HYPOTHESIS OF QUANTUM GRAVITY

RESULTING FROM A STATIC, TOPOLOGICAL UNIVERSE RESULTING FROM THE POSITIVES AND NEGATIVES OF THE STEADY STATE AND BIG BANG THEORIES

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Abstract

This hypothesis is the result of my conviction that science will oneday prove everything in space and time is part of a unification. In "A Brief History of Time", Stephen Hawking wrote,

"If everything in the universe depends on everything else in a fundamental way, it might be impossible to get close to a full solution (of the universe's puzzles) by investigating parts of the problem (such as general relativity and quantum mechanics) in isolation."

The goal: to establish a "proof of concept" to which equations can be added. It's concluded the Steady State, Big Bang, Inflation and Multiverse theories all ultimately fail and a topological model including bits (binary digits), Mobius strips, figure-8 Klein bottles and Wick rotation works better. The failed cosmologies have impressive points leading to the idea that they're all necessary stepping-stones. For example, the Big Bang is seen here as violation of the 1st Law of Thermodynamics but its supposed origin from quantum fluctuations is reminiscent of bits switching between 1 and 0. The topological hypothesis has potential to explain dark matter, dark energy, and electromagnetic-gravitational union. Finally, the article introduces what is called vector-tensor-scalar geometry - and extensions of Einstein's Gravity and Maxwell's Electromagnetism.

KEYWORDS

Gravitation, Electromagnetic-gravitational union, Dark energy, Dark matter, Cosmology, Geometry, Topology

SECTION 1: THERMODYNAMICS AND COSMIC ORIGINS

A nearly 60-year-old cosmology book, Bergamini (1964), says, “(The Steady State theory – proposed in the 1940s by Fred Hoyle, Hermann Bondi and Thomas Gold – states) new matter or energy has to be continuously created at a rate equal to the mass of one hydrogen atom in each quart of space every half-billion years. Anti-steady-state cosmologists object to this feature because scientifically there is no evidence that mass-energy can be created.”

Modern science tells us that the observations and predictions of the Big Bang successfully replace the Steady State. However, isn't the Big Bang the ultimate example of creation of mass-energy from nothing? (The 1st Law of Thermodynamics states that a body can only gain or lose heat by taking it from, or passing it to, its environment or another body - this is because energy can neither be created nor destroyed.)

A much more recent cosmology book, Lewis (2020), says, “Maybe (the Big Bang) was a true birth from nothing ...” That sentence finishes with “... or maybe our universe is the child of an older (and wiser?) pre-existing universe.” That pre-existing universe would solve the problem of creation of our universe's mass-energy, but where did the pre-universe come from? (To those who say the pre-universe came from quantum fluctuation, there's more on this topic later.) Did the pre-existing universe originate from another universe which existed before it? We can repeat these steps endlessly but sooner or later we need to ask, “Where did the first, the original, universe come from? There would have been no creation of mass-energy from nothing back then either (unless the first universe had different laws of physics – as will be seen, this isn't an option in the hypothesis presented here).

The very first universe could originate naturally from knowledge gained in some part of this (our) universe's space and time. In other words, time wouldn't be linear (rectilinear) but cyclical (curvilinear), and the very first universe would actually be our universe. The knowledge would necessarily refer to General Relativity, Special Relativity, quantum mechanics and mathematics (this includes base 2 or binary maths, and topology a.k.a. rubber-sheet geometry).

SECTION 2: BITS AND TOPOLOGY

Electronics' binary digits can be used to draw a two-dimensional computer image of a Mobius strip. Two united Mobius strips create a three-dimensional figure-8 Klein bottle, Polthier (2003), that acts as a building block of space, time, forces' bosons and matter's fermions. This creates a supersymmetry (linkage) between fermions and bosons. A recent paper, Afshordi (2017), says that in a holographic universe, all of the information in the universe is contained in two-dimensional packages trillions of times smaller than an atom. Therefore, trillions of Mobius strips could form a photon and trillions of more complex figure-8 Klein bottles could form a more complex graviton (suggesting union of electromagnetism and gravitation).

Figure 1 – the Mobius Strip, which is two-dimensional and only has one surface (source: http://www.clker.com/cliparts/3/7/a/9/1220546534781713951lummie_Mobius_Strip.svg.hi.png)

The physicist and science historian Abraham Pais wrote that “In 1924 the scientist Wolfgang Pauli was the first to propose a doubling of electron states due to a two-valued non-classical “hidden rotation”, Pais (1991). Extending the ideas of “doubling”, “two-valued” and “hidden rotation” to the Mobius strip being a basic, fundamental unit of reality; it can be seen that Pauli’s proposal has an analogy to this article. The doubled Mobius strips (doubled to form a figure-8 Klein bottle) could be produced by the two-valued binary-digit system used in electronics. The bottles possess a hidden rotation, now identified as adaptive Wick rotation, which gives a fourth dimension to space-time. This Wick rotation is consistent with Special Relativity’s slowing of time (a.k.a. time dilation) because -

SECTION 3: DARK MATTER AND DARK ENERGY

For a hundred and ten years, science has accepted the concept of space-time which was formulated by Russian-German mathematician Hermann Minkowski and unites one time dimension with three space dimensions. Today, so-called imaginary numbers (such as i , which equals $\sqrt{-1}$) describe so-called imaginary time. Imaginary time is a concept derived from special relativity and quantum mechanics. Geometrically, imaginary numbers are found on the vertical axis of the Complex Number Plane, allowing them to be presented perpendicular to the real axis of space-time as we know it. One way of viewing imaginary numbers is to consider a standard number line, positively increasing in magnitude to the right, and negatively increasing in magnitude to the left. At 0 on this x-axis (the so-called real axis), a y-axis (the so-called imaginary axis) can be drawn with "positive" direction going up - "positive" imaginary numbers then increase in magnitude upwards, and "negative" imaginary numbers increase in magnitude downwards.

