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Article

A New Approach to Modeling the Prediction of Movement Time

Chiuhsiang Joe Lin [1*], and Chih-Feng Cheng [2]

[1] National Taiwan University of Science and Technology, Taipei, Taiwan; cjoelin@mail.ntust.edu.tw
[2] National Taiwan University of Science and Technology, Taipei, Taiwan; jeffsis886@mail.ntust.edu.tw
* Correspondence: cjoelin@mail.ntust.edu.tw; Tel.: + 886-2-2737-6352

Abstract: Fitts' law predicts the human movement response time for a specific task by a simple linear formulation, in which the intercept and the slope are estimated from the task's empirical data. This research was motivated by our pilot study, which found that the linear regression's essential assumptions are not satisfied in the literature. Furthermore, the keystone hypothesis in Fitts' law, that the movement time per response will be directly proportional to the minimum average amount of information per response demanded by the particular amplitude and target width, has never been formally tested. Therefore, this study developed an optional formulation derived from fusing the findings in psychology, physics, and physiology for fulfilling the statistical assumptions. An experiment was designed to test the hypothesis in Fitts' law and validate the proposed model. To conclude, our results indicated that movement time could be related to the index of difficulty underlying the same constant amplitude. The optional formulation accompanies the index of difficulty in Shannon form robustly performs the prediction better than the traditional model across studies. Finally, a new approach to modeling movement time prediction is deduced from our research results

Keywords: Fitts' law; information theory; index of difficulty; SQRT_MT model

1. Introduction

Since Fitts' work about the speed–accuracy trade-off in rapidly-aimed movements [1] was published, many researchers have proposed different movement time prediction equations to compete with Fitts' proposal. Hoffmann et al. [2] defined one-, two-, and three-dimensional targets by whether the limitation of movement performance is in the direction of movement or perpendicular to the movement, and the depth at the target location. Readers can refer to a survey paper [3] of Fitts' law's mathematical formulations for one-dimensional targets. Soukoreff and Mackenzie published a work about the suggestions on using Fitts's law [4]. The existing formulations could be divided into two categories: information-theory-based and non-information-theory-based. There are two subcategories of the non-information-theory-based formulations. One is the theoretical formulation, which is derived from a specific theoretical argument proposed by previous authors. Works [5-9] belong to this subcategory. The other is the non-theoretical formulation, proposed by some authors without the reasoning. Some examples are Jagacinski et al.'s [10] and Kvålseth's [11] works.

Despite Fitts' law having an exceptional reputation in broad applications, there are still gaps in the literature. One of the extant issues is that there is no single and united formulation in the community. Another is that Fitts did not apply any statistical tests to validate his assumption that MT is only related to ID [1]. In addition, we found that the current Fitts' law models did not follow the statistical requirements in regression analysis, e.g., fitting the linearity adequately, independence and equal variance of residuals between predictor levels, and the normality of the residuals, when Fitts' data of 1954 were applied.

1.1. Literature review: models in Fitts' law

Fitts' law is a highly successful formulation from psychology [12] stating that the time to complete a movement (movement time, MT) depends on the distance to be covered and the spatial accuracy required [1]. The distance is defined as the movement amplitude (A) a human has to move to hit a target, and the target, with tolerance in width (W), is treated as the accuracy requirement. A logarithmic term defined by A and W is called the index of difficulty (ID) and is measured in bits, which specifies the minimum information required on average for achieving each movement. Researchers have manipulated movement amplitudes and target widths. Consequently, a specific ID value could be composed of different A and W combinations. Fitts [1] argued that ID was consistent with Shannon Theorem 17. Shannon Theorem 17 [13] describes the transmitted information capacity (C) in a communication system with bandwidth (B), signal power (S), and white noise power (N),

$$Capacity = B \times log\left(\frac{S+N}{N}\right). \quad (1)$$

Fitts' law is a regression equation, called the Canon model in this study, derived from empirical data and describes the relationship between the dependent variable (MT) and independent variable (ID) with two parameters, intercept (a) and slope (b),

$$MT = a + b \times ID. \quad (2)$$

Interestingly, this equation was proposed not by Fitts in 1954 [1] but by Welford in 1960 [14]. After Welford's work, all the research related to Fitts' law applied the linear equation, including Fitts himself [15]. Fitts defined ID as the information required to finish a task with the ID's specific movement amplitude [1]. ID has multiple definitions, and the most popular three are considered information-theory-based [16]. They are:

$$ID_{Fitts} = log_2\left(\frac{2A}{W}\right) \quad (3)$$

Fitts explained that 2A could cover the endpoints during the movement and result in a non-zero difficulty.

Another attractive metric, index of performance (I_P=MT/ID), was defined as the specific task's maximum information rate. This index was used to test Fitts' hypothesis; MT is only proportional to ID if I_P is constant for all IDs. However, Fitts argued his hypothesis was valid by checking his data visually. Even he was aware that I_P was not precisely constant [1]. Moreover, after the regression equation appeared six years later, in 1966, Fitts and Radford used slope b in Equation 2 instead of I_P . They applied the data in their previous studies and claimed a slope ranging from 90 to 110 msec/bit was a constant, again without statistical tests [17].

$$ID_{Welford} = log_2\left(\frac{A}{W} + 0.5\right). \quad (4)$$

Welford found ID_{Fitts} had drawbacks such as a negative intercept and upward curve fitting at the lower end of ID values. He thought the law chose a distance W out of a total distance extending from the starting point to the far edge of the target, equal to A+W/2. The modification in $ID_{Welford}$ preserved the advantage of non-zero ID, resulted in a near-zero intercept, and removed the upward curve (Welford, 1960).

$$ID_{Shannon} = log_2\left(\frac{A}{W} + 1\right). \quad (5)$$

MacKenzie [18] analogized A to S and W to N directly and claimed this analogy was an exact adaptation to Shannon Theorem 17. Consequently, $ID_{Shannon}$ is called ID in Shannon form. It was shown that $ID_{Shannon}$ had better performance in R-square than those of ID_{Fitts}

and $ID_{Welford}$. MacKenzie applied this adapted model in varied human–computer interaction studies [19-21]. The $ID_{Shannon}$ version of Fitts's law has gained popularity in the human–computer interaction (HCI) community since the 1990s.

One of the assumptions of the information-theory-based form is that movement is performed under a visual-feedback loop. Therefore, Schmidt et al. [22] found that the visual feedback assumption in Fitts [1] might be impossible for a movement time of less than 200 msec. Schmidt [23] found that the reaction time in response selection to correct error required at least 120 to 200 msec. Those movements, not involving visual feedback, were prestructured muscle commands called motor programs [24] or ballistic movements [25]. Based on the above results, Gan and Hoffmann's study argued that low ID movement is ballistic [5]. They indicated that Equation 2 was only valid for a visual-feedback movement when ID is higher than three. When the response was a ballistic movement, the upward curve resulted. A theoretical formulation,

$$MT = a + b \times \sqrt{A}, \tag{6}$$

was proposed to fit the ballistic movement [5]. They derived Equation 6 from the kinetics of arm movement. The square root of A replaced the A/W ratio in the logarithmic term. Subsequently, a discrete tapping experiment with four amplitudes (4, 9, 16, and 25 cm), ten ID_{Fitts} (1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, and 6.0) and ten repetitions was conducted. The R-square for IDs of less than or equal to 3 was 0.944 in Equation 6. When a constant ID was considered respectively, the R-square was over 0.992 for ID less than 3. However, the R-square ranged from 0.883 to 0.983 when ID was between 3 and 6. Gan and Hoffmann thought that the movement time was a function of movement amplitude [5].

Kvålseth [26] indicated that the ID based on a direct analogy with Shannon's Theorem 17 was not justified, and they proposed that an information-theory statistic was indeed questionable. The critical reason was that Theorem 17 involves a power ratio but no amplitude/width ratio applied in the ID. Instead of the A/W in a logarithmic term, Kvålseth proposed an alternative to Equation 2 [11],

$$MT = aA^bW^c. \tag{7}$$

When the empirical constants b and c are identical, Equation 7 can be expressed as Equation 8.

$$MT = a\left(A/_W\right)^b. \tag{8}$$

Fitts' data were applied to an analysis with Equations 7 and 8. The R-squares were 0.987 for 1-oz stylus tapping and 0.985 for 1-lb stylus tapping. Kvålseth claimed that the power law was superior to the information-theory-based models in R-square.

Although Kvålseth did not explain the power law's origin, Meyer et al. developed a stochastic optimized-sub-movement model by extending the impulse variability [8]. Another power law, that the constant b in Equation 8 is equal to ½, was proposed as follows:

$$MT = a + b \times \sqrt{A/_W}. \tag{9}$$

The square root of A/W replaced ID in the logarithmic term. Equation 9 is called the Power model in this study. After application of the alternative formulation, the R-squares were 0.974 for 1-oz stylus tapping and 0.972 for 1-lb stylus tapping of Fitts's study [1]. Goldberg [9] gave a succinct derivation for Equation 9 from the kinematic aspect.

Some works extended Fitts' law to two-dimensional [20,27-29] and three-dimensional [30-32] circumstances with a new formula. Bi et al. developed a modified model for finger-touch-based input on a screen [33]. In addition, researchers utilized Fitts' law to evaluate the operating performance of new technological devices [34-38]. Also, some fundamental studies were conducted to examine the theory of Fitts' law. Gori et al. worked on the rationale of Fitts' law on the pointing task [39], and Muller et al. concluded that the control theory complemented Fitts' law [40]. Since the Shannon form was proposed, no studies have investigated the formula for Fitts' paradigm in one-dimensional targets.

1.2. The valid version of Fitts' law

With the advances on Fitts' law, the many candidates in Equations 3 to 8 for the formulation have confused researchers. Drewes was the first researcher to appeal the union of Fitts' law formulation [41]. This issue might be caused by explaining the analogy of A and W in Shannon Theorem 17.

Fitts did not explain the relationship between A/W and S/N in his work [1]. Therefore, there was no argument about the relationship between A/W and S/N in Welford's study [14]. However, in a subsequent study on Fitts' law for a discrete tapping task, Fitts explained that A was analogous to S+N, and W/2 could be assumed to be N [15]. MacKenzie indicated that ID_{Fitts} might be inappropriate from an information perspective, and $ID_{Welford}$ went halfway to Shannon Theorem 17. Therefore, $ID_{Shannon}$ mimicked Shannon's original equation in that A/W was analogous to S/N [18].

