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# Review **Trajectory Optimization in Robotic Applications Survey of Recent Developments**

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Abstract: Trajectory Optimization (TO) is the sequence of processes that are considered in order 8 to produce the best path that mends the overall performance or reduces the consumption of the 9 resources where the restriction system remains maintained. In this survey, an inclusive review 10 of the latest advancements in modeling and optimization of trajectory generation in robotic ap-11 plications will be discussed broadly. In recent times, numerous studies have employed optimal 12 control techniques involving direct and indirect methods in order to convert the authentic Tra-13 jectory Optimization problem into a constrained parameter optimization problem. Moreover, a 14 huge variety of optimization algorithms such as Genetic Algorithms (GA), Simulated Anneal-15 ing (SA), Sequential Quadratic Programming (SQP), and Particle Swarm Optimization (PSO) are 16 used aiming to find the optimal solutions for trajectory planning. It is observable that, minimiz-17 ing the jerk, energy consumption, and execution time among the most widely used design objec-18 tives, on the other hand, the robot's joints configurations and the motor torques are the most used 19 design variables. This paper aims to review the fundamental techniques and their coincident 20 robotic applications in the field of trajectory optimization aiming to afford some steering for 21 related researchers. 22

Keywords: Robotics; Optimization; Optimal Control (OC); Trajectory planning; Dynamic Transcrip-23 tion methods 24

# 1. Introduction

An unbounded number of applications use the techniques of trajectory generation 27 aiming to speculate, plane and formalize the optimum path especially in robotics applica-28 tions which are the subject of this research. Important applications in robotics such as the 29 robots that use legs [1-4], navigation robots [5-12] and robot's manipulators [13-23] are in 30 dire need of using the techniques of optimal trajectory in order to optimize the usage of 31 these applications according to the estimates of peripheral conditions. 32

From a broader perspective, it is possible to count the problem of finding the optimal path 33 as one category of techniques that seek to find the optimal control which is interested in 34 determining the best control option to fulfill goals are required [24]. Closed circuit solu-35 tion and Open circuit solution are among the most famous solutions to the problem of 36 finding the optimal control [25]. However, although the closed circuit solution is more 37 accurate and comprehensive but it is not suitable when the studied system involves a high 38 number of degrees of freedom, a quadruped robot encompasses a 12 servo motors will 39 lead to 24 dimensions' state in control function and there is no way to deal with and solve 40the problem of optimal control using closed loop with such number of degrees of freedom. 41 As a result of this discussion we can consider the open loop solution, the control is a func-42 tion only of time, to the problem of finding the optimal control in case of robotics applica-43 tions is quite convenient [26]. 44

Historically, Brachchystochrone issue [27], a search process for the twist that gives the rapid decline when a pellet skid between two points different in height and uneven, 46 represents the starting point of human thinking in the search for the optimal path between 47



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two points. However, the scientist Johann Bernoulli presented in 1969 a revolutionary vi-48 sion based on the shape of the path to solve this problem using the calculus of variation 49 [28], the sort of math specialized to locate the maximum and minimum using the trivial 50 alteration of the functions. Subsequently, with the rise of the digital revolution, the com-51 puting devices opened a new era of making a trajectory generation theories applicable in 52 actual life problems. 53

Aviation and rocket applications were the first to use the strategies of open control 54 optimization aiming to increase the overall performance. The path of the missile is affected 55 by two factors, the first is the rocket thrust and the second is the combination of air forces 56 applied to the surface of the extruded body. Several conditions affect the second factor 57 such as launch angle, the shape of the outer surface and the rise-drop schedule. However, 58 each set of specific shape missile, required execution, and constraints system represents a 59 special case of trajectory optimization problem with specific specifications [29]. In regard 60 to jet aircraft, the trajectory optimization techniques were able to provide a special sched-61 ule used by pilots organize the speed according to the height of the aircraft [30]. Moreover, 62 Quadrotor helicopter control systems have benefited pointedly through the use of the 63 techniques of trajectory generation. An engineering team in Pennsylvania university per-64 formed a trajectory generated based model authorize the Quadcopter to maneuver over a 65 ring after giving it an initial push velocity [31]. Another stunning quadrotor application 66 based on path planning algorithms comprises two quadrocopters one of them sling a pen-67 dulum and the other pick it up [32]. In the industrial field, paramount applications use 68 optimal trajectory algorithms such as a supervision control of chemical processes in fac-69 tories [33]. When discussing robot applications, talk is incomplete without taking the tra-70 jectory planning methods into account. The rest of this research will focus on the use im-71 pact of the optimal path trajectory in the various robot fields. 72

## 2. Classification of Robotics System

There are many theories on how robots are categorized. However, the most literature 74classified the robots in terms of its applications and the type of locomotion and kinematics. 75 Both categories will be discussed about regarding robotic trajectory optimization. 76

#### 2.1. Classification According to the Applications of the Robot Systems

Unlimited number of tasks that can be executed by robots. However, robots can be 78 classified in terms of performing tasks 79

- 1. Industrial Robots: Including the robots that function in manufacturing environment 80 as the robots in [36-38]; 81
- Medical Robots: The group of robots operating in medical facilities such as surgical 2. 82 robots. Some of these robots exist in [42-43]; 83
- Service Robots: Robots used in researches are classified as service robots [8, 44]; 3.
- Space Robots: Any Robot functions beyond earth's atmosphere is space robots [45]; 4.

The following table summarizes important researches handling the trajectory opti-86 mization in robotic field and classifies these researches according to the type of the robot 87 applications.

System ap- plication	References	Remarks
Industrial Robots	Paes et al (2014)	<ul> <li>Attempt to improve the energy efficiency of the robot.</li> <li>ACCADO-toolkit used for trajectory optimization.</li> <li>The experiments carried on IRB1600 industrial ABB robot.</li> </ul>
	Abu-Dakka et al (2013)	Minimizing the execution time between two configurations.

Table 1. Classification according to the system application

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		<ul> <li>An INDIRECT approach used for the optimality.</li> </ul>
		• PUMA 560 is tested in this research.
		• Minimize the execute time and the jerk alongside the trajectory.
	Gasparetto et al (2010)	• Comparison between third-order and fifth-order B spline.
		• Gantry robot with spherical wrist is tested in this research.
		• A dynamic modeling is used to solve TO problem.
	Jin et al (2015)	<ul> <li>Hadamard's inequality used aiming to define the objective</li> </ul>
		• Staubli TX-90 used to test the optimization novel.
		Minimize execution time.
	Valente et al (2017)	• TO problem constructed by non-optimal control formulation.
		• The approach is valid for all kinematic-chain based robot.
		• Find the optimal trajectory for the MIRS robot
	Gupta et al (2015)	• The TO problem solved using several metaheuristics algorithms
	-	• Artificial Bee Colonization algorithm showed the best result.
		• DTM algorithm is used to align actual and optimum trajectory.
	Jiang et al (2017)	• PSO used to solve the optimization problematic.
Medical		MIRS robot utilized to estimate the algorithm performance
Robots		• A PUMA manipulator is to be used with custom regenerative drives
	Poya et al (2020)	• A standard robust passivity based control approach is used for
		optimal trajectories tracking
		• The optimization problem modeled to find point-to point
		trajectories maximizing energy
	$C_{2}$	<ul> <li>TO problem solved using Cross Entropy method</li> </ul>
	Celeste et al (2009)	• The robot state is defined using map-based localization.
	De Magistris et al (2017)	<ul> <li>The TO problem formulated as a Quadratic program.</li> </ul>
		Minimize the energy consumption.
Service Ro-		• Find the smooth trajectory under each foot.
bots		• TO performed on the robot combining legs and wheels.
	$M_{\rm ext} = 1.0000$	• The optimization problem formalized to be solved online using
	Marko et al (2020)	predictive control strategies
		• The method used is robust against unpredicted disturbances
		Minimize the fuel consumption.
Space Robots	Chu et al (2017)	• TO problem formulated as optimal control problem.
		Gauss Pseudospectral method used for transcription.

