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Interaction of Gravitational Field and Orbit in Sun-planet-moon System

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Abstract

Studying the old problem why the moon can move around the Sun with the orbital perturbation theory, we found that the planet and moon are unified as one single gravitational unit which orbits around the Sun on the orbit of the planet. And, the gravitational fields of both the planet and moon is interacting with the Sun as one single gravitational field. The gravitational field of the moon is limited in the unit which cannot interact with the mass out of the unit with the force of $F = G\frac{Mm}{R^2}$. The findings are further clarified by reestablishing Newton's repulsive gravity.

Key words: Interaction of gravitational field, Three-body problem, Orbital perturbation theory, Repulsive gravity

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1. Introduction

The theory of gravity is developed from astronomy observation. It is the first and main consequence that the theory of gravity is valid to understand the celestial orbit and to design artificial orbit. In the time of the geocentric theory, through observation, the orbits of some celestial bodies can be accurately predicted and the accurate ephemeris and almanac were established. Observing the shape and orbit of the celestial bodies, Nicolaus Copernicus[1] established the heliocentric theory in 1507. Based on Kepler's law,[2] Sir Isaac Newton[3] established the theory of gravity in his Principia in 1687. And, Newton established the theory of orbit. After the Newtonian theory of gravity, the orbit is not only an observed result, but can be understood with celestial mechanics and can be designed artificially.

Factually, an orbit is not only determined with $G\frac{Mm}{R^2} = \frac{mv^2}{R}$, but is always perturbed by several celestial bodies. The orbital perturbation theory was initially formulated by Newton.[3] Now, it was taught in textbooks.[4,5]

Almost in 1900, from the Newtonian theory of gravity, the Hill sphere was derived. [6,7] The sphere of influence, including the Hill sphere and Laplace sphere, is generally used to study the extrasolar system and to design the interplanetary satellite orbiter.[8,9,12-14] The Hill sphere is valid to understand the orbital stability zone of a moon around a planet.[8-14]

After 1900, Einstein's general relativity[15] and quantum mechanics were established. And, the quantum theory of gravity was presented. Generally, it is thought that Einstein's general relativity is a good theory for gravity while we are very far from having a complete quantum theory of gravity.[16]

In modern theory of gravity, gravitational field is fundamental. But, now, we have known little about the interaction of gravitational fields, especially the interaction of gravitational fields in the solar system.

In this paper, we noticed that, there is a series of problems about the orbit in the solar system. 1) There is a famous old problem:[17] Why the orbit of the Moon around the Earth is stable under the condition that calculated with Newton's law of $F = G\frac{Mm}{R^2}$, there is $F_{sm}/F_{em} \approx 2.2$, where F_{sm} and F_{em} are the force of the Sun and Earth on the Moon, respectively. 2) As the Earth is orbiting around the Sun, the Moon is also moving around the Sun. An old problem is: What is the mechanics that makes the Moon moved around the Sun?[18] 3) In 1860, Delaunay[17,19] presented that the Earth-Moon system is binary planet. The binary planet was used to explain the Moon moving around the Sun by Turner[18] in 1912. And, now the Pluto- Charon system is thought binary.[20-23] Binary star was presented in 1700s.[24] Now, the binary star/planet/blackhole is very prevailing. But, what is the mechanics that could make two planets/stars/blackholes orbiting around the barycenter of them or orbiting around each other? 4) Generally, in studying the orbit of the Moon, the Three-body problem was studied by Newton[3]. Before Poincaré', it had been studied generally.[25] Euler,[26] Lagrange[27], Jacobi[28] and Hill[6,7] had contribution to the restricted Three-body problem. Poincaré[29] published his study about the Three-body problem in 1892-99. Today, the Poincaré's equation is generally used to calculate the Three-body problem. In Poincaré's equation, the orbits in the Three-body are chaotic. But, why the orbits in the typical Three-body, such as the Sun-Earth-Moon system and Sun-Pluto-Charon system, are stable?

In the Pluto system, the mass of the Charon is almost 0.12 times that of the Pluto. This is a special case for the planet-moon system. The Charon is discovered in 1978.[30] Therefore, some of the theories and concepts that presented before 1978 need be rechecked and re-understood with the Pluto system. From the Pluto-Charon system, we presented a new problem: Calculating with $F = G\frac{Mm}{R^2}$, as the Pluto is at the perihelion of the orbit around the Sun, there is $\frac{F_{cp}}{F_{sp}} \approx 40$, where F_{sp} and F_{cp} are the gravitational force of the Sun and Charon acting on the Pluto, respectively, why the orbit of the Pluto around the Sun was not broken off by the Charon? (Now, the Pluto is excluded from the planet. But, in studying the orbits in the Pluto system,[20-23] the mechanics and dynamics are just that for other planets. So, the Sun-Pluto-moon system can be treated as other Sun-planet-moon system.)

These problems are fundamental. Although having been studied by many scientists for a long time, they are still open problems. It seems that a new line is needed for them. Here, we presented, these problems could be understood with the interaction of gravitational field. And, the orbital perturbation theory is well applied in designing the artificial orbit. It means that it is a valid theory. Therefore, here, we shall investigate other theories with the orbital perturbation theory. In Sec.2, the line of the moon moving around the Sun is studied systematically. From the orbital perturbation theory, we found, a planet-moon system is unified a one single solid gravitational unit which orbits

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around the Sun on the orbit of the planet. And, the gravitational fields of both the planet and moon is unified as one field interacting with that of the Sun while the field of the moon is limited in the unit. This is our main conclusion. In Sec.3, the Poincaré's equation for Three-body problem is compared with the orbital perturbation equation. It is shown that, the gravitational unit and the interaction gravitational field is implied in the orbital perturbation theory. And, it is wrong that a gravitational field could interact with any other ones with the force of $F = G\frac{Mm}{R^2}$. In Sec.4, it is shown that Newton's conclusion that the perturbation of the Sun to the Moon is always repulsive was well observed with modern technology from the artificial orbit. In Sec. 5, it was presented that, the Hill sphere and the orbital perturbation theory is complementary to each other. In Sec.6, we presented that all of the gravitational fields are interacting with others, no gravitational field can be isolated from others. So, the gravitational field only can be understood from the interaction field. And, by analogy to the Maxwell equation, we establish a set of equations for the interaction of gravitational field. In Sec.7, it is shown that our concept of gravitational unit is very analogous to the concept of binary planet which is currently used to explain the line of the moon moving around the Sun. But, it is concluded that the binary planet/star/blackhoole cannot exist in mechanics and dynamics. In Sec.8, we presented our result could be valid to the orbit in the galaxy.

