# FOUR DIMENSIONAL CIRCULAR MODEL TO MEASURE INTERACTION OF TWO ECONOMIES 

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#### Abstract

In the book "Everyhing Is A Circle: A New Model For Orbits Of Bodies In The Universe", and further in the Article "Distance Between Two Circles in Any Number of Dimensions is a Vector Ellipse", it has been mathematically demonstrated that "distance between points on any two different circles in any number of multiple dimensions" is equivalent to "distance of points on a vector ellipse from another fixed or moving point". Using this mathematical methodology, a method is provided in this Article as a measure for the amount of interaction between two international economies, which are two countries or economic zones, by modelling each economy in terms of a "circle in four dimensions". Based on this method, the proximity of the two economies at the end of each fiscal period, which is generally a fiscal year, is then measured by the distance between points at the end of the given period on the two circles in four dimensions, associated with these two economies in our model.


GNI, GDP, GNIpc, International Economy, Export, Import, Circle, Model

## ARTICLE

Gross National Income (GNI) ${ }^{1}$ is a country's total domestic and foreign income claimed by its residents during in a fiscal period, consisting of Gross Domestic Product (GDP) ${ }^{2}$ plus net receipts of income from abroad.

Gross Domestic Product (GDP) ${ }^{2}$ is the measure of income earned through the production of goods and services in a country during a fiscal period, or equivalently the total amount spent on goods and services less imports.
"GNI per capita"" is a country's final income in a fiscal period, divided by its population.

[^0]In the book "Everyhing Is A Circle: A New Model For Orbits Of Bodies In The Universe" ${ }^{4}$, and further in the Article "Distance Between Two Circles in Any Number of Dimensions is a Vector Ellipse", it has been mathematically demonstrated that "distance between points on any two different circles in any number of multiple dimensions" is equivalent to "distance of points on a vector ellipse from another fixed or moving point". Using this mathematical methodology, in this Article we aim to provide a measure for the amount of interaction between two international economies, in other words between two countries or economic zones, by modelling each economy in terms of a "circle in four dimensions". The proximity of the two economies at the end of each fiscal period, which is generally a fiscal year, can then be measured by the distance between points at the end of the given period on the two circles in four dimensions, corresponding to these two economies in our model.

As a background, consider a system of two circles in four-dimensions defined in terms of perpendicular coordinates $\left(\hat{\boldsymbol{u}}_{1}, \hat{\boldsymbol{u}}_{2}, \hat{\boldsymbol{u}}_{3}, \hat{\boldsymbol{u}}_{4}\right)$.

The phases $\phi_{1}(t)$ (2) and $\phi_{2}(t)$ (3), respectively, of points $\mathbf{P}_{1}$ and $\mathbf{P}_{2}$ on these two circles, moving as a function of time $t$ with respect to the centers of their own circles, and their phase difference $\phi_{0}(t)$ (4), is expressed in (2) - (4), with the moving points $\mathbf{P}_{1}$ and $\mathbf{P}_{2}$ phased with fixed but different (1) angular velocities $\omega_{1}$ (2) and $\omega_{2}$ (3), respectively, with respect to the centers of their own circles of revolution, their phase difference being $\left[\phi_{0}\left(t_{0}\right)=0\right]$ (4) at time $\left(t=t_{0}=0\right)$, the reference timestamp $t_{0}$ taken to be the point in time when points $\mathbf{P}_{1}$ and $\mathbf{P}_{2}$ have a phase of $\left[\phi_{1}\left(t_{0}\right)=\phi_{2}\left(t_{0}\right)=0\right]$ (2) - (3) simultaneously.

$$
\begin{equation*}
\omega_{1} \neq \omega_{2} \quad\left(\omega_{1} \& \omega_{2} \text { constant }\right) \tag{1}
\end{equation*}
$$

$$
\begin{array}{rlrl}
\phi_{2}(t)=\phi=\phi(t)=\omega_{2} t & ; \quad \phi_{2}\left(t_{0}=0\right)=0 & \left(\text { Phase of } \mathbf{P}_{2}\right) \\
\phi_{0}(t)=\phi_{1}(t)-\phi_{2}(t)=\left(\omega_{1}-\omega_{2}\right) t & \left.; \quad \phi_{0}\left(t_{0}=0\right)=0 \quad ; \quad \text { (Phase difference of } \mathbf{P}_{1} \text { and } \mathbf{P}_{2}\right) \tag{4}
\end{array}
$$

$$
\begin{equation*}
\phi_{1}(t)=\phi+\phi_{0}=\phi(t)+\phi_{0}(t)=\phi_{2}(t)+\phi_{0}(t)=\omega_{1} t \quad ; \quad \phi_{1}\left(t_{0}=0\right)=0 \quad\left(\text { Phase of } \mathbf{P}_{1}\right) \tag{2}
\end{equation*}
$$

