optical properties of metal oxides such as cerium oxide (CeO<sub>2</sub>) and titanium oxide (TiO<sub>2</sub>), which depend on the particle size, shape, and crystal morphology [2]. Hence, the tuning of structural and morphological properties is highly relevant due to their possible usefulness in different applications. For example, CeO<sub>2</sub> NPs are widely used due to their wide range of application in electrochemistry, such as electrode materials in supercapacitors, and medicine, due to their antibacterial properties [3]. On the other hand, TiO<sub>2</sub> NPs are highly used in photocatalysis, solar cells, biomedicine, chemical sensors, and lithium storage [2], and have been also explored for heavy metal water cleaning purposes [4].

Currently, physical techniques such as scanning electron microscopy (SEM), transmission electron microscopy (TEM), and atomic force microscopy (AFM) are used to estimate and confirm the NPs scale [5], as well as indirect methods such as powder X-ray diffraction (pXRD). In particular, pXRD analysis is widely used for determining crystallite size and lattice deformation. The information of the two latter physical parameters correlates the diffraction peak broadening with lattice strain which sources come from imperfections (stacking faults or coherency stresses) of the studied material. There exist several methods to estimate the peak broadening from XRD data [6]. However, the accuracy of these methods is questionable due to inconsistencies between them.

The above-mentioned pXRD methods used nowadays are modifications of the 1918 Scherrer method, which relates the peak broadening ( $\beta$ ) of each diffracted Bragg peak to a characteristic mean size ( $D$ ) [7]. Since that first work, new parameters have been added to better represent the physical effects that can be produced in the intensity distribution of diffracted X-rays. In 1953, the Williamson Hall (W-H) method introduced the general distribution of deformations  $\varepsilon$  [8], which takes into account the two-parameter effects of size and lattice strain. Generalized models, such as the uniform deformation model (UDM) considers the uniform stress in all crystallographic directions, considering the crystal of isotropic nature [9]. Whilst the mechanical Hooke's law considers the strain in the uniform deformation stress model (UDSM) and uniform deformation energy density model (UDEDM), the latter methods take into account the anisotropic nature of the Young's modulus of the crystal [10]. Instead, the Halper Wagner (H-W) method implies the assumption that the spreading of the peak is a symmetric Voigt function, which means that the size of the crystallite is defined by the Lorentzian and Gaussian functions [11]. Similarly, by the size-strain plot (SSP) method, assumes that the crystallite size profile is described by a Lorentz function and the deformation profile by a Gaussian function [12].

On the other hand, Rietveld method (RM) is a tool for analyzing crystalline structures [13], which consists of theoretical refinement of the structural or cell parameters, atomic displacement, anisotropy, cell stresses, shape and anisotropy effects, among others, until a condition of convergence is reached between the values of experimental diffractogram curves and the theoretical model [14]. This refinement is suitable when calculating the microstructural parameters for a multiphasic sample (percentual concentration). For example [15], where diffraction peaks overlap and systematical errors can arise from correct determination of full width at half maximum (FWHM) values, as it can be the case in the previous discussed methods. The theoretical model includes structural aspects such as: crystalline structure, spatial group, atom Wyckoff positions, etc. In addition, microstructural information are involved including crystal size and micro-deformations. The RM also includes instrumental factors such as the optical effect of XRD equipment on the width of diffraction peaks [16]. It is worth mentioning that the RM makes use of known atomic structures to generate an initial theoretical model of the structure of the phases present in the sample, so these must be previously identified. From this initial model, the method allows to refine the structural parameters, based on the analysis of least squares [17], until the model matches the experimental profile [16].

In the present work, a comparative study of the microstructural parameters of metal-oxide  $\text{CeO}_2$  and  $\text{TiO}_2$  NPs has been investigated by employing different modified models including W-H, UDM, UDSM, UDEDM, SSP, and H-W. All the methods were compared to the obtained results from RM and TEM images. Additionally, crystallite size was also estimated from  $\mu$ -Raman measurements by means of a phonon confinement model.

## 2. Materials and Methods

### 2.1. X-ray and TEM experimental details

$\text{TiO}_2$ ,  $\text{CeO}_2$ , and Lanthanum hexaboride ( $\text{LaB}_6$ ) were obtained from Sigma Aldrich, and no further purification was performed to the powder samples. The pXRD data were collected using a Rigaku diffractometer, operating with  $\text{CuK}\alpha$  radiation ( $1.5406 \text{ \AA}$ ), at 50 kV and 100 mA. The diffractograms were collected in step scanning configuration

between  $2\theta = 20^\circ$ - $100^\circ$  for  $\text{CeO}_2$  and  $\text{TiO}_2$ , with  $0.02^\circ$  and 5s per step. The crystallographic phases were identified using Match version 3 software obtaining crystallographic cards [96-434-3162] and [96-500-022] with the crystallographic information file (CIF) #9009008 and #5000223 for the  $\text{CeO}_2$  and  $\text{TiO}_2$  anatase phases, respectively. The Origin pro 9.0 software was used to estimate the FWHM using a pseudo-Voigt fitting model. By making use of the instrumental width correction ( $\beta_{inst} = 0.01^\circ$ ); the microstructural parameters were systematically obtained by using a code in Matlab version R2017b which is given in the Supplementary material. The values for microstructural parameters estimated from the code (including corrected instrumental resolution) are close in value to the obtained by the linear fit for each crystallite estimation model. Therefore, we only reported these values in Table 1.

For the RM of the diffractograms, the software Fullprof was employed, the  $\text{CeO}_2$  and  $\text{TiO}_2$  crystallographic information files (CIF) obtained from Match v3 software were used as initial parameters, which crystallographic data for  $\text{CeO}_2$  are: cubic crystalline structure, space group Fm-3m, and cell parameters  $a=5.4110 \text{ \AA}$ . And, for  $\text{TiO}_2$  anatase they were: tetragonal crystalline structure, space group I 41/amd, cell parameters  $a=3.78435 \text{ \AA}$ ,  $c=9.50374 \text{ \AA}$  for both cases the Caglioti's initial parameters were  $U=0.004133$ ,  $V=-0.007618$ , and  $W=0.006255$  and refined using the pseudo-Voigt function. Finally, the average crystallite size was determined in the Fullprof program. To do that, we first characterized the standard Lanthanum hexaboride reference material (nano  $\text{LaB}_6$ ), its measurement on the diffractometer was performed between  $2\theta = 10^\circ$ - $80^\circ$  with a step of  $0.02^\circ$ . For the  $\text{LaB}_6$  refinement, the CIF #1000055 was inserted as the initial model and the Thompson-Cox-Hastings (TCH) pseudo-Voigt Axial divergence asymmetry function was used to obtain the instrumental parameters of the equipment which is added to the instrumental resolution file (IRF), and later used to determine the average crystallite sizes of the  $\text{CeO}_2$  and  $\text{TiO}_2$ .

## 2.2. $\mu$ -Raman experimental details

Structural and vibrational features of the nano powders were analyzed by Raman spectroscopy using a confocal  $\mu$ -Raman microscope inVia™ by Renishaw. The spectrometer was configured with a 1200 grooves/mm diffraction grating and a  $\times 50$  objective with N.A. 0.75 and working distance of 0.37 mm. The excitation wavelength was set to 785 nm from a laser diode. Laser power was set to  $\sim 1 \text{ mW}$ . After identifying the main Raman peaks, we use a phonon confinement model (PCM) for the estimation of nanocrystals size [18-19].

## 3. Results and Discussion

### 3.1 p-XRD analysis

p-XRD diffractograms for  $\text{CeO}_2$  and  $\text{TiO}_2$  NPs are shown in Figure S1(a-b). For  $\text{CeO}_2$ , it indicates a monophasic phase and could be indexed to a cubic structure. The commercial  $\text{TiO}_2$  powder presents two characteristic crystal phases: rutile and anatase [20]. Figure S1 (b) shows the  $\text{TiO}_2$  diffractogram which only detected the Bragg peaks of anatase phase as there is no diffraction lines that have a rutile-phase  $\text{TiO}_2$ .