The ultraviolet catastrophe, also called the Rayleigh–Jeans catastrophe, is a failure of classical physics to predict observed phenomena: it can be shown that a blackbody - a hypothetical perfect absorber and radiator of energy - would release an infinite amount of energy, contradicting the principles of conservation of energy and indicating that a new model for the behaviour of blackbodies was needed. At the start of the 20th century, physicist Max Planck derived the correct solution by making some strange (for the time) assumptions. In particular, Planck assumed that electromagnetic radiation can only be emitted or absorbed in discrete packets, called quanta. Albert Einstein postulated that Planck's quanta were real physical particles (what we now call photons), not just a mathematical fiction. From there, Einstein developed his explanation of the photoelectric effect (when quanta or photons of light shine on certain metals, electrons are released and can form an electric current). So it appears entirely possible that another supposed mathematical trickery (the imaginary y-axis) will find practical application in the future.

The electromagnetic and gravitational waves composing space-time rotate in a cycle. The waves rotate through the vertical y-axis* that is home to so-called Dark Matter and the Dark Energy composing it, and back to the horizontal x-axis' space-time. (As NASA's measurements reveal in the next paragraph, the composition of dark matter by dark energy isn't as simple as $\text{energy}=\text{mass}$ in all cases i.e. it isn't always similar to ordinary energy composing ordinary matter via $E=mc^2$.) Since quantum mechanics says particles can be in two or more places at once, the photons and gravitons which make up the waves in space-time can be on the x- and

y-axes simultaneously and thus interfere with themselves, causing time to slow down significantly near the speed of light in a vacuum or under intense gravity.

*The dark matter/dark energy (DM/DE) residing at or near the Complex Number Plane's y axis remains in space-time's curves (in gravity) so it gravitationally affects space-time on the x axis. But this exotic mass-energy lies perpendicular (or almost perpendicular) to each dimension of our instruments, and thus electromagnetically undetectable (at least at present). 5.5 rotations, each of ~ 65.45 degrees, means there would be $5 \frac{1}{2}$ times as much dark matter as ordinary matter (or, to use NASA's number in NASA (2019), about 27% of the universe would be DM). Constant rotation keeps the x- and y-axes interactive but doesn't make more ordinary matter since the x-axis is restricted to $E=mc^2$ (the amount of available energy limits the production of matter). Mass-energy equivalence may not be $DE=DMc^2$ in every "dark" dimension. In some, there might be more "dark" energy available. It'd be possible for the universe to contain more than 5.5 times as much energy as our dimension. DE could be roughly 68% of the content of the cosmos.

Figure 2 – WICK ROTATION: "The complex plane reveals i 's special relationship with cycles via the circle of i , also known as Wick rotation. Whenever a point on the complex plane is multiplied by i , it moves a quarter rotation around the origin or center of the plane." [Figure and quote from Welch (2015)]

SECTION 4: SPACE-TIME'S CURVATURE AND FLATNESS

This paper's conclusions, though unconventional in certain ways, are supported by Albert Einstein's General Relativity Theory which concluded that space-time is curved. Curvature of space-time (from it being constructed of the curvature of Mobius strips, figure-8 Klein bottles, and Wick rotation) implies this range of allowable energies could be continuous and not restricted to certain bands. Since it's known the energy of electrons can only have discrete values, these values (and space-time's curves) must be determined by discrete pulses (possibly, the binary digits of 1 and 0). The Planck and WMAP satellites recently showed space-time to be extremely close to flat, WMAP (2012).

See the diagram below to help understand how curved space can be flat -

Figure 3 - MOBIUS DOUBLET (FIGURE-8 KLEIN BOTTLE) (source: <https://upload.wikimedia.org/wikipedia/commons/7/73/KleinBottleFigure8-01.png>) Note that the positive curvature fits together with the negative curvature to produce the outline of a doughnut which is technically flat (see the 2nd paragraph of Section 6 and the reference to Vanessa Janek). When many doublets are placed together, binary digits can fill in any gaps or voids in the same way that computers can morph a picture on a screen and extrapolate a small patch of blue sky to make a sky that's blue from horizon to horizon. Morphing by bits can also delete a single doublet's central "hole".

But the doublet doesn't become multiply connected like the doughnut. Merely the doughnut's outline is adopted – the doublet retains the property of being simply connected, a property necessary for space-time's infinity. (Informally, if an object in space consists of one piece - the constituent two Mobius strips now have the outline of one doughnut - and no longer has any "holes" that pass all the way through it, it is called simply-connected.

A flat universe that is also simply connected implies an infinite universe, Luminet (1995). (In regions of space-time that are exclusively flat, light beams travel in straight lines and can go infinite distance without ever meeting.) Since space and time are always unified, time is also infinite and the universe is eternal.

The lines beneath Fig. 3 attempt to describe "quantum foam", the Planck-scale subatomic region where apparently continuous space and time may dissolve into superstrings - possibly, strings of 1's and 0's - whose fluctuations could be the source of everything in the universe.

SECTION 5: MACROSCOPIC QUANTUM ENTANGLEMENT

The Klein bottle is a closed surface with no distinction between inside and outside. Thanks to quantum mechanics' entanglement applying on macroscopic scales,* this doesn't refer only to the surface itself. What is supposed to be outside the thickness of the figure-8 Klein bottles composing our universe [either another universe in the multiverse or exterior void, and the interior multiverse member or hole) would be the same as what exists within that surface. This results in the space-time of our universe existing everywhere and everywhen. The relativistic universe is only infinite and eternal because of macroscopic entanglement – if entanglement could be removed, the universe would be finite in time and might originate in the Big Bang. The inside and outside of the universe are continuous when it's composed of Mobius strips and figure-8 Klein bottles - there cannot be other universes outside our infinite and eternal universe, and there's no universe with different laws of physics (such a state of supposed multiple universes is called the multiverse).

