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Type II Vacuum Spacetime Admitting Closed Timelike Curves

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Abstract : We present a cyclic symmetry type II vacuum spacetime admitting closed timelike curves (CTCs) which appear after a certain instant of time, i.e., a time-machine spacetime. The various authors in past have considered the 2D and 4D flat generalization of Misner space, but in the present work, we have considered the curved spacetime a 4D generalization of the Misner space, singularity-free and asymptotically flat.

Keywords: closed timelike curves, Misner space, vacuum spacetime
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1 Introduction

One of the most interesting aspects of Einsteins theory of gravitation is that there are solutions of the Field Equations admitting closed timelike curves (CTCs). The first known spacetime with such causality violating behaviour is the Gödel universe [1] and, in fact, there are a considerable number of spacetimes in the literature that admit CTCs. The presence of CTCs, or equivalently, closed null or timelike geodesics in certain solutions of the field equations lead to the theoretical possibility of time-travel. A small sample of solutions that allow such causality violating behaviour are [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. One way of classifying such causality violating spacetimes would be to categorize the metrics as either eternal time-machine in which CTCs always exist (in this class would be [1] or [2]), or as time-machine spacetimes in which CTCs appear after a certain instant of time. In the latter category would be the ones discussed in [14, 15, 16, 17]. The most natural and simplest situation would, of course, be a vacuum solution. In fact, several vacuum spacetimes with CTCs are known. Some well known ones are the NUT-Taub metric [23], the spacetimes found by Ori [16, 17] and the Kerr-Newmann black holes [21, 22].

Many of the above models, however, suffer from one or more severe drawbacks as possible candidates for a working time-machine. For instance, in some of these solutions, the weak energy condition (WEC) is violated indicating unrealistic matter-energy content [5]. The WEC states that for any physical (timelike) observer the energy density is non-negative, which is the case for all known types of (classical) matter fields. We note that the eternal time-machine spacetimes are unrealistic models for a putative time machine. A workable model of a time machine must be a spacetime where CTCs appear at a definite instant of time.

Among the time-machine spacetimes, we mention two: the first being Ori's compact core [16] which is represented by a vacuum metric locally isometric to pp waves and second, which is more relevant to the present work, the Misner space [24]. This is essentially a two dimensional metric (hence flat) with peculiar identifications. The Misner space is interesting in the context of CTCs as it is an example of a spacetime where CTCs evolve from causally well-behaved initial conditions. The metric for the Misner space [24]

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$$ds^2 = -2dtdx - t\,dx^2 \qquad (1)$$

where $-\infty < t < \infty$ but the co-ordinate x is periodic. The metric (1) is regular everwhere as $detg = -1$ including at $t = 0$. The curves $t = t_0$, where t_0 is a constant, are closed since x is periodic. The curves $t = t_0 < 0$ are spacelike, but $t = t_0 > 0$ are timelike and the null curves $t = t_0 = 0$ form the chronology horizon. The second type of curves, namely, $t = t_0 > 0$ are closed timelike curves (CTCs). This metric has been the subject of intense study and quite recently, Levanony and Ori [25] have studied the motion of extended bodies in the 2D Misner space and its flat 4D generalizations. However, the major problem with the Misner space as a feasible time machine, namely, that the spacetime is not physically relevant, since basically, it is a cut and paste version of the Minkowski spacetime, remains, and this lacunae has been identified in [25]. A non-flat 4D spacetime, satisfying all the energy conditions, but with causality violating properties of the Misner space, primarily that CTCs evolve smoothly from an initially causally well-behaved stage, would be physically more acceptable as a time-machine spacetime.

In this paper, we shall attempt to show that causality violating curves may appear in vacuum spacetimes (which automatically satisfy the energy conditions) with comparatively simple structure.

2 Analysis of the spacetime

Consider the following metric

$$ds^2 = e^{3r}\,dr^2 + e^{-r}\,dz^2 - e^{2r}\,(t\,d\phi^2 + 2\,dt\,d\phi) - 2\,\beta\,e^{2r}\,dr\,d\phi \qquad (2)$$

where ϕ co-ordinate is assumed periodic $0 \le \phi < \phi_0$, with $\phi_0 > 0$ and $\beta > 0$ is a real number. We have used co-ordinates $x^1 = r$, $x^2 = \phi$, $x^3 = z$ and $x^4 = t$. The ranges of the other co-ordinates are $-\infty < t, z < \infty$ and $0 \le r < \infty$. The metric has signature $(+, +, +, -)$ and the determinant of the corresponding metric tensor $g_{\mu\nu}$, $det\,g = -e^{6r}$. The spacetime (2) is a vacuum solution of the Field Equations with the Ricci tensor $R_{\mu\nu} = 0$.