Hoffmann developed the derivation of these three IDs from information theory and claimed that, although ID_{Fitts} and $ID_{Welford}$ were valid, $ID_{Shannon}$ was invalid. He reanalyzed three data sets available in the literature. He concluded that $ID_{Shannon}$ did not always have the best R-square performance, and that it was even worse when W_e and $ID_{Shannon}$ were applied simultaneously [16].

MacKenzie challenged Hoffmann's arguments that the validity issue was no basis for analysis, since the formula was an analogy of human movement to an electronic signal. In addition, Hoffmann excluded the data of ID equal to one in his work. Via reanalysis with the complete ID range for the same data sets and one more application in a computer's input devices, MacKenzie showed that $ID_{Shannon}$ had the best performance in R-square, whether the set or the effective width was applied [42].

Unfortunately, in the community, there is still no common agreement on the version. Researchers apply their preferred formulations in their studies. Which version of Fitts' law should be applied is still an open issue in the community.

1.3. Statistical principles in Fitts' law

The satisfaction of statistical principles is the second inadequacy of Fitts' law. The first statistical insufficiency is related to the design of the experiment. Traditionally, Fitts' law researchers have manipulated the target's amplitude and width as independent variables, but not the crucial factor ID directly [43]. That is an uncommon situation, for the premier purpose is to investigate the relationship between the response movement time and the IDs. Such an experimental design also causes the empirical regression formulation to be built from just one A/W combination for the two extreme ID values. When the ID approaches the mean, there are more A/W combinations for a specific ID. Consequently, the resultant regressive parameters might be biased due to the smaller amount of information in extreme conditions.

Another flaw in the literature is that there is only one observation in every A/W combination. A single observation means that the statistical test of Fitts' hypothesis, i.e., "the average movement time per response would be proportional to the minimum average information per response demanded by the particular conditions of amplitude and tolerance" argued in Fitts's original work [1], cannot be tested. This hypothesis implies that the MT would be the same for tasks with the same ID value, regardless of the amplitudes. Fitts claimed his hypothesis was valid because the difference between movement times within the same ID value is tiny, but he provided no support with a formal statistical test. In addition, without repeated measures in every ID, the lack-of-fit test is impossible. Whether the formulation fits the data adequately cannot be justified statistically.

The second statistical insufficiency is regarding the essential assumption of residual normality in the regression analysis. Generally, this assumption is thought to have been satisfied in the literature. Therefore, this research found that the residuals using ID_{Fitts} in the Canon model failed the normality test of Fitts' data [1], as shown in Section 2.2.

The third statistical insufficiency is whether a linear regression formulation adequately fits the data or not, which should be formally tested by the lack-of-fit test [44]. This study found that Fitts' law in the Canon and Power models also failed the lack-of-fit test of Fitts' data [1], as shown in Section 2.2.

The fourth statistical insufficiency is the way to evaluate the quality in a regression model. Researchers applied Fitts' law in pursuit of a high R-square and claimed a good fit in the resultant formula [41]. R-square was also applied as the metric to justify which model had a better performance [11,15,16,42]. However, the idea that a value close to one represents a good fit is a common misunderstanding about R-square in regression analysis [44]. While more than one competing model exists, other metrics, such as the PRESS (prediction sum of squares), could be criteria for model selection [44,45].

Based on the fact that Fitts' law in all forms utilizes regression analysis to estimate the empirical parameters, a sound model satisfies the assumptions and principles in the regression analysis; e.g., the error term follows the normal distribution, the residuals are independent and the variance of residuals is constant between independent variables. The linearity fitting is appropriate, and evaluations of more quality indexes, such as the PRESS and not just the R-square, should be satisfied in the empirical application.

1.4. Motivation

This research was motivated by the disagreement on which version of Fitts' law should be applied in the literature. During the probing investigation, the authors found the statistical insufficiencies of past studies, which are discussed in Section 1.3. Additionally, the Canon model does not fit Fitts' data well for the ballistic movement of low ID values. This study aimed to find the solution for the mentioned gaps. Since Fitts' law regulates the human speed–accuracy trade-off in the psychological aspect, the authors investigated the relationship between MT and ID from a physiology viewpoint.

1.5. Research Purposes and Limitation

This research focused on one-dimensional targets on the input device of computers. The purposes of this research were: (1) to propose another optional equation inspired by the findings in physiological research on motor units in a movement, which have the advantage of satisfying the statistical principles and are robust in performance; (2) to apply the optional formulation for visual feedback and ballistic movement synchronously; and (3) to test Fitts' hypothesis that movements with the same ID value have the same average moving time even if the movement amplitudes and target tolerances differ [1]. This study hypothesized that movement time is positively related to ID but differs for the same ID value with varied movement amplitude and target tolerance. To the best of our knowledge, none of the purposes have been discussed in the literature.

This study's scope is limited to the information-theory-based forms and the theoretical formulation applied in Fitts' paradigm for two reasons. The first reason is that researchers could constantly develop a good matching equation, e.g., polynomial regression fitting, between dependent and independent variables. Therefore, scientists pursue a cause–effect relationship between responses and circumstances to advance their research comprehension. A non-theoretical formulation contributes little to knowledge. The second reason is that the information-theory-based forms have been successfully applied in the community for almost seventy years. The information-theory-based forms have a solid relationship between movement time and the index of difficulty.

The remainder of this paper is organized as follows. Section 2 describes this study's methodology and materials. A succinct derivation of the SQRT_MT model is presented in Section 2.1. Then the validation of the SQRT_MT model applying Fitts' data is summarized in Section 2.2. Section 2.3 describes the experimental design for testing this study's hypothesis. The SQRT_MT model is applied to analyze the experimental data. Section 3 presents the results and discussion based on the experiment. The kernel of this study, that movement time might differ for the same ID when the movement amplitude and target tolerance are varied, is demonstrated in Section 3.1. To support this study's hypothesis, Section 3.2 utilizes the results of the experiment to do regression analysis by considering just one constant movement amplitude per time. Section 3.3 discusses the valid version issue by reviewing the evidence in the literature and this study. The meanings of the intercept and the slope in the regression model are discussed in Section 3.4. Finally, Section 4 concludes this research with suggestions for researchers in Fitts' law applications.

2. Materials and Methods

2.1. The Derivation and Validation of the SQRT_MT Model

Schmidt et al. treated motor-output variability as the noise producing movement inaccuracy [24]. Muscles contracted to execute the pre-structured commands and introduced variability during movement, considered the motor-output noise. The relationships between the motor-output variability and the movement distance, the effective width, movement time, generated force, and mass to be moved in the movement accuracy were investigated. Some critical relationships were derived in the aspects of physics. First, W_e is directly proportional to the variability in the velocity and proportional to the variability in the impulse for acceleration, from Newton's second law.

$$W_e \propto \sigma_{Impulse} \propto \sigma_{velocity}.$$

Variability is related to the magnitude of force and movement time.

$$\sigma_{Force} \propto Force.$$
$$\sigma_{MT} \propto MT.$$

The proportional relationship between the variability and mean force implies that a constant coefficient of variance (CV) exists.

$$\text{CV} = \frac{\sigma_{Force}}{\mu_{Force}}. \tag{10}$$

Second, the variability of impulse is directly proportional to the movement amplitude.

$$W_e \propto \sigma_{Impulse} \propto A.$$

The variability of impulse is inversely proportional to the movement time, in which both force and movement time may vary.

$$W_e \propto \sigma_{Impulse} \propto {}^{1}/_{MT}.$$

Consequently, the fused expression could be

$$W_e \propto {}^{A}/_{MT}.$$

Such a relationship implies that MT is a function of A/W_e.

$$MT \equiv f({}^{A}/_{W_e}). \tag{11}$$

Harris and Wolpert reported that noise in the firing of motor neurons would cause trajectories to deviate from the planned path [46]. The cumulated deviations over the movement duration would lead to variability in the final position. A cost function was defined by minimizing the position's variance across repeated movements over the movement duration. They assumed that a human would select a movement trajectory with the minimum cost over the duration in the presence of signal-dependent noise. A later study [47] called this concept Task Optimization in the Presence of Signal-dependent noise, TOPS. TOPS is a critical concept that might connect open- and closed-loop movement behaviors into the same formula.

Jones et al. investigated the sources of signal-dependent noise and found that firing rate variability comes from the motor neuron pool innervating into muscles [48]. Hamilton et al. investigated the scaling of motor noise with muscle strength measured by maximum voluntary torque (MVT) [49]. The result was

$$\text{CV} = e^{-2.76} MVT^{-0.25}.$$

A simulation was conducted to investigate the relationship between CV and the number of motor units (MUN). The simulation result was

$$\text{CV} = e^{k} MUN^{-0.5}.$$

where k is a constant that varied with the level of spike noise. Fusing the results, the relationship between the MUN and MVT could be

$$MUN \propto MVT^{0.5}.$$

Harris and Wolpert applied TOPS, but the noise in the cost function was defined by CV instead of position [46]. That muscles define the cost should be activated to minimize the variability of muscle noise,

$$Cost_{TOPS} = \sum_{i=1}^{n} \sigma^2_{Force_i}.$$

where i is the number of motor units activated in the muscle. By substituting Equation 10,

$$Cost_{TOPS} = \sum_{i=1}^{n} \sigma_{Force_i}^2 = \sum_{i=1}^{n} CV^2 \mu_{Force_i}^2 = \sum_{i=1}^{n} \frac{\mu_{Force_i}^2}{MUN_i} + k_1$$
$$= \sum_{i=1}^{n} \frac{\mu_{Force_i}^2}{MVT_i^{0.5}} + k_2.$$

where k_1 and k_2 are constants estimated from empirical data. From Equation 10, CV is a dimensionless statistic; it is more likely to be normalized by the Noise/Signal (N/S) ratio. Thus, we could have the following relationship during MT,

$$\left(\frac{S}{N}\right)^2 \equiv \left(\frac{\mu_{Force}}{\sigma_{Force}}\right)^2 = CV^{-2} = (e^{2.76} MVT^{0.25})^2 \propto (MVT^{0.5}).$$

μ_{Force} could be a constant from the mean force during the movement. Thus, we have

$$\sigma_{Force}^{-2} \propto (MVT^{0.5}).$$
$$\frac{1}{\sigma_{Force}} \propto (MVT^{0.5})^{0.5}.$$

Hence, MT in the TOPS model has a relationship with S/N,

$$\frac{S}{N} \equiv \frac{\mu_{Force}}{\sigma_{Force}} \propto (MVT^{0.5})^{0.5} \propto (MT)^{0.5}. \quad (12)$$

Equation 12 implies that the square root of movement time is a function of S/N. Hence, we have,

$$\sqrt{MT} \equiv f(S/_N). \quad (13)$$

Fusing the result from physics in Equation 11, the aspect from physiology in Equation 13, and the analogy of S/N to the A/W from psychology in Equation 2 into the general form, we have,

$$\sqrt{MT} = a + b \times ID. \quad (14)$$

The ID is in the form of A/W in the logarithm, which could be ID_{Fitts}, $ID_{Welford}$, or $ID_{Shannon}$. Part of purpose one is achieved in Equation 14, which is called the SQRT_MT model in this study. There are two reasons for using the same ID type with the Canon model. The first is that we expect a linear formulation. A logarithmic transformation frequently makes the relationship between two variables linear. Second, it is beneficial to keep the same ID type as the convention. Those are easy to apply and memorize for researchers.