*“Physicists now believe that entanglement between particles exists everywhere, all the time, and have recently found shocking evidence that it affects the wider, ‘macroscopic’ world that we inhabit.” New Scientist (2004). Though the effect is measured for distances in space, the inseparability of space and time means that moments of time can become entangled too. Brukner (2004) The photons of the Cosmic Microwave Background (CMB) could be quantum-entangled with every other particle existing in space as well as time. Then the Background would be radiated from every direction in the sky without requiring a Big Bang. Particles of matter separated by billions of light years or more would interact, and experience similar temperatures and densities and curvature (or flatness) of space because of quantum entanglement. If gravitons are entangled with microwave photons (they would be if entanglement exists everywhere and everywhen), imprints in the microwave background caused by gravitational waves must be unavoidable. This recalls BICEP (Background Imaging of Cosmic Extragalactic Polarization) and the Keck Array - a series of experiments which aim to measure the polarization of the CMB. Reports stated in March 2014 that BICEP2 had detected imprints from gravitational waves but cosmic dust is now considered the most likely explanation for the detected signal by many scientists. This article predicts that imprints in the CMB from gravitational waves will one day be detected unambiguously and not interpreted as evidence of inflation.

SECTION 6: THE STATIC NON-MULTIVERSE AND PERSPECTIVE

For the note below on the figure-8 Klein bottle, I refer to Bourbaki (2005), Conway (1986), Gamelin (2001), Joshi (1983), and Spanier (1994).

"The doughnut is technically a flat Universe, but one that is connected in multiple places. Some scientists believe that large warm and cool spots in the Cosmic Microwave Background could actually be evidence for this kind of ... (doughnut/figure-8 Klein bottle) ... topology". Janek (2015)

A flat universe that is also simply connected implies an infinite universe (see under Fig. 3 - the reference to Jean-Pierre Luminet). The flat, non-big-bang universe that's being proposed therefore needs to be simply connected and not have any holes like the hole in the centre of Fig. 3's Klein bottle. This is accomplished with the paragraph in the previous section that begins with "The Klein bottle is a closed surface with no distinction between inside and outside. Thanks to quantum mechanics' entanglement applying on macroscopic scales, this doesn't refer only to the surface itself". In classical science, the lack of distinction between inside and outside

does refer only to the surface itself (potentially leaving the universe dependent on the Big Bang). When quantum mechanics comes into play, our universe becomes entangled and unified with any other part of the so-called multiverse – and there is nothing except our static universe.

A few problems associated with the Big Bang may simply be matters of perspective viz. antimatter, protogalaxies and the Lyman-alpha forest. The question of why the universe contains any matter assumes the very early universe had equal numbers of particles and antiparticles which annihilated each other. Big-Bang cosmologists have concluded there were roughly a billion and one protons for every billion antiprotons in the universe's first moments. But the problem of the amount of antimatter in the early universe almost disappears if the universe had no early period but is static and eternal. It becomes related to the "creation of mass-energy" issue. In the last paragraph of the next section (7), it is written "There is no violation of the First Law of Thermodynamics since no energy needs to be created and converted into mass. It's transformed into every known and unknown mass by the interaction of existing gravitational and electromagnetic energy via what this article calls vector-tensor-scalar geometry." This idea can hardly be called new or untested. Not only does the Steady State theory speak of creation of mass-energy but the Big Bang theory itself is often presented as the creation of everything (space, time, matter, energy).

From the Big Bang's perspective, protogalaxies formed from the initial expansion of the universe and coalesced to become the larger galaxies. In a static universe, the smaller protogalaxies would still be seen since astronomy's instruments are looking further back in time as they look at more distant regions in space-time. When focussed, say, 12 billion light years away, they'd detect these clouds of material that coalesce into galaxies. When telescopes focus far beyond that, they see nothing. This is not because there's nothing to see earlier than the Big Bang. It's because light from galaxies that may be 100 billion light years distant (or infinitely more) has been redshifted out of range of our detectors.

The Lyman alpha ($\text{Ly}\alpha$) line in hydrogen can be compared to a railway track. As you look further and further along a track, perspective naturally brings the two rails closer and closer together until they seem to meet at a point ... a "concentration". (they're actually still parallel). As we look at the distant universe and further back in time, we see more dense regions of hydrogen absorption lines (the Lyman alpha forest). Some cosmologists interpret this as evidence that the universe began in a Big Bang and 75% of the cosmos is composed of hydrogen. In a static universe, the hydrogen which spectroscopes are examining is simply becoming more and more concentrated by the laws of perspective. Perspective is any method whereby the illusion of

depth is achieved on a flat surface (railway lines appearing to meet on the horizon at what is called the "vanishing point" is one of those various methods). Penguin (2006) Perspective therefore anticipates the Planck satellite by saying we live in a flat (nearly flat) universe. The method's consistent with the reference in the first paragraph of section 2 which says that in a holographic universe, all of the information in the universe is contained in two-dimensional packages trillions of times smaller than an atom - and with this article's statements that we live in an already infinite, nearly flat universe (it also contains curves).

SECTION 7: QUANTUM FLUCTUATION AND REDSHIFT DRIFT

It's correct to point out that this article is proposing creation of mass-energy. However, there's no proposal of expanding (or contracting) space-time at a cosmic scale – and no proposal of creation from nothing. The Big Bang is often said to be creation from nothing but it's also referred to as creation from quantum fluctuations. Although many different processes might produce the general form of a black body spectrum, no model other than the Big Bang has yet explained the fluctuations. Such a model is being suggested now: these fluctuations are comparable to the switching between ones and zeros – switching between on and off – which occurs in a universe using binary pulses to produce Mobius strips, Mobius doublets (figure-8 Klein bottles) and Wick rotation.