Consider closed orbits of constant $t = t_0$, $r = r_0$ and $z = z_0$, the metric (2) reduces to one-dimensional form

$$ds^2 = -te^{2r}d\phi^2 \qquad (3)$$

These orbits are null curves for $t = t_0 = 0$, spacelike throughout for $t = t_0 < 0$, but become timelike for $t = t_0 > 0$, which indicates the presence of closed timelike curves (CTCs).

In order that the above analysis is valid, the CTCs evolve from an spacelike $t = constant$ hypersurface (and thus t is a time coordinate) [16]. This can be ascertained by calculating the norm of the vector $\nabla_\mu t$ (or by determining the sign of the component g^{tt} in the inverse metric tensor $g^{\mu\nu}$). We find from (2) that

$$g^{tt} = e^{-3r}\,(te^r + \beta^2) \qquad (4)$$

Thus a hypersurface $t = constant$ is spacelike provided $g^{tt} < 0$ for $t = t_0 < 0$, but become timelike provided $g^{tt} > 0$ for sufficiently large $t = t_0 > 0$. Here we choose r_0 ($r = r_0$, a constant) and β is sufficiently small positive number such that the above condition is satisfied.

For cyclic symmetry metric, consider the Killing vector $\eta = \partial_\phi$ which has the normal form

$$\eta^\mu = (0,1,0,0) \qquad (5)$$

Its co-vector is

$$\eta_\mu = e^{2r}\,(-\beta,-t,0,-1) \qquad (6)$$

(5) satisfies the Killing equation $\eta_{\mu;\nu} + \eta_{\nu;\mu} = 0$. For a cyclically symmetric metric, the norm $\eta^\mu \eta_\mu$ of the Killing vector form spacelike, closed orbits [26, 27]. We note that

$$\eta^\mu \eta_\mu = -te^{2r} \tag{7}$$

which is spacelike for $t < 0$, closed orbits (Periodicity of ϕ). As $r = 0$ doesn't represent a symmetry axis, the spacetime is multiple-connected like the Misner space. Hence the spacetime (2) is cyclically symmetric without a symmetry axis.

3 Classification of the spacetime and the kinematical properties

For classification of the spacetime (2), we can construct the following set of null tetrads $(k, l, m, \bar{m})$ where k, l are real and m, $\bar{m}$ are complex vector in the form

$$k_\mu = (0,1,0,0) \tag{8}$$

$$l_\mu = e^{2r}\left(\beta, \frac{t}{2}, 0, 1\right) \tag{9}$$

$$m_\mu = \frac{1}{\sqrt{2}}\left(e^{\frac{3r}{2}}\, 0, i e^{-\frac{r}{2}}, 0\right) \tag{10}$$

$$\bar{m}_\mu = \frac{1}{\sqrt{2}}\left(e^{\frac{3r}{2}}\, 0, -i e^{-\frac{r}{2}}, 0\right) \tag{11}$$

where $i = \sqrt{-1}$. The set of null tetrad above are such that the metric tensor for the line element (2) can be expressed as

$$g_{\mu\nu} = -k_\mu l_\nu - l_\mu k_\nu + m_\mu \bar{m}_\nu + \bar{m}_\mu m_\nu \tag{12}$$

The vectors (8)—(11) are null vectors and are orthogonal

$$\begin{aligned} k^\mu k_\mu = l^\mu l_\mu = m^\mu m_\mu = m^\mu m_\mu = 0 \\ k^\mu m_\mu = k^\mu \bar{m}_\mu = l^\mu m_\mu = l^\mu \bar{m}_\mu = 0 \end{aligned} \tag{13}$$

except for $k_\mu k^\mu = -1$ and $m_\mu \bar{m}^\mu = 1$. Using this null tetrad above we have calculated the five Weyl scalars

$$\Psi_2 = \frac{e^{-3r}}{2} \quad \text{and} \quad \Psi_4 = -\frac{\beta}{4} e^{-r} \tag{14}$$

are non-vanishing, while $\Psi_0 = \Psi_1 = \Psi_3 = 0$. This clearly shows that the metric (2) is of type II in the Petrov classification scheme. The metric (2) is free-from curvature singularities. The curvature invariant known as Kretchsmann scalar is given by

$$R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} = 12 e^{-6r} \tag{15}$$

The Kretschmann scalar, the Weyl scalar Ψ_2, Ψ_4 vanish at $r \longrightarrow \infty$ indicating asymptotic flatness of the spacetime (2).