The two circles can be defined in the simplest case to have vector radii $\overrightarrow{\boldsymbol{r}}_{1}(5)-(6)$ and $\overrightarrow{\boldsymbol{r}}_{2}(7)$, with constant magnitudes $r_{1}$ (8) and $r_{2}$ (8), respectively, which are radius vectors at point $\mathbf{P}_{1}$ phased at $\left(\phi+\phi_{0}\right)$ (2) and $\mathbf{P}_{2}$ phased at $\phi$ (3) on the two circles, respectively, phased apart by a time $(t)$-dependent angle $\phi_{0}$ (4). Note that $\beta_{1}(5)-(6)$ is taken to be the constant inclination angle between these two circles in the $\hat{\boldsymbol{u}}_{1}-\hat{\boldsymbol{u}}_{3}$ dimension plane, and $\beta_{2}$ (5) - (6) is taken to be the constant inclination angle between these two circles in the $\hat{\boldsymbol{u}}_{2}-\hat{\boldsymbol{u}}_{4}$ dimension plane.

$$
\begin{align*}
& \overrightarrow{\boldsymbol{r}}_{1}=\overrightarrow{\boldsymbol{r}}_{1}\left(\phi+\phi_{0}\right)= \hat{\boldsymbol{u}}_{1} r_{1} \operatorname{Cos}\left(\phi+\phi_{0}\right) \operatorname{Cos} \beta_{1}+\hat{\boldsymbol{u}}_{2} r_{1} \operatorname{Sin}\left(\phi+\phi_{0}\right) \operatorname{Cos} \beta_{2}  \tag{5}\\
&+\hat{\boldsymbol{u}}_{3} r_{1} \operatorname{Cos}\left(\phi+\phi_{0}\right) \operatorname{Sin} \beta_{1}+\hat{\boldsymbol{u}}_{4} r_{1} \operatorname{Sin}\left(\phi+\phi_{0}\right) \operatorname{Sin} \beta_{2} \\
& \overrightarrow{\boldsymbol{r}}_{1}=\overrightarrow{\boldsymbol{r}}_{1}(\phi)=\left(\hat{\boldsymbol{u}}_{1} r_{1} \operatorname{Cos} \beta_{1} \operatorname{Cos} \phi_{0}+\hat{\boldsymbol{u}}_{2} r_{1} \operatorname{Cos} \beta_{2} \operatorname{Sin} \phi_{0}+\hat{\boldsymbol{u}}_{3} r_{1} \operatorname{Sin} \beta_{1} \operatorname{Cos} \phi_{0}+\hat{\boldsymbol{u}}_{4} r_{1} \operatorname{Sin} \beta_{2} \operatorname{Sin} \phi_{0}\right) \operatorname{Cos} \phi+  \tag{6}\\
&\left(-\hat{\boldsymbol{u}}_{1} r_{1} \operatorname{Cos} \beta_{1} \operatorname{Sin} \phi_{0}+\hat{\boldsymbol{u}}_{2} r_{1} \operatorname{Cos} \beta_{2} \operatorname{Cos} \phi_{0}-\hat{\boldsymbol{u}}_{3} r_{1} \operatorname{Sin} \beta_{1} \operatorname{Sin} \phi_{0}+\hat{\boldsymbol{u}}_{4} r_{1} \operatorname{Sin} \beta_{2} \operatorname{Cos} \phi_{0}\right) \operatorname{Sin} \phi \\
& \overrightarrow{\boldsymbol{r}}_{2}=\overrightarrow{\boldsymbol{r}}_{2}(\phi)=\hat{\boldsymbol{u}}_{1} r_{2} \operatorname{Cos} \phi+\hat{\boldsymbol{u}}_{2} r_{2} \operatorname{Sin} \phi  \tag{7}\\
& \overrightarrow{\boldsymbol{r}}_{1} \cdot \vec{r}_{1}=r_{1}^{2} \quad ; \quad\left|\overrightarrow{\boldsymbol{r}}_{1}\right|=r_{1}=\sqrt{\boldsymbol{r}_{1} \cdot \overrightarrow{\boldsymbol{r}}_{1}} \quad ; \quad \overrightarrow{\boldsymbol{r}}_{2} \cdot \overrightarrow{\boldsymbol{r}}_{2}=r_{2}^{2} \quad ;\left|\overrightarrow{\boldsymbol{r}_{2}}\right|=r_{2}=\sqrt{\overrightarrow{\boldsymbol{r}}_{2} \cdot \overrightarrow{\boldsymbol{r}}_{2}} \tag{8}
\end{align*}
$$

The centers of these two circles in four-dimensions are displaced by a constant vector $\vec{\ell}$ (9) with magnitude $\ell(10)$ at each phase $\phi$, defined in terms of perpendicular coordinates $\left(\hat{\boldsymbol{u}}_{1}, \hat{\boldsymbol{u}}_{2}, \hat{\boldsymbol{u}}_{3}, \hat{\boldsymbol{u}}_{4}\right)$.

