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INTEGRATION FRAMEWORK FOR MODELING TOLLS CONSIDERING DIFFERENT SUBPOPULATIONS WITH DIFFERENT VALUES OF TIME

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ABSTRACT

Unique analytical challenges arise when drivers, who face a route choice between a toll lane and a set of free lanes, have different *values of time*. The most complex situation is one in which multiple sub-populations of drivers exist, each with their own unique mean and coefficient of variation of value of time. This situation, when imbedded within a larger network cannot be tackled using existing planning models, and consequently is usually only approximated. This paper examines these different approximations, the resulting numerical solutions and the implications of these approximations on the estimate of the number of expected toll lane users. The paper also shows how this problem can be solved using a combined traffic assignment/simulation model. The first part of this paper develops an analytical formulation for solving the toll lane scenario using the "value of time" representations range from the simplest to the most complex. It is shown that one of the most critical issues is a determination of who the marginal users are of the toll lane, at each level of usage, as the perceived disutility of the last marginal toll lane user depends dynamically upon that driver's value of time. Analytical formulations based on these different approximations are then solved numerically in the second part of the paper. These numerical solutions show that significant different lane use estimates result, depending upon the representation of value of time. Consequently, it is clear that solving this problem with the fewest approximations is both of theoretical and practical importance. The third part of the paper illustrates the solution to the toll lane problem, with each level of approximation, using a combined traffic assignment/simulation model. The simulated resulting estimates of the toll lane usage for each case matches both the relative and absolute trends found in analytical solutions. However, the solution using the assignment/simulation model is not only much faster and simpler to obtain, but is also scalable both in size and complexity. The additional complexities, that are associated with a less approximate representation of value of time, should therefore be incorporated in all future assessments of toll lane facilities, be they analyzed analytically or through simulation.

Keywords: Tolls, INTEGRATION software, microscopic traffic simulation, traveler value of time

1. INTRODUCTION

An increasingly common traffic network situation is one in which drivers, traveling from A to B, have at least two main types of route choices, one which is tolled and one which is not. An example of such a situation is illustrated in Figure 1a, where a multi-lane freeway splits at several locations to provide 3 lanes that are free of toll and 1 lane that involves a toll. While there are some unique traffic engineering issues associated with the weaving that can take place, when there are multiple breaks in the structure that divides the toll lanes from the free ones, many important equilibrium issues can

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already be analyzed by looking at single diverge sections. Such a sample section is illustrated in Figure 1b and is utilized as the medium by which the background to this toll lane problem is described.

1.1 Background

If traffic demand is low, and the distance and speed limits are similar on both types of lanes, there exists little incentive for drivers to utilize the toll lane alternative. However, as the traffic demand on the toll alternative increases, its speed will decrease. This will make its travel time become longer, making the free lanes less attractive. This sets up a condition in which it may become attractive to some, but not necessarily all drivers, to start utilizing the toll lane. The timing of when drivers start to use the toll lanes and the number of drivers who elect to use the toll lane will depends primarily on 2 factors, namely the “value of the toll” and the “value of time” of the drivers involved.

The “value of the toll” and “value of time” are closely related quantities in this problem, as a higher toll value accompanied with a higher value of time will result in the same perceived disutility and therefore path impedance (Ben-Akiva et al., 1993). Consequently, it is ratio of the “value of the toll” to the “value of time” that is most important. In addition, one should also note that for the same toll value, those with the highest “value of time” will be least influenced by the toll. They will therefore also be the first ones to start using the toll lanes when the free lanes become busy and slower. In contrast, those with the lowest value of time perceive the toll as having the highest disutility or travel time equivalency. They will therefore be the last ones to switch to the toll lane, if at all.

The decisions, of those drivers who have a lower value of time, are also influenced by the decisions of drivers with a higher value of time. Specifically, any switch over of the latter drivers from the free lanes to the toll lanes will leave the free lanes less congested for those with a lower value of time who stay in the free lanes. This may leave it undesirable for those with a low value of time to also switch to the toll lane. However, if the percent of drivers with a high value of time is relatively small, or if the total flow from A to B is very high, even some of those with a low value of time will perceive themselves as being better off if they use the toll lanes. The issue of toll lane usage therefore requires one to answer two interrelated questions, namely: (1) What group or sub-population of drivers will utilize the toll road, and (2) What fraction of this group or sub-population will utilize the toll road.