Although the SQRT_MT model is similar to the Power model in Equation 9 at first glance, the two models' meanings are different. The SQRT_MT model is inspired from physiological research, including *in vivo* studies. In contrast, the Power model is given without any reasoning at the beginning. Later researchers tried to explain Equation 9 from kinetics and kinematics theoretically. In spite of believing people could optimally move without consciousness, it might be hard to prove that this ideational condition is identical to human behavior. In addition, the unit of the Power model is time, but in the SQRT_MT model, it is the square root of time.

2.2. Validation of the SQRT_MT Model: Results and Discussion

Fitts' data [1] were applied to validate the proposed Equation 14. The lack-of-fit test for adequate fitting, the Anderson-Darling (AD) test for residual normality, the residual plot for the constant variance and independent assumptions, and R-square and PRESS for model selection were utilized to evaluate model quality. A PRESS close to the sum square of error (SSE) supports the regression formulation's validity [44]. However, the unit of the dependent variable in the SQRT_MT model is the square root of a microsecond, whereas in the Canon and Power models, is the microsecond. Researchers cannot compare PRESSs in these models directly. The ratio, PRESS/SSE, makes the metric unit-free, like R-square. Consequently, this study applied both R-square and PRESS/SSE as the model selection indices. Since PRESS is always larger than SSE, a ratio close to one implies a suitable formulation.

Table 1 presents the performance of 1-oz stylus tapping in the Canon, Power, and SQRT_MT models. The SQRT_MT models applying $ID_{Welford}$ and $ID_{Shannon}$ satisfied the straight line fitting. In addition, all three Canon models, the Power model, and the SQRT_MT model applying ID_{Fitts} were significant in the lack-of-fit test. Such results imply that the five models did not fit the linearity adequately. In the ID type effect, $ID_{Shannon}$ had the best fit, and ID_{Fitts} did the worst in both the Canon and the SQRT_MT models. In addition, the SQRT_MT model was better than the Canon model in the lack-of-fit, no matter which ID type was applied.

In the residual normality performance, the Canon model applying ID_{Fitts} was the only significant (p-value = 0.014 < 0.05) formulation. Also, the $ID_{Welford}$ in the Canon model passed the test marginally (p-value = 0.057). The residuals of other models in Table 1 satisfied the normal distribution sufficiently.

Although all the models in Table 1 provided considerably high R-squares (ranging from 0.966 to 0.992) and low PRESS/SSE ratio (ranging from 1.27 to 1.78), the failures in the linearity fitting and the residual normality excluded the validity in the application of three Canon models and the Power model. The effects of ID type on R-square and the PRESS/SSE in the Canon and SQRT_MT models were consistent with the lack-of-fit test. $ID_{Shannon}$ had the best performance, followed by $ID_{Welford}$ and then ID_{Fitts}. Overall, the SQRT_MT model performed better (means: 0.990 in R-square and 1.41 in PRESS/SSE) than the Canon (means: 0.978 in R-square and 1.65 in PRESS/SSE) and the Power model (means: 0.971 in R-square and 1.78 in PRESS/SSE) in the quality indexes of model selection.

Table 1. Performance of 1-oz stylus tapping in the Canon, the Power, and the SQRT_MT models using Fitts' data [1]. The SQRT_MT model performed better than the Canon and Power models in all statistics.

Model	ID	a	b	R^2	PRESS/SSE	Lack-of-fit	AD Test's P-value
Canon	ID_{Fitts}	12.8	94.67	0.966	1.71	< 0.001	0.014
Canon	$ID_{Welford}$	65.4	103.79	0.980	1.65	0.004	0.057
Canon	$ID_{Shannon}$	27.7	111.54	0.987	1.58	0.023	0.265
Power	$\sqrt{A/W}$	121.3	82.40	0.971	1.78	0.001	0.542
SQRT_MT	ID_{Fitts}	9.793	2.405	0.986	1.58	0.022	0.432
SQRT_MT	$ID_{Welford}$	11.162	2.625	0.991	1.37	0.155	0.899
SQRT_MT	$ID_{Shannon}$	10.239	2.812	0.992	1.27	0.221	0.936

Table 2 presents the performance of 1-lb stylus tapping in the Canon, Power, and SQRT_MT models. The pattern in Table 2 was almost the same as that in Table 1, except that all models passed the normality test. Only the SQRT_MT model with $ID_{Welford}$ and $ID_{Shannon}$ passed the lack-of-fit test again. $ID_{Shannon}$ achieved better performance in R-square, PRESS/SSE, and linearity than did $ID_{Welford}$ and ID_{Fitts}. Similarly, the SQRT_MT model performed better (means: 0.988 in R-square and 1.37 in PRESS/SSE) than the Canon (means: 0.973 in R-square and 1.53 in PRESS/SSE) and the Power models (means: 0.971 in R-square and 2.00 in PRESS/SSE) in the quality indexes of model selection.

Comparing the results in Tables 1 and 2, the R-square increased slightly, and the PRESS/SSE improved in the SQRT_MT model with $ID_{Welford}$ and $ID_{Shannon}$. Although $ID_{Shannon}$ performed better than $ID_{Welford}$ in the two model selection indexes, the difference in PRESS/SSE might be more critical than that in R-square for these cases. The R-squares for $ID_{Welford}$ and $ID_{Shannon}$ were 0.991 vs. 0.992 in 1-oz stylus tapping and 0.990 vs. 0.992 in 1-lb stylus tapping. The difference in R-squares was pretty small. Instead, the PRESS/SSEs for $ID_{Welford}$ and $ID_{Shannon}$ were 1.37 and 1.27 and 1.34 and 1.27 in 1-oz and 1-lb stylus tapping, respectively. As mentioned, the PRESS could be helpful in model validation. More improvement in the PRESS near the SSE meant a more adequately suitable formulation. Such a result implied that the model selection might not depend on the R-square only.

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Table 2. Performance of 1-lb stylus tapping task with the Canon, Power, and SQRT_MT models using Fitts' data [1]. The results were consistent with the 1-oz stylus tapping task.

Model	ID	a	b	R^2	PRESS/SSE	Lack-of-fit	AD Test's P-value
Canon	ID_{Fitts}	-6.2	104.80	0.960	1.61	0.001	0.379
Canon	$ID_{Welford}$	51.7	114.99	0.975	1.53	0.007	0.506
Canon	$ID_{Shannon}$	9.7	123.65	0.983	1.45	0.038	0.353
Power	$\sqrt{A/W}$	112.8	91.56	0.971	2.00	0.004	0.700
SQRT_MT	ID_{Fitts}	9.556	2.586	0.983	1.51	0.032	0.182
SQRT_MT	$ID_{Welford}$	11.022	2.825	0.990	1.34	0.240	0.702
SQRT_MT	$ID_{Shannon}$	10.023	3.028	0.992	1.27	0.480	0.161

Furthermore, many cases in Tables 1 and 2 failed the lack-of-fit test even though their residuals satisfied the normality. The possible reasons could be the violation that the residuals are independent, or constant variance in the predictor variables. Figure 1 shows the worst case in the 1-oz stylus tapping, when the Canon model with ID_{Fitts} was applied. The left graph is the normality plot using the AD test, in which the residuals show an S pattern along the straight line. Therefore, the residual normality was not satisfied in the model. In the right graph, as the ID departed away from the central value 4, the residual increased gradually. The scatters' shape was liked a parabola with an opening upwards. Consequently, the independent assumptions underlying the linear regression did not exist.

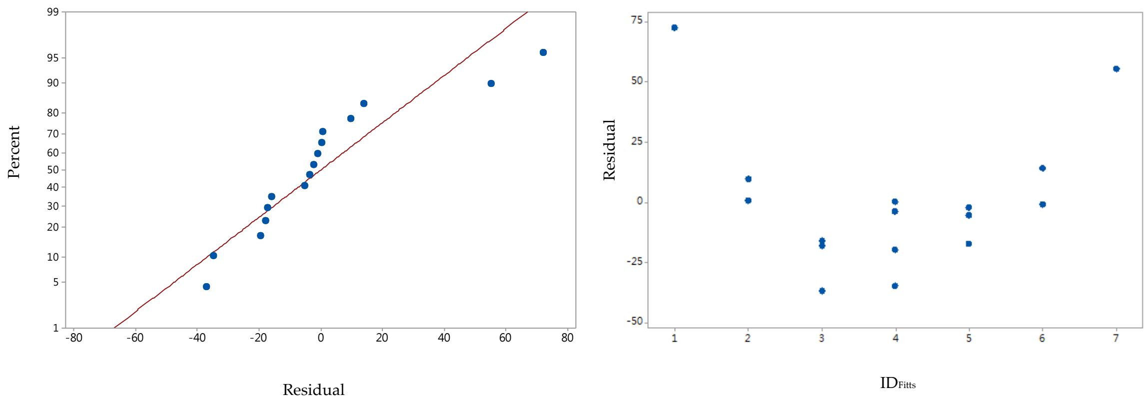

Figure 1. The normality plots of the residuals in the 1-oz stylus tapping task using the Canon model with ID_{Fitts}. The left side is residual vs. response percentage in AD test; the right side is residual vs. ID_{Fitts}. Both plot patterns show violation of the normality assumption.