2.2. Classification According to the type of locomotion of the robot systems

Robot systems could be classified in terms of the movement manner as follow:

- Stationary Robots: This type of robot is installed in fixed place. Mainly, there is an arm can manipulate around the fixed part of the robot [13-23];
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- 2. Wheeled Robots: Characterized as being able to maneuver through wheels [5-12];
- **3.** Legged Robots: Includes four main varieties; bipeds, quadrupeds, hexapods and octopods [1-4];
- Swarm Robots: Independent robots with the same form and design, characterized by being able to perform functions in a harmonious and participatory manner [48-50];
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The following table summarizes important researches handling the trajectory optimization in robotic field and classifies these researches according to the type of the robot locomotion. 101

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System locomotion type	References	Remarks
	Menasri et al (2015)	<ul><li>TO problem formulated by bilevel optimization technique.</li><li>GA utilized aiming to solve the optimization problem.</li><li>The technique is valid for redundant manipulator.</li></ul>
	Gregory et al (2012)	<ul> <li>TO problem formulated as an optimal control problem.</li> <li>Finding the optimal energy consumption.</li> <li>Two Revolute planar manipulator used for the experiments.</li> <li>The TO problem modeled with Holonomic Constraints.</li> <li>Indirect method is used to achieve the discretization.</li> </ul>
	Böck et al (2016)	<ul> <li>Optimal control formula is used to model the TO problem.</li> <li>PSOPT simulator utilized to solve the OC problem.</li> <li>Fmincon is used to solve the static optimization problem.</li> <li>The algorithms presented are valid at Staubli TX60L.</li> </ul>
	Wu et al (2016)	<ul><li>Minimize the Jerk of the 10 DOF serial articulated robot.</li><li>TO problem formulated using alternative method.</li><li>GA applied to catch the global solution of the TO problem.</li></ul>
	Shareef et al (2014)	<ul><li>Finding the optimum trajectory energy consumption.</li><li>TO problem is solved as dynamic program.</li><li>The validity of the technique tested at DELTA robot.</li></ul>
Stationary Robots	Kucuk (2017)	<ul> <li>TO problem formulated by non-optimal based model.</li> <li>PSO utilized to find the minimum time smooth trajectory.</li> <li>Cubic spline scheme used for interpolation.</li> <li>PUMA robot used to test the techniques.</li> </ul>
	Števo et al (2014)	<ul> <li>Minimizing time, energy consumption and joints rotation.</li> <li>TO problem modeled by alternative formula.</li> <li>GA is used to find the optimal trajectory.</li> </ul>
	Elshabasy et al (2017)	<ul> <li>Minimize the power consumption.</li> <li>Planar redundant robot used to test the OT Techniques.</li> <li>Hybrid method consisting of GA and Fmincon is utilized.</li> </ul>
	Cao et al (2016).	<ul> <li>RRR redundant robotic arm stated for testing the algorithm.</li> <li>TO problem formulated by geometric techniques.</li> <li>The optimization proposed using hybrid technique.</li> <li>Crossover, Mutation PSO along with interior point method.</li> </ul>
	Zachary et al (2019)	<ul> <li>Trajectory optimization based on direct collocation method.</li> <li>Approximate invariant funnel method used along the trajectory in order to achieve the optimality.</li> </ul>
	Matteo et al (2019)	• Kinematic decoupling through via points used to generate the optimal trajectory of the redundant serial manipulator
	Poya et al (2020)	<ul> <li>A PUMA manipulator used with custom regenerative drives</li> <li>A standard robust passivity based control approach is used for optimal trajectories tracking</li> <li>The optimization problem modeled to find point-to point trajectories maximizing energy</li> </ul>
	Walambe et al (2016)	<ul> <li>TO problem solved by spline based approach.</li> <li>The prototype car modeled as nonholonomic system.</li> <li>Differential flatness used to define the steer control line.</li> </ul>
Wheeled Robots	Celeste et al (2009)	<ul> <li>Cross Entropy method utilized aiming to solve TO problem.</li> <li>Geometric map is used to define the mobile robot positions.</li> </ul>

Table 2. Classification according to the type of the robot locomotion

	Kalmár-Nagy (2016)	<ul> <li>path optimal solutions have done using SNOPT function</li> <li>Dynamic inversion based method presented for TG</li> </ul>
		• TO problem formulated using optimal control based scheme.
	Li et al (2015)	<ul> <li>Minimize the execution time.</li> </ul>
		• The optimization is solved using dynamic program.
		Improve Bspline of paths constructed by noisy control points
	Gilimyanov et al (2008)	• The improvement is reached by varying the control points.
	5	• The measurements error is found by solve standard QP
		TO problem modeled as non-control optimization problem
	Vale et al (2014)	• Maximize the clearance of the obstacles.
	× ,	• Suitable for dangerous environment.
		Multi-objective path planning.
		Minimize the path length
	Mac et al (2017)	Maximize the smoothness.
		• Dijkstra's algorithm used to find a collision-free trajectory.
		• PSO is utilized to obtain the optimized trajectory.
		• Ant colony optimization algorithm used in this research.
		• Turning point optimization algorithm proposed to solve the
	Hui et al (2019)	trajectory optimization.
		• For easier tracking control of the mobile robot B-spline path
		smoother is presented.
		• 8 DOF humanoid robot.
	Sarkar et al (2015)	• Optimize the gait of the robot.
		• TO problem formulated geometrically.
		• GA is used to find the optimality.
		• Lagrange-Euler method used to model the dynamic system.
	Sun et al (2015)	<ul> <li>Optimize the trajectory of bipedal walking robot.</li> </ul>
		<ul> <li>TO problem modeled as Optimal Control problem.</li> </ul>
		Exact penalty function algorithm used to solve OC problem.
	Aoustin et al (2013)	<ul> <li>Planar biped robot with four-bar knees.</li> </ul>
		• A sthenic criteria is used to solve the TO problem.
	Koch et al (2012)	• Humanoid robot HRP2 with 36 DOF and 30 actuators.
		• TO modeled as an optimal control.
		Minimize torque squared.
		Maximize forward velocities.
Legged Robots		The OC problem solved by framework MUSCODII
Leggen novels		<ul> <li>Generate optimized walking trajectories for Bipedal robot</li> </ul>
	Chen et al (2016)	• The cost function defined as a sthenic criterion.
		• SQP is used to solve the optimization problem.
	De Magistris et al (2017).	Minimize the energy consumption.
		• Quadratic program used to formulate the TO problem.
	Lim et al (2014)	<ul> <li>Efficient trajectory stair walking for humanoid robot.</li> </ul>
		Genetic algorithm is used to generate OT.
		Trajectory Optimization of Biped Robot performed online
		• Wrench Polytope (AWP) and the Feasible Wrench Polytope
	Romeo et al (2018)	(FWP) introduced to study the stability and the actuation
		consistency of a given motion planning.
		• Feasibility factor adapted based on AWP and FWP to improve
		the the process of trajectory optimization
	In-Seok et al (2019)	Humanoid used in this research

		• The control input generated by capture point method
		• Sliding mode controller used to follow the zero moment point
	$W_{allows a b al}(2020)$	Dynamic obstacle avoidance considered
	wolfgang et al (2020)	Harmonic potential field used for collision avoidance
	Amoin $a = 1 (2010)$	Biped robot is used in this research
	Amir et al (2019)	<ul> <li>Orthogonal Collocation used for trajectory optimization</li> </ul>
		Clutter environment is considered.
		<ul> <li>Optimize the trajectory of multi-robot</li> </ul>
	Das et al (2016)	• An improved PSO is used.
		GSA algorithm used to improve exploration performance
		Minimize the energy consumption.
	Asma et al (2017)	Multiple robot trajectory generation system.
Swarm Robots		• Dynamic Distributed PSO used to find the optimality.
	Das et al (2016)	Multi robot path planning
		• An improved gravitational search is proposed for TO
	Oleiwi et al (2015).	Multi robots and multi objective path planning
		• Trajectory for each robot generated in independent manner
		• Modified GA with A* algorithm used for optimal results.

The trajectory optimization studies varied in terms of the nature of the analysis into 104 experimental research, simulation research and combination of the experimental and simulation research. In the following table, the classification according to the type of research 106 illustrated as follow 107

Table 3. Classification according to type of Research

type of Research	References	Remarks
	Gadaleta et al (2017)	• Optimize the Energy consumption of the industrial robots.
		Propose simulation interface in Delmia robotic environment
		• Automatic compute the motion parameters of the trajectory.
		Minimize the time and the jerk along the robot trajectory
	Gasparetto et al (2008)	<ul> <li>Cubic spline used for trajectory generation</li> </ul>
		• Proposed algorithm tested in iterative based simulator.
		Robot based inspection system.
		<ul> <li>Minimize the path along the trajectory.</li> </ul>
	Ulrich et al (2016)	• TO problem converted to salesman problem.
		Christof ides heuristic used to solve salesman problem.
		• 3DCreate simulator software used to test the algorithm.
	Hossain et al (2015)	• Path planning for a mobile robot.
		Dynamic unknown environment.
		<ul> <li>Minimize the path between two configurations.</li> </ul>
Simulation		Non-optimal control formulation.
		<ul> <li>Bacterial foraging optimization stated for optimality.</li> </ul>
	Pellegrinelli et al (2015)	Minimize the energy consumption.
		<ul> <li>Pick-and-place assembly robot operation is investigated.</li> </ul>
		• The results presented using simulation software.
		Optimize the Trajectory of the arm manipulator.
	Baghli et al (2017)	<ul> <li>TO problem solved using Ant Colony algorithm.</li> </ul>
	Ç ( , ,	• The stated technique tested using Matlab environment.
	Chehote et al. (2014)	Optimize varied parameters along the trajectory.
	Snenata et al (2014)	Ensure safe navigation around the obstacles.