### 3.2 Scherrer method

Scherrer obtained his equation for the ideal condition of an ideal parallel, infinitely narrow, and monochromatic X-ray beam diffracting on a monodisperse powder of cubic-shaped crystallites [21]. The broadening of the diffracted Bragg peak in the nanocrystals is due to the crystallite size and the intrinsic strain effects. This broadening often consists of one physical and instrumental broadening parts, this last one can be corrected with the following relationship [22]:

$$\beta_D = [\beta_{\text{measured}}^2 - \beta_{\text{instrumental}}^2]^{1/2} \quad (1)$$

where  $\beta_D$  is the corrected peak broadening. The instrumental broadening and physical broadening of the sample have been measured as FWHM. So, with the Scherrer method we can calculate the average particle size ignoring the contribution of the strain, the average crystallite size is calculated by the following equation:

$$D = \frac{K\lambda}{\beta_D \cos\theta} \quad (2)$$

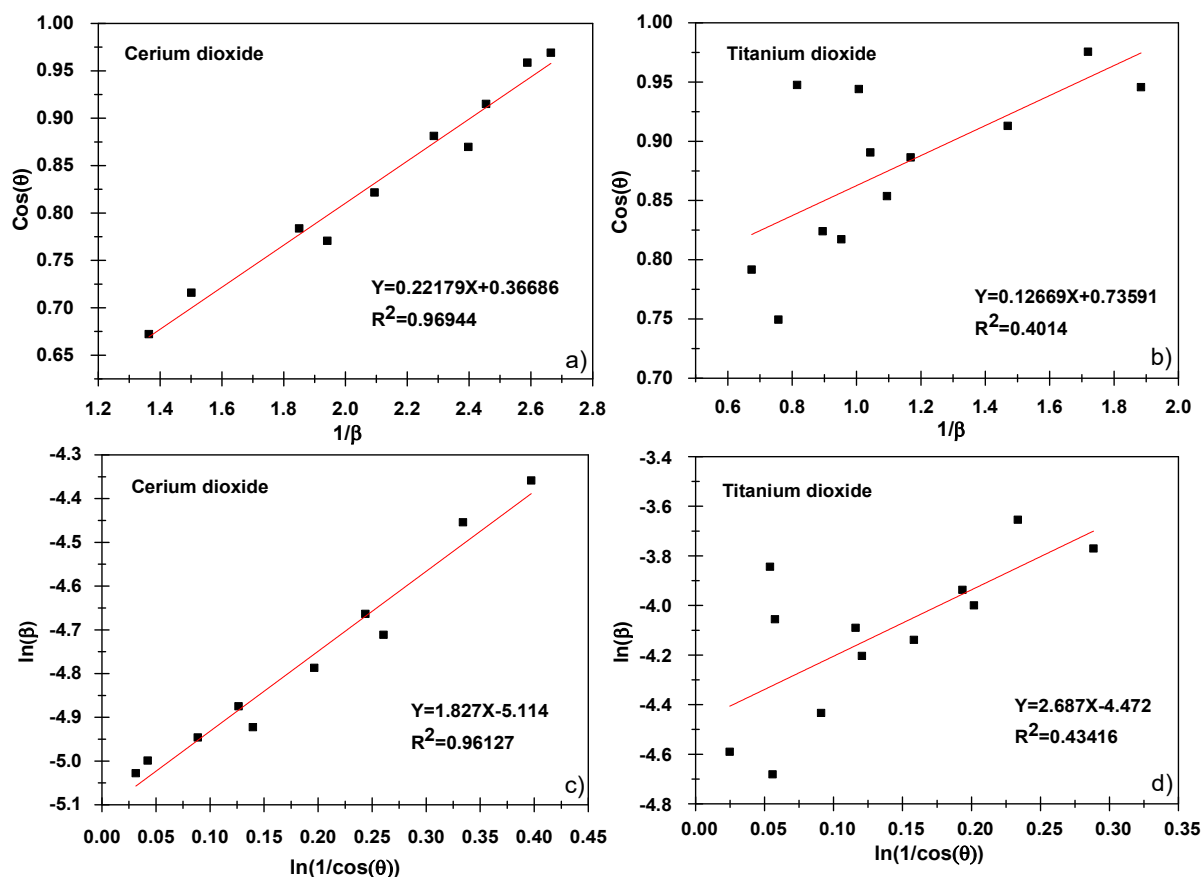


Figure 1: Scherrer plot (a,b) and Modified Scherrer plot (c,d) of CeO<sub>2</sub> and TiO<sub>2</sub> NPs.

where  $K$  is the morphological parameter or shape factor for spherical particles equal to  $0.94 \text{ nm}^{-1}$ , the wavelength ( $\lambda$ ) of the radiograph is  $1.54056 \text{ \AA}$  for  $\text{CuK}\alpha_1$  radiation, the Bragg diffraction angle ( $\theta$ ) and the FWHM is rewritten as  $\beta_D$  and expressed in radians. The plot  $1/\beta_D$  vs  $\cos\theta$  shown in Figure 1a gave  $R^2$  values higher for the CeO<sub>2</sub> NPs than TiO<sub>2</sub> NPs (Figure 1b). To have accurate results it is important to highlight that the Scherrer equation can only be used in: i) for average sizes up to 100-200 nm, ii) sample and signal/noise ratio, because the broadening of the XRD peak decreases as the crystallite size increases and it is difficult to separate the broadening from the peak [21].

### 3.3 Monshi method

Monshi [6] introduced some modifications to the Scherrer equation. Scherrer's equation has been seen to show an increment in the nanocrystalline size values as the  $d_{hkl}$  (diffracted planes distance) values decrease and the  $2\theta$  values increase, as  $\beta \cos\theta$  cannot be kept constant. The modified Scherrer equation depends on the fact that the crystallite size

is obtained during each main peak and the error in crystallite size assessment is reduced, due the advantage of reducing the sum of absolute values of errors  $\sum(\pm\Delta\ln\beta)^2$  [23].

$$\beta_D = \frac{K\lambda}{D\cos\theta} = \frac{K\lambda}{D} \frac{1}{\cos\theta} \quad (3)$$

$$\ln\beta_D = \ln\left(\frac{K\lambda}{D}\right) + \ln\left(\frac{1}{\cos\theta}\right) \quad (4)$$

Plot  $\ln\left(\frac{1}{\cos\theta}\right)$  vs  $\ln\beta_D$ , we can observe a straight line with a slope of around one and an intercept of  $\ln\left(\frac{K\lambda}{D}\right)$  from which it was calculated the average crystallite size, see Figures 1 (c) and (d). As we can see in Figure 1 (d), the value of  $R^2$  increases with respect to the Scherrer equation. This is because if there are different  $N$  peaks in the range of  $2\theta = 20^\circ$ - $100^\circ$  it is assumed that all these peaks should represent equal values for the crystallite size. But as seen in the development of this research for each peak a different value was obtained and there is a systematic error in the results for each peak, this correction gave us a decrease in the average crystallite sizes obtaining 24.1 nm for  $\text{CeO}_2$  and 12.7 nm for  $\text{TiO}_2$  NPs, respectively.

### 3.4 W-H method

In comparison to the Scherrer formula, the W-H method considers the effect of strain-induced in the XRD peak broadening and can be used for the calculation of intrinsic strain separated from crystallite size. As already mentioned, the broadening of the physical line of the XRD peak occurs due to the size and micro-deformation of the nanocrystals. Therefore, the total broadening can be written as [22]:

$$\beta_{total} = \beta_{size} + \beta_{strain} \quad (5)$$

where,  $\beta_{size}$  is the broadening due to size and  $\beta_{strain}$  the broadening due to strain. In the next section we will analyze the crystallite size and micro-deformation using the modified W-H equation as UDM, USDM, and UEDM.