The universe's redshift could be seen not as galaxies receding from each other, but in Einsteinian terms of all space-time being a gravitational field in which gravitational redshift causes electromagnetic waves to become increasingly redshifted as distance increases. When light – not only the visible spectrum but all forms of electromagnetism – enters a gravitational well, it's deflected a very small amount (General Relativity calculates the refraction around the Sun to be 1.75 arcseconds). It's attracted to the mass at the centre of the well then climbs out. After escaping from billions of gravity wells as it passes galaxies and stars and planets during its journey to Earth, light from a galaxy billions of light years away (which might begin its travels as blue light) expends much energy and could be shifted to its red phase. It would then be gravitationally redshifted enough to explain the redshift of distant galaxies without invoking the Big Bang and the supposed recessional velocities of those galaxies.

Redshift drift refers to the phenomena that redshift of cosmic objects is a function of time. The observed wavelength of light received from any distant source of photons – or source of electromagnetic or gravitational disturbance whose shock wave excites photons and gravitons in the space between Earth and the source – drifts from higher to lower frequencies. According

to a calculation by astronomer Allan Sandage, this drift is one part in 10^{13} per day. Sandage (1962) An expanding universe that originated in a Big Bang would obviously explain this drift. But if this article's proposed topological model does indeed work better, it must be able to provide an explanation of drift that's just as feasible.

As noted at the beginning of this section, "It's correct to point out that this article is proposing creation of mass-energy. However, there's no proposal of expanding (or contracting) space-time at a cosmic scale – and no proposal of creation from nothing." It's proposed that creation of mass-energy continually produces a greater number of gravitational wells for light to escape from - gravitationally redshifting it more today than yesterday and less today than tomorrow. There is no violation of the First Law of Thermodynamics since no energy needs to be created and converted into mass. It's transformed into every known and unknown mass by the interaction of existing gravitational and electromagnetic energy via what this article calls vector-tensor-scalar geometry.

SECTION 8 - VECTOR-TENSOR-SCALAR (VTS) GEOMETRY

SUBSECTION 8.1 - GRAVITY, THE HIGGS AND JETS

"Dust grains assemble by chemical bonding. Once they are sand or gravel sized, how they continue to stick is a mystery. Metre-sized rocks should spiral into the star rapidly due to disc drag (the gas orbits a little slower than the rocks as a pressure gradient partially supports it). Once rocks somehow get past these barriers, they collide with each other in a chaotic and random way assembling the planets." Australian National University (2012-2019)

The following method of building planets is preferred to collisions between rocks and dust in the disc because most planetary systems seem to outweigh the protoplanetary discs in which they formed, leaving astronomers to re-evaluate planet-formation theories. Astronomy (2019)

Fig. 4 – VTS (VECTOR-TENSOR-SCALAR) GEOMETRY - Interaction of Gravitation and Electromagnetism Produces a Momentum in Gravitons and Photons (and a Pressure Which is Known as Mass). VTS Geometry inspired by Einstein (1919)

A vector is a quantity which possesses both magnitude and direction. Two such quantities acting on a point may be represented by two adjoining sides of a parallelogram, so that their resultant is represented in magnitude and direction by the diagonal of the parallelogram (AD and CD, for example, can symbolize the electromagnetic and gravitational vectors ... while the resultant green diagonal of DB substitutes for the interaction of those two forces). A scalar variable is representable by a position on a line, having only magnitude e.g. the red dot on the diagonal, symbolic of the Higgs boson. A tensor is a set of functions which, when changing from one set of coordinates to another, are transformed in a precisely defined manner (e.g. changing from the coordinates of AD and CD to those of the green diagonal, or of the red dot, is a transformation performed in a particular way). Adapted from Macquarie (2001)

Two sides thus illustrate the graviton's spin 2 and the photon's spin 1. The resultant diagonal represents the interaction of the sides/vectors ($1 \div 2 =$ the spin $\frac{1}{2}$ of every matter particle). Tensor calculus changes the coordinates of the sides and diagonal into the coordinates of a single (scalar) point on the diagonal. This scalar point is associated with particles of spin 0. Klauber (2018) If the mass produced during the photon-graviton interaction (the energy and momentum of photons and presently hypothetical gravitons produces a pressure we call mass*) happens to be $125 \text{ GeV}/c^2$, its union with spin 0 produces the Higgs boson. $125 \text{ GeV}/c^2$ united with spin 0 means the central scalar point of the Higgs boson is related to the vector of the graviton's spin 2, and the Higgs field is therefore united with the supposedly unrelated gravitational field (together with the latter's constant interaction with the electromagnetic field).

*Material from a star could fall onto a neutron star, heating it up and causing it to emit radiation. Then the energy and momentum of the photons and presently hypothetical gravitons

would be the interaction of electromagnetism (the charged particles and strong magnetism) with the neutron star's powerful gravity. This results in wave-particle duality. The heating could produce gravitational and electromagnetic radiation which would produce the mass and quantum spin of subatomic particles - instead of only radiation being emitted, jets of matter would be emitted too (normally, the matter would be emitted as beams or jets from the neutron star's magnetic poles).