$$
\begin{gather*}
\vec{\ell}=\hat{\boldsymbol{u}}_{1} \ell_{u_{1}}+\hat{\boldsymbol{u}}_{2} \ell_{u_{2}}+\hat{\boldsymbol{u}}_{3} \ell_{u_{3}}+\hat{\boldsymbol{u}}_{4} \ell_{u_{4}}  \tag{9}\\
|\vec{\ell}|=\ell=\sqrt{\vec{\ell} \cdot \vec{\ell}} ; \quad \ell^{2}=\vec{\ell} \cdot \vec{\ell}=\ell_{u_{1}}^{2}+\ell_{u_{2}}{ }^{2}+\ell_{u_{3}}^{2}+\ell_{u_{4}}{ }^{2} \tag{10}
\end{gather*}
$$

The vector distance between points $\mathbf{P}_{1}$ phased at $\left(\phi+\phi_{0}\right)$ (2) and $\mathbf{P}_{2}$ phased at $\phi$ (3) on the two respective circles in four dimensions, at any value of the phase $\phi$ (3), can be equivalently expressed as $\overrightarrow{\boldsymbol{d}}(\phi)$ (11) in terms of vectors $\overrightarrow{\boldsymbol{a}}$ (12) and $\overrightarrow{\boldsymbol{b}}$ (13), defined utilizing $\overrightarrow{\boldsymbol{r}}_{1}$ (5) - (6) and $\overrightarrow{\boldsymbol{r}}_{2}$ (7), with magnitudes $a$ (14) and $b(15)$, respectively, and $\operatorname{Dot}^{\operatorname{Product}^{6}}(\overrightarrow{\boldsymbol{a}} \cdot \overrightarrow{\boldsymbol{b}})$ (16).

$$
\begin{equation*}
\overrightarrow{\boldsymbol{d}}(\phi)=\overrightarrow{\boldsymbol{r}}_{1}(\phi)-\overrightarrow{\boldsymbol{r}_{2}}(\phi)+\vec{\ell} \quad \text { where } \quad \overrightarrow{\boldsymbol{r}}_{1}(\phi)-\overrightarrow{\boldsymbol{r}_{2}}(\phi)=\overrightarrow{\boldsymbol{a}} \operatorname{Cos} \phi+\overrightarrow{\boldsymbol{b}} \operatorname{Sin} \phi \tag{11}
\end{equation*}
$$

$$
\begin{align*}
& \overrightarrow{\boldsymbol{a}}=\hat{\boldsymbol{u}}_{1}\left(r_{1} \operatorname{Cos} \beta_{1} \operatorname{Cos} \phi_{0}-r_{2}\right)+\hat{\boldsymbol{u}}_{2} r_{1} \operatorname{Cos} \beta_{2} \operatorname{Sin} \phi_{0}+\hat{\boldsymbol{u}}_{3} r_{1} \operatorname{Sin} \beta_{1} \operatorname{Cos} \phi_{0}+\hat{\boldsymbol{u}}_{4} r_{1} \operatorname{Sin} \beta_{2} \operatorname{Sin} \phi_{0} \\
& \Rightarrow \overrightarrow{\boldsymbol{a}}=\left[\overrightarrow{\boldsymbol{r}}_{1}(\phi)-\overrightarrow{\boldsymbol{r}}_{2}(\phi)\right](\phi=0)  \tag{12}\\
& \overrightarrow{\boldsymbol{b}}=-\hat{\boldsymbol{u}}_{1} r_{1} \operatorname{Cos} \beta_{1} \operatorname{Sin} \phi_{0}+\hat{\boldsymbol{u}}_{2}\left(r_{1} \operatorname{Cos} \beta_{2} \operatorname{Cos} \phi_{0}-r_{2}\right)-\hat{\boldsymbol{u}}_{3} r_{1} \operatorname{Sin} \beta_{1} \operatorname{Sin} \phi_{0}+\hat{\boldsymbol{u}}_{4} r_{1} \operatorname{Sin} \beta_{2} \operatorname{Cos} \phi_{0} \\
& \Rightarrow \overrightarrow{\boldsymbol{b}}=\left[\overrightarrow{\boldsymbol{r}}_{1}(\phi)-\overrightarrow{\boldsymbol{r}}_{2}(\phi)\right]\left(\phi=\frac{\pi}{2}\right)  \tag{13}\\
& \overrightarrow{\boldsymbol{a}} \cdot \overrightarrow{\boldsymbol{a}}=a^{2}=r_{1}^{2}-2 r_{1} r_{2} \operatorname{Cos} \beta_{1} \operatorname{Cos} \phi_{0}+r_{2}^{2} \quad ; \quad|\overrightarrow{\boldsymbol{a}}|=a  \tag{14}\\
& \overrightarrow{\boldsymbol{b}} \cdot \overrightarrow{\boldsymbol{b}}=b^{2}=r_{1}^{2}-2 r_{1} r_{2} \operatorname{Cos} \beta_{2} \operatorname{Cos} \phi_{0}+r_{2}^{2} \quad ; \quad|\overrightarrow{\boldsymbol{b}}|=b  \tag{15}\\
& \overrightarrow{\boldsymbol{a}} \cdot \overrightarrow{\boldsymbol{b}}=r_{1} r_{2}\left(\operatorname{Cos} \beta_{1}-\operatorname{Cos} \beta_{2}\right) \operatorname{Sin} \phi_{0} \tag{16}
\end{align*}
$$

Using this mathematical basis we have described through (1) - (16), the following "circular model in four dimensions" with the given parameter mappings is being proposed, to provide a simple mathematical expression for two individual economies, as well as a measure for the amount of interaction between these two international economies.