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2. Orbital perturbation theory and the lines of moons moving around the Sun

As a moon is orbiting around a planet, it is also moving around the Sun. But, till now, the line of the moon moving around the Sun has not been systematically studied. Here, it is noted that, there are two typical lines of the moon moving around the Sun as shown in Fig.1. And, between the two kinds of typical lines there are many other kinds of the lines for the moon moving around the Sun. A crucial problem was presented[18]: What is the mechanics that makes the moon moved around the Sun?

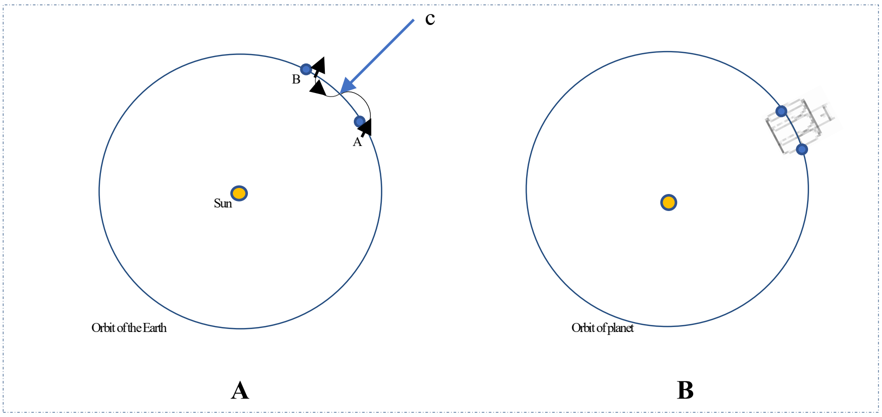

Fig.1. The line of a moon moved around a star. There are two typical kinds of lines as shown in **A** and **B**. Between the two kinds of lines, there are many other different kinds of lines.

A. The orbit of the moon around the planet and that of the planet around the star are in a same plane. The black line is that the Moon is moved around the Sun. As the Earth is orbiting around the Sun from point A to B, the Moon just orbited a period around the Earth. The direction of the orbit of the Moon around the Earth in AC are different from that in CB.

B. The orbit of the satellite around a planet is vertical to that of the planet around the Sun. The line of the moon moving around the Sun is helix.

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Fig.1 shows that relative to the direction of the moon moving around the Sun, the direction of the orbit of the moon around the planet is varied in all directions. For example, in Fig.1A, as the Moon moves from point C to B, the direction of the Moon orbiting around the Earth is contrary to that of the Moon moving around the Sun. And, in Fig.1B the two directions are vertical to each other. And, the velocity of the Moon around the Sun is almost 30 km/s. It is almost equal to that of the Earth while that of the Moon around the Earth is almost 1.023km/s. No any force that is out of the planet-moon system could make the moon moved around the Sun in such a case. The only reason is that a planet-moon system is one single gravitational unit that is moved by the planet orbiting around the Sun. A moon cannot independently move around the Sun. It is a part of a planet-moon gravitational unit that orbits around the Sun.

There are two features for a planet-moon union orbiting the Sun. First, the orbit of a planet-moon unit around the Sun is only determined with the velocity v_p and mass m_p of the planet and the gravitational force of the Sun $\mathrm{G}\frac{M_S m_p}{R^2}$, i.e., $\mathrm{G}\frac{M_S m_p}{R^2} = m_p \frac{v_p^2}{R}$, where s and p denote the Sun and planet respectively. (For convenience, in this work, we assume all orbits are circular and on a same plane.) The mass and velocity of the moon cannot affect the orbit. The evidence is that the orbit of the Pluto around the Sun is not affected by the Charon although the calculated attractive force of the Charon on the Pluto could be larger than 40 times that of the Sun. Second, the force out of a planet-moon unit cannot affect the orbit of the moon around the planet and the line of the moon around the Sun. The evidence is that the Moon is not moved to the Sun although the calculated gravitational force of the Sun on the Moon is almost 2.2 times that of the Earth on the Moon.

The planet-moon unit orbiting around the Sun could be well understood with the orbital perturbation theory and the Hill sphere.

The Hill sphere usually deals with the stability of the orbit of the moon around a planet. It is approximately written as[8]

$$r_H \approx (1-\mathrm{e})\mathrm{a}\sqrt[3]{\frac{m}{3M}}, \tag{1}$$

Where r_H is the Hill radius, M and m are the mass of the Sun and Earth, a and e are the semi-major axis and eccentricity of the orbit of the Earth, respectively.

The Hill sphere means that, inside the radius of r_H, the Earth dominates the gravity. And, the condition for that a moon orbits around a planet in a stable way is that only if the moon lies always within the Hill sphere.

The Pluto-Charon system shows another problem for the theory of gravity. Because of $\frac{F_{cp}}{F_{sp}} \geq 40$, if the formula $\mathrm{F} = \mathrm{G}\frac{Mm}{R^2}$ was valid for the gravitational force of the Charon acting on the Pluto, the orbit of the Pluto around the Sun should be broken off in a short time. Because the orbit of the Pluto around the Sun is stable, the Charon cannot act on the Pluto with the force $\mathrm{F} = \mathrm{G}\frac{Mm}{R^2}$. It means that, out of the Hill sphere, the moon cannot have the force $\mathrm{F} = \mathrm{G}\frac{Mm}{R^2}$ on other bodies.

From the orbital perturbation theory,[4] the force of the Sun and Moon on an artificial satellite orbiting around the Earth was exactly known.

In the N-body system, the orbit of an artificial satellite around the Earth is determined with:

$$g_{total} = G\frac{M_E}{r^3}\boldsymbol{r} + \sum_{i=1}^{n} Gm_i(\frac{\boldsymbol{r_i}}{r_i^3} - \frac{\boldsymbol{r_j}}{r_j^3}) \quad (2)$$

Where M_E is the mass of the Earth, r is the distance between the Earth and the satellite. *i* is the *i*th body, m_i is the mass of *i*th body. For the Sun-Earth-Moon system, *i*=2, i.e., the Sun and Moon. r_i is the distance between the satellite and the *i*th body, r_j is the distance between the Earth and *i*th body, **r**, $\boldsymbol{r_i}$ and $\boldsymbol{r_j}$ are vectors.

Eq.(2) shows that the Earth is a central mass which determines the orbit of the Moon around the Earth, while the Sun only can have a perturbative force on the orbit.