#### 3.4.1 UDM method

The UDM assumes that the deformation is uniform along the crystallographic direction, so this model considers the deformation, which is isotropic nature and is also known as isotropic deformation model (ISM). The intrinsic deformation affects the physical broadening of the XRD profile and the broadening  $\beta_s$  is related to the effective stress and Bragg angle by the equation [24]:

$$\beta_{strain} = 4\varepsilon\tan\theta \quad (6)$$

where the deformation  $\varepsilon$  can be calculated from the expression  $\varepsilon = \frac{\beta_{hkl}}{4\tan\theta}$ . Therefore, the total broadening  $\beta_{hkl}$  representing the FWHM of a diffracted peak due to the contribution of the lattice strain  $\beta_{strain}$  and the size of the  $\beta_{size}$  crystallites in a particular peak that can be expressed as:

$$\beta_{hkl} = \beta_{strain} + \beta_{size} \quad (7)$$

$$\beta_{hkl} = \frac{K\lambda}{D\cos\theta} + 4\varepsilon\tan\theta \quad (8)$$

Eq. (8) can be mathematically represented by:

$$\beta_{hkl} \cos \theta = \frac{K\lambda}{D} + 4\epsilon \sin \theta \quad (9)$$

From the slope of the straight line between  $4\sin \theta$  and  $\beta_{hkl} \cos \theta$  the strain can be estimated and the average crystallite size can be estimated by extrapolation of the Y-intercept Eq. (10); see Figures 2 (a) and (b) .

$$D = \frac{K\lambda}{\text{intercept (y)}} \quad (10)$$

By taking the  $\beta_{inst} = 0.01^\circ$ , we obtained the average crystallite size of the y-intercept from the linear fit. Values of 28.9 nm for the CeO<sub>2</sub> and 16.3 for TiO<sub>2</sub> NPs were obtained, respectively. In principle, this method is not realistic at all due to the consideration of isotropic nature.

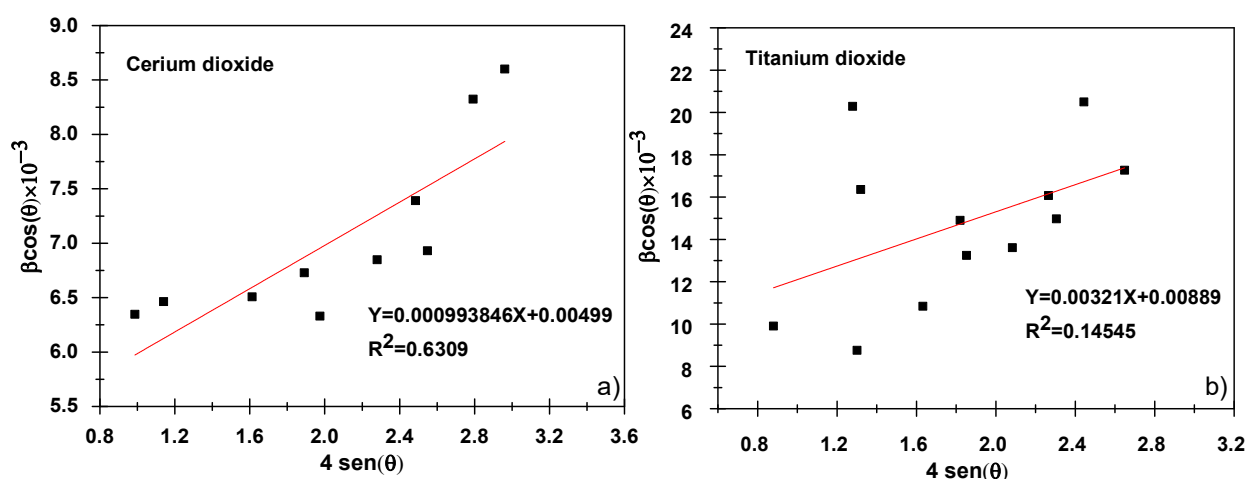


Figure 2: The W-H analysis of CeO<sub>2</sub> (a) and TiO<sub>2</sub> (b) NPs, assuming UDM model.

### 3.4.2 USDM method

This model takes into account the uniform deformation stress for a more realistic crystalline system where the anisotropic nature is considered. In addition to the uniform deformation energy density, the anisotropic nature of the Young's modulus of the crystal is more realistic [2].

Remembering Hooke's law, within the elastic limit, there is a linear proportionality relationship between strain ( $\epsilon$ ) and stress ( $\sigma$ ), the constant of proportionality being the modulus of elasticity or simply Young's modulus and is given by  $\sigma = y_{hkl} \epsilon$ , where the constant of proportionality is the modulus of elasticity or Young's modulus, denoted by  $y_{hkl}$ . Isolating the strain  $\epsilon = \sigma/y$  and substituting in Eq. (9) we have:

$$\beta_{hkl} \cos \theta = \frac{K\lambda}{\beta_D} + \frac{4\sigma \sin \theta}{y_{hkl}} \quad (11)$$

In Eq. (11),  $y_{hkl}$  depends on the crystallographic direction perpendicular to the set of planes (hkl) or the Miller indices. Next, the expressions for the cubic and tetragonal crystal system that are in relation to the elastic compliances constants ( $s_{ij}$ ) and stiffness constants ( $c_{ij}$ ) are presented.

## i. Cubic crystal

For a cubic crystal, Young's modulus is calculated using the following equation:

$$y_{hkl} = \frac{9B_0G_v}{3B_0 + G_v} \quad (12)$$

where:

$$G_v = \frac{1}{5}(3C_{44} + 2C') \quad (13)$$

$$C' = \frac{1}{2}(C_{11} - C_{12}) \quad (14)$$

$$C_{11} + 2C_{12} = 3B_0 \quad (15)$$

The elasticity constants  $C_{44}$ ,  $C_{11}$ ,  $C_{12}$  for cubic crystals are 71, 380, 275 GPa respectively. Replacing this data in Eqs. (13), (14) and (15) for the calculation of Young's modulus in Eq. (12) giving a value of 187.5 GPa [25].

Young's modulus ( $y_{hkl}$ ) is in the direction perpendicular to the crystalline lattice plane set (hkl), and for a cubic crystal it is represented by the following equation:

$$\frac{1}{y_{hkl}} = S_{11} - 2 \left[ (S_{11} - S_{12}) - \frac{1}{2}S_{44} \right] \left[ \frac{h^2k^2 + k^2l^2 + l^2h^2}{(h^2 + k^2 + l^2)^2} \right] \quad (16)$$

$$S_{11} = \frac{C_{11} + C_{12}}{(C_{11} - C_{12}) \cdot (C_{11} + 2C_{12})} \quad (17)$$

$$S_{12} = \frac{-C_{12}}{(C_{11} - C_{12}) \cdot (C_{11} + 2C_{12})} \quad (18)$$

$$S_{44} = \frac{1}{C_{44}} \quad (19)$$

where the value of elasticity stiffness constants  $C_{11}$ ,  $C_{12}$ ,  $C_{44}$  for cubic  $\text{CeO}_2$  is  $455.06 \times 10^9$ , 188.7,  $81.48 \times 10^9 \frac{\text{N}}{\text{m}^2}$  respectively [26]. Using these values of elasticity constants, we can calculate the elastic compliances values as  $2.904 \times 10^{-12}$ ,  $-8.513 \times 10^{-13}$  and  $1.227 \times 10^{-11}$ , respectively. Therefore, the value of Young's modulus for each peak was calculated taking as average 262.9 GPa, this value is higher than the value calculated in Eq. (12) because the (hkl) planes are taken into account.

## ii. Tetragonal crystal

Young's modulus is given by the following relation [2]:

$$\frac{1}{y_{hkl}} = \frac{S_{11}(h^4 + k^4) + (2S_{12} + S_{66})h^2k^2 + (2S_{13} + S_{44})(h^2 + k^2)l^2 + S_{33}l^4}{(h^2 + k^2 + l^2)^2} \quad (20)$$

where  $S_{11}$ ,  $S_{12}$ ,  $S_{13}$ ,  $S_{33}$ ,  $S_{44}$  are elastic strain for  $\text{TiO}_2$  anatase, their values are  $5.1 \times 10^{-12}$ ,  $-0.8 \times 10^{-12}$ ,  $-3.3 \times 10^{-12}$ ,  $10.7 \times 10^{-12}$ ,  $18.5 \times 10^{-12}$ , and  $16.7 \times 10^{-12} \text{N/m}^2$ , respectively [27]. Using these elastic strains, the value of Young's modulus for each peak was calculated by taking as average 127 GPa.