SUBSECTION 8.2 - BOSONS AND PLANET/BLACK-HOLE FORMATION

It must be remembered that referring to space alone is incomplete. Living in space-time, it's necessary to add some sentences about the time factor. The photon must interact with the graviton to produce the mass of the weak nuclear force's W and Z bosons. To produce their quantum spin, the photon's spin 1 needs to react with the graviton's spin 2. That is, the photon's turning through one complete revolution needs to be combined with the graviton's being turned through two half- revolutions*. Incorporating the time factor as a reversal of time (Richard Feynman, 20th-century winner of the Nobel Prize in Physics, used reversal-of-time to explain antimatter) in the middle of the interaction: a gravitonic half revolution is subtracted from the photonic full revolution then the graviton's time-reversal adds a half revolution ($1 - \frac{1}{2} + \frac{1}{2} =$ the spin 1 of W and Z bosons). The strong nuclear force's gluon's quantum spin of 1 could arise in the same way as the spin 1 of weak-force bosons. Every reaction in this section except one may be explicable purely by the retarded** portions of waves interacting. The masslessness of gluons might be produced by retarded and advanced waves cancelling. They neutralize each other, producing a mass of zero and relating gluons to the Higgs boson whose zero quantity is its quantum spin.

*Professor Stephen Hawking writes, Hawking (1988) -

"What the spin of a particle really tells us is what the particle looks like from different directions."

Spin 1 is like an arrow-tip pointing, say, up. A photon has to be turned round a full revolution of 360 degrees to look the same.

Spin 2 is like an arrow with 2 tips - 1 pointing up, 1 down. A graviton has to be turned half a revolution (180 degrees) to look the same.

Spin 0 is like a ball of arrows having no spaces. A Higgs boson looks like a dot: the same from every direction.

Spin $\frac{1}{2}$ is logically like a Mobius strip, though Hawking doesn't specifically say so. This is because a particle of matter has to be turned through two complete revolutions to look the same, and you must travel around a Mobius strip twice to reach the starting point.

***"When we solve (19th-century Scottish physicist James Clerk) Maxwell's equations for light, we find not one but two solutions: a 'retarded' wave, which represents the standard motion of light from one point to another; but also an 'advanced' wave, where the light beam goes backward in time. Engineers have simply dismissed the advanced wave as a mathematical curiosity since the retarded waves so accurately predicted the behavior of radio, microwaves, TV, radar, and X-rays. But for physicists, the advanced wave has been a nagging problem for the past century." Kaku (2009)

The interacting gravity and electromagnetism produce mass e.g. they can form a Higgs boson or the strong/weak nuclear forces' bosons as well as matter. On a cosmic level - if gravitational and electromagnetic waves focus on a protoplanetary disc surrounding a newborn star, the quantum spin of the particles of matter in the disc ($1/2$) could imprint itself on the waves' interaction and build up a planet layer by layer from vector-tensor-scalar geometry's $1 \div 2$ interaction. If the waves focus on a region of space where there's no matter, the opposite interaction occurs and the graviton's spin 2 is divided by the photon's spin 1 to produce $2 \div 1$. The mass produced has the spin inherent in each of the gravitons composing spacetime - and could be an alternative, or complementary, method to supernovas for producing the gravitational waves making up black holes.

SECTION 9 – NOVEL RESULTS DERIVED FROM EINSTEIN’S GRAVITY

SUBSECTION 9.1 - OCEAN TIDES

A lot has been said about gravitation e.g. regarding dark matter and dark energy, curves and flatness in space-time, rotation, redshift, VTS geometry. It’s thus appropriate to say a bit more about it. Albert Einstein thought of gravity as a push caused by the warping and curvature of space-time, not as a pull. How, then, can repelling or pushing gravity account for the apparent attraction of ocean tides towards the Moon? I believe Galileo’s idea that the Earth’s movements slosh its water needs to be joined with the idea of Isaac Newton and Johannes Kepler that the moon causes the tides.

"If a barge (carrying a cargo of freshwater) suddenly ground to a halt on a sandbar, for instance, the water pushed up towards the bow then bounced back toward the stern, doing this several times with ever decreasing agitation until it returned to a level state. Galileo realized that the Earth's dual motion—its daily one around its axis and its annual one around the sun—might have the same effect on oceans and other great bodies of water as the barge had on its freshwater cargo." Tyson (2002)

Gravity’s apparent attraction can be summarised by the following - the momentum of the gravitons (united with far more energetic photons) carries objects towards Earth’s centre at 9.8 m/s or 32 ft/s. The volume of the oceans on Earth is estimated at nearly 1.5 billion cubic kilometres. Van Nostrand (2008) All this water is being pushed towards Earth’s centre at 32 feet per second every second. But the seafloor prevents its descent. So there is a recoil. This recoil is larger during the spring tides seen at full and new moon because Sun, Earth and Moon are aligned at these times.

The previous paragraph’s alignment of Sun, Earth and Moon refers to their being lined up where the gravitational current is greatest (in the plane where planets and moons are created*) - and to more of the gravitational waves travelling from the outer solar system being captured (absorbed**) by the solar and lunar bodies, and less of them being available on Earth to suppress oceanic recoil (there are still enough to maintain the falling-bodies rate of 32 feet per second per second). At the neap tides of 1st and 3rd quarter; the sun, earth and moon aren’t lined up but form a right angle and our planet has access to more gravitational waves, which suppress oceanic recoil to a greater degree. We can imagine the sun and moon pulling earth’s

water in different directions at neap tide but suppression is a more accurate description. If variables like wind/atmospheric pressure/storms are deleted, this greater suppression causes neap tides which are much lower than spring tides.

*A similar narrow plane, the consequence of gravitational currents, may be responsible for the orbits of many of the dwarf satellite galaxies of the Milky Way and Andromeda.

**In the final paragraph of Section 8 was this sentence, "On a cosmic level - if gravitational and electromagnetic waves focus on a protoplanetary disc surrounding a newborn star, the quantum spin of the particles of matter in the disc (1 / 2) could imprint itself on the waves ..." Such interaction with matter in a disc implies that gravity does not simply penetrate everything but is absorbed and re-radiated.