Notice that, even though the residuals followed the normality requirement, this did not guarantee the satisfaction of the linearity. Figure 2 presents the residual analysis in the Power model for the 1-oz data. The p-value in the AD test was 0.542, and the left graph shows that the scatters fitted the straight line well. However, the right graph shows a parabola with an opening downwards. The data violated the independent assumptions underlying the linear regression again. The lack-of-fit assumes that observations in a response variable for a given predictor variable are (1) normally distributed and (2) independent, and that (3) the distribution of the response variable has constant variance. These assumptions could be checked visually with the residuals plot. Figure 2 implied that even though the residual followed the normal distribution, it might violate the independent assumption.

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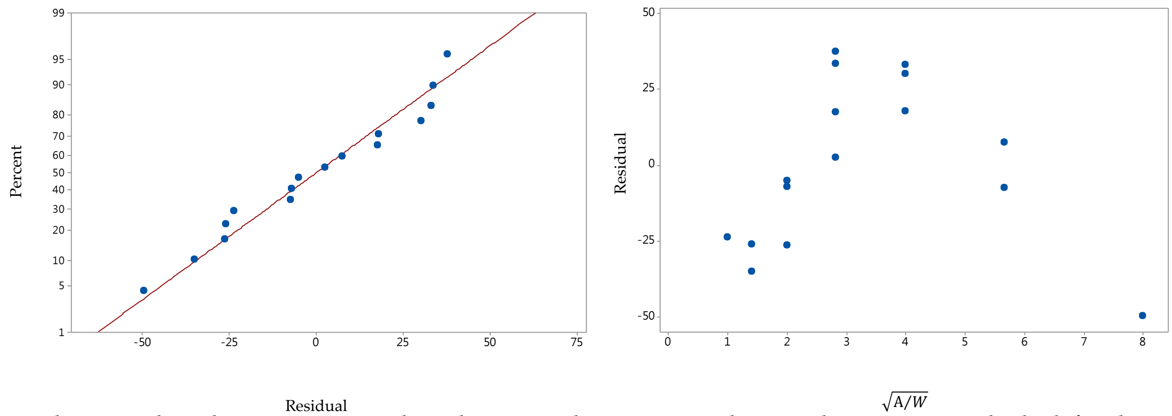

Figure 2. The normality plots of the residuals in the 1-oz stylus tapping task using the power model. The left side is residual vs. response percentage in AD test; the right side is residual vs. $\sqrt{A/W}$. The residuals changed the plus-minus sign compared with Figure 1, and the plot pattern showed the violation of the independent assumption.

Figure 3 demonstrates the advantage of the SQRT_MT model in the residuals. The normality plot using the AD test showed the scatters perfectly fitted along the straight line in the left graph. The scatter pattern of residual vs. ID also implied that the independent requirement and the constant variance underlying the linear regression were not violated.

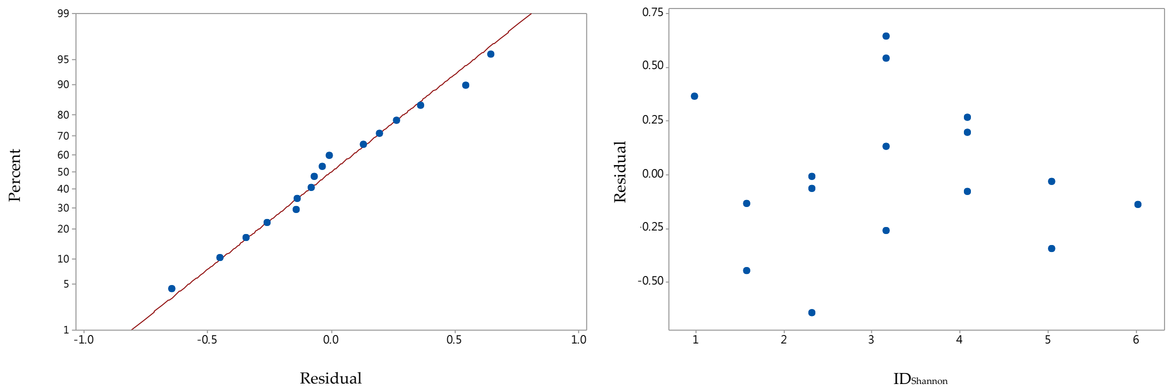

Figure 3. The normality plots of the residuals in the 1-oz stylus tapping task using the SQRT_MT model with $ID_{Shannon}$. The left side is residual vs. response percentage in AD test; the right side is residual vs. $ID_{Shannon}$. The normality assumption was satisfied.

This study hypothesized that MT is related to ID with a constant amplitude limitation; however, this hypothesis cannot be tested by Fitts' data, as mentioned in the literature review. Nevertheless, a trial study using Fitts' data is possible. Table 3 presents the performance of 1-oz stylus tapping with constant amplitude considered. Compared with the pooled amplitude's performance in Table 1, the much worse performance in the PRESS/SSE was a distinct difference. The overall mean ratio was 5.71. The maximum ratio, 7.33, occurred at the 16-inch amplitude in the Power model. In contrast, the minimum ratio was 3.06 at the 8-inch amplitude in the SQRT_MT model with $ID_{Welford}$. Although all models passed the residual normality assumption and had considerable R-squares, the high PRESS/SSE ratio still indicated that the model was not appropriate.

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Table 3. Performance of 1-oz stylus tapping by considering the constant amplitude used Fitts' data [1]. The performance in all statistics was good except the PRESS/SSE.

Amplitude = 2 inches						
Time	ID	a	b	R^2	PRESS/SSE	AD Test's P-value
MT	ID_{Fitts}	90.0	70.5	0.941	6.58	0.062
MT	$ID_{Welford}$	113.7	85.2	0.962	6.58	0.099
MT	$ID_{Shannon}$	67.1	98.6	0.973	6.68	0.139
Power	$\sqrt{A/W}$	52.9	117.8	0.991	6.90	0.134
SQRT_MT	ID_{Fitts}	10.797	2.135	0.962	6.57	0.079
SQRT_MT	$ID_{Welford}$	11.525	2.574	0.979	6.52	0.085
SQRT_MT	$ID_{Shannon}$	10.129	2.974	0.987	6.54	0.101
Amplitude = 4 inches						
Time	ID	a	b	R^2	PRESS/SSE	AD Test's P-value
MT	ID_{Fitts}	-4.5	95.50	0.981	5.97	0.553
MT	$ID_{Welford}$	49.7	105.43	0.987	5.59	0.553
MT	$ID_{Shannon}$	9.4	114.77	0.991	5.16	0.638
Power	$\sqrt{A/W}$	46.2	110.75	0.995	4.46	0.351
SQRT_MT	ID_{Fitts}	8.668	2.6420	0.991	4.90	0.610
SQRT_MT	$ID_{Welford}$	10.181	2.9120	0.994	3.97	0.255
SQRT_MT	$ID_{Shannon}$	9.078	3.166	0.996	3.20	0.206
Amplitude = 8 inches						
Time	ID	a	b	R^2	PRESS/SSE	AD Test's P-value
MT	ID_{Fitts}	-35.5	101.50	0.994	5.91	0.566
MT	$ID_{Welford}$	38.9	106.87	0.996	5.62	0.603
MT	$ID_{Shannon}$	12.3	111.87	0.997	5.23	0.549
Power	$\sqrt{A/W}$	122.3	82.55	0.992	7.19	0.25
SQRT_MT	ID_{Fitts}	9.129	2.4902	0.998	3.87	0.186
SQRT_MT	$ID_{Welford}$	10.960	2.6177	0.999	3.06	0.616
SQRT_MT	$ID_{Shannon}$	10.314	2.7409	0.999	3.14	0.691
Amplitude = 16 inches						
Time	ID	a	b	R^2	PRESS/SSE	AD Test's P-value
MT	ID_{Fitts}	-779.9	114.30	0.993	6.58	0.052
MT	$ID_{Welford}$	16.0	117.31	0.994	6.57	0.058
MT	$ID_{Shannon}$	-2.1	120.25	0.995	6.56	0.065
Power	$\sqrt{A/W}$	211.4	65.68	0.995	7.33	0.314
SQRT_MT	ID_{Fitts}	9.801	2.4479	0.999	6.57	0.075
SQRT_MT	$ID_{Welford}$	11.860	2.5114	0.999	6.56	0.062
SQRT_MT	$ID_{Shannon}$	11.475	2.5737	0.999	6.58	0.056

Because each ID had only one observation, the lack-of-fit test could not be implemented. Generally, we use all the observations to develop a regression equation to predict these observations themselves. The PRESS avoids this dilemma by predicting each observation based on a model developed using all other observations. The PRESS is always larger than SSE because a case deleted in fitting can never be as good as a case included. Consequently, the PRESS/SSE ratio is a supplement to evaluate whether the model fits the observations adequately in this situation.

Figure 4 presents the scatter relationship between the MT and the ID underlying the same amplitude. The upward curve mentioned by Welford [14] existed in every amplitude, which might explain the high PRESS/SSE ratio.

On the other hand, the R-square was higher in the constant amplitude in Table 3 than in the pooled amplitude in Table 1, except at the 2-inch amplitude. The curvature in the 2-inch amplitude was more remarkable than the others in Figure 4, which might explain

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the lower R-square. Such a distinct curvature might have resulted from ballistic movement at the low ID [14]. The effect of ID type was the same as that in the pooled amplitude. $ID_{Shannon}$ had the best performance, followed by $ID_{Welford}$ and ID_{Fitts}, at each amplitude. Overall, the SQRT_MT model had a better R-square than the Power and the Canon models at every amplitude.