Figure S2 (a-b) shows a plot between  $4\sin\theta/y_{hkl}$  vs  $\beta_{hkl}\cos\theta$ , a linear fit was done where the slope represents the strain, and the average crystallite size was calculated of the intersection with the axis obtaining a value of 27.5 nm for  $\text{CeO}_2$  and 13.4 nm for  $\text{TiO}_2$ . As



mentioned, in contrast to the above method, this model uses the corresponding Young's modulus.

### 3.4.3 UEDM method

It has been seen that the UDM model assumes an homogeneous crystal which is isotropic in nature. This homogeneity and isotropy is no longer justified for a real crystal. Since a crystal is anisotropic, the W-H equation must be modified by anisotropic terms [8]. This modified model is the USDM model which assumes a linear relationship between stress and strain, according to Hooke's law. But, in real crystals, the isotropic nature and linear proportionality between stress and strain cannot be considered, because there are several defects, such as dislocations and agglomerations that create imperfections in almost all crystals.

Thus, we have the UEDM which considers the deformation of crystals, the uniform anisotropic deformation of the lattice in all crystallographic directions, and the cause of that uniform anisotropic deformation of the lattice is the deformation energy density ( $u$ ). Therefore, the proportionality constants associated with the stress-strain relationship left to be independent. The strain energy (energy per unit volume) as a function of strain is given by Hooke's law as:

$$u = \frac{(\varepsilon^2 y_{hk})}{2} = \frac{\sigma^2}{2y_{hk}} \quad (21)$$

where stress and strain are related as  $\sigma = \varepsilon \times y_{hkl}$ , so the intrinsic strain can be written as a function of energy density.

$$\varepsilon = \sigma \sqrt{\frac{2u}{y_{hk}}} \quad (22)$$

The W-H equation is modified in the UEDM by:

$$\beta_{hkl} \cos \theta = \frac{K\lambda}{D} + 4 \sin \theta \left( \frac{2u}{y_{hk}} \right)^{1/2} \quad (23)$$

Plot of Eq (23), with the term  $4 \sin \theta \left( \frac{2u}{y_{hkl}} \right)^{1/2}$  along X-axis and  $\beta_{hkl} \cos \theta$  along Y-axis corresponding to each diffraction peak where the density of energy is obtained from the slope and the average crystallite size is got from the y-intercept of the linear fitting. Figure S2 (c-d) represents the UEDM fitted plots for CeO<sub>2</sub> and TiO<sub>2</sub> NPs, respectively. The points are from experimental data and the fitted data are shown with a straight line. The calculated parameters of this model are summarized in Table 1. With this method the estimated crystallite size was 28.5 and 15.1 nm, with R<sup>2</sup> values equal to 0.681 and 0.129 for CeO<sub>2</sub> and TiO<sub>2</sub>, respectively. This fit is not as good comparison to previous methods. The strain values obtained for these samples were  $1.287 \times 10^{-3}$ ,  $2.42 \times 10^{-3}$  and the stress values were  $2.87 \times 10^4$ ,  $2.18 \times 10^4$  for CeO<sub>2</sub> and TiO<sub>2</sub>, respectively.

### 3.5 SSP method

This method has a better result for isotropic broadening, since at higher diffracted angles, the XRD data are of lower resolution and the peaks overlap [28]. Figure 3 (a) and (b) show the linear-regression plots obtained for CeO<sub>2</sub> and TiO<sub>2</sub> NPs, respectively.



The SSP is one of the methods that considers the XRD peak profile to be a combination of the Lorentzian and the Gaussian functions. In this assumption, the strain profile is shown by the Gaussian function and the size of the crystallites by the Lorentz function.

$$\beta_{hkl} = \beta_L + \beta_G \quad (24)$$

where  $\beta_L$  and  $\beta_G$  are the broadening peak due to the Lorentz and Gauss functions, respectively. The SSP equation is presented below:

$$(d_{hkl}\beta_{hkl}\cos\theta)^2 = \frac{K\lambda}{D} (d_{hkl}^2\beta_{hkl}\cos\theta) + \frac{\varepsilon^2}{4} \quad (25)$$

In this particular method, less value is given to high angle diffraction data as an advantage it gives less weight to these, where the precision is usually lower [11]. This is because, at higher angles, XRD data are of lower quality and peaks are generally highly overlapped at higher diffracting angles [22]. It was clearly observed that the average crystallite size obtained for both CeO<sub>2</sub> and TiO<sub>2</sub> NPs are smaller compared to Scherrer's method, this difference can be given by the deformation. Therefore, when using a method that does not consider stress, it can give us inaccurate results [29]. It seems that the reason why the crystalline size decreases with this method is due to the deformation of the compounds, that is uniform and hence the peak broadening is reduced.

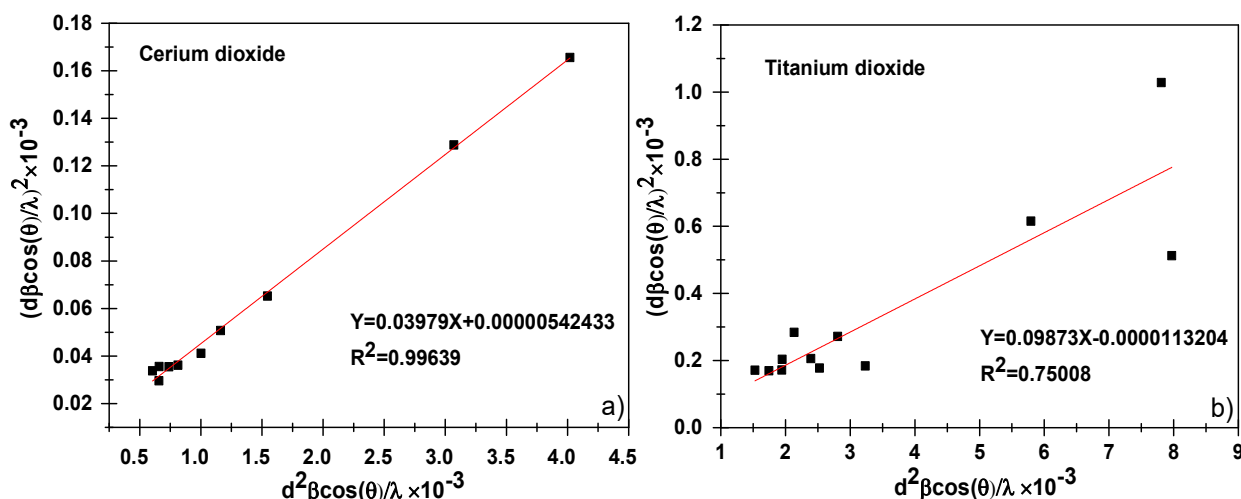


Figure 3: The SSP of CeO<sub>2</sub> (a) and TiO<sub>2</sub> NPs (b).

### 3.6 H-W method

In the above method, the XRD peak profile size extension has been assumed as a Lorentzian function, while strain broadening, as a Gaussian function. But actually, the XRD peak is neither Lorentzian function nor Gaussian function, as XRD peak region matches well with the Gauss function, whereas its tail falls off too fast matched and; on the other hand, the profile tails fit quite well with the Lorentz function but does not full the total area of the Bragg diffracted peak [22]. That is why H-W method is proposed, which is based on the assumption that peak broadening is a symmetric Voigt function which is a convolution of Lorentz and Gauss's function. Hence, for Voigt function, the full width at half maximum of the physical profile can be written by H-W method as:

$$\beta_{hkl}^2 = \beta_L \beta_{hk} + \beta_G^2 \quad (26)$$

where,  $\beta_L$  and  $\beta_G$  are the full width at half maximum of the Lorentzian and Gaussian function. This method gives more weight to Bragg peaks in the low angle range and middle angle, where overlap of diffractant peaks is low. Now, the relationship between the size of the crystallite and the lattice strain according to the H-W method is given by [22]:

$$\left(\frac{\beta_{hkl}^*}{d_{hkl}^*}\right)^2 = \frac{1}{D} \times \frac{\beta_{hkl}^*}{d_{hkl}^{*2}} + \left(\frac{\varepsilon}{2}\right)^2 \quad (27)$$

where  $\beta_{hkl}^* = \beta_{hkl} \times \frac{\cos\theta}{\lambda}$  and  $d_{hkl}^* = 2 \times d_{hkl} \times \frac{\sin\theta}{\lambda}$ , plot  $\frac{\beta_{hkl}^*}{d_{hkl}^{*2}}$  vs  $\left(\frac{\beta_{hkl}^*}{d_{hkl}^*}\right)^2$  is shown in the Figure 4 [28]. The slope of the plotted straight line provides the average crystallite size; these have been calculated from Figures 4 (a) and (b) obtaining values of 10.4 and 5.1 nm for CeO<sub>2</sub> and TiO<sub>2</sub> NPs, respectively. Also, the intercept gives the intrinsic strain of the nanocrystals. We can observe that the H-W method is more accurate. In Figures 4 (a) and (b), we can regard a greater approach of the fit line to the experimental data for both NPs as a result of the integral broadening to a Voigt function.

