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Matroid And Its Relations

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Abstract

In this article, the connections amid matroid and other notions have been studied. The structure of matroid could be a reflection of some other structure in lattice theory, group theory, other algebraic structure, graph theory, combinatorics and enumeration theory.

Keywords: Edge, Groupoid, Named and unnamed graphs, Matroid
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1 Preliminary On The Concept

I'm going to refer to some books which are cited to the necessary and sufficient material which are covering the introduction and the preliminary of this outlet so look [**Ref.** [1], **Ref.** [2], **Ref.** [3], **Ref.** [4]] where **Ref.** [1] is about the textbook, **Ref.** [2] is common, **Ref.** [3] has good ideas and **Ref.** [4] is kind of disciplinary approaches in the good ways. Further references could be referred and could be addressed in **Refs.** [5–11].

2 Definition And Its Clarification

Definition 2.1. (Matroid)
Let E is a given set and B is an arbitrary set is including the some subsets of E. If there's three conditions for these two sets, then there's matroid which is corresponded to (E, B). So (E, B) is the characteristic of the **matroid**. Three conditions are the following:

- The set which has no member, is belonging to B.
- If $b \in B$, then any of subset of b, is belonging to B.
- If $b_1, b_2 \in B$ and b_1 has more members than b_2 then there's the member of b_1, when it's added to b_2, it makes the new member of B.

3 Relationships And Its illustrations

Theorem 3.1. *Let (E, B) be a matroid. Then there is a*

(*i*) *corresponded poset.*

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(*ii*) *decreasing property.*

(*iii*) *power set.*

(*iv*) *chain.*

(*v*) *totally ordered set.*

(*vi*) *meet-lattice.*

Proof. (*i*) There is a corresponded poset $(B, \subseteq)$.

(*ii*) By second principle in the Definition (2.1), every member of a matroid has the decreasing property when $(B, \subseteq)$. Because All subsets of a member is the member so $(B, \subseteq)$ has the decreasing property on its members with the relation of $\subseteq$.

(*iii*) If $E \in B$, then B is power set of E.

(*iv*) Every member of B makes the chain.

(*v*) If $E \in B$, then B is power set of E. $(B, \subseteq)$ is totally ordered set in the way that, all members are comparable.

(*vi*) Every subsets of members have the minimum in B which is $\emptyset$.

□

4 Results And Its Beyond

Theorem 4.1. *Let (E, B) be a matroid and I is the maximum independent set in the term of order $\subseteq$. Then*

(*i*) *cardinality of I equals with components of its graph.*

(*ii*) *I has representatives which are the partitions for B.*

(*iii*) *I is the maximum minimal set.*

(*iv*) *I is neither poset nor totally ordered set.*

(*v*) *$\subseteq$ on I is only reflexive.*

(*vi*) *I isn't $\emptyset$.*

Proof. Obvious. □

Theorem 4.2. *Let (E, B) be a matroid and B is one chain. Then $E \in B$ so B is power set of E.*

Proof. Obvious. □

Theorem 4.3. *Let (E, B) be a matroid and $|E| = n$. Then $1 \leq |B| \leq 2^n$.*

Proof. Obvious. □

Theorem 4.4. *Let (E, B) be a matroid and $|B| = 2^n$. Then B is power set of E.*

Proof. Obvious. □

Theorem 4.5. *Let (E, B) be a matroid and $|B| = 1$. Then B is power set of $\emptyset$.*

Proof. Obvious. □

Theorem 4.6. *Let (E, B) be a matroid. Then $(B, \max, \min)$ is an algebraic structure.*

Proof. Obvious. □

Theorem 4.7. *Let (E, B) be a matroid. Then $(B, \max)$ is an abelian group.*

Proof. Obvious. □

Theorem 4.8. *Let (E, B) be a matroid. Then $(B, \min)$ is an abelian semi-group.*

Proof. Obvious. □

Theorem 4.9. *Let (E, B) be a matroid. If $B = E$, then $(B, \min)$ is an abelian monoid.*

Proof. Obvious. □

Theorem 4.10. *Let (E, B_1) and (E, B_2) be matroids. Then $(E, B_1 \cup B_2)$ and $(E, B_1 \cap B_2)$ are matroids.*

Proof. Obvious. □

Theorem 4.11. *Let (E, B_1) and (E, B_2) be matroids. If $B_1 \subseteq B_2$, then $(E, B_2 - B_1)$ is matroid.*

Proof. Obvious. □

Theorem 4.12. *Let (E, B_1) be matroid. then (E, B^c) is matroid when B^c is complement of B in the matter of power set of E.*

Proof. Obvious. □

Theorem 4.13. *Let $\mathcal{B}$ be a set of all subsets of E which they construct matroid on E. If $E_1 \subseteq E$, then $\mathcal{B}_1 \subseteq \mathcal{B}$.*

Proof. Obvious. □

Theorem 4.14. *Let $\mathcal{B}$ be a set of all B of E which they construct matroid on E. Then $P(E) \in \mathcal{B}$ where $P(E)$ is the power set of E.*

Proof. Obvious. □

Theorem 4.15. *Let $\mathcal{B}$ be a set of all B of E which they construct matroid on E. If $|E| = n$, then $|\mathcal{B}| = n^n$.*

Proof. Obvious. □

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