- Each economy is represented by a circle in four dimensions, namely Economy 1 and Economy 2 correspond to Circle 1 and Circle 2 in four dimensions, respectively.
- Population of Economy 1 is $p_{1}(17)$, which is used as the radius $r_{1}$ (8) of Circle 1 , and the population of Economy 2 is $p_{2}(17)$, which is used as the radius $r_{2}$ (8) of Circle 2.

$$
\begin{equation*}
r_{1}=p_{1} \quad ; \quad r_{2}=p_{2} \tag{17}
\end{equation*}
$$

- For simplicity, the start of every fiscal period is accepted to be a timestamp $t_{0}$ point when points $\mathbf{P}_{1}$ and $\mathbf{P}_{2}$ on the two circles have a phase of $\left[\phi_{1}\left(t_{0}\right)=\phi_{2}\left(t_{0}\right)=0\right]$ (2) - (3) simultaneously, hence their phase difference being $\left[\phi_{0}\left(t_{0}\right)=0\right]$ (4) at time $\left(t=t_{0}=0\right)$.
- At the end of each fiscal period $\left(t=t_{\text {fiscal }}\right)$, which is generally a fiscal year, the state of Economy 1 is taken to be positioned at point $\mathbf{P}_{1}$ phased at $\phi_{1}\left(t_{\text {fiscal }}\right)$ (18) on Circle 1, and the state of Economy 2 is taken to be positioned at point $\mathbf{P}_{2}$ phased at $\phi_{2}\left(t_{\text {fiscal }}\right)$ (19) on

Circle 2, their phase difference at the end of fiscal period $\left(t=t_{\text {fiscal }}\right)$ being $\phi_{0}\left(t_{\text {fiscal }}\right)$

$$
\begin{gather*}
\phi_{1}\left(t_{\text {fiscal }}\right)=\phi+\phi_{0}=\phi_{2}\left(t_{\text {fiscal }}\right)+\phi_{0}\left(t_{\text {fiscal }}\right)=\omega_{1} t_{\text {fiscal }} \quad\left(\text { Phase of } \mathbf{P}_{1} \text { at } t=t_{\text {fiscal }}\right)  \tag{18}\\
\phi_{2}\left(t_{\text {fiscal }}\right)=\phi=\phi\left(t_{\text {fiscal }}\right)=\omega_{2} t_{\text {fiscal }} \quad\left(\text { Phase of } \mathbf{P}_{2} \text { at } t=t_{\text {fiscal }}\right)  \tag{19}\\
\phi_{0}\left(t_{\text {fiscal }}\right)=\phi_{1}\left(t_{\text {fiscal }}\right)-\phi_{2}\left(t_{\text {fiscal }}\right)=\left(\omega_{1}-\omega_{2}\right) t_{\text {fiscal }} \quad\left(\text { Phase difference of } \mathbf{P}_{1} \text { and } \mathbf{P}_{2} \text { at } t=t_{\text {fiscal }}\right) \tag{20}
\end{gather*}
$$

- Gross National Income (GNI) ${ }^{1}$ of each economy at the end of a fiscal period $\left(t=t_{\text {fiscal }}\right)$ is taken as the area swept by the radius vector of the corresponding circle pointing at sweeping points on the given circle corresponding to that economy in four dimensions. In other words, $(G N I)_{1}(21)$ of Economy 1 is equal to area $A_{1}(21)$ swept by radius vector $\overrightarrow{\boldsymbol{r}}_{1}$ (5) - (6) pointing at point $\mathbf{P}_{1}$ on Circle 1 from time $\left(t=t_{0}=0\right)$ until time $\left(t=t_{\text {fiscal }}\right)$, and $(G N I)_{2}$ (22) of Economy 2 is equal to area $A_{2}$ (22) swept by radius vector $\overrightarrow{\boldsymbol{r}}_{2}$ (7) pointing at point $\mathbf{P}_{2}$ on Circle 2 from time $\left(t=t_{0}=0\right)$ until time $\left(t=t_{\text {fiscal }}\right)$. This definition leads to an expression of $(G N I)_{1}(21)$ of Economy 1 as $k_{1}$ (21) times the area of Circle 1, which is defined in terms of the population $p_{1}(17)$ of Economy 1, and an expression of $(G N I)_{2}(22)$ of Economy 2 as $k_{2}$ (22) times the area of Circle 2, which is defined in terms of population $p_{2}(17)$ of Economy 2, for fiscal period $\left(t=t_{\text {fiscal }}\right)$, where $k_{1}$ (21) and $k_{2}$ (22) are positive real numbers. The coefficient $k_{1}$ (21) can thus be obtained using $(G N I)_{1}$ (21) and $p_{1}$ (17) values, and the coefficient $k_{2}$ (22) can be obtained using $(G N I)_{2}$ (22) and $p_{2}(17)$ values.