<ul> <li>Non Dominated Sorting CA used for the optimality.</li> <li>Mattab is used to demonstrate the validity.</li> <li>An INDIRECT approach</li> <li>Abu-Dakka et al (2013)</li> <li>PUMA 560</li> <li>C+u used to determine numerical results</li> <li>6 DOF open chain manipulator.</li> <li>SQP algorithm used to find the optimality.</li> <li>Minimize the energy and the jerk.</li> <li>Fifth-order-Bspline used to generate the trajectory.</li> <li>Mattab environment used to inplement the simulation.</li> <li>Trajectory Optimization of two Legged Robots performed online</li> <li>Feasibility factor adapted based on Wrench Polytope (AWP) and the Feasible Wrench Polytope (FWP) to improve the the process of trajectory optimization</li> <li>Quadrupedal robots with actuated wheels considered</li> <li>Solve components of the base and the feet trajectories based on a linear formulation of the zero moment point balance criterion</li> <li>The optimizer based on quadratic programming</li> <li>Biped robot is used in this research</li> <li>Orthogenal Collocation used for trajectory optimization</li> <li>Legged robot considered</li> <li>The approach susceptible to uncertainty</li> <li>Lis suitable for utilizing robots on rough terrain and the corresponding risk-sensitive objectives bandled for contact-implicit trajectory optimization.</li> <li>Experimental</li> <li>Paes et al (2014)</li> <li>RB1600 industrial ABB robot.</li> <li>ACCADO-tookkit used for trajectory optimization.</li> <li>Experimental</li> <li>Spline based approach to solve TO problem.</li> <li>Simulation carried out on SNOT thooks.</li> <li>Spline based approach to solve TO problem.</li> <li>Simulation for finding OT toobsen.</li> <li>Spline based approach to solve the proposed algorithm.</li> <li>Laboratory experiments used to un SNOT toolse.</li> <li>Foropsed algorithm carried out using Laprange-Buler method Simulation for finding OT t</li></ul>			
Mallab is used to demonstrate the validity.     An INDIRECT approach     Abu-Dakka et al (2013)     PUMA 560     C++ used to determine numerical results     Gasparetto et al (2007)     Minimize the energy and the jerk.     Gasparetto et al (2007)     Minimize the energy and the jerk.     Fifth-order-Bapline used to generate the trajectory.     Mallab environment used to implement the simulation.     Trajectory optimization to two Legged Robots performed online     Feasibility factor adapted based on Wrench Polytope (AWP) and     the Feasibility factor adapted based on Wrench Polytope (AWP) and     the Feasibility factor adapted based on Wrench Polytope (AWP) and     the Feasibility factor adapted based on Wrench Polytope (AWP) and     the Feasibility factor adapted based on Wrench Polytope (AWP) and     the feasible Wrench Polytope (FWP) to improve the the process of     trajectory optimization of the zero moment point balance criterion     The optimizer based on quadratic programming     Amir et al (2019)     Eliped robot is used in this research     Amir et al (2019)     Eliped robot is used in this research     Amir et al (2012)     The approach susceptible to uncertainty     Luke et al (2014)     Elegged robot considered     The suproach susceptible to uncertainty     The uncertainty modeled from the terrain and the corresponding     risk-sensitive objectives handled for contact-implicit trajectory     optimization.     Experiments placed to find the energy consumptions.     Minimizing the execution time and the jerk     Cantry robot with spherical wrist.     To algorithm implemented using simulation software.     Results tested experimentally using laboratory robot.     Softre Compared adject work for proposed algorithm.     Laboratory experiments used Staubil TX-90 robot.     Softre Compared adject dusing Dynamic inversion based method.     MATLAB simulation used to run SNOPT toolbox.     Four wheeled Ommidirectoral verifice resperiments     Froposed adjorithm carried out using Tadjece Publics     So			<ul> <li>Non Dominated Sorting GA used for the optimality.</li> </ul>
Abu-Dakka et al (2013)       • A INDIRECT approach         Abu-Dakka et al (2013)       • PUMA 560         Casparetto et al (2007)       • G DOF open chain manipulator.         SQP algorithm used to find the optimality.       • Minimize the energy and the jerk.         Fifth-order-Bspline used to generate the trajectory.       • Matlab environment used to implement the simulation.         Romeo et al (2018)       • Trajectory Optimization of two Legged Robots performed online         Yvain et al (2019)       • Quadrupedal robots with actuated wheels considered         Solve components of the base and the feet trajectories based on a linear formulation of two Legged Robots performed online         Amir et al (2019)       • Biped robot is used in this research         I Luke et al (2021)       • Biped robot considered         I Luke et al (2021)       • Dorhogonal Collocation used for trajectory optimization         Luke et al (2019)       • RB1600 industrial ABB robot.         Gasparetto et al (2010)       • RB1600 industrial ABB robot.         Gasparetto et al (2010)       • Spring robot with spherical wrist.         Gasparetto et al (2016)       • Spring robot with spherical wrist.         Gasparetto et al (2016)       • RB1600 industrial ABB robot.         J in et al (2015)       • Spring robot with spherical wrist.         Gasparetto et al (2016)       • Spring robot with spherical wrist.			Matlab is used to demonstrate the validity.
Abu-Dakka et al (2013) <ul> <li>PUMA 560</li> <li>C++ used to determine numerical results</li> <li>6 DOF open chain manipulator.</li> <li>SQP algorithm used to find the optimality.</li> <li>Minimize the energy and the jerk.</li> <li>Fifth-order-Bspline used to generate the trajectory.</li> <li>Matuba environment used to implement the simulation.</li> <li>Trajectory Optimization of two Legged Robots performed online</li> <li>Feasibility factor adapted based on Wrench Polytope (AWP) and the Feasible Wrench Polytope (AWP) and the Feasible Wrench Polytope (FWP) to improve the the process of trajectory optimization</li> <li>Quadrupedal robots with actuated wheels considered</li> <li>Solve components of the base and the feet trajectories based on a linear formulation of the zero moment point balance criterion</li> <li>The optimizer based on quadratic programming</li> <li>Biped robot is used in this research</li> <li>Orthogonal Collocation used for trajectory optimization</li> <li>Legged robot considered</li> <li>The approach susceptible to uncertainty</li> <li>It is suitable for utilizing robots on rough terrain</li> <li>The approach susceptible to uncertainty modeled from the terrain and the corresponding risk-sensitive objectives handled for contact-implicit trajectory optimization.</li> <li>Experiments place to find the energy consumptions.</li> <li>Minimizing the execution time and the jerk</li> <li>Castry robot with spherical wrist.</li> <li>Castry robot with spherical wrist.</li> <li>Proposed algorithm implemented using simulation software.</li> <li>Results tested experimental using laboratory robot.</li> <li>Dynamic model used to solve TO problem.</li> <li>Simulation carried out to verify the proposed algorithm.</li> <li>Laboratory experiments us</li></ul>		Abu-Dakka et al (2013)	An INDIRECT approach
e         C+ used to determine numerical results           6 DOF open chain manipulator.         5QP algorithm used to find the optimality.           SQP algorithm used to find the optimality.         Minimize the energy and the jerk.           Fifth-order-Bspline used to generate the trajectory.         Matlab environment used to implement the simulation.           Romeo et al (2018)         Trajectory Optimization of two Legged Robots performed online           Yvain et al (2019)         Quadrupedal robots with actuated wheels considered           Solve components of the base and the feet trajectories based on a linear formulation of the zero moment point balance criterion           The optimizer based on quadratic programming           Biped robot is used in this research           Orthogonal Collocation used for trajectory optimization           Luke et al (2021)         Legged robot considered           The approach susceptible to uncertainty           Luke et al (2014)         It is suitable for utilizing robots on rough terrain           The uncertainty modeled from the terrain and the corresponding risk-sensitive objectives handled for trajectory optimization.           Experimental         Casparetto et al (2010)           Gasparetto et al (2010)         It RB 1600 industrial ABB robot.           Gasparetto et al (2011)         Norminity specical wrist.           Gasparetto et al (2016)         Spline based approach to solve TO problem. <td rowspan="2"></td> <td>• PUMA 560</td>			• PUMA 560
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• The uncertainty modeled from the terrain and the corresponding risk-sensitive objectives handled for contact-implicit trajectory optimization.         • IRB1600 industrial ABB robot.         • ACCADO-toolkit used for trajectory optimization.         • Experiments placed to find the energy consumptions.         • Minimizing the execution time and the jerk         • Gasparetto et al (2010)         • Gasparetto et al (2010)         • Jin et al (2015)         • Jin et al (2015)         • Simulation carried out to verify the proposed algorithm.         • Laboratory experiments used Staubil TX-90 robot.         • Spline based approach to solve TO problem.         • Suit2Drft radio controlled vehicle used for experiments.         • Proposed algorithm carried out using matlab simulation.         • SNOPT function used to solve the TO problem.         • TG solved using Dynamic inversion based method.         • Four wheeled Omnidirectional vehicles used in experiments         • Four wheeled Omnidirectional vehicles used in experiments         • Optimize the gait of the biped robot.         • Ga is used to find the TO.         • Dynamic system modeled using Lagrange-Euler method         • Simulation for finding OT for biped is done in Matlab.         • Purposed algorithm terried out experimentally using 8 DOF biped         • The Used Biped developed by bioloid premium kit. <td></td> <td>Luke et al (2021)</td> <td>It is suitable for utilizing robots on rough terrain</td>		Luke et al (2021)	It is suitable for utilizing robots on rough terrain
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<ul> <li>Results carried out experimentally using 8 DOF biped</li> <li>The Used Biped developed by bioloid premium kit.</li> </ul>		Sarkar et al (2015)	• Simulation for finding OT for biped is done in Matlab.
The Used Biped developed by bioloid premium kit.			• Results carried out experimentally using 8 DOF biped
			• The Used Biped developed by bioloid premium kit.