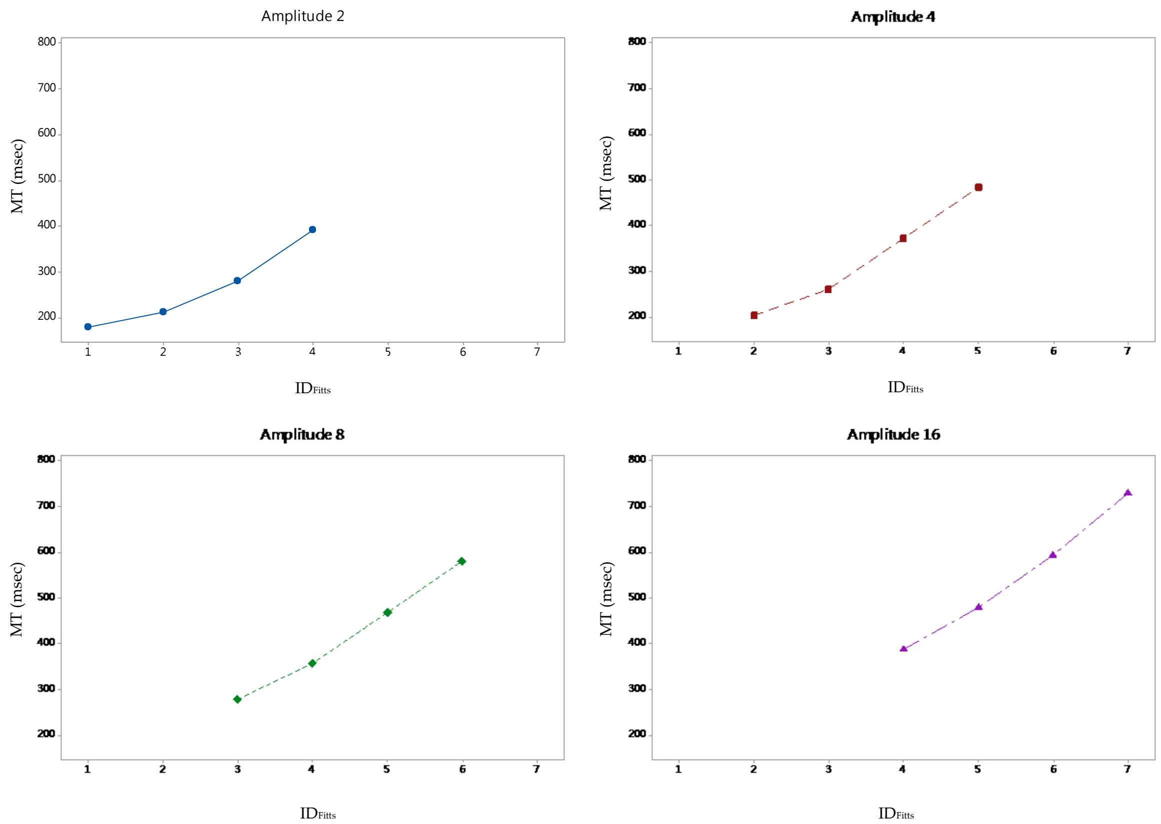

Figure 4. The scatter plot of the movement time vs. ID_{Fitts} in the 1-oz stylus tapping task with a constant amplitude. All the plots showed an upward curve that caused the PRESS to be much worse than the SSE.

Table 4 shows the performance of 1-lb stylus tapping with a constant amplitude. Equally, $ID_{Shannon}$ had the highest R-square, followed by $ID_{Welford}$ and ID_{Fitts}, in the Canon and SQRT_MT models at each amplitude. Instead, the Power model had better R-square than the SQRT_MT and the Canon models at every amplitude except the 16-inch amplitude. However, both the maximum and minimum PRESS/SSE ratios occurred at the 16-inch amplitude. The Power model had the worst performance, with a ratio of 6.80. In contrast, the SQRT_MT model with ID_{Fitts} performed the best, with a ratio of 2.97.

The upward curve is a design flaw in that there is only one observation in each predictor variable. Due to this flaw, every model in Tables 3 and 4 with a high PRESS/SSE ratio was not appropriate for the regression analysis. The SQRT_MT model's advantage was not shown at a constant amplitude; however, the SQRT_MT model performed excellently in the pooled amplitude in Tables 1 and 2.

If there is strong evidence that the linear regression is not appropriate, the next step in the regression analysis is to conduct a transformation. The Box–Cox procedure automatically executes the family of power transformations on the response variable. Generally, the user specifies a numerical range for the parameter lambda: the response variable's power. When the lambda is equal to 0.5 for the Canon model, it is identical to the proposed SQRT_MT model, Equation 14, in this study. Nevertheless, the physiological aspect's inspiration makes the SQRT_MT a causality and not just a statistical relation between MT and ID.

Table 4. Performance of 1-lb stylus tapping at a constant amplitude using Fitts' data [14]. The performance in all statistics was good except the PRESS/SSE.

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Amplitude = 2 inches						
Time	ID	a	b	R^2	PRESS/SSE	AD Test's P-value
MT	ID_{Fitts}	86.5	73.8	0.927	6.52	0.145
MT	$ID_{Welford}$	111.1	89.3	0.949	6.57	0.161
MT	$ID_{Shannon}$	62.1	103.4	0.963	6.70	0.191
Power	$\sqrt{A/W}$	46.7	123.9	0.985	7.36	0.330
SQRT_MT	ID_{Fitts}	10.747	2.208	0.952	6.56	0.094
SQRT_MT	$ID_{Welford}$	11.495	2.664	0.970	6.59	0.119
SQRT_MT	$ID_{Shannon}$	10.045	3.081	0.980	6.70	0.175
Amplitude = 4 inches						
Time	ID	a	b	R^2	PRESS/SSE	AD Test's P-value
MT	ID_{Fitts}	3.5	97.0	0.962	6.58	0.047
MT	$ID_{Welford}$	58.1	107.3	0.972	6.56	0.067
MT	$ID_{Shannon}$	16.6	116.9	0.978	6.58	0.088
Power	$\sqrt{A/W}$	51.8	113.73	0.998	6.18	0.539
SQRT_MT	ID_{Fitts}	9.162	2.607	0.981	6.54	0.166
SQRT_MT	$ID_{Welford}$	10.641	2.878	0.988	6.49	0.124
SQRT_MT	$ID_{Shannon}$	9.539	3.133	0.982	6.45	0.141
Amplitude = 8 inches						
Time	ID	a	b	R^2	PRESS/SSE	AD Test's P-value
MT	ID_{Fitts}	-95.4	120.7	0.981	6.47	0.206
MT	$ID_{Welford}$	-7.3	127.1	0.984	6.45	0.240
MT	$ID_{Shannon}$	-39.4	133.2	0.987	6.44	0.276
Power	$\sqrt{A/W}$	88.45	99.2	0.999	4.07	0.368
SQRT_MT	ID_{Fitts}	8.055	2.858	0.994	6.40	0.281
SQRT_MT	$ID_{Welford}$	10.150	3.006	0.996	6.32	0.384
SQRT_MT	$ID_{Shannon}$	9.400	3.150	0.997	6.18	0.487
Amplitude = 16 inches						
Time	ID	a	b	R^2	PRESS/SSE	AD Test's P-value
MT	ID_{Fitts}	-80.2	121.90	0.997	6.12	0.510
MT	$ID_{Welford}$	22.3	125.09	0.998	5.95	0.551
MT	$ID_{Shannon}$	3.0	128.18	0.998	5.73	0.536
Power	$\sqrt{A/W}$	232.2	69.90	0.990	6.83	0.163
SQRT_MT	ID_{Fitts}	10.240	2.5255	0.999	2.97	0.829
SQRT_MT	$ID_{Welford}$	12.367	2.5903	0.999	3.22	0.419
SQRT_MT	$ID_{Shannon}$	11.973	2.6539	0.999	3.85	0.202

In summary, the SQRT_MT model demonstrates better accomplishments in the statistical requirements for the regression analysis than do the Canon and the Power models. Also, $ID_{Shannon}$ achieves better performance than do $ID_{Welford}$ and ID_{Fitts}. With $ID_{Shannon}$ applied, the SQRT_MT model might be the robust option for Fitts' law application.

This study's first purpose was to propose an optional model to the Canon model with satisfaction of the assumption and the statistical principle of regression analysis for researchers, and it has been achieved by the succinct derivation and validation using historical data in the literature. The results in Tables 1 and 2, and Figure 3, also demonstrated that purpose 2, applicability for ballistic movement, was also achieved.

2.3. Design of Experiment for Study Purpose 3: MT is related to ID with a constant amplitude limitation

A 24 inch/full-HD-resolution projected capacitive touch monitor (model: Nextech NTSP240) and an optical mouse (model: ASUS MM-5113) were applied in the experiment. The experimenter developed specific software to show the targets and record hit positions and durations. A conservative and straightforward rule proposed by Zhai et al. [50] was used to remove hitting outliers. Figure 5 shows the set-up of the experiment.

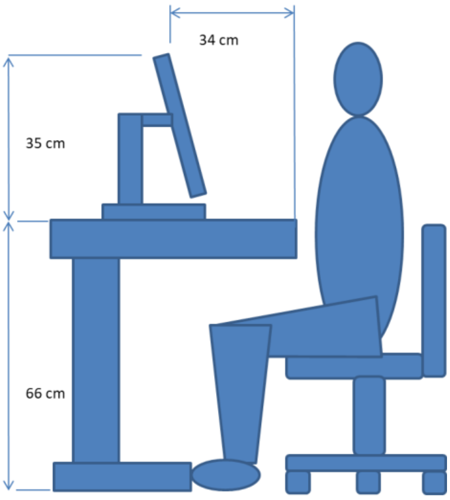

Figure 5. The apparatus set-up in this study. The screen was located on a 66 cm high desk and angled backward; the top edge of the screen was 35 cm from the desktop in the vertical direction, and the bottom edge of the screen was 34 cm from the desk edge near the participant. Participants sat on a chair and adjusted the seat height until they felt comfortable operating the apparatus.

Twelve students (5 males and 7 females) served as the participants. They were 25.3 ± 2.6 (mean/standard deviation) years old and 164.7 ± 7.3 cm in height. All of them were right-handed with no history of upper arm injury. Figure 6 shows the procedure of the experiment. In each treatment, the participants were asked to tap the targets 25 times. The timing began at the first hit and ended at the twenty-fifth hit. Twenty-four response times of twelve participants in the same treatment were averaged as MT. Session one was designed as a training session to increase the participants' familiarity with the tasks. Data acquired from sessions two and three were used for the analysis.

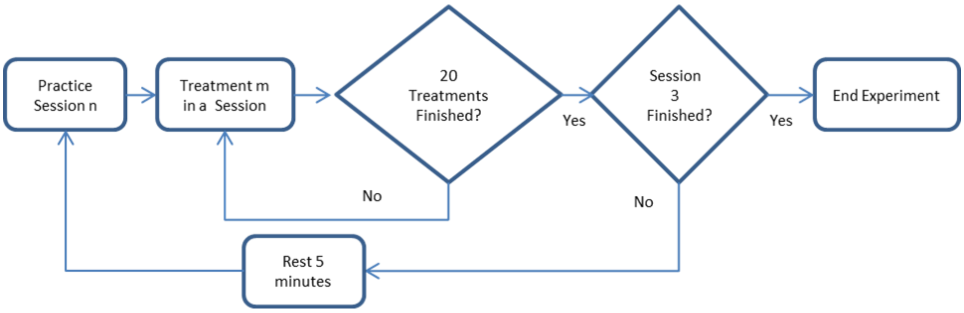

Figure 6. The procedure of the experiment implemented in this research. Every participant began each session with a practice task, A128/ID2. There were three sessions for each participant. Twenty treatments were randomly conducted in a session. After a session finished, the participant was allowed to rest for five minutes or until they felt they had recovered from any fatigue before beginning the next session.