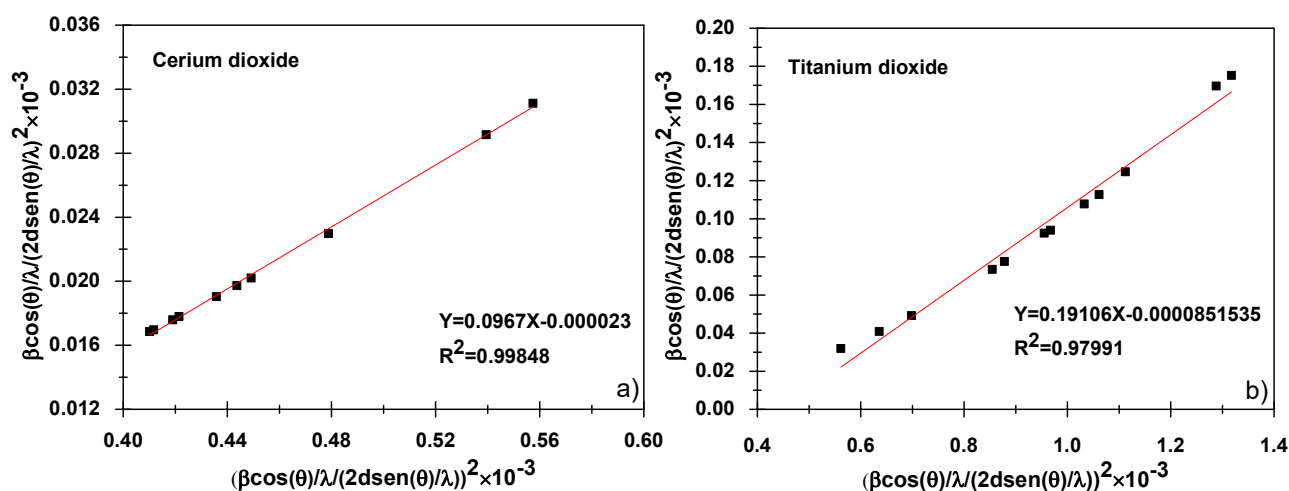


Figure 4: H-W plot of CeO<sub>2</sub> (a) and TiO<sub>2</sub> (b).

By comparing the results of the crystallite size and lattice strain, shown in the Table 1, we can see that the H-W method shows a decrease in crystallite size as well as lattice strain, a common feature between the W-H and H-W method is that the dispersion of data points increases with increased lattice strain, which would indicate that lattice strain is anisotropic [30], but in our case you see a decrease in the dispersion of the points and also a decrease in deformation so you could say that lattice strain is isotropic in nature.

**Table 1.** Microstructural parameters of the CeO<sub>2</sub> and TiO<sub>2</sub> NPs obtained by several crystalline size analysis methods.

		Samples	CeO <sub>2</sub>	TiO <sub>2</sub>
Scherrer method		D (nm)	20.8	10.5
		R <sup>2</sup>	0.969	0.401
Modified Scherrer method		D (nm)	24.1	12.7
		R <sup>2</sup>	0.961	0.434
Williamson-Hall method	UDM	D(nm)	28.9	16.3
		$\epsilon \times 10^{-3}$	0.994	3.21
		R <sup>2</sup>	0.631	0.146
	USDM	D (nm)	27.5	13.4
		$\epsilon \times 10^{-3}$	9.852	28.4
		$\sigma$ (MPa) $\times 10^2$	2.19	2.56
		R <sup>2</sup>	0.674	0.076
	UDEDM	D (nm)	28.5	15.1
		$\epsilon \times 10^{-3}$	1.287	2.422
		$\sigma$ (MPa) $\times 10^4$	2.87	2.18
		U (TJm <sup>-3</sup> )	0.018	0.026
		R <sup>2</sup>	0.681	0.129
Size-Strain Plot method		D (nm)	18.9	7.6
		$\epsilon \times 10^{-3}$	4.653	6.751
		R <sup>2</sup>	0.996	0.75
Halder Wagner method		D (nm)	10.4	5.1
		R <sup>2</sup>	0.999	0.979
		$\epsilon \times 10^{-3}$	9.584	18.62
Rietveld Refinement		D (nm)	12 (4)	14.1 (4)
		$\epsilon$	20.41	137.66
$\mu$ -Raman		D (nm)	14.5	11.5
TEM		D (nm)	10-20	5.2-16.3

The R<sup>2</sup> values are important to differentiate among all of the studied linear methods (see Table 1). We obtained only positive values of R<sup>2</sup> for all of the crystallographic phases, one method is more accurate if the R<sup>2</sup> is near 1 or in other words, data points of x-y are more touching the fitting line [31]. The H-W method is more accurate, suggesting that this model better fits the XRD diffractograms presented in this study.

### 3.7. Rietveld Refinement

Figures 5 (a) and (b) shows the refined diffractogram of CeO<sub>2</sub> and TiO<sub>2</sub> NPs using the TCH profile functions, and the refined parameters are displayed in Table 2. For CeO<sub>2</sub> NPs, the RM confirms the cubic structure of the CeO<sub>2</sub> NPs where the characteristic peaks are very close to the fluorite-structured CeO<sub>2</sub> crystal and no peak of any other phase was detected, indicating the high purity of the sample. For TiO<sub>2</sub>, the corresponding tetragonal structure of the TiO<sub>2</sub> NPs shows the presence of anatase phase only. The weighted profile residual ( $R_{wp}$ ) and the profile residual factor ( $R_p$ ) were taken into account to follow the progress and as indicator of the refinement improvement. The goodness of refinement,  $\chi^2$  (chi-squared), indicates the statistical error. In both samples, small values of 1.94 and 2.25 were obtained using the TCH profile for TiO<sub>2</sub> and CeO<sub>2</sub> NPs, respectively.. For the estimation of the mean crystallite size the TCH profile was used, for which an IRF file is

yielded by fitting the XRD diffractogram of the LaB<sub>6</sub> nano-standard, which subtracts the instrumental broadening. The mean apparent size was 12 and 14.1 nm, for the CeO<sub>2</sub> and TiO<sub>2</sub> nanocrystallites, respectively, and of 40.5 nm for the LaB<sub>6</sub> standard (see Figure S3 (a-d) and Table S1).