	Minimize the energy consumption.
DeMagistris et al (2017)	<ul> <li>Walking gait is to be optimized</li> </ul>
	<ul> <li>HRP-4 humanoid used to validate the proposed algorithm.</li> </ul>
	• FEM simulation to model the contact e ground-sole contact.
	Legged robot considered.
	• The optimization solver has the ability to freely evolution between
Jan et al (2019)	open, closed, and sliding contact states along the path.
	• The trajectory optimization method can detect stepping motions
	without predefined contact schedule
	• Ant colony optimization algorithm used in this research
	• Turning point optimization algorithm proposed to solve the
Hui et al (2019)	trajectory optimization
•	• For easier tracking control of the mobile robot B-spline path
	smoother is presented.
• • Poya et al (2020)	• A PUMA manipulator used with custom regenerative drives.
	• A standard robust passivity based control approach is used for
	optimal trajectories tracking.
	• The optimization problem modeled to find point-to point
	trajectories maximizing energy.
	Wheeled-legged robot considered.
	• TO performed on the robot combining legs and wheels.
Marko et al (2020)	• The optimization problem formalized to be solved online using
	predictive control strategies.
	• The method used is robust against unpredicted disturbances.

# 3. Trajectory Optimization System Description

Generally speaking, the problem of finding the optimal trajectory defined as the 110 search process for the quixotic path of the system that expressed in terms of continuous 111 parameters using collocation of arithmetic models [24]. Trajectory optimization problem 112 divided in terms of the mathematic formulated type into optimal control model as [45] 113 and alternative formulations mainly solved using heuristic algorithms. However, Trajec-114 tory planning problem overwhelmingly treated as an open control problem interested in 115 detecting the superior options for control function in which the control subject only to the 116 time. 117

#### 3.1. Problem Statement

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The general form of the objective statement of the path planning program stated in 119 the subsequent equation 120

$$f(u, s(u), c(u)) = f_1(u_I, u_F, s(u_I), s(u_F)) + \int_{u_I}^{u_F} f_2(\tau, s(\tau), c(\tau)) d\tau$$
(1)

The objective statement characterizes the eligible path, it possesses two main portions, the 121 first function statement part  $f_1$  known as boundary statement describes the quality of the 122 trajectory for the particular beginning and termination point. The second term illustrates 123 a quantum varying over the path, mainly the integration of the engine torque. In the Equa-124 tion (1), the decision variables  $u_I$  and  $u_F$  represent the time at the beginning and the end 125 of the trajectory, while the functions s(u) and c(u) illustrates the state of the dynamic 126 system and the control as a function of time respectively. We can observe that the purpose 127 statement has two continuous time functions as decision variables. Subsequently, the min-128

imization process happens over the space of functions making the optimization of the tra-129 jectory generation a difficult task. In control theory, the statement shown in Equation (1) 130 referred as a cost function. 131

# 3.2. Constraints system

The statement of purpose function described in equation (1) has a system of constraints consists of 7 inequalities and 1 equality called state equation in control theory references. 134 However, the various applications that employ the trajectory optimization algorithms use 135 all or part of them. The constraints are described in the following table 136

Table 4: The system of constraints that the main objective statement

Constraints	Description
$J(u_{I}, u_{F}, s(u), c(u)) \le 0$	Nonlinear inequality constraint on boundary
$u_L \le u_I < u_F \le u_H$	Start and end time inequality boundary
$s(u_I)_{\min} \leq s(u_I) \leq s(u_I)_{\max}$	Premier configuration inequality constraint
$s(u_F)_{\min} \leq s(u_F) \leq s(u_F)_{\max}$	Ultimate configuration inequality constraint
$\dot{s}(u) = p(u, s(u), c(u))$	Continual dynamic equality constraint
$k(u, s(u), c(u)) \leq 0$	Constraints placed along the path
$s(u)_{\min} \le s(u) \le s(u)_{\max}$	Continual constraint on the configuration
$c(u)_{\min} \le c(u) \le c(u)_{\max}$	Continual constraint on the control

#### 3.3. Transcription Techniques

Providing a solution to the trajectory generation problem involves special operations 139 called transcription processes ensure changing the continuous shape functions of the 140problem statement into another form of a finite set of variables. Whatever these processes 141 are, they fall into a major picture includes two central classes called direct and indirect 142 methods. 143

# 3.3.1. Indirect methods

consider the following objective function

$$f(u_{F},s(u_{F})) \tag{2}$$

Given that, the condition at the initial time  $s(u_1)$  is known. We need to find the control 146 c(u) that makes the objective function optimal, such that the control function subject to 147 the dynamic constraint 148

$$\dot{s}(u) = p(u,s(u), c(u)) \tag{3}$$

And boundary conditions

$$J(u_F, s(u_F), c(u_F)) = 0$$
 (4)

We can observe clearly that the control function and the state function must satisfy the 151 dynamic constraint over the time interval  $u_{I} \leq u \leq u_{F}$ . That what makes the control opti-152 mization problem different than the conventional optimization. Now, we can consider the 153 explanation of the Lagrange multiplier to formulate the following preference 154

$$\hat{f} = [f + \delta_1^T J]_{u = u_F} + \int_{u_I}^{u_F} \delta_2^T(u) \{ \dot{s}(u) - p(u, s(u), c(u)) \} .du$$
(5)

In equation (5),  $\delta_1$  is the Lagrange variable for the boundary constraint, while  $\delta_2(u)$  is 155 the Lagrange multiplier for dynamic constraint, commonly mentioned as costa variable. 156 To define the optimality, the following conditions must be satisfied 157

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$$\dot{\delta_2} = -H_S^T \tag{6}$$

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$$-H_{\rm C}^{\rm T}=0$$
(7)

The conditions in the set of equations (6) and (7) are derived based on the definition of 159 Hamiltonian [34] and mostly mentioned as costa equations and control equations respec-160 tively. In previous equations,  $H_S$  and  $H_C$  represent the partial derivatives in terms of 161 Hamiltonian. 162

$$H = \delta_2^T p(u,s(u), c(u))$$
(8)

Another set of conditions should be fulfilled called transversely conditions as follows

$$\delta_2(\mathbf{u}_{\mathrm{F}}) = \left[f + \delta_1^{\mathrm{T}} \mathbf{J}\right]_{\mathrm{S}_{u=u_{\mathrm{F}}}}^{\mathrm{T}}$$
(9)

$$0 = \left[ \left[ f + \delta_1^{\mathrm{T}} J \right]_{\mathrm{u}} + H \right]_{\mathrm{u} = \mathrm{u}_{\mathrm{F}}}$$
(10)

$$\delta_2(\mathbf{u}_{\mathrm{I}}) = 0 \tag{11}$$

Essentially, the concept of the indirect methods based on generate and derive the required 164 and adequate conditions for optimal trajectory problem, the costa and the control equation 165 in Equation (6) and (7) in addition to the transversely conditions (9), (10) and (11). Poste-166 riorly, apply one of the numerical discretization algorithms on the previously mentioned 167 necessary conditions along with the state equations, boundary condition and other system 168 constraints such as path constraints is required. Lastly, the optimality may be found using 169 some of nonlinear optimization algorithms. 170

#### 3.3.2. Direct methods

The concept of direct methods established on applying some numerical analysis in 172 order to discretize the cost function in addition to the accompanying constraints directly 173 with no need to find any of costa, control or transversely conditions. Subsequently, we 174 can perform nonlinear program algorithms to find the required optimality. For more de-175 tailed information, reviewing [24] is highly recommended.