For an artificial satellite, the distance between the satellite and the Sun is almost equal to that between the Sun and Earth, i.e., $r_i \approx r_j$, the gravitational acceleration of the Sun on the satellite approximately is[31]

$$g_{perturb} = -G\frac{M_s}{R^3}r \quad (3)$$

Where M_s is the mass of the Sun, R is the distance between the Sun and Earth, *r* is the distance between the satellite and Earth.

It is well known that, the perturbation force of the Sun on the satellite is very little. For a low orbit satellite, it is less than the force of the light pressure of the Sun on the same satellite. Usually, $g_{perturb}$ is on the level of $10^{-7}ms^{-2}$. (In Eq.(3), the variation of the direction of r is omitted. Here, only the level of the magnitude of the force need be known. Eq.(3) can well express the level of magnitude of the force.)

For a spacecraft orbiting around the Earth, the orbit is determined with the gravity of the Earth with $\mathrm{g} = G\frac{M_e}{r_{es}^2}$ while it is perturbed by the Sun and Moon with $g_{perturb} = -G\frac{M_i}{R_i^3}r_{es}$, where *i* denote the Sun or Moon. Because the perturbation force is very little, the orbit is stable.

In Fig.2, as a spacecraft is out of the Hill sphere of the Moon, from the perturbation theory of the orbit[4] we know, the total gravitational acceleration by the force of both the Earth and Moon on it is

$$g_{total} = G\frac{M_e}{r_{es}^2} + G\frac{M_m}{R^3}r_{es} \tag{4}$$

Where M_e and M_m are the mass of the Earth and Moon respectively, R is the distance between the Earth and Moon and r_{es} is the distance between the Earth and spacecraft.

It is emphasized, Eq.(4) is well-confirmed. The theory of orbital perturbation is a well-understood and well-developed theory[4]. It was used to the artificial orbit. The acceleration by the force of the Earth and Moon on an artificial satellite around the Earth is just the Eq.(4).

It is interesting that as this spacecraft is inside the Hill sphere of the Moon, the total acceleration by the Earth and the Moon on it become as:

$$g_{total} = G\frac{M_m}{r_{ms}^2} + G\frac{M_e}{R^3}r_{ms} \tag{5}$$

Where r_{ms} is the distance between the Moon and the spacecraft.

We also emphasize, Eq.(5) is also well-confirmed. The total force of the Sun and Earth on a satellite orbiting around the Earth just is $g_{total} = G\frac{M_e}{r_{es}^2} + G\frac{M_s}{R^3}r_{es}$, where s and e denote the Sun and Earth, respectively. The reason that the force of the Sun on a satellite orbiting around the Earth is only the perturbation force is that this satellite is inside the Hill sphere of the Earth just as in Eq.(5) the spacecraft is inside the Hill sphere of the Moon.

Comparing Eq.(4) to Eq.(5), it is shown that, for the same spacecraft, the central mass with the central force is varied with that the spacecraft is inside or out of the Hill sphere.

From the Hill sphere and Eqs.(4) and (5), understanding with the interaction of gravitational field, we concluded that, the gravitational field of the Moon could be limited into the Hill sphere as shown in Fig.2. As a field of a body is limited or trapped, it means that it cannot extend freely. Therefore, it could not interact with other ones with $\mathrm{F} = \mathrm{G}\frac{Mm}{R^2}$ in infinite space.

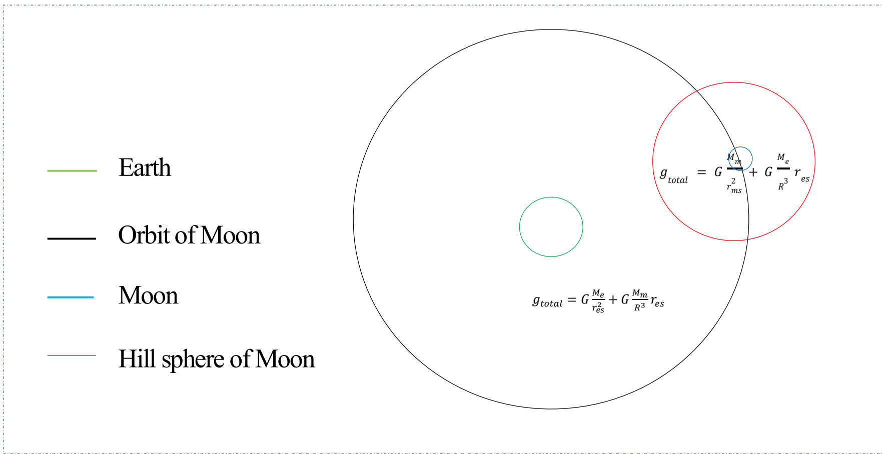

Fig. 1. The Hill sphere and the limited gravitational field. As a spacecraft is out of the Hill sphere of the Moon, the total gravitational acceleration by the force of both the Earth and Moon on it is $g_{total} = G\frac{M_e}{r_{es}^2} + G\frac{M_m}{R^3}r_{es}$; As inside the Hill sphere of the Moon, it becomes $g_{total} = G\frac{M_m}{r_{ms}^2} + G\frac{M_e}{R^3}r_{es}$. It means that the field of the Moon is limited into the Hill sphere of the Moon.

Therefore, there are two observations for the limited field in the Earth-Moon system: 1) The field of the Moon is trapped or limited in the Hill radius of the Moon. It cannot extend infinitely. In another words, a gravitational field can be trapped into a limited zone by a large one. 2) Because of trapped or limited into the Hill radius zone, the force of the moon acting on the planet is not $\mathrm{F} = \mathrm{G}\frac{Mm}{R^2}$. It also the perturbation force, just as the force of the Moon on an artificial satellite around the Earth is only the perturbation force for that this satellite is out of the Hill sphere of the Moon.

Therefore, there are the conclusions for the gravitational unit. 1) A primary gravitational unit is usually made up of a planet and a moon. It is inside the Hill sphere of the planet as shown in Fig. 3. As the distance between the planet and the moon is less than $\frac{1}{2}r_H$ (r_H is the Hill radius of the planet), the orbit of the moon is stable[10]. 2) A gravitational unit has an orbit around the Sun and the orbit is only determined with the velocity and mass of the planet. 3) The fields of the planet and moon are unified as one single field to interact with the Sun. In this case, the direct action of the Sun on the planet is $g_s = G\frac{M_s}{R^2}$ while on the moon only is $g_{perturb} = G\frac{M_s}{R^3}r$. 4) The gravitational field of the moon is limited or trapped into the Hill sphere. It cannot have the action of $\mathrm{F} = \mathrm{G}\frac{m}{R^2}$ on the planet. 5) The primary gravitational units can be combined to a larger unit, for example, the Sun-planets-moons unit contains several planet-moon units.