$$
\begin{array}{lll}
(G N I)_{1}=A_{1}=k_{1}\left(\pi r_{1}^{2}\right)=k_{1} \pi p_{1}^{2} & \Rightarrow & k_{1}=\frac{(G N I)_{1}}{\pi r_{1}^{2}}=\frac{(G N I)_{1}}{\pi p_{1}^{2}} \\
(G N I)_{2}=A_{2}=k_{2}\left(\pi r_{2}^{2}\right)=k_{2} \pi p_{2}^{2} & \Rightarrow & k_{2}=\frac{(G N I)_{2}}{\pi r_{2}^{2}}=\frac{(G N I)_{2}}{\pi p_{2}^{2}} \tag{22}
\end{array}
$$

- Using the definitions for $(G N I)_{1}(21)$ and $(G N I)_{2}$ (22), the "GNI per capita" of the two economies, which is Gross National Income (GNI) ${ }^{1}$ of each economy divided by its population, can be calculated and expressed as $(\text { GNIpc })_{1}$ (23) for Economy 1 and $(\text { GNIpc })_{2}$ (24) for Economy 2, for the given fiscal period.

$$
\begin{align*}
& (G N I p c)_{1}=\frac{(G N I)_{1}}{p_{1}}=\frac{A_{1}}{p_{1}}=k_{1} \pi p_{1}  \tag{23}\\
& (G N I p c)_{2}=\frac{(G N I)_{2}}{p_{2}}=\frac{A_{2}}{p_{2}}=k_{2} \pi p_{2} \tag{24}
\end{align*}
$$

- Based on the given definitions for $(G N I)_{1}$ (21) and $(G N I)_{2}$ (22), we can conclude that the radius vector $\overrightarrow{\boldsymbol{r}}_{1}$ (5) - (6) pointing at point $\mathbf{P}_{1}$ on Circle 1 sweeps a total angle of $\phi_{1}\left(t_{\text {fiscal }}\right)$ (25) between time $\left(t=t_{0}=0\right)$ and time $\left(t=t_{\text {fiscal }}\right), k_{1}$ (21) times traversing $(2 \pi)$ radians, and the radius vector $\overrightarrow{\boldsymbol{r}}_{2}(7)$ pointing at point $\mathbf{P}_{2}$ on Circle 2 sweeps a total angle of $\phi_{2}\left(t_{f i s c a l}\right)$ (26) between time $\left(t=t_{0}=0\right)$ and time $\left(t=t_{\text {fiscal }}\right), k_{2}$ (22) times traversing $(2 \pi)$ radians. This leads to the definitions in (25) - (26), from which we are able to calculate angular velocities $\omega_{1}(25)$ and $\omega_{2}(26)$, as well as frequencies $f_{1}(25)$ and $f_{2}$ (26), for the two circles corresponding to the two economies in this model.

$$
\begin{align*}
& \phi_{1}\left(t_{\text {fiscal }}\right)=\phi+\phi_{0}=2 \pi k_{1}=\frac{2(G N I)_{1}}{p_{1}^{2}}=\omega_{1} t_{\text {fiscal }} \Rightarrow \omega_{1}=\frac{2 \pi}{T_{1}}=2 \pi f_{1}=\frac{2 \pi k_{1}}{t_{\text {fiscal }}} \Rightarrow f_{1}=\frac{1}{T_{1}}=\frac{k_{1}}{t_{\text {fiscal }}}  \tag{25}\\
& \phi_{2}\left(t_{\text {fiscal }}\right)=\phi=2 \pi k_{2}=\frac{2(G N I)_{2}}{p_{2}^{2}}=\omega_{2} t_{\text {fiscal }} \Rightarrow \omega_{2}=\frac{2 \pi}{T_{2}}=2 \pi f_{2}=\frac{2 \pi k_{2}}{t_{\text {fiscal }}} \Rightarrow f_{2}=\frac{1}{T_{2}}=\frac{k_{2}}{t_{\text {fiscal }}} \tag{26}
\end{align*}
$$

- For the given fiscal period, we take the "amount of Gross National Income (GNI) ${ }^{1}$ of Economy 1 obtained in Economy 2", defined as $(G N I)_{1,2}$ (27), to be the projection of $(G N I)_{1}(21)$ onto plane of Circle 2, where the angle $\beta_{1}(5)-(6)$ is the constant orbital inclination angle between Circle 1 and Circle 2 in the $\hat{\boldsymbol{u}}_{1}-\hat{\boldsymbol{u}}_{3}$ dimension plane. Similarly, for the given fiscal period, we take the "amount of Gross National Income (GNI) ${ }^{1}$ of