1.2 Focus of the Paper

In this paper the problem to be considered in one in which there is a constant toll on one lane, but where the value of time is different for different drivers. Non-homogeneity of the driver types is one example of the source of such potential differences in value of time. For example, commuters, business travelers and freight vehicles may be present in the same traffic stream, where each sub-population may have a different mean value of time. In addition, within each sub-population there may be further variations about the mean value of time for that sub-population. In other words, not all commuters, business travelers or drivers of trucks will have the same value of time as their peers within the same sub-population.

This paper examines the following questions:

1. if there is variability in value of time, how should this be analyzed? , and
2. if analyzed appropriately, are results different for different approximations?

It should also be noted that there is an important distinction between condition 1, when there are known variations in the value of time of a sub-population, and condition 2 when there is uncertainty as to the mean “value of time” of a sub-population. In terms of condition 1, we need to find out what deterministic percentile, within the sub-population, will use the toll lane. In contrast, in condition 2 one needs to determine the probability that a sub-population will have a certain value of time, and to assess if for this value of time they will or will not use the toll lane. This paper examines condition 1.

Finally, the current problem considers that neither the tolls or the traffic demand vary as a function of time. It also assumes that the toll levels for a given day are specified a priori and are known in advance by all drivers. These additional levels of complexity have been addressed using the combined assignment/simulation that is referenced at the end of this paper. However, they are not considered in this paper as there is insufficient space in this paper to address the associated analytical formulations in order to first deal with the issues arising from differences in value of time. The treatment of dynamics in tolls, demand or queues all depend upon the ability to be able to properly represent differences in value of time, as is discussed in this paper, making this paper a logical first building block.

1.3 Objectives of the Paper

The first objective of this paper is to illustrate how variability in “value of time” can be represented at different levels of potential aggregation/approximation within analytical formulations of the toll lane problem. For example, one can ignore all variability and use a single mean value for all drivers. Alternatively, one can consider that the fleet consists of multiple sub-populations, and consider both the different mean values of time and associated coefficients of variations for each of the sub-populations. This paper will examine how one can deal with the full range of these alternatives.

Clearly the latter extensions increase the level of complexity of the problem, but it is not clear (at least at the outset) by how much. It is also not clear if the additional complexity significantly changes the final solution, namely, the estimate of how many drivers will likely use the toll lanes and how much total toll will be collected. The second objective of this paper is therefore to illustrate how different the final solutions become.

Finally, it will be shown that the same numerical solutions can also be obtained using a combined traffic assignment/simulation model. While the use of a traffic assignment/simulation model makes it possible to solve this problem for larger networks and more complex situations, the fourth objective is simply to illustrate that the simulation model can provide consistent results for small networks where bench mark comparisons to analytical solutions are possible. Such validation of the traffic assignment/simulation model is not possible for larger problems.

The above objectives are pursued using four cases to represent the alternative ways in which the “value of time” can be represented. These cases are described next in the following section of the paper that describes the study’s experimental design.

2. Experimental Design

The experimental design consists of four elements; namely the different methods for representing the value of time, the speed-flow characteristics of the network being studied, the demand levels being applied to the network, and the different methods utilized to solve the resulting problems, as described next.

2.1 Value of Time Representation: Four Cases

In the simplest situation, all drivers can be considered to have the same mean value of time and there is considered to be no individual variation about this mean value. In this situation, one simply needs to find out what fraction of drivers will switch from the free lanes to the toll lanes, without

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needing to know which specific sub-population these drivers belong to. This is situation referred to as case I, and is illustrated in Figure 2a.

Case I is by far the most common way in which toll lanes or roads are evaluated at present, and is presented in this paper a benchmark relative to which the solutions for the other cases can be compared. It should be noted here that Case I is commonly represented in the 4 step planning process by simply increasing the length of the tolled link or by reducing its free speed.

The second case, which is also illustrated in Figure 2a, is one in which the total driver population consists of multiple sub-populations, each with their own unique mean value of time. However, within each sub-population all drivers are considered to have identical values of time. In this case one needs to find which sub populations will utilize the toll road and which will not. Furthermore, within Case 2 one usually also finds that, while certain sub-populations are entirely on either the free lanes or the toll lanes, one sub-population will typically be split between the two types of lanes.

Case II can be approximated within the four-step planning process by representing the single toll lane as a number of lines, one for each sub-population. This approximation or “trick” becomes difficult, however, when the level of congestion on the single real toll lane becomes a factor in establishing the attractiveness of this lane, as the use of separate links for each sub-population generally precludes the modeling of the collective congestion their interactions produce.