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Instead of the traditional manipulation of target width as an independent variable to vary the ID values in the same amplitude, this research fixed ID values at 2, 3, 4, 5, and 6 (ID2, ID3, ID4, ID5, and ID6) within the amplitude. The movement amplitudes were set at 128, 256, 512, and 1024 (A128, A256, A512, and A1024) device-independent pixels. One device-independent pixel is equal to a square 0.265 mm in width. We treated the index of difficulty under a specific amplitude, ID(A), as a factor in this research. Accordingly, the target widths (W) were determined by the ID and A. Every participant completed the experiment in three sessions on the same day. There were 20 randomized treatments, 20 ID(A), in each session.

This study challenged Fitts' argument, that MT is only related to ID regardless of amplitude, based on the reported results in the literature. Instead, this research hypothesized that MT was related to ID, but the relationship should be considered under the same amplitude. Thus, the null hypotheses are,

$$H_0: \mu_{MT_{ID(A)}} = \mu_{MT_{ID(A')}} \quad \forall A \neq A'. \tag{15}$$

Thirty tests ($5 \text{ IDs} \times C_2^4 \, paired \; amplitudes = 30$) were planned to test this research's hypothesis for the third purpose. The Dunn-Šidák procedure, one of the *a priori* tests suggested by Kirk (Kirk, 2013), was applied in this study. The familywise confidence level was 0.95. Consequently, the individual confidence level was 0.9983.

To explore the effects of amplitude and target width, these two variables were applied in analysis of variance (ANOVA) with a 5% significant level. Tukey's test with a 5% familywise significant level was utilized for the significant factors in ANOVA.

The experiment data were used to compare the coefficient parameters and performance indexes of the Canon, Power, and SQRT_MT models. Model performance was assessed by the residuals' normality, the adequate fitting, and the criteria for model selection.

3. Results and Discussion

3.1. Hypothesis Test on MT is only related to ID

One of Fitts' hypotheses [1] is as follows: "The average time per response will be directly proportional to the minimum average amount of information by the particular conditions of amplitude and tolerance." The statement implies that the average movement time will be identical for the same ID even with varied movement amplitude and target width. Fitts thought the logarithmic term of amplitude/width ratio was the minimum average amount of information. To achieve the third purpose of this study, testing the hypothesis that MT was only related to ID, as claimed by Fitts, this study designed an experiment with repetition in each ID underlying each amplitude. Table 5 presents the *a priori* test of the hypothesis mentioned in Equation 15. Generally, 26 out of 30 planned contrasts differed significantly. Consequently, MT might not be related only to ID. The possible reason might be the confounding factor, ID, as Guiard [43] indicated. The proper solution to this problem is to disaggregate ID into two individual factors, A and W.

Since the ID cannot be the only dominator of MT, this study was interested in the effects of amplitude and target width on MT. An ANOVA showed both A ($F_{3,33} = 148.49$) and W ($F_{7,77} = 40.18$) were significant, with p-values less than 0.001. Table 6 presents the post-hoc test for the amplitude. All the p-values of the Tukey pairwise comparisons were less than 0.001. This result implied that the MT depends on varied amplitude: A longer amplitude requires more MT. Likewise, Table 7 demonstrates the post-hoc test for the target width. Almost all p-values of the Tukey pairwise comparisons were less than 0.001, except that of 256-128, which was 0.002. The result implied that MT depends on varied target width, as well. But MT is negatively related to target width; a smaller target width requires more MT.

In Table 5, all six planned contrasts were significant at ID2 except (128, 256). When the difference between the paired amplitude was not huge, as in the cases of (128, 256), the contrast in MTs was just marginally not significant, since the lower bound of the simultaneous 95% confidence interval was near zero. In contrast, when the difference between the amplitudes increased, the difference in MT tended to be more significant. In the easiest task, the contribution of movement amplitude to MT is more than that of target width. For example, for target widths of 64 and 512, and the amplitudes are 128 and 1024 pixels in the 2(128)-2(1024) contrast. The absolute mean difference in MT of the 128-1024 contrast in the movement amplitude was 298.3 msec, and that of 64-512 in the target width was 128.3 msec. Similarly, this study found that the contribution of amplitude was more than that of width in all six planned contrasts at ID2. The smallest target width was 64 pixels at ID2, which was still wide enough for participants to hit the target quickly. The trade-off effect of speed vs. accuracy was not evident in the easiest task.

Table 5. Dunn-Šidák simultaneous test for the planned contrasts, indicating that most the same ID does not have identical MT.

Difference of ID(A) Levels	Difference of Mean	Standard Error of Difference	Simultaneous 95% Confidence Interval	Significant Difference
2(128)-2(1024)	-158.9	11.6	(-195.9, -121.9)	Yes
2(256)-2(1024)	-130.5	11.6	(-167.5, -93.5)	Yes
2(512)-2(1024)	-54.1	11.6	(-91.1, -17.1)	Yes
2(256)-2(128)	28.4	11.6	(-8.6, 65.4)	No
2(512)-2(128)	104.7	11.6	(67.7, 141.7)	Yes
2(512)-2(256)	76.4	11.6	(39.4, 113.4)	Yes
3(128)-3(1024)	-149.0	11.6	(-186.0, -112.0)	Yes
3(256)-3(1024)	-137.5	11.6	(-174.5, -100.5)	Yes
3(512)-3(1024)	-76.8	11.6	(-113.8, -39.8)	Yes
3(256)-3(128)	11.5	11.6	(-25.5, 48.5)	No
3(512)-3(128)	72.2	11.6	(35.2, 109.2)	Yes
3(512)-3(256)	60.7	11.6	(23.7, 97.7)	Yes
4(128)-4(1024)	-105.2	11.6	(-142.2, -68.2)	Yes
4(256)-4(1024)	-105.9	11.6	(-142.9, -68.9)	Yes
4(512)-4(1024)	-65.8	11.6	(-102.8, -28.8)	Yes
4(256)-4(128)	-0.7	11.6	(-37.7, 36.3)	No
4(512)-4(128)	39.4	11.6	(-2.4, 76.4)	Yes
4(512)-4(256)	40.1	11.6	(-3.1, 77.1)	Yes
5(128)-5(1024)	-126.1	11.6	(-163.1, -89.1)	Yes
5(256)-5(1024)	-90.2	11.6	(-127.2, -53.2)	Yes
5(512)-5(1024)	-53.0	11.6	(-90.0, -16.0)	Yes
5(256)-5(128)	36.0	11.6	(-1.0, 73.0)	No
5(512)-5(128)	73.1	11.6	(36.1, 110.1)	Yes
5(512)-5(256)	37.1	11.6	(0.1, 74.1)	Yes
6(128)-6(1024)	-145.3	11.6	(-182.3, -108.3)	Yes
6(256)-6(1024)	-95.0	11.6	(-23.0, -58.0)	Yes
6(512)-6(1024)	-54.3	11.6	(-91.3, -17.3)	Yes
6(256)-6(128)	50.3	11.6	(13.3, 87.3)	Yes
6(512)-6(128)	91.1	11.6	(54.1, 128.1)	Yes
6(512)-6(256)	40.8	11.6	(3.8, 77.8)	Yes

Individual confidence level = 99.83%

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Table 6. Tukey simultaneous tests for differences of means in amplitude. The larger the amplitude, the longer the MT was.

Difference of Amplitude Levels	Difference of Mean	Standard Error of difference	Simultaneous 95% Confidence Interval	T-value	P-value
256-128	80.0	5.5	(65.8, 94.2)	14.47	< 0.001
512-128	184.9	5.9	(169.7, 200.2)	31.16	< 0.001
1024-128	298.3	6.5	(281.7, 314.9)	46.09	< 0.001
512-256	104.9	5.4	(91.0, 118.8)	19.32	< 0.001
1024-256	218.3	5.9	(203.7, 233.5)	36.79	< 0.001
1024-512	113.4	5.5	(99.2, 127.6)	20.52	< 0.001

Individual confidence level = 98.93%

Table 7. Tukey simultaneous tests for differences of means in width. The smaller the width, the longer the MT was.

Difference of Width Levels	Difference of Mean	Standard Error of difference	Simultaneous 95% Confidence Interval	T-value	P-value
8-4	-34.7	10.3	(-160.0, -88.3)	-3.38	< 0.001
16-4	-88.3	9.9	(-275.8, -200.8)	-8.91	< 0.001
32-4	-159.9	9.8	(-352.6, -278.6)	-16.34	< 0.001
64-4	-223.0	9.8	(-427.1, -353.1)	-22.80	< 0.001
128-4	-274.6	10.6	(-480.7, -400.5)	-25.92	< 0.001
256-4	-304.1	11.3	(-521.3, -435.5)	-26.81	< 0.001
512-4	-351.3	13.1	(-589.2, -489.9)	-26.77	< 0.001
16-8	-53.7	7.6	(-139.8, -82.6)	-7.11	< 0.001
32-8	-125.2	7.3	(-216.3, -160.7)	-17.05	< 0.001
64-8	-188.4	7.3	(-290.8, -235.2)	-25.65	< 0.001
128-8	-239.9	8.1	(-344.0, -283.0)	-29.80	< 0.001
256-8	-269.5	9.2	(-386.2, -316.3)	-29.17	< 0.001
512-8	-316.7	11.3	(-455.4, -369.5)	-27.92	< 0.001
32-16	-71.5	6.3	(-101.1, -53.5)	-11.37	< 0.001
64-16	-134.7	6.3	(-175.6, -128.0)	-21.40	< 0.001
128-16	-186.2	6.9	(-228.6, -176.1)	-26.85	< 0.001
256-16	-215.8	8.1	(-270.6, -209.6)	-26.80	< 0.001
512-16	-263.0	10.6	(-341.3, -261.2)	-24.83	< 0.001
64-32	-63.1	5.7	(-96.1, -52.9)	-11.06	< 0.001
128-32	-114.7	6.3	(-148.9, -101.2)	-18.23	< 0.001
256-32	-144.2	7.3	(-190.6, -135.0)	-19.64	< 0.001
512-32	-191.5	9.8	(-261.0, -187.0)	-19.57	< 0.001
128-64	-51.6	6.3	(-74.3, -26.7)	-8.19	< 0.001
256-64	-81.1	7.3	(-116.1, -60.5)	-11.05	< 0.001
512-64	-128.3	9.8	(-186.5, -112.4)	-13.12	< 0.001
256-128	-29.6	7.6	(-66.4, -9.2)	-3.91	0.002
512-128	-76.8	9.9	(-136.4, -61.4)	-7.74	< 0.001
512-256	-47.2	10.3	(-100.0, -22.3)	-4.60	< 0.001