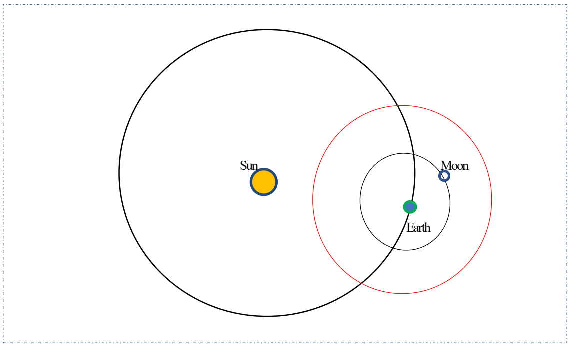

Fig. 3. The Hill sphere and gravitational union. The red circle is the Hill sphere. The Earth-Moon system orbits around the Sun on the orbit of the Earth as one single gravitational unit. The force of the Sun on the Earth or that of the Earth on the Moon is $\mathrm{F} = \mathrm{G}\frac{Mm}{R^2}$ while the Sun on the Moon is $g_{perturb} = G\frac{M_s}{R^3}r$.

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3. Poincaré's equation for Three-body problem and the understanding about gravitational field

Continuing a series of previous studies for the Three-body problem,[25-28] in 1892-1899, Poincaré[29] published his equation for the Three-body problem.

Denote the three masses by M_i, where i = 1, 2 and 3, the positions of them with respect to the origin of a Cartesian coordinate system by the vectors $\boldsymbol{R_i}$, and define the position of one body with respect to another by $\boldsymbol{r_{ij}} = \boldsymbol{R_j} - \boldsymbol{R_i}$, where $\boldsymbol{r_{ij}} = -\boldsymbol{r_{ji}}$, j = 1, 2, 3 and $i \neq j$, Poincaré's equation is [25]

$$M_i \frac{d^2 \boldsymbol{R_i}}{dt^2} = G \sum_{j=1}^{3} \frac{M_i M_j}{r_{ij}^3} \boldsymbol{r_{ij}} \quad (6)$$

Comparing Eq.(6) to Eq.(2), we found, in Eq.(2) there is a central force from a central mass. It is different from the perturbative force by other bodies. While in Eq.(6), the force of anybody interacting with another one is always $\mathrm{F} = \mathrm{G}\frac{Mm}{R^2}$. So, in Eq.(6), no orbit in a Three-body system could be stable. (Even the orbit of artificial satellite around the Earth is unstable. The orbit of a real artificial satellite is only acted by the Sun and Moon with the perturbative force. So, Poincaré's restricted Three-body problem is questioned.) So, Eq.(6) is invalid to understand why a real orbit, including the orbit of the Moon or an artificial satellite around the Earth, is stable.

We noticed, in Eq.(2), the central mass with the central force for the Moon is a result of observation. For example, for the Sun-Earth-Moon system, it only can be an observation that the central mass for the Moon orbiting around is the Earth. Conversely, if calculated with $\mathrm{F} = \mathrm{G}\frac{Mm}{R^2}$, the central mass could be the Sun for the calculated force of the Sun on the Moon is 2.2 times that of the Earth. It means that, in the perturbation theory, the interaction of gravitational field was taken as a condition implied in Eq.(2). It defines that the Sun cannot have the force $\mathrm{F} = \mathrm{G}\frac{Mm}{R^2}$ on the Moon. But, in the Poincaré's equation, no observation about the real orbit was considered. It is a pure mathematics derivation based on $\mathrm{F} = \mathrm{G}\frac{Mm}{R^2}$. The Sun-Earth-Moon system and Sun-Pluto-Charon system are typical Three-body problem. Their orbits are stable. And, if applied Poincaré's equation on the orbits of the two system, the calculated orbits should be chaotic. So, Poincaré's conclusion[29] about the Three-body problem is wrong.

The Poincaré's equation for Three-body problem is a strange emergence. First, no orbit of the celestial body is chaotic. A broken orbit also is predictable. So, Poincaré's equation cannot related with any real orbit. Second, the orbits of the typical Three-body system are stable. It is invalid to use the Poincaré's equation to understand these orbits. Third, Poincaré's equation is invalid to design an artificial orbit. It is very clear, the Poincaré's equation is nonsense in physics. But, there have been a big lots of works for the Poincaré's equation and now every year a lot of this kind of works are being published. This strange emergence need be understood and explained with the theory of scientific communication.

Comparing the Poincaré's equation with the orbital perturbation theory, it is clearly shown that, there are two different understanding about the gravitational field. It is noted that, the Poincaré's equation is based on the assumption that a gravitational field could interacted with any other one with the force $F = G\frac{Mm}{R^2}$. This assumption is prevailing in current theory of gravity. With this understanding in the Poincaré's equation, the observation of the real orbit is omitted. It only is based on mathematics derivation. Now, it presented that, physics may be lost in mathematics which results in that the development of theoretical physics is stopped.[32] We think, the Poincaré's equation is a case that physics is lost in mathematics.

In the understanding in the orbital perturbation theory, Newton used the mathematics to describe observation. But, in the age of Newton, the concept of gravitational field was not developed. Newton cannot state the interaction of field clear. Therefore, after Newton, the wrong understanding about the gravitational field is prevailing.

4. Orbital perturbation equation and repulsive gravity

Under the condition of Fig.4, as the Moon is at point M_1, the orbit of the Moon is perturbed by the Sun with:

$$g = GM_S[\frac{1}{(r-R)^2} - \frac{1}{R^2}] \quad (7)$$

where s and e denote the Sun and Earth, r and R are the distances between the Sun and Earth and between the Moon and Earth, respectively.