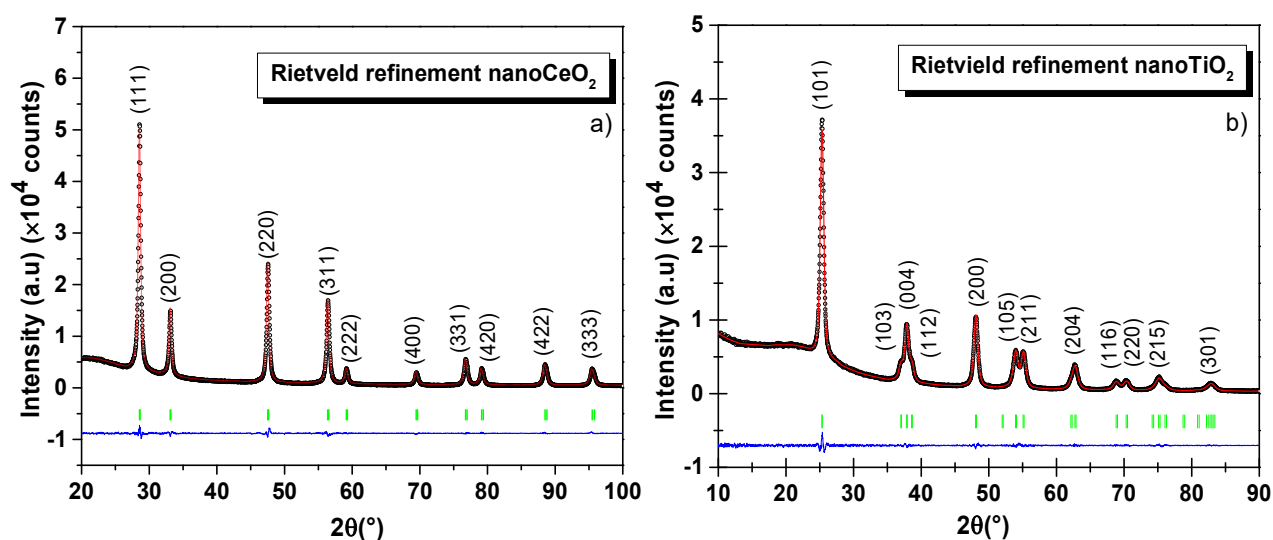


Figure 5. Rietveld refinement of XRD diffractograms using the TCH function for CeO<sub>2</sub> (a) and TiO<sub>2</sub> NPs (b). The observed experimental diffractograms are given by the red lines ( $I_{obs}$ ), the black lines ( $I_{cal}$ ) are calculated diffractograms, and the residual lines are shown in blue color. The refinement parameters are reported in Table 2.

### 3.8 Electron density modeling

Electronic density measures the probability of finding an electron in a certain region of the atom. The higher the concentration of electrons at a given point, the higher the electronic density. Therefore, depending of the material will exhibit certain characteristics. It can be seen on a scale composed of rainbow colors, red to indicate the region of highest electronic density and blue for that region where there are fewer electrons. Figures 6a and 6b show that there is no formation of ionic bonds using as an indicator the red color that are the covalent bonds. The Vesta program was used with a resolution of 0.6 Å for both samples.

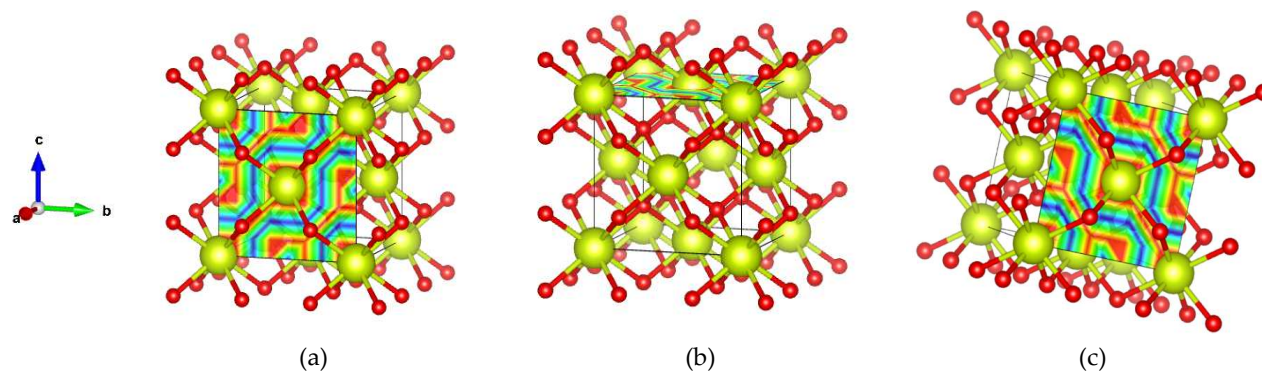
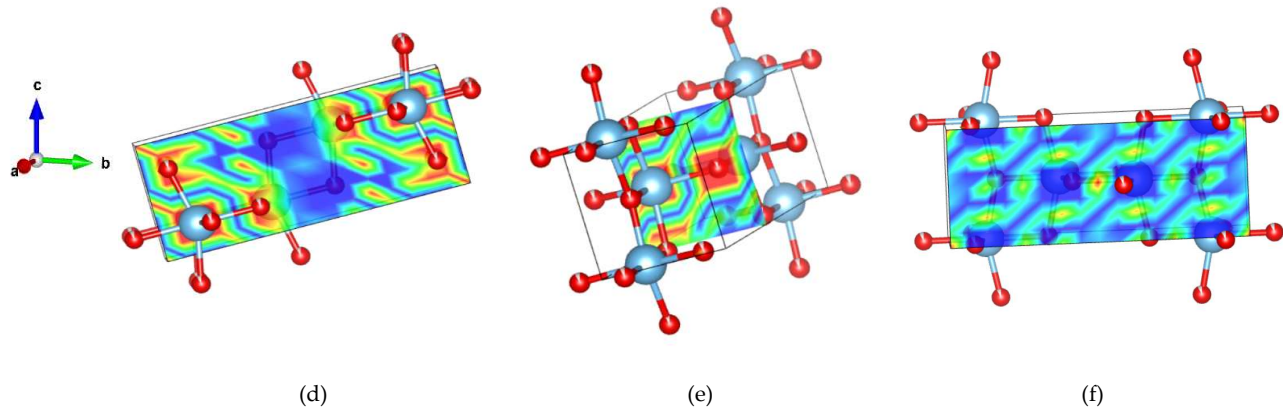


Figure 6a. Electron density model with 0.6 Å resolution of nanoCeO<sub>2</sub> for the (hkl) lattice planes (1 0 0) (a), (0 0 1) (b) and (0 1 0) (c).



**Figure 6b.** Electron density model with 0.6 Å resolution of nanoTiO<sub>2</sub> for the (hkl) lattice planes (1 0 0) (a), (0 0 1) (b) and (0 1 0) (c).

### 3.9. $\mu$ -Raman analysis

For spherical particles with a diameter  $D$  and no vibration mode degeneration, the Raman intensity can be written as shown in Eq. (28)

$$I(\omega) \propto \int_{BZ} d^3q \frac{|C(q,D)|^2}{(\omega - \omega(q))^2 + \left(\frac{\Gamma}{2}\right)^2} \quad (28)$$