Figure 5 – TIDE SCHEMATIC - Does explanation of spring and neap tides require combining the theories of Newton, Kepler and Galileo? - public domain image from https://en.wikipedia.org/wiki/File:Tide_schematic.svg

Let's apply this article's concept of gravity to a few other instances -

SUBSECTION 9.2 - M-SIGMA

The M-sigma relationship was only discovered in 2000 and is observational, meaning scientists noticed it first and are now trying to understand the cause. M refers to the mass of a galaxy's central black hole, and sigma stands for the speed at which stars fly about in the galaxy's bulge. The bigger the black hole, the faster the stars move - the greater is their velocity dispersion. Astronomy (2016)

Gravitational waves would explain the simultaneous increase in black-hole mass / increase in stellar velocity dispersion. Some of the ocean waves passing an island are refracted - when they enter shallow water, they're refracted by friction with the mass of the seabed. They change direction and head towards the island, breaking onto its beaches. Similarly, gravitational waves are refracted and focus on the centre of a mass. In this case, the mass the waves are headed toward is the black hole, where they help form its composition (and increase the black-hole mass).

General Relativity proposes that the space-time composing the cosmos IS gravitation. Gravitational waves not only compose space-time but also so-called "imaginary" space-time (which is described with imaginary numbers such as $i=\sqrt{-1}$, exists on the Complex Number Plane's y-axis, can interact with our dimension on the x-axis, and is the possible domain of what are called dark matter and dark energy). The linear motion of waves headed towards the central black hole and striking stars' sides during the journey is converted into increased (and perpendicular) velocity of the stars since the gravitational waves of imaginary time are simultaneously at 90 degrees to each dimension of the gravitational waves of familiar space-time (recall how we can picture imaginary time as another kind of time in the vertical direction when familiar time is a horizontal line, and also recall that x-axis space-time and y-axis space-time interact).

SUBSECTION 9.3 - GEYSERS ON SATURN'S MOON ENCELADUS

"A small water jet on Enceladus, an icy moon of Saturn, spews its fiercest eruptions when the moon is farthest from the planet, a new study suggests, but the overall gas output doesn't increase much during that time. The study points to a mystery in Enceladus' plumbing." Howell (2016)

In 1919, Albert Einstein submitted a paper to the Prussian Academy of Sciences asking "Do

gravitational fields play an essential role in the structure of elementary particles?" If so, gravitational waves from deep space would focus on the centre of a planet's mass. When Enceladus is near Saturn, it would also be close to increased activity of the waves. The increased push from them would suppress emission of dust-sized water-ice grains, which is 3 times greater at the moon's farthest point because suppression is reduced there. Gas emission is also increased. Since this is not 3 times more, but only 20% more, a plumbing problem would be causing the discrepancy.

SUBSECTION 9.4 - A BRIEF HISTORY OF GRAVITY

In three dimensions, the gravitational force drops to $1/4$ if one doubles the distance. In four dimensions it would drop to $1/8$, and in five dimensions to $1/16$. The positive direction on the x-axis (representing the length, width and depth of "real" space-time) is an extension of the negative direction on x (this may be called the 5th space dimension or complex space-time). Therefore, real gravity is perpetually amplified by complex gravity. Using science's figures, the amplification equals $1/4$ multiplied by $1/4$ i.e. doubling the distance in 5 space dimensions causes gravity to become $1/16$ as powerful. It is not $1/4$ multiplied by $-1/4$ since numbers have the same property regardless of direction on the Complex Number Plane (they increase in value). To conserve this sameness, the second one must be $+1/4$ if the first one is $+1/4$. Alternatively, the gravity's strength is reduced 4 times and this number is multiplied by another 4 to reduce it 16 times overall. In the 4th space dimension/2nd time dimension represented by the imaginary axis, this y-axis is half the distance (90 degrees) from the real x-axis that the complex x-axis is (the complex is removed 180 degrees). So gravitational weakening from doubling distance in 4 space dimensions = (reduction of 4 times multiplied by another reduction of 4 times) / 2, for an overall reduction of 8 times to a strength of $1/8$.

SUBSECTION 9.5 - INFORMATION THEORY CONQUERS A RED GIANT

In about 5 billion years the Sun is supposed to expand into a red giant and engulf Mercury and Venus and possibly Earth (the expansion would probably make Earth uninhabitable in less than 1 billion years). It's entirely possible that there may not even be a red giant phase for the Sun. This relies on entropy being looked at from another angle - with the apparent randomness in quantum and cosmic processes obeying Chaos theory, in which there's a hidden order behind apparent randomness. Expansion to a Red Giant could then be described with the Information Theory vital to the Internet, mathematics, deep space, etc. In information theory, entropy is defined as a logarithmic measure of the rate of transfer of information. This definition

introduces a hidden exactness, removing superficial probability. It suggests it's possible for information to be transmitted to objects, processes, or systems and restore them to a previous state - like refreshing (reloading) a computer screen. Potentially, the Sun could be prevented from becoming a red giant and returned to a previous state in a billion years (or far less) - and repeatedly every billion years - so Earth could remain habitable permanently.

SECTION 10 – NOVEL RESULTS FROM MAXWELL'S ELECTROMAGNETISM

SUBSECTION 10.1 - NEWTONIAN GRAVITY AND RETARDED/ADVANCED

Albert Einstein's equations in the theory of General Relativity say gravitational fields carry enough information about electromagnetism to allow James Clerk Maxwell's equations to be restated in terms of these gravitational fields. This was discovered by the mathematical physicist George Yuri Rainich. Rainich (1925) Following the phrasing of the opening sentence, Einstein's Gravity was discussed first – in the previous section – and Maxwell's Electromagnetism is the subject of this section.