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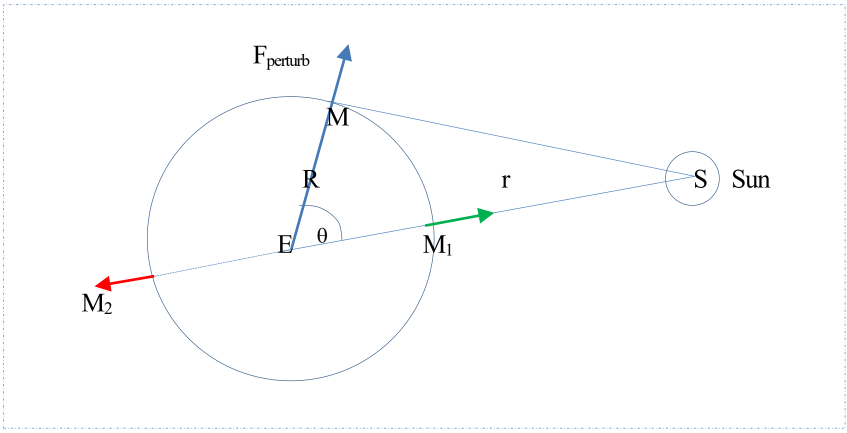

Fig. 4. The direction of the perturbative force. For convenience, assuming that the Sun, Earth and Moon and the orbit of the Moon around the Earth are on a same plane. E is the Earth, and the orbit of the Moon is circle. M is the Moon which is orbiting around the Earth. The perturbative force is directed along line connected with the Earth and Moon. The Sun, M_1, E and M_2 are on a straight line. r and R are the distance between the Sun and Earth and between the Moon and Earth respectively.

From Eq.(7), there is

$$g = \frac{GM_S}{r^3}\boldsymbol{R} \tag{8}$$

For the Moon moving on the orbit, Eq.(8) becomes: [31]

$$g = -\frac{GM_S}{r^3}R(3\cos^2\theta - 1) \tag{9}$$

It is stressed that Eq.(9) is well applied in designing the orbit of artificial satellite. Now, the force on an artificial satellite can be measured with the precision of 10^{-8}m/s^2.[33] So, Eq.(9) was well measured in practice.

It is noted that, in Eq.(9), the direction of the force of the Sun on the Moon along the line EM_2 (red arrow) is contrary to that along EM_1 (green arrow) which is the direction that the Sun attracts the Moon. This is a clear observation.

Factually, Newton formulated that, the perturbation of the Sun on the Moon is always repulsive.[3,5] Today, we can easily prove with measurement that Newton is right. At point M_1, the direction of the perturbative force is contrary to the attractive force of the Sun. And, besides along the green arrow, no other perturbative force is directed along the attractive force of the Sun. From Eq.(9) we know, the force of the Sun on the Moon is along the line connected with the center of the Earth and the Moon. It is clear that the Moon is repulsed away from the center of the Earth along the same direction. In the artificial orbit, this force has been measured with the precision of 10^{-8}m/s^2.[33] It is certainly measured that the direction of the tidal force of the Sun on an artificial satellite is just as that shown in Fig. 4. But, Newtonian repulsive gravity has not been understood in current theory of gravity. Now, it is thought that the repulsive gravity is impossible. So, we think, now it is the time to reestablish Newtonian theory of repulsive gravity. Generally, the perturbative force could be rewritten as

$$\mathcal{g} = -2g\frac{\boldsymbol{R}}{r} \tag{10}$$

where $g = \frac{GM_S}{r^2}$ is the gravitational acceleration of the Sun, r and R are the distance between the Earth and the Sun and between the Earth and Moon, respectively. $\boldsymbol{R}$ is a vector. The sign " $-$ " means that the direction of $\mathcal{g}$ is contrary to that of g.

It is noted that, now, it is an observed result that the perturbative force is repulsive gravity. The reason that results in the repulsive gravity has not been known in current theory. A new theory is needed to know the reason.

5. Hill sphere and gravitational interaction

In history, the Hill sphere was derived from Newtonian theory of gravity through the formula analogous to that Poincaré used to derive the equation for the Three-body problem. Because Poincaré's equation is invalid, this derivation also is questioned. In this derivation, the condition for the Hill sphere is that, for mass M and m, it is needed that m/M is very little. But, in the Pluto-Charon system, there is $\mathrm{m/M} \approx 0.12$. It is clear, the Hill sphere is valid to

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system. So, the Hill sphere need be re-understood with this system. It is clear, the Hill sphere is a valid theory in understanding the celestial orbit and in designing artificial interplanetary orbit. So, we prefer to believe that, the Hill sphere is an observed result.

Here, we present that, the Hill sphere could be better understood with the orbital perturbation theory. We think, the Hill sphere and the orbital perturbation theory are complementary for each other. Both of them are needed to understand the orbits of the Three-body problem. The Hill sphere studies the stability zone of the orbit while the orbital perturbation theory studies the force of everybody on the orbit. It is clear, the stability zone is determined with the force.

It is noted that, in current theory of gravity, the Hill sphere and the orbital perturbation theory are independent of each other. It is in this work, it is first presented that, the force inside and out of the Hill sphere is determined with orbital perturbation theory. Currently, it is known, the Hill sphere means that, inside the Hill radius of r_H of the Moon, the Moon dominates the gravity. But, it is not clear what are the force inside and out of the radius. The orbital perturbation theory determined that the total force of the Earth and Moon on a satellite is Eq.(4) or (5). But, it is not clear what is the distance (radius) that the force of Eq.(4) or (5) can function. So, we think, the Hill sphere and the orbital perturbation theory are two sides of a coin. The orbital perturbation theory determined that the total force is Eq.(4) or Eq.(5). While the Hill radius determined the distance that Eq(4) or Eq.(5) can function. i.e., inside the Hill radius Eq.(5) functions while out of the Hill radius Eq.(4) functions. For the reason, the Eq.(1) and (9) could be combined as

$$g = -2g\frac{\boldsymbol{R}}{r}, \ \mathrm{R} \le r_H \tag{11}$$

where r_H is the Hill radius which is determined in Eq.(1).

In Eq.(11), because the perturbative force is measured with high precision, the force in a Hill sphere is clear. As shown in Fig.5, in the Hill sphere of the Earth, the Earth dominates the gravitational force. The force of the Sun in the Hill sphere is only the perturbative force of $g = -2g\frac{\boldsymbol{R}}{r}$. The force $g = -2g\frac{\boldsymbol{R}}{r}$ is a repulsive force which repulses the Moon from away the center of the Earth along the Earth-Moon line. There should not be the force of $\mathrm{F} = \mathrm{G}\frac{M_S}{r^2}$ along the Sun-Moon line.

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As sentenced above, this is a well observed result. It was well known that, the force of the Sun on the Moon is the perturbative force. In this sense, the perturbative force and the Hill sphere are the two surface of the same interaction. The Hill radius determines the distance that, in the Sun-Earth system, the perturbative force can function while the perturbative force determines that what is the force in the Hill sphere.