# 3.3.3. Direct methods against Indirect methods

The indirect methods invented formerly by the Russian scientist Lev Pontryagin in 178 1956 [35] and has made a quantum leap in the applications of space manufacturing due to 179 its high precision. However, despite its super accurate results, its implementation showed 180 great difficulties. Here are some awkwardness, the process of finding the partial deriva-181 tives of Hamiltonian  $H_S$  and  $H_C$  stands a clear challenge and a difficult task. Moreover, 182 indirect method is showing a great inflexibility, deriving all the equations and conditions 183 are required each time we need pose the problem. There are other difficulties for this 184 method mentioned in literatures. An indirect method is used mainly in space science. In 185 contrast, direct methods are relatively modern and has a high degree of flexibility and 186 strength. However, direct methods are less precise than its predecessor and not suitable 187 for applications that required very high degrees of sensitivity such as launching a space-188 craft into space. In the field of robot's application, direct methods are very suitable and 189 sufficient. 190

#### 4. Solution Techniques

Whether the transcription method is direct or indirect, the discretization is done us-192 ing one of these two algorithms; shooting methods and collocation methods. Shooting 193 methods depend on simulation generally Explicit Runge-Kutta integration scheme such 194

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as Heun's method. Collocation methods depends on function approximation generally 195 implicit Runge-Kutta integration scheme such as trapezoidal methods. Direct methods 196 sometimes handled as a dynamic program. Shooting methods sectioned into two main 197 categories; single shooting simulates the entire trajectory and multiple shooting which 198 segments the trajectory into several shorter paths. The collocation methods could be var-199 ied in terms of using the spline interpolation schemes that represent the trajectory. The 200 global collocation use High-order splines to represent the entire trajectory different from 201 local colocation that could use spline of a different order to represent each segment of the 202 trajectory. Moreover, Dynamic programming could be used to solve the OC problem, it 203 does not clearly distinct the transcription and the optimization. As an alternative, it does 204 a system of iterative onward and backward passes along the trajectory. The following fig-205 ure represent the Framework of the procedures stated for solve the trajectory optimization 206 problems 207



Figure 1. Schematic of the Trajectory Optimization System







Figure 3. Schematic of the direct method procedure

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Figure 4. Schematic of the Dynamic programming

The following table describes the trajectory optimization procedure regarding several recent trajectory optimization researches in the field of the robot application 217

Table5: Description of the trajectory optimization procedure regarding several recent researches

References	Trajectory optimization procedure
	Minimize the cycle time and energy consumption.
$C_{10}$ (2019)	<ul> <li>A novel methodology proposed to solve the TO problem.</li> </ul>
Giorieux et al (2016)	<ul> <li>TO problem formulated using alternative techniques.</li> </ul>
	• AMPL software used to optimize the problem after transcription procedure.
	<ul> <li>TO problem formulated as an optimal control problem.</li> </ul>
Liu et al (2017)	<ul> <li>Slack convex feasible set method (SCFS) proposed to handle TO problem.</li> </ul>
	Minimize computation time and convergence faster.
$M_{0.01} = 1.(2015)$	<ul> <li>A new biogeography PSO algorithm proposed</li> </ul>
WIO et al (2015)	<ul> <li>Finding the optimality of the paths by AVBN.</li> </ul>
	<ul> <li>Solve the TO problem using Dynamic program.</li> </ul>
Kala et al (2012)	• Dynamic obstacles considered.
	<ul> <li>Additional processing called acceleration nodes to modify the result.</li> </ul>
	<ul> <li>Minimize the deburring process for industrial robot.</li> </ul>
Abola at al $(2016)$	<ul> <li>Proposed technique based on A* to find the optimal path.</li> </ul>
Abele et al (2016)	<ul> <li>TO problem transferred to salesman problem.</li> </ul>
	<ul> <li>OpenRAVE matlab interface used to solve salesman problem.</li> </ul>
	• EEG controlled robotic arm considered.
Roy et al (2016)	<ul> <li>GA proposed to solve TO of the EEG arm.</li> </ul>
	<ul> <li>The simulated arm simulated using GA in Matlab platform.</li> </ul>
	<ul> <li>Trajectory generating technique for free-floating type planetary robot.</li> </ul>
Wang et al (2015)	<ul> <li>7 (DOF) redundant manipulator considered.</li> </ul>
	PSO is stated to solve the TO problem.
Propertial $(2014)$	Direct method for optimal control
1 des et al (2014)	<ul> <li>Sequential quadratic program algorithm used for the optimality</li> </ul>
	<ul> <li>SSGA stated to optimize the path between 2 neighboring positions</li> </ul>
Abu-Dakka et al (2013)	<ul> <li>PGA to optimize a sequence of neighboring positions</li> </ul>
	Clamped cubic spline aims to subject the various constraints
	<ul> <li>An inverse kinematics process to find sequence of joint sequence.</li> </ul>
Gasparetto et al (2010)	<ul> <li>SQP algorithm works for the optimality</li> </ul>
	Cubic spline algorithm and b-spline are selected to construct the path
	<ul> <li>The TO have done by minimizing the observation matrix</li> </ul>
$J_{in ot al} (2015)$	<ul> <li>Hadamard's relation to represent the trajectory</li> </ul>
Jiii et al (2013)	<ul> <li>The trajectory represented as a finite sum of sinusoidal function</li> </ul>
	<ul> <li>Dynamic constraints handled using weighted least square algorithm.</li> </ul>
Valente et al (2017)	<ul> <li>Starting and ending points of the trajectory are known.</li> </ul>

	<ul> <li>Generate several paths and then chose the best</li> <li>The optimality based on Sine-jerk motion and kinematic limits</li> </ul>
	Initial guess is given using a via points algorithm
Gleeson et al (2016)	<ul> <li>Robot studio ABB RABID to find the optimal time for the trajectory</li> </ul>
	Novel trajectory generation presented
	Discretize the optimization problem
Gasparetto et al (2007)	• SQP optimization process performed for the optimal results
	<ul> <li>Interpolation process have done using B-spline algorithm</li> </ul>
$C_{max}$ of al $(2012)$	<ul> <li>TO formulated as conventional unconstrained problem.</li> </ul>
Gregory et al (2012)	<ul> <li>Euler–Lagrange algorithm used to solve the optimization problem.</li> </ul>
Pick at al (2016)	<ul> <li>Discretize the path into segments describe the position and orientation</li> </ul>
bock et al (2016)	<ul> <li>The optimization at each segments using matlab function (fmincon).</li> </ul>
$W_{11} = 1 (2016)$	<ul> <li>Direct method for optimal control problem</li> </ul>
wu et al (2016)	Genetic algorithm is used.
Sharoof at al $(2014)$	<ul> <li>Nonlinear variation used to formulate a TO problem.</li> </ul>
Shareer et al (2014)	<ul> <li>Convex programing used for optimization.</li> </ul>
Hassan at $a1(2017)$	<ul> <li>Dynamic side of the TO problem handled using calculus of variation</li> </ul>
Tiassail et al (2017)	<ul> <li>GA performed to obtain the optimal path between two configurations</li> </ul>
Kucuk (2017)	<ul> <li>Particle swarm optimization to allocate the optimum result</li> </ul>
Kucuk (2017)	Cubic spline interpolation used
	<ul> <li>Minimize power consumption by the joint actuators.</li> </ul>
Števo et al (2014)	Parabolic trajectory discretizing
	Two evolutionary algorithm based are applied
Flshabasy et al (2017)	<ul> <li>The dynamic parameters are discretized</li> </ul>
Lisitabasy et al (2017)	Genetic algorithm applied on two different objective functions.
	<ul> <li>The dynamic configuration is handling using Lagrange multiplier.</li> </ul>
Cao et al (2016)	<ul> <li>PSO performed, the result applied as an initial guess for (IPM)</li> </ul>
	IPM is the interior point method
Hank (2016)	Hybrid method presented
	<ul> <li>Reactive navigation procedure and near optimal time TO simulation.</li> </ul>
Haddad et al (2007)	Handle the problem of existing the dynamic configuration
	Simulated annealing algorithm is used to find the optimal result
Walambe et al (2016)	• B spline approximation algorithm is used for segment the path
	Flatness of derivation is used to obtain the optimal results
Celeste et al (2009)	• Calculus of variation used to handle the dynamic property.
	Cross Entropy algorithm utilized to recognize the optimal solution
Pandev et al (2017)	• A fuzzy logic controller used for trajectory planning.
	Evolutionary based wind driven applied as feedback on the controller
Kalmár-Nagy (2016).	Dynamic of inversion used to handle the dynamic properties
	Path optimal solutions has done using SNOPT function
Li et al (2015)	• IPM-based algorithm to solve the problem of optimal control.
	Hamiltonian conditions is used
Gilimvanov et al (2008)	<ul> <li>Improvement on B-spline approximation to suit the path curvature</li> </ul>
	<ul> <li>The discretized problem uses SQP algorithm to find the optimality.</li> </ul>
	• The optimization problem formalized to be solved online using predictive control
Marko et al (2020)	strategies.
	<ul> <li>The method used is robust against unpredicted disturbances.</li> </ul>
	• A standard robust passivity based control approach is used for optimal trajectories
Pova et al (2020)	tracking.
- 5, a ct ar (2020)	<ul> <li>The optimization problem modeled to find point-to point trajectories maximizing</li> </ul>
	energy.
Hui et al (2019)	Ant colony optimization algorithm used for the optimality