Economy 2 obtained in Economy 1", defined as $(G N I)_{2,1}(28)$, to be the projection of $(G N I)_{2}(22)$ onto plane of Circle 1 , where the angle $\beta_{2}(5)-(6)$ is the constant orbital inclination angle between Circle 1 and Circle 2 in the $\hat{\boldsymbol{u}}_{2}-\hat{\boldsymbol{u}}_{4}$ dimension plane.
$(G N I)_{1,2}=(G N I)_{1} \operatorname{Cos} \beta_{1}=k_{1} \pi p_{1}^{2} \operatorname{Cos} \beta_{1}\left[\right.$ Amount of $(G N I)_{1}$ obtained in Economy 2]
$(G N I)_{2,1}=(G N I)_{2} \operatorname{Cos} \beta_{2}=k_{2} \pi p_{2}^{2} \operatorname{Cos} \beta_{2} \quad\left[\right.$ Amount of $(G N I)_{2}$ obtained in Economy 1]

- As a result, for the given fiscal period, the definitions in (27) - (28) would allow us to determine the values of $\beta_{1}(29)$ and $\beta_{2}(30)$ in this two-circle system in four dimensions, which we use to model the inter-relation of Economy 1 and Economy 2.

$$
\begin{align*}
& \beta_{1}=\operatorname{Cos}^{-1}\left[\frac{(G N I)_{1,2}}{(G N I)_{1}}\right]=\operatorname{Cos}^{-1}\left[\frac{(G N I)_{1,2}}{k_{1} \pi p_{1}^{2}}\right] \Rightarrow\left\{\begin{array}{l}
\operatorname{Cos} \beta_{1}=\frac{(G N I)_{1,2}}{(G N I)_{1}} \\
\left.\operatorname{Sin} \beta_{1}=\sqrt{1-\left[\frac{(G N I}{} \frac{(G N I, 2}{}\right)_{1}}\right]^{2}
\end{array}\right.  \tag{29}\\
& \beta_{2}=\operatorname{Cos}^{-1}\left[\frac{(G N I)_{2,1}}{(G N I)_{2}}\right]=\operatorname{Cos}^{-1}\left[\frac{(G N I)_{2,1}}{k_{2} \pi p_{2}^{2}}\right] \Rightarrow\left\{\begin{array}{l}
\operatorname{Cos} \beta_{2}=\frac{(G N I)_{2,1}}{(G N I)_{2}} \\
\operatorname{Sin} \beta_{2}=\sqrt{1-\left[\frac{(G N I)_{2,1}}{(G N I)_{2}}\right]^{2}}
\end{array}\right. \tag{30}
\end{align*}
$$

- In order to express the import-export relation of the two economies for the given fiscal period, in the corresponding two-circle model in four dimensions, we define $\ell_{1 \rightarrow 2}$ as the amount of exports from Economy 1 to Economy 2, or equivalently as the amount of imports to Economy 2 from Economy 1, and we define $\ell_{2 \rightarrow 1}$ as the amount of exports from Economy 2 to Economy 1, or equivalently as amount of imports to Economy 1 from Economy 2. In vector notation, we define $\vec{\ell}$ (31) vector corresponding to $\vec{\ell}$ (9) vector between centers of the two circles in four-dimensions, for convenience taken to be in $\hat{\boldsymbol{u}}_{3}$ dimension only, whose magnitude $\ell$ (32) is determined by the absolute value of the difference $\left(\ell_{2 \rightarrow 1}-\ell_{1 \rightarrow 2}\right)$.

$$
\begin{align*}
& \vec{\ell}=\hat{\boldsymbol{u}}_{3}\left(\ell_{2 \rightarrow 1}-\ell_{1 \rightarrow 2}\right)  \tag{31}\\
& \ell=|\vec{\ell}|=\left|\ell_{2 \rightarrow 1}-\ell_{1 \rightarrow 2}\right| \tag{32}
\end{align*}
$$

- As a result, based on our "two-circle model in four dimensions", the amount of interaction between Economy 1 and Economy 2 at the end of each fiscal period $\left(t=t_{\text {fiscal }}\right)$ can be measured by the distance $\overrightarrow{\boldsymbol{d}}$ (33) vector to point $\mathbf{P}_{1}$ on Circle 1 positioned at angle of $\phi_{1}\left(t_{\text {fiscal }}\right)$ (25), from point $\mathbf{P}_{2}$ on Circle 2 positioned at angle of $\phi_{2}\left(t_{\text {fiscal }}\right)$ (26), based on $\overrightarrow{\boldsymbol{d}}(\phi)$ (11). In determining $\overrightarrow{\boldsymbol{d}}$ (33), we use the value of radius vector $\overrightarrow{\boldsymbol{r}}_{1}$ (5) with magnitude $r_{1}$ (17) pointing at point $\mathbf{P}_{1}$ on Circle 1 at $\phi_{1}\left(t_{\text {fiscal }}\right)$ (25), the value of radius vector $\overrightarrow{\boldsymbol{r}}_{2}$ (7) with magnitude $r_{2}$ (17) pointing at point $\mathbf{P}_{2}$ on Circle 2 at $\phi_{2}\left(t_{\text {fiscal }}\right)$ (26), the $\vec{\ell}$ (31) vector from center of Circle 2 to center of Circle 1, as well as values of $\beta_{1}$ (29) and $\beta_{2}(30)$, all obtained via the known values of $p_{1}, p_{2},(G N I)_{1}$, $(G N I)_{2},(G N I)_{1,2}$, and $(G N I)_{2,1}$ at the end of each fiscal period $\left(t=t_{\text {fiscal }}\right)$ for these two international economies. The magnitude $d$ (34) of $\overrightarrow{\boldsymbol{d}}$ (33) is a scalar measure for the amount of interaction between these two international economies, and calculated by taking the Dot Product ${ }^{6}$ of $\overrightarrow{\boldsymbol{d}}$ (33) vector with itself.