Using the null tetrad (8)—(11), we have calculated the optical scalars [27], namely, the *expansion*, the *rotation* and the *shear* and they are found to be

$$\begin{aligned} \theta &= \frac{1}{2} k^\mu_{;\mu} = 0 \quad , \\ \omega^2 &= \frac{1}{2} k_{[\mu;\nu]} k^{\mu;\nu} = 0 \quad , \\ \sigma \bar{\sigma} &= \frac{1}{2} k_{(\mu;\nu)} k^{\mu;\nu} - \theta^2 = 0 \end{aligned} \tag{16}$$

and the null vector (8) satisfy the geodesies equation

$$k_{\mu;\nu} k^\nu = 0 \tag{17}$$

Hence the spacetime is non-diverging, have shear-free null geodesies congruence. We have determined the Newmann-Penrose spin coefficients [27] for metric (2) and they are

$$\begin{aligned} \nu &= -\frac{\beta}{2\sqrt{2}} e^{\frac{r}{2}}, \\ -\tau &= \pi = \frac{e^{-\frac{3r}{2}}}{\sqrt{2}}, \\ \beta &= 3\alpha = -\frac{3}{4\sqrt{2}} e^{-\frac{3r}{2}}, \\ \gamma &= -\frac{1}{4}, \\ \kappa &= \rho = \sigma = \tau = \mu = \lambda = \pi = \epsilon = 0 \end{aligned} \tag{18}$$

where the symbols are same as in [27].

4 Conclusion

Our primary motivation in this paper is to write down a metric for a spacetime that incorporates the Misner space and its causality violating properties and to classify it. The solution presented here is Ricci flat, cyclicly symmetric metric (2) and serves as a model of time-machine spacetime in the sense that CTCs appear at a definite instant and the energy conditions are satisfied. Most well-known CTC spacetimes violate one or more of these caveats and the spacetime discussed here thus falls in the category of a miniscule number of those which may be termed as true time-machine spacetimes.

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Research Journal of Contemporary Concerns Vol. 10 & 11 (B), 2016

CONTENTS

- A Review On Microbial Inoculants For Fertilizer Use Efficiency In Crop Production 3
  Hari Prasanna Deka Boruah
- Silica Gel Supported Tetrabutyl Ammonium Chlorochromate: An Effecient Reagent For Regeneration Of Carbonyl Compounds From Their Oximes At Room Temperature 29
  S.B. Hazarika and D.K. Kakati
- Biosynthesis Of Silver Nanoparticles From Leaf Extracts Of *Azadirachta Indica*, *Ocimum Sanctum* and *Citrus Limon* 33
  Sangeeta Agarwal and Dipika Konwar
- Metamorphism In The Precambrian Rocks Around Changsari-Agyathuri, North Guwahati, Kamrup District, Assam 38
  J. Medhi
- Herpetofaunal Assemblage Of Deeporbeel Ramsar Site Of Assam, India 52
  Saibal Sengupta and Jayaditya Purkayastha, Madhurima Das and Binode K. Baruah
- Accumulation Pattern Of Toxic Metals In The Filamentous Green Alga *Microspora Quadrata* Hazen. : An Experimental Approach 58
  Mautushi Das and P. Ramanujam
- Protective Effect Of *Nelsonia Canescens* (Lam.) Spreng On the Damaged Liver of Laboratory Animal Model Induced By Acute And Chronic Toxicity Of Carbon Tetrachloride 67
  B. Chowdhury and J.C. Kalita
- Role Of Gravity Components On The Formation Of Adventitions Roots On Stem Cuttings Of *Chrysanthemum Coecineum* L Plants 81
  Mrinal Kumar Das
- Agrodiversirty Along The Altitudinal Gradient In East Siang District Of Arunachal Pradesh, Eastern Himalaya 94
  J Y Yumnam, O P Tripathi and M L Khan
- Effect Of Calcium Metal On Growth And Physiology Of Rice During Water Stress 104
  Hrishikesh Upadhyaya, Sourav Sen, Panchami Roy and Mrinal Kanti Bhattacharya
- Study On The Wild Mushrooms Of Dhubri District Of Assam 112
  Manashi Baruah and S.R. Patgiri
- Effect Of Water Stress On Growth Efficiency And Productivity Of Tomato (*Lycopersicum Esculentum*. L.) 117
  Goyir Dabi and Milu R. Das
- Life Form Classification And Biological Spectrum Of Baridua Forest Range In Ri Bhoi District, Meghalaya 122
  S.I. Bhuyan, P. Ramwa, I. Laskar and B.N. Baruah
- Association Of The Arbuscular Mycorrhizal Fungi With *Cinnamomum Tamala* (Buch.-Ham.) T.Nees & Eberm 127
  Sufian A. Tapadar, Ternyak Kamchi, A. Hussain and D.K. Jha
- Influence Of Phylloplane Microflora Of Tea, *Camellia Sinensis* (L).O.Kuntze ON Germination Of *Exobasidium Vexans* Massee. 139
  S. Debnath and P.K. Baruah
- Thermo-Diffusion And Diffusion-Thermo Effects In A MHD Free Convective Flow Through Porous Medium Between Two Long Vertical Wavy Walls 148
  H. Deka, D. Sarma and K. Sarma
- Biosensors With DNA Templated CdS Nanoparticle 165
  A.M.P. Hussain
- Type II Vacuum Spacetime Admitting Closed Timelike Curves 171
  Faizuddin Ahmed