	<ul> <li>Turning point optimization algorithm proposed to solve the trajectory optimization</li> </ul>
	• For easier tracking control of the mobile robot B-spline path smoother is presented.
	• The optimization solver has the ability to freely evolution between open, closed, and
In $at al (2019)$	sliding contact states along the path.
Jan et al (2019)	• The trajectory optimization method can detect stepping motions without predefined
	contact schedule
	<ul> <li>The approach susceptible to uncertainty</li> </ul>
Luke et al (2021)	<ul> <li>The uncertainty modeled from the terrain and the corresponding risk-sensitive</li> </ul>
	objectives handled for contact-implicit trajectory optimization.
Amir et al (2019)	<ul> <li>Orthogonal Collocation used for trajectory optimization</li> </ul>
	• Solve components of the base and the feet trajectories based on a linear formulation of
Yvain et al (2019)	the zero moment point balance criterion
	<ul> <li>The optimizer based on quadratic programming</li> </ul>
$\mathbf{P}_{\text{opp}(\alpha)}$ of al (2018)	• Feasibility factor adapted based on Wrench Polytope (AWP) and the Feasible Wrench
Komeo et al (2018)	Polytope (FWP) to improve the the process of trajectory optimization
Walfgang at al (2020)	<ul> <li>Harmonic potential field used for collision avoidance</li> </ul>
woligang et al (2020)	Dynamic obstacle avoidance considered
In Sock at al $(2010)$	<ul> <li>The control input generated by capture point method</li> </ul>
III-Seok et al (2019)	<ul> <li>Sliding mode controller used to follow the zero moment point</li> </ul>
$\mathbf{P}_{i}$	<ul> <li>Guaranteed Sequential Trajectory Optimization algorithm is proposed to solve</li> </ul>
Riccardo et al (2019)	trajectory optimization problems for control-affine systems with drift.
	• Differential geometric approach for optimizing trajectories proposed on a Riemannian
Michael et al (2020)	manifold with obstacles. Hence, The optimization problem based on a metric and
	collision function
	• The trajectory optimization done by direct transcription with ellipsoidal disturbances
Zachary et al (2018)	and linear feedback algorithm using approximate invariant funnels along the trajectory

## 5. Design Objectives of the Trajectory Optimization in robotic applications

Finding the best solutions that optimize the design objectives considered as the main 220 purpose of the optimization processes. However, regarding the trajectory optimization in 221 robotic application, one can observe three main design objectives. Some researches con-222 cerning single objective, but the most studies consider multi-objective design. In what fol-223 lows, there is a description of the most used design objectives of the trajectory optimiza-224 tion in robotic field. Minimizing the time that required to execute some task or to displace 225 the hand frame of the robot from one place to the other considered one of the most used 226 design objectives of the trajectory optimization in robotic application are: 227

- Minimize the energy consumed during the displacements: among the most used design objectives of the trajectory optimization in robotic application;
- Minimize the total execution time: Minimizing the time that required to execute some task or to displace the hand frame of the robot from one place to the other considered one of the most used design objectives of the trajectory optimization in robotic application;
- Minimize the Jerk of the robot actuator: In robotic field, the jerk of the robot thought
   as one of the most problem facing some kind of the problem. Subsequently, minimiz ing the jerk is very important to retain the stability of the robot;
   236
- 4. Minimize the Path length: One of the most important design objective in the field of the mobile robots; 238
- Minimizing the Joint rotation: in some cases, especially in manipulator it is important 239 to minimize the rotation of the joint in order to make the path less power consuming; 240
- 6. Minimize the Fuel Consumption: crucial characteristics in case of aerospace robots; 241

7. Maximize the Clearance of the obstacles: when the trajectory planning system con-242 sider the holonomic constraints, make the obstacles more observable to the vision sys-243 tem is very important; 244

Table	6.	Typical	Design	Objectives
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Objective	Units	References	Remarks
		Paes et al	Dynamic model used to find energy-optimal trajectory so that the energy consumption measured for pick-place motion.
		Elshabasy et al	
	Joule	De Magistris et al	Minimize the nerver concurrentian of materiainte
TYPE-1		Gadaleta et al	Minimize the power consumption of motor joints
		Pellegrinelli et al	
	NINA	Gleeson et al	Torques utilized as an estimator of the consumed nerver
	INIVI	Koch et al	Torques utilized as an estimator of the consumed power
		Abu-Dakka et al	An indirect technique utilized to find the optimality.
		Gasparetto et al	Presetting of the execution time is required.
		Jin et al	Hadamard's inequality is used to define the objective function
		Valente et al	The minimization by a novel multivariable optimization approach
TYPE-2	Sec	Böck et al	The execution time minimized along the trajectory
111 2 2		Shareef et al	Optimal time trajectory whereas satisfying all the torques constraints
		Kucuk	Minimize the time-optimal trajectory for serial and parallel robot.
			De Magistris et al
		Li et al	
		Gasparetto et al	Low-jerk trajectories could be performed more swiftly and precisely.
TYPE-3	mm/s <sup>3</sup>	Wu et al	The jark defined as reat mean square
		Gasparetto et al	The jerk defined as foot mean square
		Mac et al	PSO is utilized for minimizing the path length.
TYPE-4	mm	Ulrich et al	Minimize the path length for the based inspection system.
		Hossain et al	Bacterial foraging optimization stated for optimality.
TYPE-5	Deg	Števo et al	Suitable to reduce the undesirable manipulation.
TYPE-6	KG	Chu et al	The tenacity of the OC method is to lead the lander with the lowest quantity of fuel consumed.
TYPE-7	units	Vale et al	Minimize the space of the mobile robot to the adjacent obstacles.

#### 6. Design Variables of the Trajectory Optimization in robotic applications

Design variables is the numerical input that is permissible to change during the op-247 timization procedure. In what follow a description of the most used design variables of 248 the trajectory optimization in robotic field. 249

- Joint Motor torque: rotation joints could be limited so that the solid can revolve only 1. 250 to a definite point, the motor so that will attempt to execute at a specific speed; 251
- Joint Angle: defines the position and the orientation of the local reference frame of the 2. 252 robot at any moment;
- Joint Velocity: very important variables when we are using the inverse kinematics 3. 254 through the optimization procedure of the robot;
- 4. Joint Acceleration: has special importance when studying the control system of the 256 robot; 257

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- Joint position: significant for defining the system of the robot; 5.
- 6. Joint Jerk: allocating the jerk of the motor joint significant to minimalize the jerk of the 259 robot; 260

Variable	Units	References	Remarks	
		Paes et al		
	Nīma / Airma a	Abu-Dakka et al	The motor torques predicted by the dynamic model.	
TVDE 1	MIII/AIIIIS	Jin et al		
1 1 FE-1		Gleeson et al	The squared torque of each joints in the robot	
	NM	Gregory et al	Two joint Scara robot with holonomic constraint.	
	WATT	Wu et al	EAMA robot	
	TYPE-2 Radian	Paes et al	Dynamic model written in terms of the joint angles	
TVDE 0		Abu-Dakka et al	Used to model the rebet system	
1 1 FE-2		Jin et al	Used to model the robot system.	
		Menasri et al	Redundant manipulator.	
TYPE-3	Rad/sec	Gasparetto et al	The trajectories of the joints considered as a system of the kinematic configurations	
		Gasparetto et al	Used to model the robot system.	
TYPE-4	Deg/sec <sup>2</sup>	Gasparetto et al	The kinematics of the joint considered for the trajectory	
TYPE-5	mm	Valente et al	The locations of the joints in the robot chain.	
TYPE-6	rad/sec <sup>3</sup>	Gasparetto et al	Six Joints open chain manipulator considered.	

Table 7. Typical Design Variables System

#### 7. Design Constraints of the Trajectory Optimization in robotic applications

Constrained optimization is a procedure of minimize or maximize an objective re-263 garding some variables in the attendance of functional or behavioral limitation on those 264 variables. In what follow there is a description of the most used design variables of the trajectory optimization in robotic field.

- 1. Constraints on Joint Velocity: Place a suitable boundary on velocity of the joint could 267 very important to ensure that the robot working in safe situation. 268
- Constraints on Joint Acceleration: Avoiding some undesirable behavior, the accelera-2. tion of the joint motion bounded.
- Constraints on Jerk: It is not appropriate to have high jerk in the robot while perform-3. ing tasks. For this reason, constraints placed on the joint motor jerk.
- Dynamic Constraint: Continual dynamic equality constraint to ensure that the dy-4. 273 namic system working efficiently. 274
- Holonomic constraints: If the robot system considers the obstacles, the holonomic con-5. 275 straint should be stated. 276
- Constraint Joint Torque: Maximum torque bounds on the motor interpreted as con-6. 277 tinuous bound on control. 278
- 7. Constraints on joint position: In order to avoid some singularity cases, a bound should 279 be placed on the joint position. 280

Constraint	Units	References	Remarks
TYPE-1	Rad/sec	Paes et al	TO calculated in which the velocity doesn't overdo 80% of the maximum joint angle
		Gasparetto et al	Upper bounds on velocity are considered.