$$
\begin{align*}
& \overrightarrow{\boldsymbol{d}}=\overrightarrow{\boldsymbol{r}}_{1}\left[\phi_{1}\left(t_{\text {fiscal }}\right)\right]-\overrightarrow{\boldsymbol{r}}_{2}\left[\phi_{2}\left(t_{\text {fiscal }}\right)\right]+\vec{\ell} \\
& =\hat{\boldsymbol{u}}_{1}\left\{r_{1} \operatorname{Cos}\left[\phi_{1}\left(t_{\text {fiscal }}\right)\right] \operatorname{Cos} \beta_{1}-r_{2} \operatorname{Cos}\left[\phi_{2}\left(t_{\text {fiscal }}\right)\right]\right\} \\
& +\hat{\boldsymbol{u}}_{2}\left\{r_{1} \operatorname{Sin}\left[\phi_{1}\left(t_{\text {fiscal }}\right)\right] \operatorname{Cos} \beta_{2}-r_{2} \operatorname{Sin}\left[\phi_{2}\left(t_{\text {fiscal }}\right)\right]\right\} \\
& +\hat{\boldsymbol{u}}_{3}\left\{r_{1} \operatorname{Cos}\left[\phi_{1}\left(t_{\text {fiscal }}\right)\right] \operatorname{Sin} \beta_{1}+\left(\ell_{2 \rightarrow 1}-\ell_{1 \rightarrow 2}\right)\right\} \\
& +\hat{\boldsymbol{u}}_{4} r_{1} \operatorname{Sin}\left[\phi_{1}\left(t_{\text {fiscal }}\right)\right] \operatorname{Sin} \beta_{2} \\
& =\hat{\boldsymbol{u}}_{1}\left\{p_{1} \frac{(G N I)_{1,2}}{(G N I)_{1}} \operatorname{Cos}\left[\frac{2(G N I)_{1}}{p_{1}^{2}}\right]-p_{2} \operatorname{Cos}\left[\frac{2(G N I)_{2}}{p_{2}^{2}}\right]\right\} \\
& +\hat{\boldsymbol{u}}_{2}\left\{p_{1} \frac{(G N I)_{2,1}}{(G N I)_{2}} \operatorname{Sin}\left[\frac{2(G N I)_{1}}{p_{1}^{2}}\right]-p_{2} \operatorname{Sin}\left[\frac{2(G N I)_{2}}{p_{2}^{2}}\right]\right\} \\
& +\hat{\boldsymbol{u}}_{3}\left\{p_{1} \sqrt{1-\left[\frac{(G N I)_{1,2}}{(G N I)_{1}}\right]^{2}} \operatorname{Cos}\left[\frac{2(G N I)_{1}}{p_{1}^{2}}\right]+\left(\ell_{2 \rightarrow 1}-\ell_{1 \rightarrow 2}\right)\right\}  \tag{33}\\
& +\hat{\boldsymbol{u}}_{4} p_{1} \sqrt{1-\left[\frac{(G N I)_{2,1}}{(G N I)_{2}}\right]^{2}} \operatorname{Sin}\left[\frac{2(G N I)_{1}}{p_{1}^{2}}\right] \\
& \left\{p_{1} \frac{(G N I)_{1,2}}{(G N I)_{1}} \operatorname{Cos}\left[\frac{2(G N I)_{1}}{p_{1}^{2}}\right]-p_{2} \operatorname{Cos}\left[\frac{2(G N I)_{2}}{p_{2}^{2}}\right]\right\}^{2} \\
& +\left\{p_{1} \frac{(G N I)_{2,1}}{(G N I)_{2}} \operatorname{Sin}\left[\frac{2(G N I)_{1}}{p_{1}^{2}}\right]-p_{2} \operatorname{Sin}\left[\frac{2(G N I)_{2}}{p_{2}^{2}}\right]\right\}^{2} \\
& d=|\stackrel{\rightharpoonup}{\boldsymbol{d}}|=\sqrt{\overrightarrow{\boldsymbol{d}} \cdot \stackrel{\rightharpoonup}{\boldsymbol{d}}}=\left\{+\left\{p_{1} \sqrt{1-\left[\frac{(G N I)_{1,2}}{(G N I)_{1}}\right]^{2}} \operatorname{Cos}\left[\frac{2(G N I)_{1}}{p_{1}^{2}}\right]+\left(\ell_{2 \rightarrow 1}-\ell_{1 \rightarrow 2}\right)\right\}^{2}\right.  \tag{34}\\
& +\left\{p_{1} \sqrt{1-\left[\frac{(G N I)_{2,1}}{(G N I)_{2}}\right]^{2}} \operatorname{Sin}\left[\frac{2(G N I)_{1}}{p_{1}^{2}}\right]\right\}^{2}
\end{align*}
$$