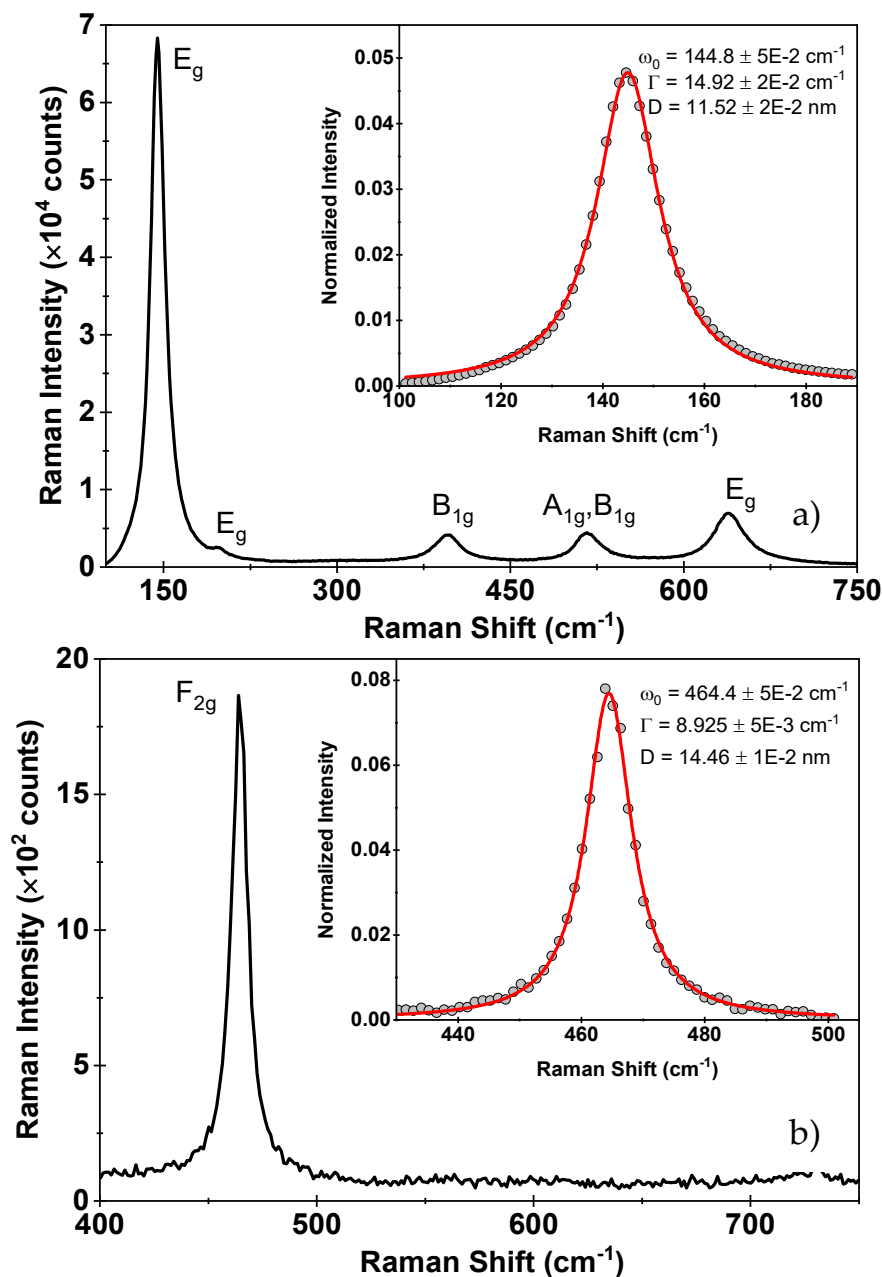
Here,  $q$  is the phonon wave vector ranging in the BZ from  $-\pi/a$  to  $\pi/a$ , with  $a$  the lattice parameter. The volume differential  $d^3q$  under the spherical symmetry approximation is written as  $d^3q = 4\pi q^2 dq$ .  $\Gamma$  is the intrinsic mode line width of the Lorentzian response centered at  $\omega(q)$ , with  $\omega(q)$  the phonon dispersion relation,  $C(q,D)$  are the Fourier coefficients of a weighting function describing the confinement, i.e. enforcing the decay of the phonon wave function to a minimal value close to the nanocrystal boundary.

$$|C(q,D)|^2 = \left( -\frac{q^2 D^2}{8\beta} \right) \quad (29)$$

According to previous reports, for particle diameters greater than  $D \geq 10$  nm [17, 18, 32], a Gaussian confinement weighting function is a good approximation, and thus the Fourier coefficients are written as depicted in Eq. (29). The confinement factor  $\beta$  can vary from 1 to  $2\pi^2$  for the Richter confinement model [33] and Campbell model, respectively [34]. Here, for the sake of simplicity we fix  $\beta = 1$ . Previous reports suggest that the dispersion relation for anatase TiO<sub>2</sub> NPs near the E<sub>g</sub> band can be modeled as  $\omega(q) = \omega_0 + \Delta(1 - \cos(qa))$  with  $\Delta = 20 \text{ cm}^{-1}$ . For the case of CeO<sub>2</sub> NPs, we simply approximate the dispersion relation with a polynomial function of order 3,  $\omega(q) = \omega_0 + A_1 q + A_2 q^2 + A_3 q^3$ . Distinct reports put  $\omega_0$  for the F<sub>2g</sub> mode between 464 and 466 cm<sup>-1</sup>. Here the best fitted value is obtained for  $\omega_0 = 464.4 \text{ cm}^{-1}$  in agreement with Spainer et al [32]. However, our analysis did not include strain effects.

In order to fit Eq. (28) we normalize the Raman intensity to the peak area. We do this with the experimental data and with Eq. (28). The fitting procedure is based on the minimization of the unbiased squared error with free parameters  $\omega_0$ ,  $L$  and  $\Gamma$ .

Figure 7 a and b, depict the Raman spectra of the TiO<sub>2</sub> and CeO<sub>2</sub> NPs, respectively. For the TiO<sub>2</sub> NPs the E<sub>g</sub> and B<sub>1g</sub> bands are identified. The A<sub>1g</sub> band cannot be resolved. The analysis is performed with the E<sub>g</sub> band centered at 144 cm<sup>-1</sup>. The characteristic F<sub>2g</sub> band at 464 cm<sup>-1</sup> of the CeO<sub>2</sub> particles is observed and used for the analysis. Both peaks were fitted using Eq. (29) after normalization. The estimated NPs size are 11.5 nm and 14.5 nm for TiO<sub>2</sub> and CeO<sub>2</sub>, respectively with a fitting error close to 0.01 nm. Additional error source could be attributed to a particle size distribution not considered in this analysis.



**Figure 7.** Raman spectrum of TiO<sub>2</sub> (a) and CeO<sub>2</sub> (b) nanopowders, respectively. Inset graphs denotes the fit after Eq. 29. Best fitted parameters are shown.



### 3.10. TEM analysis and comparison

Commercial nanoceria NPs have average TEM particle sizes in the range of 15-25 nm [35]. The nanoceria are agglomerated and present a particle size distribution (PSD). While the crystallite estimation methods including Rietveld refinement, gave values between 10 and 20 nm, which is reasonable agreement with TEM determination and hence suggesting that the Scherrer, W-H, SSP, and H-W methods are accurate suggesting the presence of PSD. On the other hand, donut-like TiO<sub>2</sub> NPs shown in Figures 8(a) and (b) shows a particle size of ca. 20 nm with mean pore sizes of 5-7 nm. By comparing with the size estimation methods, the crystallite sizes are in the interval of 5.2 to 16.3 nm. Then the anatase TiO<sub>2</sub> NPs is composed of these individual crystallites. It can be concluded that in the case of PSD, the NPs are formed by nanocrystallites with anisotropic size behavior, then a crystallite size distribution is expected. By comparing all the methods, it can be said that the crystallite sizes obtained by the W-H method are more accurate, but an overestimation of 35% with respect to Scherrer method is obtained, as shown in TiO<sub>2</sub> NPs, where W-H crystallite sizes are on average 3-5 nm times higher than the sizes of the Scherrer equation [36]. Moreover, the H-W method underestimate the crystallite size obtained from Rietveld method in ca. 5 nm. This suggest that the strain contribution must be considered in the sample with exotic morphologies as compared with nanoceria.

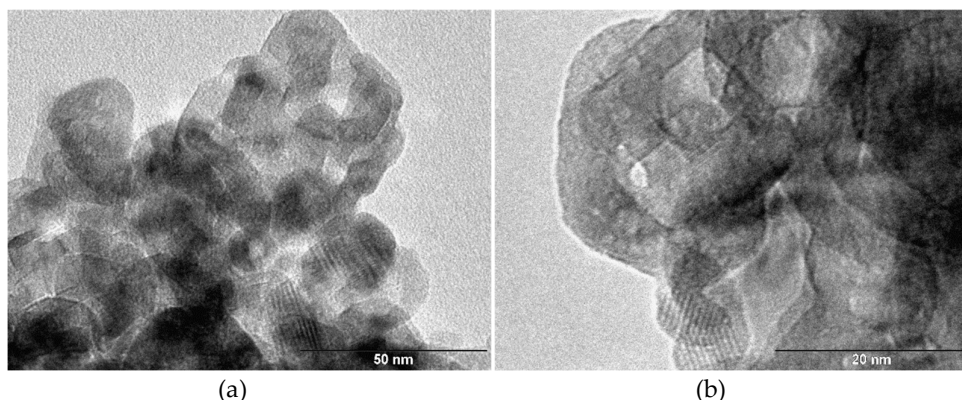


Figure 8. TEM image of nano-TiO<sub>2</sub> with bar length of 50 nm (a) and bar length of 20 nm (b).