#### Table 8. Typical Design Constraints

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	_	Shareef et al	Angular velocity constraints the Joint motion
Dog/soc		Gasparetto et al	Kinematic limits on the velocity
	Deg/sec	Kucuk	Kinematic constraints placed on the joint velocity
	Rad/sec <sup>2</sup>	Paes et al	accelerations to visualize the resulting trajectory
TYPE-2	Deglese	Gasparetto et al	- Vincenatic hounds on the joint acceleration
	Deg/sec <sup>2</sup>	Kucuk	- Kinematic bounds on the joint acceleration
TYPE-3	rad/sec <sup>3</sup>	Valente et al	Jerk ranges, depending on the type of joint and on its position in
	rud/bee	valence et ui	the kinematic chain
TYPE-4	1/s	Gleeson et al	Continuous Equality constraint along the path.
TYPE-5	-	Gregory et al	The hand frame determined in presence of Obstacles
TYPE-6	NM	Shareef et al	joint torques can be changed impractical for real applications
TYPE-7	degree	Kucuk	Kinematic constraints placed on the joint position

# 8. Simulation Platforms used for Design the Trajectory in robotics applications

In what follow there is a description of the most used simulation platform that used 283 for the trajectory optimization in robotic field. 284

- 1. Matlab: well-known multi-paradigm numerical computing environment;
- Robot Studio: designed by ABB Company. Provides the tools to increase the profitability of the robot;
- 3. PSOPT simulator: respectable simulator achieves a variance, fast-moving the progress and conservation progressions of construction robots;
- 4. PUMA Simulator: the code is based on '3D Puma Robot Demo' from Don Riley;
- 5. Delmia Simulator: numerical Industrial determinations engineering invention and efficacy by preparation, simulating, and forming global assembly practices;
- 6. 3DCreate simulator: create complex robot models;
- 7. ACCADO Simulator: powerful simulator designed by ACCADO Company;
- FEM simulator: the robot model could be computed in real-time to obtain the actuation required to orientate and position as desired;
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- 9. IPOPT Simulator: visual studio used to simulate the robots;

**Table 9.** Typical Simulation Platforms

Simulation Platforms	References	Remarks
	Baghli et al	TO problem solved using Ant Colony algorithm in matlab environment.
TYPE-1	Gasparetto et al	Matlab environment used to implement the simulation.
	Kalmár-Nagy et al	MATLAB simulation used to run SNOPT toolbox.
TYPE-2	Paes et al	The resultant trajectory applied in RAPID-code
TYPE-3	Böck et al	PSOPT simulator used to solve the optimal control problem
TYPE-4	Kucuk et al	Optimal Trajectory Algorithm implemented using PUMA simulator platform.
TYPE-5	Gadaleta et al	Automatic compute the motion parameters of the trajectory
TYPE-6	Ulrich et al	Robot based inspection system.
TYPE-7	Paes et al	IRB1600 industrial ABB robot.
TYPE-8	De Magistris et al	FEM simulation to model the contact e ground-sole contact.
TYPE-9	Lietal	Utilize IPM software platform

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9. Frequently used Optimization Algorithms applied to the Trajectory Optimization	299
In what follow there are description of the most used algorithms applied for the trajectory optimization in robotic field.	300 301
1. Sequential Quadratic Program: Iterative technique aims to solve the NOP. Functional when the objective along with the constraints are continuously differentiable;	302 303
2. Genetic algorithm: Metaheuristic stimulated by the development of regular selection to resolve complex optimization problem;	304 305
3. Dynamic Programing: This algorithm technique utilized to resolve the OC problem, it does not clearly distinct the transcription and the optimization. As an alternative, it	306 307
<ul><li>does a system of iterative onward and backward passes along the trajectory;</li><li>4. Particle swarm optimization: Population based stochastic to solve the optimization problem;</li></ul>	308 309 210
<ol> <li>Artificial Bee Colonization algorithm: ABC is an Optimization technique built on the foraging performance of bee swarm, utilized to solve optimization problem:</li> </ol>	310 311 312
<ol> <li>Artificial Ant Colony optimization algorithm: AAC is an optimization technique built on the foraging performance of ant colony, utilized to solve optimization problem;</li> </ol>	313 314
7. Predictive Control strategies: The optimization problem formalized to be solved online using predictive controller;	315 316
Table 10. Typical Optimization Algorithms	317

Algorithm	References	Remarks	
	Paes et al	ACCADO-toolkit used to solve the optimal control problem using SQP algorithm.	
	Jin et al	MATLAB Optimization toolbox Fmincon	
TYPE-1	Böck et al	Fmincon is used to solve the static optimization problem.	
	Gilimyanov et al	Standard SQP is used to minimize measurements errors.	
	Chen et al	SQP is used to solve optimization problem.	
	Menasri et al	Bilevel optimization technique	
	Wu et al	GA utilized to find the trajectory planning for EAMA	
	Števo et al		
TVDE 2	Lim et al	GA used to find the TO problem.	
I IPE-2	Sarkar et al	-	
	Hassan et al	NSGA platform used to apply GA	
	Elshabasy et al	Hybrid method consisting of GA and Fmincon is utilized.	
	Oleiwi et al	Modified GA with A* algorithm used for optimal results	
TVDE 2	Shareef et al	DD to colors the optimal control method	
TTPE-3	Li et al	Hybrid method consisting of GA and Fmincon is utilized.         Modified GA with A* algorithm used for optimal results         DP to solve the optimal control problem.	
	Kucuk	PSO utilized to find the minimum time smooth trajectory.	
	Cao et al	Crossover, Mutation PSO along with interior point method	
TVDE 4	Jiang et al	PSO applied to find the MIRS optimization problem	
1 Y I <sup>2</sup> E-4	Mac et al	PSO used to resolve the TO problem	
	Das et al	An enhanced PSA is used for multiple-robots.	
	Asma et al	Dynamic Distributed PSO used to find the optimality.	
TYPE-5	Gupta et al	Artificial Bee Colonization algorithm showed the best result for MIRS medical robot.	
TYPE-6	Hui et al	Ant colony optimization algorithm used in this research so that easier tracking control of the mobile robot B-spline path smoother is presented.	

TYPE-7 Marko et al The method used is robust against unpredicted disturbances.

# 10. Discussion

Robotic applications have been greatly enhanced by the use of the trajectory optimi-320 zation technologies. In this research we reviewed many of the most recent trajectory opti-321 mization techniques that used in the field of the robotic applications. However, Stationary robots including industrial and manipulator, mobile robots and walking robots got super 323 attention. 324

Concerning the stationary robot, we note that the most prominent discussion topics 325 are how we can design a robotic trajectory with optimal energy efficiency, implementation 326 time and the jerk of the robot. The authors in [15] used the indirect method OC problem 327 to find the optimality of the power consumed path of the two revolute planar robot in 328 2012. In [36] the writers utilized the ACCADO-toolkit for trajectory optimization. The tra-329 jectory optimization problem solved as a dynamic problem in [18]. [21] A method is pro-330 posed to solve the trajectory optimization formulated as non-optimal control problem by 331 utilizing Genetic algorithm in 2014. The authors in [14] found a valid optimal trajectory 332 for redundant manipulator robot. [62] TO problem solved the by converting it to salesman 333 problem. In [16] PSOPT simulator used to solve the optimal control problem in 2016. [23] 334 Proposed optimization using hybrid technique utilize Crossover, Mutation PSO along 335 with interior point method. In 2017 [22] stated a Hybrid method consisting of GA and 336 Fmincon to improve the trajectory optimization for redundant manipulators. In [60] a sim-337 ulation interface in Delmia robotic environment proposed to solve TO problem. In [65] 338 TO problem solved using Ant Colony algorithm. 339

One of the works in the domain of the stationary robot that could considered as a 340 prospected task in the future is to use a modernistic interpolation technique such as trig-341 onometric spline in order to discretize the problem. Moreover, considering the dynamic 342 holonomic constraints in the stationary robot trajectory generation could be a prospected 343 work. Utilizing Ant Colony optimization algorithm along with another optimization pro-344 gram to solve the problem of the trajectory optimization also need to test and compare 345 with another optimization program. 346

With regard to mobile robots, we can notice that the most trending topics discussed 347 as a TO problem is to solve the optimal time implementation of traveling time and to 348 minimize the trajectory between two mobile robot locations. In [8] the authors applied 349 Cross Entropy method to solve TO problem in 2009. Moreover, in [11] dynamic program 350 is used to solve optimal control based scheme problem to Minimalize the execute time of 351 the wheeled robot. Bacterial foraging optimization stated for optimize the time between 352 two configurations in unknown dynamic environment in [63]. The authors in [7] solved 353 the TO problem by spline based approach where 354

Differential flatness used to define the steer control line. The research in [10] pro-355 posed a new technique called Dynamic inversion based method to solve the TG. Dijkstra's 356 algorithm used to find a collision-free trajectory in [48] where PSO is utilized to obtain the 357 optimized trajectory. 358

In the domain of the mobile robots, the researcher is working to improve the re-359 sponses to the dynamic obstacles along the path. Moreover, considering the electromag-360 netic constraints is considered as one of prospected work in the future. 361

Regarding the legged robots, we can observe clearly that the most researches focused 362 on designing the optimal trajectory that satisfies minimum energy consumption and max-363 imum forward velocity. Moreover, optimizing the gait of the walking robot has an espe-364 cial attention. In [55] TO problem designed as an optimal control where the OC problem 365 solved by framework MUSCODII. The authors in [56] used a sthenic criteria is used to 366 solve the TO problem. Genetic algorithm is used to generate OT in [1]. The researchers in 367 [53] utilized exact penalty function algorithm used to solve OC problem. The researches 368

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in this domain are working on enhance the ability of the walking robots to avoid the dynamic obstacles along the trajectory.