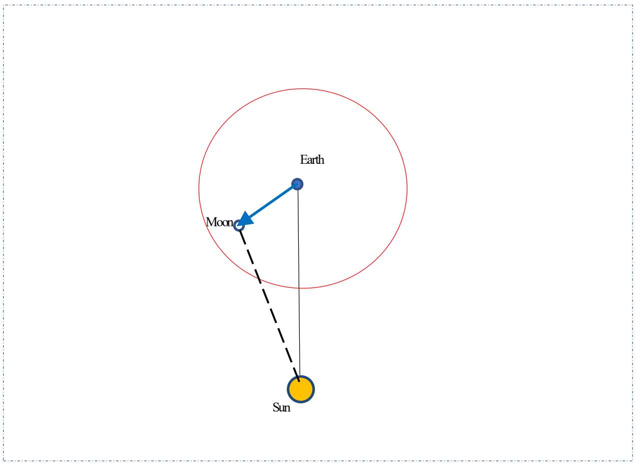

Fig. 5. **Hill sphere and the force of the Sun on the Moon.** In the Hill sphere, the Earth dominates the gravitational force on the Moon. The force of the Sun on the Moon is $\mathscr{g}_s = -2g_s\frac{\boldsymbol{R}}{r}$ as shown in the blue arrow, where $g_s = G\frac{M_s}{r^2}$. $\mathscr{g}_s$ means that the force of the Sun repulses the Moon away from the center of the Earth. There is no force along the line connected the Sun and Moon

In current theory of gravity, it is generally accepted that, in the Hill sphere of the Earth, the Earth dominates the gravity on the Moon. While it is unclear what is the force of the Sun on the Moon. However, this force need be known. Here, we may clarify this force.

Now, the Hill sphere of some bodies in the solar system was probed.[34,35] In technology, the Hill sphere for the Sun-planet-moon system can be accurately measured. This measurement should accurately show how the gravitational field of the moon is limited and how the gravitational fields of both the planet and moon is unified.

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6. Interaction of gravitational field

Now, we have known little about the interaction of gravitational field. Even we have not had a line to observe the field.

Till now, only the one single gravitational field, which is not interacting with other one, has been studied. It usually is believed that all the gravitational field can extend infinitely. But, factually, none of the gravitational field is isolating from other ones. All of them are always interacting with others. It is different from the electric field. There is the isolated electric field which is not interacting with other ones. So, the gravitational field only can be understood with the interacting fields.

In the Newtonian theory of gravity, the gravitational force is $F = G\frac{Mm}{R^2}$. It clearly showed that, the gravitational force is an interaction between two fields of M and m. If the gravitational force is propagated with the gravitational field, the variation of the field shall result in the variation of the force. On another hand, as the force is varied, the field also correspondently is varied. The orbit is determined with the gravitational force. Thus, the variation of the field can be observed through the orbit.

In the above, from the orbit, we showed three observations for the interaction of multi-field. First, it is the equation of orbital perturbation. Second is the perturbative force. Third is the Hill sphere. From the three observations, the interaction of gravitational field could be described with a set of equations.

$$\begin{cases} g = g_{center} + \sum_{i=1}^{n} \mathcal{g}_i \\ g_{center} = G\frac{M_{center}}{R^2} \\ \mathcal{g}_i = -2g_i\frac{\boldsymbol{R}}{r_i}, R \le r_H \\ r_H \approx (1-e)a\sqrt[3]{\frac{M_{center}}{3M}} \end{cases} \tag{12}$$

In Eq.(9), g_{center} is the gravitational acceleration from the force of the center mass M_{center} on the mass m which is orbiting around the M_{center} with the radius R. M_{center} and m is unified to a gravitational unit. r_i is the distance between M_{center} and m_i. $g_i = G\frac{m_i}{r_i^2}$. $\mathcal{g}_i$ is the perturbation of m_i to the m which repulses the

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m away from the center of the M_{center} along the direction of r_i. M is the mass that M_{center} is orbiting around. Just as that the force the Sun on the Moon is the perturbative force, the force of M on m also is the perturbative force.

It is clear, in Eq.(9), the four equations are related with each other as a whole. The first equation is general to describe the interaction of multi-field of the $\mathrm{M}_{center} - \mathrm{m} - \mathrm{m}_i$ system. The second equation shows that the center mass M_{center} and the mass m unified as a gravitational unit. The third equation shows that the force of m_i on m is repulsive. The fourth equation determines the radius of the gravitational unit and repulsive gravity.

Eq.(12) is analogous to Maxwell equations for the electromagnetic interaction. The Maxwell equations is established from the well-developed electromagnetic theories, including the Coulomb law, the current law, the Biot-Savart law and the Faraday's law of induction. Our equations also is established from the well-developed theories of gravity. But, in physics, our equation is different from Maxwell equations. In the Maxwell equations, there are two different kinds of fields, i.e., the electric and magnetic fields. In our equations, there is only one kind of field.

Our equations is different from Einstein's field equation. Till now, Einstein's field only has had the spherically symmetric solution for one field.[36] It cannot be used to understand the interaction of multi-field. It was known by many people that the orbit perturbed by multibody cannot be studied with Einstein's field equation. So, the Newtonian theory of gravity is the unique theory to understand and to design the celestial and artificial orbit.[37]

It is worth of emphasizing that Eq.(12) is well confirmed. In a simple word, it is needed to design an artificial orbit. In current theory of gravity, they are separated independently. In our work, they are related with the interaction of gravitational field. It is very interesting, as we list the equations in current theory of gravity in one page, we shall can obtain new results. For example, as the orbital perturbation equation and the Poincaré's equation for Three-body problem are listed in one page, we can easily find the difference between them which clearly shows that the Poincaré's equation is valid to the real orbit. And, as the orbital perturbation equation and the Hill sphere listed in one page, we can easily find that the force of the Earth on the Moon is different from that of the Sun on the Moon. It means that it is the time to establish a set of equations for the theory of gravity from current theory.

7. Binary planet and gravitational unit

To explain the motion of the Moon around the Sun, it was presented that the Earth-Moon system is binary planet.[18] Therefore, the binary planet is factually another kind of gravitational unit. The conclusions analogous to that deduced from our concept of gravitational unit, such as the gravitational field of the moon could be limited inside the unit, can be deduced from the binary planet. The Pluto and its moons orbit around the barycenter of the Pluto-Charon system factually just means that the Pluto cannot act on its moons with the force of $\mathrm{F} = \mathrm{G}\frac{M}{R^2}$. So, in the binary planet, not only the field of the moons is limited, but also that of the planet is done so. It seems that, the concept of binary planet is a strong evidence for our result. But, we found, in mechanics and dynamics, the binary planet/star/ blackhole cannot exist.