Individual confidence level = 99.74%

The similar results to the ID2 occurred at ID3, ID4, and ID5. With the increase in difficulty, the effect of width was gradually revealed. The 3(256)-3(128), 4(256)-4(128), and 5(256)-5(128) contrast was the only non-significant contrast at ID3, ID4, and ID5 again. But two contrasts, 4(512)-4(256) and 4(512)-4(128), were marginally significant at ID4. Equally, 5(512)-5(256) and 5(512)-5(128) were marginally significant at ID5. All the adjacent amplitudes that were less than 128 pixels were not significant at IDs 2 to 5. However, all six planned contrasts were significant at ID6. Only 6(512)-6(256) and 6(256)-6(128), but not 6(512)-6(128), were marginally significant at ID6. Generally, the significance increased gradually with the difference of adjacent amplitudes increased. Such results imply that a reduced target width in a harder task causes participants to employ a speed–accuracy trade-off policy.

This study reanalyzed Fitts' data (Fitts, 1954) by considering amplitude and width as factors. The ANOVA results indicated that both amplitude ($F_{3,9}$ = 101.08) and width ($F_{3,9}$ = 98.56) had p-values less than 0.001 in the 1-oz stylus tapping. Equally, amplitude ($F_{3,9}$ = 77.40) and width ($F_{3,9}$ = 81.40) had p-values less than 0.001 in the 1-lb stylus tapping. All factors' levels were significantly different, since they were in different groups, according to the Tukey pairwise comparison test (Table 8). Thus, MT increases as A increases or W decreases. Although the apparatus in this research differed from that in Fitts', the effects of amplitude and the target width on movement time were consistent. This raises a new question that we may have to examine carefully. Fitts' law fuses two significant factors into a single factor. Could this factor indicate that the effects of the two factors on the response time are still an unknown issue? Fortunately, we could overcome this issue by considering Fitts' law with an underlying constant amplitude, since we would like to reject the null hypothesis. In this way, the only factor we have to investigate is the target width.

In summary, this study suggests applying Fitts' law with an underlying constant amplitude. Based on our findings: (1) Target width is a significant factor in MT. This result is consistent with Hoffmann and Sheikh's findings [51]. When the width is sufficiently narrow, people extend the MT due to the speed–accuracy trade-off. (2) The fact that amplitude affects MT significantly is consistent with Accot and Zhai [52]. (3) This paper is the first to develop a statistical inference about Fitts' law's implication that MT is not only related to ID in the literature. Our results support Gan and Hoffmann's argument that amplitude affects MT and is independent of the index of difficulty [5].

Table 8. Tukey simultaneous tests for the effects of amplitude and target width effect on MT in 1-oz and 1-lb stylus tapping from Fitts' data [1] had consistent results with this study.

		1-oz stylus					1-lb stylus				
A*	Sample Size	Mean MT	Grouping				Mean MT	Grouping			
16	4	548.8	A				590.3	A			
8	4	421.3		B			447.8		B		
4	4	329.8			C		343.0			C	
2	4	266.3				D	271.0				D
W*	Sample Size	Mean MT					Mean MT				
0.25	4	546.8	A				586.5	A			
0.50	4	429.3		B			444.3		B		
1.00	4	327.5			C		346.8			C	
2.00	4	262.5				D	274.5				D

*: units in inches.

3.2. Validation of the SQRT_MT Model for a Constant Amplitude

The robustness of the proposed SQRT_MT model was verified with the data of this research. Since in this research, the practice effect was more significant in session one than in sessions two and three, unlike the results in the literature [53,54], the regression analysis was processed by excluding the data from session one.

Table 9 presents the regression analysis using data with all the amplitudes. Although all the models fitted the linearity well and the PRESS/SSE ratio was almost perfect, around 1.11, the R-square, which was around 0.80, was somewhat lower than the reported value of 0.95 in the literature. $ID_{Shannon}$ produced a higher R-square than $ID_{Welford}$ and ID_{Fitts} in both the Canon and SQRT_MT models. Also, the SQRT_MT model always had a better R-square than the Canon model for each ID type. The Power model's R-square value was between those of the SQRT_MT model and the Canon model. Nevertheless, none of the models satisfied the residual normality requirement.

Table 9. Regression analysis using data for all amplitudes. The R-square was much lower than the value in the literature, possibly due to the equal observations in each ID. Additionally, the normality in all models was significant.

Model	ID	a	b	R^2	PRESS/SSE	Lack-of-fit	AD Test's P-value
Canon	ID_{Fitts}	379.2	72.22	0.797	1.11	0.417	0.026
Canon	$ID_{Welford}$	423.7	78.10	0.802	1.11	0.597	0.014
Canon	$ID_{Shannon}$	397.4	83.53	0.806	1.11	0.744	0.009
Power	$\sqrt{A/W}$	451.7	68.05	0.809	1.10	0.902	0.011
SQRT_MT	ID_{Fitts}	19.656	1.471	0.820	1.11	0.771	0.017
SQRT_MT	$ID_{Welford}$	20.570	1.589	0.822	1.11	0.915	0.010
SQRT_MT	$ID_{Shannon}$	20.041	1.697	0.824	1.11	0.978	0.010

Figure 7 illustrates the results. The scatters in the left graph, distributed straightly along the regression line, satisfied the linearity adequately in the lack-of-fit test. A straight line passed through the dots' center at each ID, making the PRESS deviate from SSE slightly. The equal observations in each ID differed from Fitts' data pattern [1], which caused the worse R-square. However, the variance of the fitted value increased as the ID decreased. Such a result implies a violation of the constant variance assumption in the regression analysis. The right graph in Figure 7 presents the regression analysis under a constant amplitude. The slope of these regression lines was different, which explained the unsatisfied residual normality.

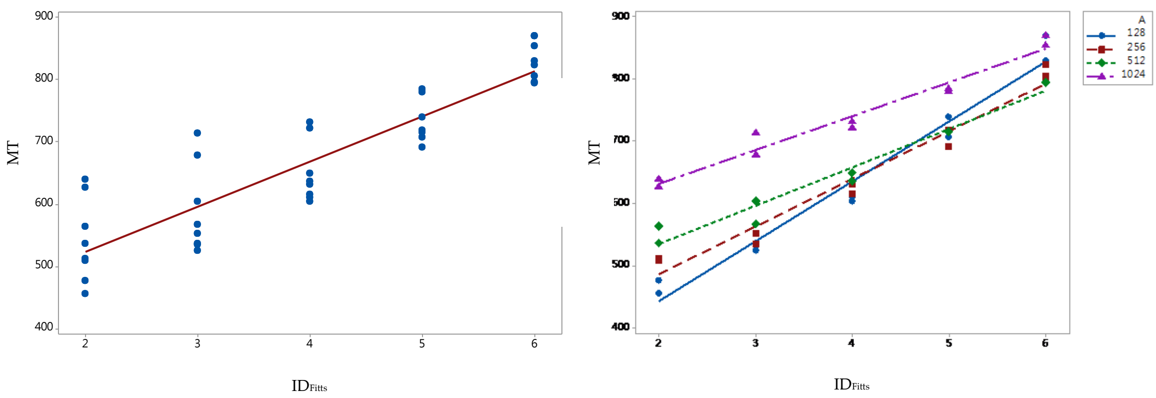

Figure 7. The scatter plot with the regression line. The regression line in the left graph used ID_{Fitts} in the Canon model with all observations. It implied the variance of varied ID was not equal. Under a constant amplitude for regression analysis, the right graph showed that the slope of each line was different.

It was expected that the residual normality would not be satisfied. Table 5 implied that a specific ID's MTs with amplitude varied would be entirely different from one another at the low ID. When the adjacent amplitudes' difference at a more difficult ID is short enough, the MT does not differ much. With increases in ID, the narrow width extends the MT due to participants' speed–accuracy trade-off to compensate for the gap due to the amplitude. Such a result implies the unequal slope of the regression lines in the varied constant amplitude. The unequal slope also implies that there might exist an interaction between amplitude and target width. The right graph in Figure 7 supported the existence of such an interaction.

This study's hypothesis, that MT is related to the ID underlying a constant amplitude, was validated in Section 3.1. A further regression analysis considering the constant amplitude was executed. Table 10 presents the parameters and test statistics required for an appreciable linear regression line considering the varied constant amplitude. Generally, the results were almost perfect, except that the Canon model applying ID_{Fitts} failed the lack-of-fit test at the amplitude of 256 pixels.

Table 10. Regression analysis with the amplitude separated, but one failed the normality.