Near the end of Section 8, it was quoted that "When we solve (19th-century Scottish physicist James Clerk) Maxwell's equations for light, we find not one but two solutions: a 'retarded' wave, which represents the standard motion of light from one point to another; but also an 'advanced' wave, where the light beam goes backward in time." Thanks to George Yuri Rainich, it can be reasoned that gravitational – as well as electromagnetic – waves possess both advanced and retarded components. Advanced waves are usually discarded because they are thought to violate the causality principle: waves could be detected before their emission. On one level, I can appreciate that reasoning. But ultimately, I think it's an error that should be replaced by Isaac Newton's idea of gravity and the modern idea of quantum mechanics' entanglement. 17th century scientist Isaac Newton's idea of gravity acting instantly across the universe could be explained by the ability of gravitational waves to travel back in time. They thereby reach a point billions of light years away not in billions of years, but the advanced part of a gravitational wave would already be at its destination billions of years before it left its source*, and its journey is apparently instant.

*Arriving at its destination billions of years before it left its source is an absurd impossibility if we cling to the traditional view of time flowing in one direction from cause to effect. But it's plausible if we accept the Block Universe theory which developed from Special Relativity's non-

simultaneity of events for different observers. In the Block Universe, all time coexists (the entire past, the present, and every point in the future all exist at once). Time can be visualized as a Cosmic DVD where our brains and consciousnesses take the place of the DVD player's laser. Everything in the Cosmic DVD's time exists at once** but we're only aware of an extremely limited number of events at any instant (these make up our present). Gravitational waves arriving billions of years prior to emission can be compared to playing part of the Cosmic DVD in reverse. Waves travel from a later frame of the cosmic movie to an earlier frame.

** If different parts of this manuscript are combined, they show all time exists at once in reality. All mass is composed of gravitational and electromagnetic waves, according to vector-tensor-scalar geometry. Both types of waves possess retarded and advanced components which entangle all masses. Wick rotation (time) is built into the Mobius strips and figure-8 Klein bottles composing electromagnetism's photons and gravitation's gravitons. Therefore, all time (the entire past and present and future) is united into one thing just as all space and all mass are united into one thing. VTS geometry says space and mass are united and physics already accepts that space and time are united. If space, mass, time, electromagnetism, and gravitation are all aspects of the same thing; that suggests the theory of quantum gravity truly exists. Mathematical equations would be just another aspect of the one thing - a tool - which people deem necessary to prove quantum gravity.

1's and 0's composing electromagnetic and gravitational waves would compose both "advanced" waves going back in time and "retarded" waves going forward in time. The retarded components with +x motion in time can obviously cancel the advanced components with -x motion in time, producing entanglement.

Stars and galaxies etc. send us retarded light which, through spectroscopy, gives an approximate measurement of how long that light has been travelling (the distance to the astronomical body). The light includes an advanced component that reaches back into the past, producing a measurement that significantly exceeds the real distance. The farther away a star or galaxy is, the more the advanced part of waves from it will reach into the past, giving us a greater inaccuracy regarding its true distance. This increase is analogous to redshift increasing with distance. We might call it readshift - re(tarded) ad(vanced) shift.

When a dinosaur dies, the advanced gravitational and electromagnetic waves composing its particles would continue traveling back in time. By the time its bones or fossilized remains, or the surrounding rocks, were subjected to modern science's dating methods; those advanced waves might have gone so far back in time that the dating method says the dinosaur died 100

million years ago or more. Radioactive dating is thus a form of (advanced) gravitational-wave detection, just as LIGO - the Laser Interferometer Gravitational-wave Observatory - picks up (retarded) gravitational waves. Technology based on the way noise-cancelling headphones work might provide a more accurate reading of when the dinosaur lived. The headphones increase the signal-to-noise ratio by incorporating a microphone that measures ambient sound (noise), generating a waveform that is the exact negative of the ambient sound, and mixing it with any audio signal the listener desires. Generating a waveform that's the exact opposite of the advanced waves emitted by the deceased dinosaur should, at least partially, neutralize the advanced waves and restrict measurement to the retarded waves associated with the animal's decay.

Advanced waves also cause living creatures to age faster than they would without those waves, by extending the creatures' reach into the past (this is equivalent to having lived longer). Neutralising the advanced waves should dramatically increase the health and lifespan of humans and all other species if it doesn't adversely affect anatomy and physiology i.e. if the retarded waves which go forward in time are sufficient for normal structure and function.

SUBSECTION 10.2 -

ANTIGRAVITONS ARE QUANTA OF DARK ENERGY + IMMORTALITY

Figure 6 – MOBIUS MATRIX (Mobius equals a,b,c,d,e array)

Mathematics has three types of numbers - real, imaginary and complex. Real numbers are exemplified by 0, the positive numbers used in counting, and negative numbers. On a two dimensional "Complex Plane", Real Numbers are on the horizontal plane and Imaginary Numbers such as $i=\sqrt{-1}$ are on the vertical plane. Complex Numbers can be easily identified as a combination of Real Numbers and Imaginary Numbers. Olivia (2011) Retarded gravitational and electromagnetic waves that go forwards in the horizontal plane of space-time can be termed real. Advanced waves that go backwards in space-time may be considered complex. The imaginary numbers of the vertical direction could describe waves in an "imaginary spacetime" - spacetime above the horizontal plane might be called Superspace or Hyperspace while spacetime below that plane would be Subspace.

Width a is perpendicular to the length (b or e) which is perpendicular to height c . How can a line be drawn perpendicular to c without retracing b 's path? By positioning it at d , which is then parallel to (or, it could be said, at 180 degrees to) a . d is already at 90 degrees to length b and height c . d has to be at right angles to length, width and height simultaneously if it's going to include the Complex Plane's vertical "imaginary" axis in space-time (the "imaginary" realm is at a right angle to the 4 known dimensions of space-time, which all reside on the horizontal real plane). In other words, d has to also be perpendicular to (not parallel to) a . This is accomplished by a twist, like on the right side of the Mobius strip, existing in the particles of matter composing side a . In other words, a fundamental composition of matter is mathematics' topological Mobius, which can be depicted in space by binary digits creating a computer image. The twist needs to be exaggerated, with the upper right of the Mobius descending parallel to side " a " then turning perpendicular to it at approximately the level of the $=$ sign, then resuming being parallel. Thus, $90+90$ (the degrees between b & c added to the degrees between c & d) can equal 180, making a & d parallel. But $90+90$ can also equal 90, making a & d perpendicular. (Saying $90+90=90$ sounds ridiculous, but it has similarities to the Matrix [of mathematics, not the action-science fiction movie] in which X multiplied by Y does not always equal Y times X . The first 90 plus the second 90 does not always equal the second 90 plus the first 90 because $90+90$ can equal either 180 or 90).