In 1870, it was claimed that the binary star was observed.[24] The binary system was taught in current textbooks of celestial mechanics.[4] It was believed that the Pluto-Charon system is binary planet. And, it was worked out that, the Pluto and its moons orbit around the barycenter of the Pluto-Charon system. [20-23] It is different from other moons that orbit around a planet. The binary star is at a very distant place from us. Compared to the Pluto-Charon system, it is much more difficult to have accurate and precision observation. So, we think, the binary star/planet could be better observed and understood from the Pluto-Charon system.

How to show the orbits of the moons of the Pluto in a figure? It is a very easy to know whether or not the Pluto-Charon system is binary planet although it was currently confused.[20-23] As the orbits in the Sun-Pluto-moons are shown in a single figure, it shall be clarified at once.

The orbit of the Sun-planet-moons system is well-known. For the Sun-Pluto-moons system, we have Fig.6A. The Pluto orbits around the Sun while the moons around the Pluto. Fig.6A may be accepted by everyone.

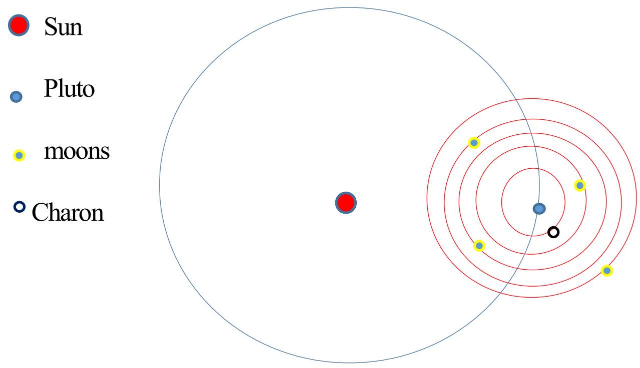

Fig.6A. The orbits of the Pluto and its moons. The Pluto orbits the Sun while all of its moons (including the Charon) orbit around the Pluto.

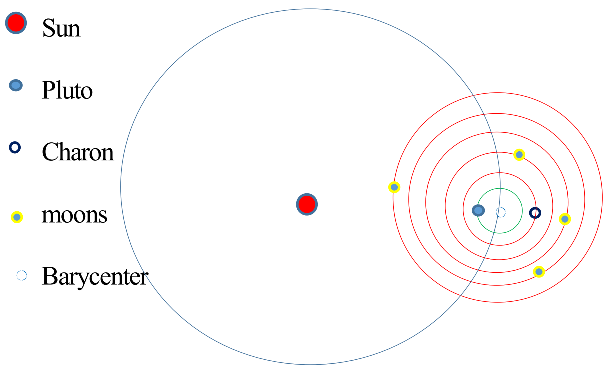

Fig.6B. The current thought of The orbits of the Pluto and its moons. The Pluto, Charon and other four little moons orbit the barycenter of the Pluto-Charon system. And, the Pluto is not on the orbit around the Sun

But, for the Pluto-Charon system, if it was true that, "Pluto's motion is the result of the combination of its motion around the Sun, and its motion around the barycenter of its system"[20] and the other four little moons orbit around the barycenter,[20-23] what a figure can we have?

In Fig. 6A, the moons cannot orbit around the barycenter of the Pluto-Charon system. And, under the condition that the Pluto is moving on the orbit around the Sun, that the four little moons orbit around the barycenter of the Pluto-Charon system cannot be shown with a figure.

If as currently believed that "Pluto's motion is the result of the combination of its motion around the Sun, and its motion around the barycenter of its system" [20], the figure of the Pluto system orbits around the Sun should be Fig. 6B. If it should be so, it is the barycenter that is on the orbit of the Pluto-Charon system around the Sun. While the Pluto is not on this orbit. And, the direction of the Pluto's motion is varied and can be contrary to the direction of this orbit.

However, there is no evidence for Fig. 6B. All observations showed that the Pluto is on the orbit around the Sun. The direction of the Pluto's motion is just along the direction of the orbit as shown in Fig. 6A.

The direction of the Pluto moving around the Sun can be easily observed. It was worked out that the period of the Pluto-Charon orbiting their barycenter is 6.38723 Earth days.[21] The variation of the direction of the Pluto's motion relative to the orbit around the Sun is also in a period of almost 6. 38723 days. Till now, no observation has shown such a variation.

It was observed that, the orbital parameters determined for this system show that the two bodies evolve in an almost circular orbit with a 6.387-day period. And, the brightness of Pluto varies by some tens of percent with a period of 6.387 days.[20] But, in mechanics and dynamics, there is no explanation for that why and how the Pluto-Charon system can be binary planet.

In fact, there is no mechanics and dynamics for the binary star/planet. Usually, the binary star/planet also is called as "double star/planet". There are two current descriptions about the binary star. First, the two stars/planets orbit around the barycenter of the two stars/planets. If the Pluto and Charon are orbiting around the barycenter as shown in Fig.3. From $G\frac{Mm}{R^2} = \frac{mv^2}{R}$ we know, the orbital velocity of the Pluto should be larger than that of the Charon for that the distance between the Pluto and the barycenter is less than that between the Charon and the barycenter. Therefore, the distance between the Pluto and Charon should be varied by the velocities of the Pluto and Charon orbiting around the barycenter. Correspondently, it should make that the distance between the barycenter and

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the Pluto/Charon varied chaotically. In this case, the orbits of the Pluto and Charon also should be chaotic. And, the orbits of other little moons also should be chaotic. However, this is contradicted with observation. So, we think, only the Fig. 6A is valid for the orbits of the Pluto and Charon. It is well known that, the most part of stars has an orbit around a center analogous to that the Pluto orbits around the Sun. The lonely star is rare and cannot exist in long time for it shall be captured or collided by other ones. So, we think, our conclusion for the orbits of the Pluto and Charon is suitable for the binary star. The second description for the binary star is that, the two stars orbit around each other. It also is contradicted with current mechanics. As two stars are orbiting around each other, from $G\frac{Mm}{R^2} = \frac{mv^2}{R}$ we know, the orbital velocity of the star with larger mass is less than that with less mass. It also results in that the distance between the two stars is chaotic which should make the orbits of them chaotic. It also contradicted with observation.