## 11. Summary of this research

In this research, we illustrated the most recent techniques used in the field of the 372 robotics system that solve the trajectory optimization problem, especially in the last few 373 years. In addition to the traditional techniques that used in the past, which solved the TO 374 problem of the robotics system as an OC problem, many heuristic optimization techniques 375 are used for this aim. Several new optimization hybrid techniques proposed recently gave 376 a superiority over the traditional technique, Crossover, Mutation PSO along with interior 377 point technique gave a remarkable enhancement in the field of the stationary robots. 378 Moreover, a Hybrid method consisting of GA and Fmincon to solve TO problem proved 379 to be efficient. PSO, AC and bacterial foraging optimization algorithms demonstrated 380 competence in various trajectory optimization robotic fields. Also, many researches 381 solved the TO problem using dynamic program. This technique developed to become dy-382 namic inversion based which stated in 2015. Exact penalty function, Cross Entropy 383 method, sthenic criteria and Spline based approach all achieved good results and were 384 used several times in different fields of the robotic trajectory generation. In terms of recent 385 TO computer platform, there are several environments stated recently, MUSCODII frame-386 work, PSOPT simulator, Delmia Simulator, 3D Create simulator, FEM simulator, IPOPT 387 Simulator and ACCADO-toolkit used to solve TO problem. In these days, the robotic re-388 searches trying to develop the used techniques in order to achieve more stable trajectory 389 optimization. In this meaning, the researches concerning in satisfying real-time trajectory 390 generation technique, considering constraint systems that reflex the real environment 391 such as dynamic holonomic and electromagnetic constraints, testing new hybrid optimi-392 zation techniques aiming to improve the results and inventing new software platforms 393 that make interaction between the user and the robot system more robust and superior. 394 More gab highlighting considered in the table 11, where the optimization techniques, con-395 straints, computer platforms, objective functions are illustrated. 396

In the table 12, several trajectory optimization techniques are illustrated in terms of 397 the strengths and weaknesses aiming to compare recent developed trajectory optimiza-398 tion techniques in the field of the robotics. Further explanation details illustrate some re-399 cent researches exist in the appendix. 400

		Stationary	Wheeled	Legged
	Optimal control	✓	✓	√
10 Problem formulation style	Non-Optimal control	√	✓	$\checkmark$
The manifestion of the impact	Direct	√	✓	$\checkmark$
Transcription techniques	IN-Direct	✓	0	0
	SQP	$\checkmark$	✓	✓
	GA	√	✓	$\checkmark$
	PSO	√	✓	0
Trajectory Optimization Techniques	Dynamic Program	✓	✓	0
	AC	√	0	0
	Bacterial foraging	0	0	$\checkmark$
	Salesman model	√	0	0
	Exact penalty function	0	✓	0
	Dynamic inversion	✓	✓	0
	Spline based approach	√	✓	0
	Cross Entropy	√	✓	0

Table 11. Trajectory generation in robotic field gab highlighting

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	sthenic criteria	$\checkmark$	0	$\checkmark$
Amiding abota los	Dynamic	0	$\checkmark$	$\checkmark$
Avoiding obstacles	Non Dynamic	$\checkmark$	$\checkmark$	$\checkmark$
	Matlab	$\checkmark$	$\checkmark$	$\checkmark$
	Robot Studio	✓	0	0
	PSOPT simulator	0	0	✓
	PUMA Simulator	✓	0	0
Software Platform	Delmia Simulator	$\checkmark$	0	0
	3DCreate simulator	$\checkmark$	0	0
	ACCADO Simulator	$\checkmark$	0	0
	FEM simulator	0	0	$\checkmark$
	IPOPT Simulator	0	$\checkmark$	0
	Constraints on Joint Velocity	$\checkmark$	$\checkmark$	✓
	Constraints on Joint Acceleration	✓	✓	✓
	Constraints on Jerk	$\checkmark$	0	0
Constraints	Dynamic Constraint	0	$\checkmark$	✓
	Holonomic constraints	$\checkmark$	$\checkmark$	$\checkmark$
	Constraint Joint Torque	$\checkmark$	$\checkmark$	$\checkmark$
	Constraints on joint position	$\checkmark$	0	0
	Energy consumed	$\checkmark$	$\checkmark$	$\checkmark$
	Total execution time	$\checkmark$	$\checkmark$	✓
	Jerk of the robot actuator	$\checkmark$	0	0
Minimization Objectives	Path length	$\checkmark$	$\checkmark$	$\checkmark$
,	Joint rotation	√	$\checkmark$	0
	Fuel Consumption	0	$\checkmark$	0
	Clearance of the obstacles	0	$\checkmark$	0

Table 12. Explanation details illustrate some recent researches

Technique	Highlights	Strength	Weakness	Reference
Non-invasive identification strategy for industrial.	Improve the energy efficiency of the robot.	Reducing the energy expense up to 5 percent compared to the methods used by ABB software	Further research is needed to integrate a robot stand-still option in the optimization procedure	Paes et al
A smooth trajectory generation model for stationary Robot.	Minimize execution time of kinematic chain based robot	The trajectory generation method is general-purpose, and could be applied to any type of robotic kinematic chain	Further research is needed by considering signals from some integrated sensors.	Valente et al
New method for trajectory planning of robot manipulators	Minimize the total execution time and the squared jerk of the manipulator	Efficient algorithms to achieve a good execution time results compared with another procedure presented in the research.	Further works could be done by testing modernistic interpolation techniques such as trigonometric spline.	Gasparetto et al
Bilevel optimization Trajectory planning of	Minimize the manipulation (displacement) of the robot.	Grant a new procedure to find the optimal free obstacles path for the	Considering the mobile obstacles dynamically that could be found on the path	Menasri et al

redundant		redundant	could be a good prospective	
manipulator		manipulators.	work.	
Trajectory plan optimization for EAMA	Minimize the Jerk of the 10 DOF serial articulated robot	This work achieved a trajectory between initial and final configuration with concussion in its lower limits compared to pre- optimization procedure.	Further stabilization improvement is one of the most important prospective works in order to develop the presented work.	Wu et al
Gripper mechanism path planning optimization	Minimize the actuator displacement of the gripper manipulator	Modeling and optimization the trajectory of the end frame of the gripper manipulator	Combine another gripper features in the optimization process.	Hassan et al
Optimization of Robotic Arm Trajectory Using GA	Minimize the manipulation time and energy consumption.	The optimized trajectory was relatively swift and low power consumption	Another properties could be integrated in order to reach better results	Elshabasy et al
Hybrid approach for autonomous navigation of mobile robots	Near optimal time trajectory generation simulation	Achieved in propose and test a new hybrid trajectory tracking	More work could be done regarding outdoor environment with dynamic obstacles	Hank et al
Swift2Drift car model for wheeled mobile robot.	Minimize the path length	A good technique uses flatness of derivation to obtain the optimal results.	Outdoor environment with dynamic holonomic constraint will be the author future work	Walambe et al
Smoothing Curvature of Trajectories Constructed by Noisy Measurements	Minimize the variation of control points	New improvement on the B-spline approximation algorithms to suit the path curvature	Further experiments regarding another kinds of physical obstacles	Gilimyanov et al

we can observe clearly that, the optimization trajectory in the robotic field became a trend
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in the last few years, where many researches are doing a great effort in order to improve
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the traditional trajectory optimization techniques aiming to make these algorithms more
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suitable for a recent types of robotics and as closer as conceivable to effort in actual time
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situation environment. In the following figure, summarization of the recent design and
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optimization methodologies for trajectory optimization in the robotic field.
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Figure 5. Recent design and optimization methodologies for Trajectory optimization in the robotic field

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# 12. Conclusion

This research offers an inclusive evaluation and critical compare of the newest strategy and optimization methods of the TG in the Robotic system environment. Aiming to find the optimal Path planning in the Robotic field, various assessment parameters such as Joint Velocity, angle, acceleration and jerk are illuminated and summed up. The chosen of some of these considerations is fundamentally to achieve an optimal mixture for the 432

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energy expended, total execution time, Jerk of the robot and the Path length. Moreover,
Dynamic and Holonomic constraints have an influence on the TO problem. According to
this review, it is noticed that utilize of the Hybrid Heuristic techniques in the optimization
problematic increase the precious of optimization results paralleled to utilize the Ordinary
Trajectory Optimization approaches.

Most of the researches for the TO in Robotic system are accomplished based on OC 438 methods including Heuristic and Gradient Base method. Heuristic methods has the capa-439 bility to exploration for global and local optimum and offer a set of optimal consequences 440 with a smaller amount of computational time. Also, Fusion algorithms have lately been 441 widely practical for the Trajectory optimization of the robotic system. Moreover, software 442 PC tools are similarly utilized broadly for scheming of Trajectory Optimization Tech-443 niques. Conversely, using recent methods such as artificial algorithms and hybrid algo-444rithms offer more precise optimization outcomes than software tools as they have the fa-445 cility to resolve multi-objective optimization problems. This research examines the key 446 root research approaches and their equivalent applications in Robotic Trajectory optimi-447 zation aiming to let more specialists recognize the present research status and also offer 448 some directing for relevant studies. 449

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