- Having determined the scalar radii $r_{1}$ (17) and $r_{2}$ (17) based on populations $p_{1}$ and $p_{2}$ of the two economies, respectively, as well as the angular velocities $\omega_{1}(25)$ and $\omega_{2}(26)$ for the given fiscal period between time $\left(t=t_{0}=0\right)$ and time $\left(t=t_{\text {fiscal }}\right)$, we can also
calculate the "specific energy"" (35) - (36) of each economy in terms of "kinetic energy per mass" as in physics, as a measure for the "energy density ${ }^{8 "}$ (35) - (36) of each economy, providing a means for comparison of income capacities of Economy 1 (35) and Economy 2 (36) in terms of $(G N I p c)_{1}$ (23) and $(\text { GNIpc })_{2}$ (24), respectively. Here, we use $\left(E=\frac{1}{2} m v^{2}\right)$ for kinetic energy of each economy system, where $m$ is taken as the virtual mass and $(v=r \omega)$ as the velocity of the system. As a result, the ratio of Gross National Income $(\mathrm{GNI})^{1}$ of each economy to its population at the end of each fiscal period $\left(t=t_{\text {fiscal }}\right)$, namely "GNI per capita"", provides a measure for the income capacity of that economy, for comparison with other international economies. Further, we understand that twice the "GNI per capita" of an economy is analogous to the velocity $v$ of an economy for a unit fiscal period, such as a fiscal year, in this circular model in four dimensions that we use.

$$
\begin{aligned}
& \omega_{1}=\frac{2(G N I)_{1}}{t_{\text {fiscal }} p_{1}^{2}} \Rightarrow v_{1}=r_{1} \omega_{1}=\frac{2(G N I)_{1}}{t_{\text {fiscal }} p_{1}} \Rightarrow \frac{E_{1}\left(t_{\text {fiscal }}\right)}{m_{1}}=\frac{\left(\frac{1}{2} m_{1} v_{1}^{2}\right)}{m_{1}}=\frac{1}{2} v_{1}^{2}=\frac{2(G N I)_{1}^{2}}{t_{\text {fiscal }}^{2} p_{1}^{2}}=\frac{2(G N I p c)_{1}^{2}}{t_{\text {fiscal }}^{2}} \\
& \omega_{2}=\frac{2(G N I)_{2}}{t_{\text {fiscal }} p_{2}^{2}} \Rightarrow v_{2}=r_{2} \omega_{2}=\frac{2(G N I)_{2}}{t_{\text {fiscal }} p_{2}} \Rightarrow \frac{E_{2}\left(t_{\text {fiscal }}\right)}{m_{2}}=\frac{\left(\frac{1}{2} m_{2} v_{2}^{2}\right)}{m_{2}}=\frac{1}{2} v_{2}^{2}=\frac{2(G N I)_{2}^{2}}{t_{\text {fiscal }}^{2} p_{2}^{2}}=\frac{2(G N I p c)_{2}^{2}}{t_{\text {fiscal }}^{2}} \text { (36) }
\end{aligned}
$$

## CONCLUSIONS

In this Article, a method is provided as a measure for the amount of interaction between two international economies, which are two countries or economic zones, by modelling each economy in terms of a "circle in four dimensions". Based on this method, the proximity of the two economies at the end of each fiscal period, which is generally a fiscal year, is then measured by the distance between points at the end of the given period on the two circles in four dimensions, associated with these two economies in our model. The radius of the circle corresponding to each economy is taken to be the population of the economy, whereas the

Gross National Income (GNI) ${ }^{1}$ of each economy at the end of a fiscal period is taken as the area swept by the radius vector of the corresponding circle for the given economy. For every fiscal period, the portion of Gross National Income (GNI) ${ }^{1}$ of each economy obtained in the other economy is used to determine the orbital inclination angles between the circles corresponding to these two economies in our model. Further, the import-export relations between these two economies are used to determine the distance between the centers of the corresponding circles of the two economies. As a consequence, angular velocities and frequencies of the two-economy system are determined, leading to an understanding that twice the "GNI per capita" of an economy is analogous to the velocity of that economy for a unit fiscal period, such as a fiscal year, in this circular model in four dimensions that we use, whose square provides a measure for the income capacity of each economy in terms of "energy density ${ }^{8}$ " of the system, for comparison.

## AUTHOR CONTRIBUTIONS

Aslı Pınar Tan is the only author contributing to this Article.

## DATA AVAILIBILITY

Data sharing is not applicable to this article as no data were created or analyzed in this study.

## DECLARATION OF INTEREST STATEMENT

The Author declares that there is no conflict of interest.

## KEYWORDS

GNI, GDP, GNIpc, International Economy, Export, Import, Circle, Model

## ABOUT THE AUTHOR

Aslı Pınar Tan's findings in this Article are part of all her findings as a result of her personal theoretical studies and research over the years independent from any institution or university,
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