$90+90=180$ corresponds to the familiar composition of space-time (the presently hypothetical graviton) and $90+90=90^*$ corresponds to the antigraviton. That is, shock waves from space-time disturbances excite already-existing gravitons and antigravitons (and their photonic counterparts) to various frequencies. Dark energy is here postulated to be extra-dimensional, therefore the extra-dimensional antigraviton (extradimensional because it's described by $90+90$ [$90+90-90$] = 90) can be the quantum unit of dark energy. Antigravitons - and their extension, dark energy - can thus be mistaken for "repelling gravitation" which causes the universe to

expand from the Big Bang. In reality, dark energy and antigravitons would be involved in the production of dark matter - just as energy and gravitons are involved in the production of matter (recall Albert Einstein's $E=mc^2$ and Subsection 8.1's VTS Geometry). Antigravitons, like gravitons, are an arrangement of figure-8 Klein bottles with each bottle being composed of two Mobius strips (the former described by $90+90=90$, the latter by $90+90=180$). Trillions of Mobius strips can be added to form photons - or those trillions can be separated into pairs, with each pair forming a figure-8 Klein bottle and trillions of Klein bottles forming a graviton (see Section 2).

*Since these numbers are also describing space-time (although an unfamiliar aspect of it), this is actually $90+90-90=90$ since it follows the same pattern as $1+1/2-1/2=1$ describing the spin 1 of W and Z bosons in Subsection 8.2. James Clerk Maxwell's electromagnetic equations admit two solutions for light (retarded and advanced waves), and George Yuri Rainich's mathematics suggests the same two solutions apply to gravitational waves. Besides following the pattern $90+90-90=90$, $90+90=90$ might possibly also follow the pattern in Subsection 9.4 which discusses the weakening of gravity when distance is doubled in the 4th space dimension/2nd time dimension represented by the imaginary axis - it would be $(90+90)/2=90$. In 3-dimensional (3-D) space, gravity includes gravitons and retarded waves (mainly). In connected 4-D space, it consists of antigravitons and advanced waves which contribute to the 3rd dimension. Regeneration of bodies and brains might be achieved by regular excitation of antigravitons composing advanced waves which reach into the past to a time when a person's body/brain was younger and healthier. When regeneration has been attained, the advanced waves could be neutralised to prevent them reaching further into the past which would be equivalent to having lived longer (see Subsection 10.1).

SUBSECTION 10.3 - INTERGALACTIC ROBOTS AND NEO-TELESCOPES

Unifying gravitation and electromagnetism has this consequence: A 2009 electrical-engineering experiment at America's Yale University, together with the ideas of Albert Einstein, tells us how we could travel to other stars and galaxies. An electrical engineering team at Yale demonstrated that, on silicon-chip and transistor scales, light can attract and repel itself like electric charges or magnets. Li (2009) This is the Optical Bonding Force. For 30 years until his death in 1955, Einstein worked on his Unified Field Theory with the aim of uniting electromagnetism (light is one form of this) and gravitation. Achievement of this means the quantum components (gravitons) of gravity/spacetime-warps between spaceships and stars could mimic the Optical Force and be attracted together, thereby eliminating distance (this, possibly acting in partnership with repulsion, could produce a wormhole, or shortcut between folds in space and

time). If the gravitons are superposed and entangled, distances between both points in space and points in time are totally eliminated.

As stated in a robotics lesson, "the time variable t varies from 0 to 1, that is, $0 \leq t \leq 1$ ". MATLAB (2019) Therefore, this article's logic states that 0 may be equal to 1 (division by 1 is accepted, so why isn't division by 0?) Since time is permanently united with space in physics, $0=1$ in space-time too. This is consistent with a proposed future theory of physics called Quantum Gravity; where Quantum Mechanics is united with General Relativity, Einstein's theory of gravity. A possible path to attainment of quantum gravity is realizing that all objects and events on Earth and in space-time are just one thing - like 0 equalling 1, and like the objects in a computer image seeming to be a lot of separate objects but really just being one thing (strings of binary digits). A spacecraft sitting on its launchpad can be assigned $t=0$, and its destination $t=1$. Since $0=1$, reaching the destination takes the same time as reaching the launchpad from the craft's position on the launchpad (travel is instant). Robot motion can also be instant and not require interpolation, which is making the end of a robot arm move smoothly from A to B through a series of intermediate points. Of course, this is nonsense if viewed from CLASSICAL mechanics. We need a mindset immersed in QUANTUM mechanics which has been extended to macroscopic entanglement.

To finish on another technological note, the thing that's really needed to learn about the universe is better telescopes. Of course - when I say "better" telescopes, I'm referring to new technology and telescopes that do far more than merely intercept what my hypothesis calls "light waves" for convenience. To be precise, "light waves" would be disturbances in space-time which excite photons to visible-light frequencies. A photon is a quantum of excitation of the electromagnetic field. That field fills all space and so do its quantum modes. (This hypothesis says excited gravitons form gravitational waves.) The new-technology telescopes would use cancelling retarded and advanced waves to create entanglement of the scope with the universe $10^{1,000,000}$ light years away (or infinitely further).

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