It is emphasized, in visual, the Pluto and Charon appears as binary. Observed at the Charon, it appears that the Pluto was orbiting around the Charon just as that, observed at the Earth, the Sun was orbiting around the Earth. As we assume that the Earth is stationary, the line of the Sun moving around the Earth just is a cycle. It exactly appears as an orbit of the Sun around the Earth. In geometrics, as a body moves within a constant distance from another body, observed at any one of the two bodies, the line of another body moving is a circle. Therefore, observed at an artificial satellite around the Earth, the line that the Earth moving just is a circle around the satellite. It is certain, no one should thank that the Earth and the satellite is binary. So, for two stars in a very distance from us, the orbit of the main star orbit around a center is difficult to be observed, it also appears as that the two stars are orbiting around each other. But, after Copernicus heliocentric theory, we know it is unsuitable to say that the Sun is orbiting around the Earth. It is worth noting that, before Copernicus heliocentric theory, many celestial orbits and accurate ephemeris were well predicted with the geocentric theory. But, we cannot say that the geocentric theory is right. Therefore, although the binary star/planet is an observed result, it need be re-understood. So, we think, binary star/planet may be a misunderstanding by the geometric picture of the orbital motion.

We know, the four small moons were discovered in the predicted dynamical stability zoo as the Pluto-Charon system was treated as binary.[23] But, we do not think it could be an evidence that the binary system can be exist in mechanics and dynamics. It is noted, in mathematics, the difference between the dynamical stability zoo of the Pluto-Charon system and that of the Pluto is little.

Today, binary star is very fashioning. It was reported that the multi-star system was observed.[38] It is believed that there are the double black hole and double neutron star. And, the LIGO and VIRGO's detections of gravitational waves are based on the assumption of the double black hole[39] and double neutron star.[40] So, it is very important to clarify the binary system.

8. Orbits in a galaxy and gravitational unit

It is clear the Sun-planets-moons system is one gravitational unit orbiting around the center of the Milk Way. The orbit of a star-planets-moons system is dominated by the mass of the center of the Milky Way. In this case, the field of the planets and the moons are limited. And, the other stars cannot have the force of $g = \mathrm{G}\frac{M}{r^2}$ on the orbit of this star around the center of the Milky Way. If it was not so, the force of the Milk Way gravitating the stellar system should be determined with $\sum\overrightarrow{g_\imath}$, where $\overrightarrow{g_\imath} = \mathrm{G}\frac{m_i}{r_i^2}\overrightarrow{r_\imath}$, m_i is the mass of a star, planet or moon and *i* is the number of the planet, star and moon in the Milk Way, r_i is the distance between this star and m_i, $\overrightarrow{r_\imath}$ is a vector. Approximately, the total mass of the Milky Way is 5.8×10^{11} times that of the Sun[41] and the mass of center of it is only 4.5×10^{6} times that of the Sun.[42] If the Star (or stellar system) was dominated by $\sum\overrightarrow{g_\imath}$, it should result in that the star (stellar systems) in different locations of the Milk Way could not be orbited around the center of the Milk Way for that the barycenter of mass of the Milk Way is determined with $\sum r_i m_i/\sum m_i$. For the stars in different locations of the Milk Way, the barycenter of the mass is different. Therefore, the only condition for the stars orbiting around the center of the Milk Way is that the gravitational field of these stars can be limited. The star (stellar system) is not gravitated by $\sum\overrightarrow{g_\imath}$, but only by the center of the Milk Way with $g = \mathrm{G}\frac{M}{R^2}$. In another words, the center of the Milk Way dominates the core of the stellar systems, this core dominates the stars, and the star dominates the planets. The mass of cores, stars and planets of a stellar system cannot affect the orbit of another stellar system.

We know, the dark matter was presented from the Galaxy rotation curves.[43] But, a recent observation[44] shows that, the orbits of stars and other matter in the spiral galaxy are dominated by the Newtonian law of gravity. The two observations are contradicted with each other. So, we stressed that, as there are problems to observe and to explain the orbits of the Earth-Moon and Pluto-Chron system, we need valid theory and much more accurate and precession observation to have valid knowledge for the orbits in a galaxy. It is noted that, in current theory for the galaxy dynamics, the baryonic mass of a galaxy (the sum of its stars and gas) correlates with the amplitude of the flat rotation velocity.[45] But, we think, the orbital perturbation theory and the Hill sphere are valid to the orbital velocity in a galaxy.

9. Conclusion

Newton formulated the orbital perturbation theory and the repulsive gravity.[3,5] Therefore, he factually laid the foundation for developing the theory of interaction of gravitational field. But, in Newton's time, the theory of field had not been known. And, till now, it is prevailingly accepted that any mass can interact with another one with the force of $F = G\frac{Mm}{r^2}$. It resulted in that the theory for the interaction of gravitational field cannot be developed. After Newton, the theory about gravity was developed in these aspects. First, in 1900, the theory of field for the electricity and magnetism was established. The electromagnetic interaction was described with the Maxwell equation. It led to that the gravitational interaction could be described with the field by analogy to the electromagnetic field. Second, the Hill sphere was presented and generally applied.[6-14] It factually shows some of features of the interaction of gravitational field. Third, the artificial orbit become a general project which have been exploited by many nations. The gravitational force on an artificial satellite can be measured with high precision. Based on these developments, a new theory of gravity could be developed.

In our work, almost all of the current theories about orbit are investigated with the orbital perturbation theory. Our main conclusion is that the planet and moon is unified as one single gravitational unit and the field of the moon is limited in the unit which cannot interacted with other ones out of the unit with $F = G\frac{Mm}{r^2}$. We repeat to emphasize that this conclusion was implied in the orbital perturbation equation. Therefore, our conclusions are based on the well-developed and well-applied theory. Now the gravitational force with the precision of 10^{-8}m/s^2 perturbing to the artificial orbit can be measured.[33] So, our conclusions are well confirmed experimentally and observationally. In another hand, we show that, the theory that is contradicted with the orbital perturbation theory, such as the Poincaré's equation for Three-body problem, is invalid. Therefore, our results should be fundamental to the theory of gravity. Especially, that the interacting gravitational field could be limited and that the perturbation of the Sun on Moon should be repulsive gravity should lead to remodel the theory of gravitational field.

By analogy to the solar system, Ernest Rutherford[46] presented his atom model. Our observation shows that the planet and moon also unified as one single solid unit by gravitational field while the planet and moon are separated in space. It is analogous to that the nuclei and electrons are unified as an atom. The space for the interaction of gravitational field is much larger than that for the electromagnetic field, some of the new features of the interaction of field should be discovered from the interaction of gravitational field.

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