				Amplitude 128			
Model	ID	a	b	R^2	PRESS/SSE	Lack-of-fit	AD Test's P-value
Canon	ID_{Fitts}	251.4	95.99	0.971	1.70	0.081	0.697
Canon	$ID_{Welford}$	310.5	103.81	0.978	1.71	0.153	0.636
Canon	$ID_{Shannon}$	275.5	111.04	0.983	1.72	0.259	0.694
Power	$\sqrt{A/W}$	347.7	90.48	0.988	1.87	0.522	0.238
SQRT_MT	ID_{Fitts}	17.184	1.926	0.985	1.69	0.236	0.649
SQRT_MT	$ID_{Welford}$	18.379	2.080	0.989	1.69	0.460	0.527
SQRT_MT	$ID_{Shannon}$	17.685	2.222	0.991	1.71	0.700	0.357
				Amplitude 256			
Model	ID	a	b	R^2	PRESS/SSE	Lack-of-fit	AD Test's P-value
Canon	ID_{Fitts}	332.6	76.71	0.964	1.70	0.030	0.423
Canon	$ID_{Welford}$	379.7	83.03	0.972	1.69	0.057	0.540
Canon	$ID_{Shannon}$	351.4	88.86	0.978	1.67	0.098	0.542
Power	$\sqrt{A/W}$	408.9	72.64	0.989	1.53	0.451	0.536
SQRT_MT	ID_{Fitts}	18.687	1.575	0.979	1.68	0.090	0.812
SQRT_MT	$ID_{Welford}$	19.661	1.702	0.985	1.65	0.185	0.893
SQRT_MT	$ID_{Shannon}$	19.089	1.819	0.988	1.62	0.322	0.581
				Amplitude 512			
Model	ID	a	b	R^2	PRESS/SSE	Lack-of-fit	AD Test's P-value
Canon	ID_{Fitts}	410.2	61.90	0.965	1.65	0.202	0.910
Canon	$ID_{Welford}$	448.3	66.95	0.972	1.63	0.332	0.605
Canon	$ID_{Shannon}$	425.7	71.61	0.977	1.61	0.484	0.547
Power	$\sqrt{A/W}$	472.5	58.30	0.979	1.54	0.623	0.166
SQRT_MT	ID_{Fitts}	20.200	1.292	0.979	1.62	0.445	0.250
SQRT_MT	$ID_{Welford}$	21.002	1.396	0.983	1.59	0.678	0.361
SQRT_MT	$ID_{Shannon}$	20.536	1.491	0.985	1.57	0.851	0.335
				Amplitude 1024			
Model	ID	a	b	R^2	PRESS/SSE	Lack-of-fit	AD Test's P-value
Canon	ID_{Fitts}	522.6	54.30	0.966	1.54	0.189	0.309
Canon	$ID_{Welford}$	556.4	58.60	0.969	1.53	0.232	0.194
Canon	$ID_{Shannon}$	536.9	62.59	0.971	1.53	0.262	0.111
Power	$\sqrt{A/W}$	578.4	50.78	0.967	1.59	0.199	0.297
SQRT_MT	ID_{Fitts}	22.553	1.092	0.977	1.51	0.220	0.273
SQRT_MT	$ID_{Welford}$	23.239	1.177	0.977	1.52	0.218	0.120
SQRT_MT	$ID_{Shannon}$	22.852	1.256	0.976	1.53	0.200	0.188

All the models strongly satisfied the residual normality. The PRESS/SSE ratio, which ranged between 1.51 and 1.89, also performed well. For the effect of ID type on the PRESS/SSE ratio, all three IDs were compatible. However, the SQRT_MT model had a minor advantage over the Canon model. The Power model provided the best and the worst ratio twice, respectively.

In the R-square performance, the values ranged from 0.964 to 0.991, and the mean was 0.978. The maximum occurred at the amplitude of 128 pixels when the SQRT_MT model with $ID_{Shannon}$ was applied. In contrast, the minimum happened at the amplitude of 256 pixels when the Canon model with ID_{Fitts} was applied. In the four amplitudes, the value of $ID_{Shannon}$ was the highest or second to the highest, with a difference of 0.001. This research utilized a heuristic procedure for model selection in Table 10. The procedure was as follows: (1) Both the lack-of-fit and the residual diagnostics must be satisfied. (2) Consider both the PRESS/SSE and the R-square simultaneously, and take a trade-off policy, such as the difference between the PRESS/SSE within 0.01 and the R-square of less than 0.1. Researchers could set the threshold values themselves. Overall, the SQRT_MT model performed slightly better in R-square than did the Power and the Canon models at the four constant amplitudes.

As mentioned in Section 2.2, the regression formulations underlying a constant amplitude using Fitts' data did not satisfy the linearity due to only one observation being conducted for each predictor variable's level, which caused the scatters to resemble a curve. The experiment in this study remedied this flaw, and the results in Table 9 implied the last piece of the puzzle for this research's argument, that Fitts' law should consider IDs underlying a constant amplitude, was finished. Also, the SQRT_MT model with $ID_{Shannon}$ performed excellently and robustly with both the data from the experiment in this study and the historical data in the literature, suggesting that it might be an excellent option for the Fitts' law application.

3.3. Model selection for application

Although the Power law's proponents emphasized that the information-theory-based formulation was an invalid analogy to Shannon Theorem 17 and derived the power law from human's optimal movement theoretically, this ideal optimal behavior might be challenging to validate in the real world. Also, the Power model resulted in a higher R-square in Fitts' data compared with the information-theory-based formulation using ID_{Fitts} [8,26]. Nevertheless, the Canon model, based on information theory, remains mainstream in Fitts' law applications. The possible reason might be the magic of information theory, which provides an attractive character explaining the information transmitted in a specific task. This character helps evaluate or compare various devices' performances, not just times to complete the same job. Oppositely, the square root of the A/W ratio is meaningless. Consequently, the Canon models are popular among researchers and industrial practitioners.

Fitts was not entirely consistent on the analogy to the information theory. As Fitts himself stated, "The index of difficulty is not an exact measure of information" [55]. Also, the statement "the analogy with Shannon's Theorem 17 is not exact" was printed in note 3 in Fitts and Peterson [15]. On the other hand, Fitts tried to rationalize this analogy aggressively even though he knew this analogy was problematic. Fitts defined ID_{Fitts} as the only information amount necessary for a movement with the amplitude and the target width specified by ID_{Fitts} [1]. ID_{Fitts} involves the amplitude and the width as metrics in length, but not in power like Shannon Theorem 17. However, Fitts still utilized ID as the information in bits for various human movement analyses.

Since the first alternative to ID appeared in 1960 [14], the valid version issue has existed. Fitts and Peterson [15] criticized the use of $ID_{Welford}$ and suggested that the choice between ID_{Fitts} and $ID_{Welford}$ should rest on heuristic considerations, since neither of them had been derived formally from a theory. In the same manner, $ID_{Shannon}$ was not a formal derivation, either. MacKenzie also had similar thoughts, stating, "It is an engineering issue to choose a higher correlation formulation" and "the only concern was utility," in his reply to Hoffmann's challenge on the valid version issue [42].

This research focused on the facts about Fitts' law in the literature that we have mentioned but were ignored by researchers. Fitts [15] clearly stated that the analogy to Shannon Theorem 17 is not proper [55], the ID is not formally derived from the information theory [11,15,17,26,42], and the selection between candidates relies on researchers' heuristic consideration. The above results imply that it is more suitable to treat all the Fitts' law models simply as a prediction model for the response time. Based on the results of this research listed in Table 10, all the available options in Fitts' law have excellent performance in a proper experiment design. This research agreed with Fitts [17] and MacKenzie's argument [42]; researchers could choose their preferred model heuristically. Therefore, the valid version should not be an issue in the community.

To summarize, the approach to modeling the prediction of movement time by applying a constant amplitude with repetition in each predictor variable and proper diagnostics for the linear regression's assumptions should be considered in Fitts' law. Researchers can select any predicting formulation they prefer: the Canon, the Power, or the SQRT_MT model.

3.4. Meaning of the intercept and the slope

Some researchers have claimed that the non-zero intercept was caused by a modeling error [24,56] or resulted from not using $ID_{Shannon}$ [18,21]. Those authors might think that movement time should be zero when ID is zero. However, ID is never zero in the real world [15]. Fitts designed the ID to be more than zero [1]. No matter how easy a movement is, there is a slight difficulty. Zero difficulty would imply a total lack of movement. That might be the reason Fitts added the constant multiplier, 2, in the definition of ID.

Other reasons, such as the time required for movement of zero distance [21], the dwell time on the target in reciprocal tapping [17], and submovements such as the pressing time of a mouse button [21], do not explain the zero intercept, either. The movement time is always measured from the departure from the starting point to the moment the target is hit. The time required for motor unit recruitment before the movement occurs, i.e., time for zero distance, might be impossible to record by the apparatus in the reported literature.

Theoretically, whether the intercept equals zero or not should be determined by a t-test based on empirical data. Many possible meanings of the intercept have been reviewed in the literature [4,50]. Fitts designed the ID never to be zero by multiplying by 2 in the logarithmic term. Also, neither $ID_{Welford}$ nor $ID_{Shannon}$ is zero, theoretically. Since the intercept is a derived value from the regression analysis and there is no theoretical reason to be zero, its calculation and explanation must obey the statistical principle. Kunter et al. [44] indicated that the intercept does not have any particular meaning if a horizontal axis variable equal to zero does not exist. Fitts' law satisfies this kind of case. All of the arguments about the zero intercept might be unnecessary.

The most attractive index, the slope "b" in Equation 2, was proposed to underlie the analogy to Shannon Theorem 17. However, this explanation would not work if the intercept, "a" in Equation 2, were not zero. Unfortunately, "a" is seldom zero in the empirical data.

Moreover, we had argued that all the Fitts' law models should be treated as one kind of simple prediction equation for the response time with the supporting evidence in the literature in Section 3.3. In normal circumstances, ID is not the required information, nor is the information transmission rate sloped.

In short, the intercept is meaningless because the predictor variable, ID, is never zero in the real world. Similarly, the slope is the rate of change in movement time for the unit change in ID.

4. Conclusions

To conclude, the present study is primary research on Fitts' paradigm. Still, it reveals some essential requirements and assumptions of the linear regression analysis that were not satisfied in the past research. The major contributions of this research are as follows: (1) This is the first study to conduct a formal hypothesis test for the essential assumption, that MT is related only to ID, in the literature. The results indicate that MT is related to the ID underlying a constant amplitude. Researchers might apply Fitts' law for varied amplitudes, respectively. (2) The SQRT_MT model provides excellent results in all the required statistics of linear regression. The proposed SQRT_MT model, fusing the findings in psychology, physics, and physiology, in combination with $ID_{Shannon}$, might be a robust option for Fitts' law application. In addition, it is a simple and easy-to-apply model that tolerates both ballistic and visual-feedback movement concurrently. (3) The approach to modeling the prediction of movement time should utilize repetition in each predictor variable, and proper diagnostics for the linear regression's assumptions should be considered. (4) The results of the experiment in this research show that all the available models are excellent prediction equations. Researchers can choose their preferred model for their investigations.

Although this study's three purposes were achieved, there are still limitations in this research. (1) The SQRT_MT model was demonstrated to tolerate the ballistic and the visual feedback movement concurrently with the historical data in the literature, but it is believed that no ballistic movement was present in this study, since the shortest MT in this study was much longer than 200 msec. (2) The linearity fitting and the residuals' assumptions were strongly satisfied in the results of the experiment, too. However, the repetition in each predictor level underlying a constant amplitude is two. Future research is suggested to improve the two limitations to enhance the SQRT_MT model's validity.

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