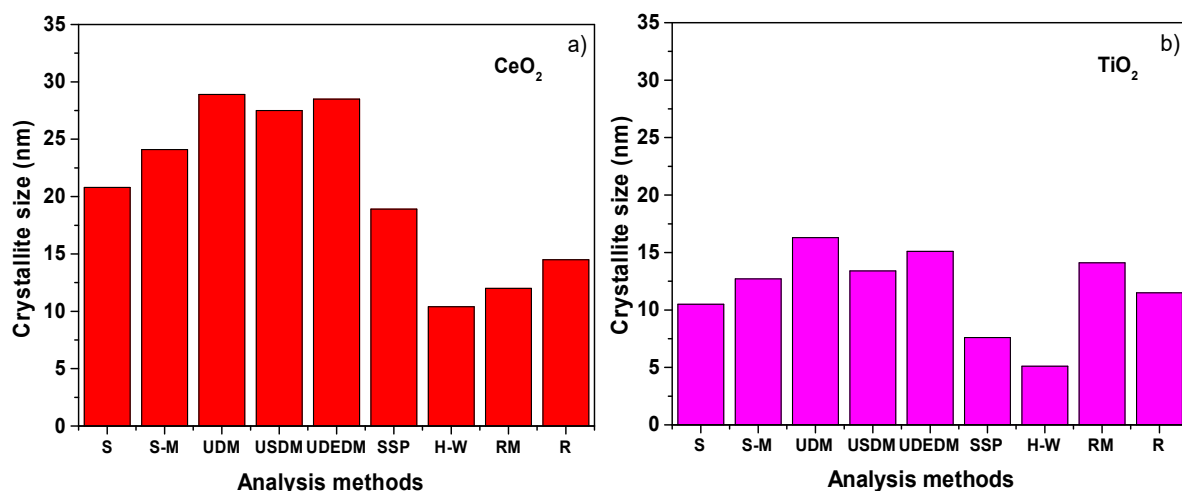


Figure 9. Comparison of the average crystallite size obtained by different estimation methods S (Scherrer method), RM (Rietveld Refinement), R ( $\mu$ -Raman) for CeO<sub>2</sub> (a) and TiO<sub>2</sub> NPs (b).



Figures 9 (a) and (b) shows the the crystallite size values as a function of the different methods based on the analysis of the line profiles using the p-XRD, showing a variation except in the three W-H models, as reflected in the data fit shown in the previous Figures. In the case of the H-W method, it shows a decrease in the crystallite size, which confirms the nanostructured character presented in the sample. On the other hand, the RM is consistent with the size strain method. This provides reliability in the method and allows its use to obtain average crystallite sizes, being one of the most important part in the characterization of nanomaterials.

**Table 2.** Rietveld refinement parameters of CeO<sub>2</sub> and TiO<sub>2</sub> samples using the Fullprof program: cell parameters, cell volume and agreement factors.  $R_p$  (%) and  $R_{wp}$  (%) are the profile residual and the weighted profile residual factors, respectively, used to verify the Rietveld refinement quality. The goodness of fit, chi-square ( $\chi^2$ ).

Refinement Parameters	nanoCeO <sub>2</sub>		nanoTiO <sub>2</sub>	
	TCH	Pseudo Voigt	TCH	Pseudo Voigt
<i>a</i> (Å)	5.408	5.409	3.784	3.784
<i>b</i> (Å)	5.408	5.409	3.784	3.784
<i>c</i> (Å)	5.408	5.409	9.502	9.504
$\alpha$ (Å)	90	90	90	90
$\beta$ (Å)	90	90	90	90
$\gamma$ (Å)	90	90	90	90
<i>V</i> (Å <sup>3</sup> )	158.14	158.28	136.03	136.11
FWHM parameters				
U	0.085	0.098	2.708	6.218
V	-0.589	-0.030	-1.784	-3.384
W	0.014	0.105	0.775	1.129
Global average size (nm)	12.0 (4)		14.1 (4)	
$R_p$ (%)	5.40	4.83	7.11	6.86
$R_{wp}$ (%)	5.42	5.01	6.33	6.19
$\chi^2$	2.25	1.93	1.94	1.99

4. Conclusion

In this work, an exhaustive analysis of XRD data for CeO<sub>2</sub> and TiO<sub>2</sub> NPs are presented using crystallite size estimation methods, among them: the Scherrer method, the Monshi method, the W-H model, the UDM, UDEDM, SSP, and H-W method, where all of them suggested an important isotropic broadening contribution, assuming Lorentzian and Gaussian profile contributions to estimate the crystallite size and microdeformation physical parameters. However, the method of Scherrer and W-H have less precision for the determination of the crystallite size for these metallic nanooxides. Furthermore, the crystallite size was calculated using the RM. Fort that, the IRF function was considered and obtained from the refinement of the standard (nano LaB<sub>6</sub>). By employing the RM, it was possible to carry out the refinement of CeO<sub>2</sub> and TiO<sub>2</sub> nanopowders corroborating the phases of cubic nanoCeO<sub>2</sub> and TiO<sub>2</sub> anatase using the TCH profile and hence allowing calculation of microstructural parameters. After comparing all the presented models, it was found that the average crystallite sizes determined by the SSP and H-W methods are close to the results obtained in RM, in case of the CeO<sub>2</sub>, but for the TiO<sub>2</sub> the Monshi method was the closest model. Then, we can say that the average crystallite size of CeO<sub>2</sub> and TiO<sub>2</sub> NPs are in a range of 10-15 nm as also corroborated by TEM analysis. Thus, suggesting

that each NP is made up of two or at least three crystallites. Crystallite size determined by Raman analysis are in agreement with this particle crystallite distribution. Therefore, we have presented in detail XRD characterization that strongly correlates with Raman and TEM analysis. This perspective can be used in future works in order to analyze and estimate accurately the crystallite size distribution presented in NPs as prepared by different physical and chemical methods as well.

**Supplementary Materials:** Figure S1. Pure p-XRD pattern of CeO<sub>2</sub> (a) and TiO<sub>2</sub> NPs (b). Figure S2. The modified W-H analysis of CeO<sub>2</sub> (a) and TiO<sub>2</sub> NPs (b), assuming USDM. The modified W-H analysis of CeO<sub>2</sub> (c) and TiO<sub>2</sub> NPs (d), assuming UDEDM. Figure S3. Rietveld refinement using the TCH function for LaB<sub>6</sub> (a). The observed experimental diffractograms are given by the red lines ( $I_{obs}$ ), the black lines ( $I_{cal}$ ) are calculated diffractograms, and the residual lines are shown in blue color. Electron density model with 0.4 Å resolution of LaB<sub>6</sub> for the (hkl) lattice planes (1 0 0) (b), (0 0 1) (c), and (0 1 0) (d). Supplementary Matlab code R2017b.

**Author Contributions:** Conceptualization, Y.C.H., A.F. M.A., J.P.V. Y.B.R., J.A.G. and J.A.R.G.; methodology, Y.C.H., A.F. M.A., J.P.V. Y.B.R., J.A.G. and J.A.R.G.; software, Y.C.H., A.F. M.A., J.P.V. Y.B.R., J.A.G. and J.A.R.G.; formal analysis, Y.C.H., A.F. M.A., J.P.V. Y.B.R., J.A.G. and J.A.R.G.; investigation, Y.C.H., A.F. M.A., J.P.V. Y.B.R., J.A.G. and J.A.R.G.; resources, Y.C.H., A.F. M.A., J.P.V. Y.B.R., J.A.G. and J.A.R.G.; data curation, Y.C.H., A.F. M.A., J.P.V. Y.B.R., and J.A.G.; writing—original draft preparation, Y.C.H., A.F. M.A., J.P.V. Y.B.R., J.A.G. and J.A.R.G.; writing—review and editing, Y.C.H., A.F. M.A., J.P.V. Y.B.R., J.A.G. and J.A.R.G.; visualization, Y.C.H., A.F. M.A., J.P.V. Y.B.R., J.A.G. and J.A.R.G.; supervision, J.A.R.G.; project administration, J. A.R.G.; funding acquisition, J.A.R.G. All authors have read and agreed to the published version of the manuscript.

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**Data Availability Statement:** The simulated data of the present research can be provided upon reasonable request at the email [juan.ramos5@unmsm.edu.pe](mailto:juan.ramos5@unmsm.edu.pe).

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**Conflicts of Interest:** The authors declare no conflict of interest.

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