Case III considers that all drivers belong to a single sub-population, with a single mean value of time. However, as illustrated in Figure 2b, about this single mean there exists a distribution of individual values of time with a known coefficient of variation. The question then becomes who, within this single sub-population will use the toll road, as those with a higher value of time will tend to switch first.

Finally, Figure 2b also illustrates a Case IV, which is a hybrid of Cases II and III. Specifically, it considers a scenario with multiple sub-populations, each with their own unique mean value of time and associated coefficient of variation. It is unclear how Cases III and IV would be accurately represented in the 4 step planning process. Consequently, given that Case IV is commonly approximated using Case I, the comparison between the cases is focused more on how the same real condition could be approximated 4 different ways, than on modeling 4 different real conditions.

For comparison purposes, Figure 2c illustrates the use of a log-normal versus a normal distribution for Case 3b versus Case 3a. Also, Figure 2c illustrates the use of a single normal distribution, as per Case 3a versus the use of a composite of 3 normal sub-populations, as per Case 4b. Finally, the different assumptions, that are associated with Cases 1 to 4, are summarized in Table 1, together with the specific numeric values that will be utilized and the numerical solutions.

2.2 Characteristics of the Network

The network for this study was illustrated in Figure 1b. It consists of four free lanes, which then it spit in a toll lane on the top and three free lanes on the bottom. The length of both routes is set in this example to be exactly the same, namely 1.0 km. Finally, the traffic is considered to flow from the west to the east.

The traffic flow characteristics for all lanes, except for the application of toll, are listed in Table 2. Specifically, one can note that the input data for the toll lane and the free lanes are identical, and that the dependency of speed on demand is based on the well-known Greenshield's model. This parabolic speed-flow curve is provided as Equation 1. Given that travel time is simply distance over speed, and considering only the free-flow speed solution, one can find the travel time on any lane as shown in Equation 2.

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$$q = av^2 + bv \tag{1}$$

Where q is the traffic flow (veh/h) and v is the speed (km/h).

$$t = \frac{2ad}{\sqrt{b^2 + 4aq} - b} \tag{2}$$

Where d is the distance in (km) and t travel time (hours).

The toll on the toll lane is set to be $0.10 for all of the drivers, and is constant over time. It is also to be paid as soon as a vehicle enters the toll lane.

c. Demand Levels

While the traffic demand levels are ultimately varied, the initial solutions and simulations are first performed using a total demand level of 6000 vehicles per hour from A to B. In the absence of queuing and in the presence of conservation of flow, the total demand D must equal to the sum of the flows on paths 1 and 2, as shown in Equation 3.

$$q = q_t + q_f \tag{3}$$

Where q is the total demand from A to B (veh/h), q_t is the flow on toll lane (veh/h), q_t is the flow on free lanes (veh/h).

It is assumed in this paper that the total demand q is both known in advance, constant over time, and that it is a deterministic quantity. Subsequently sensitivity analyses will systematically vary the demand from 0 to 7500 veh/h.

It should be noted that this paper only considers scenarios where there is no queuing present in the network at equilibrium. Solutions involving queuing have been developed, but these are beyond the scope of this paper as they involve time-dependent or dynamic analyses. It should be noted that such time-dependencies can arise during over-saturated conditions, even for constant capacities and demand, as the first vehicles joining a queue will experience different travel times than subsequent vehicles joining that queue. Furthermore, analyses based on the average queue size provide solutions different than those derived when the actual individual queue delays for each vehicle are considered. For all of these reasons, the non-queued solutions developed in this paper do not automatically apply to conditions where queuing may occur. Instead, the analysis in this paper provides a critical building block for such more advanced analyses.

d. Solution Approaches

This paper considers two main solution approaches. The first of these involves the numerical solution to an analytical formulation of the problem. These numerical solutions were obtained using simple spreadsheets and can be easily duplicated by the reader. The second of these involves the simulation of these same scenarios using traffic assignment techniques imbedded within the INTEGRATION traffic assignment/simulation model. These solutions were actually easier to obtain, and can be obtained for much more complex networks than the ones presented in this paper. However, they cannot be duplicated as easily by the reader as they require access to the INTEGRATION traffic simulation model.

Absent from this paper is a closed form analytical solution for all Cases, as the complexities of the analytical expressions that need to be solved for even this simple network prohibit such a solution. However, following the presentation of the numerical solution to these analytical expressions, it is possible to illustrate through back substitution that the found solutions do satisfy the conditional equalities that need to be solved for.

e. Assumptions

The following is a summary of the main assumptions implicit in the subsequent derivations and computations:

- The toll lane is physically separated from the free lanes by means of concrete barriers.
- The travel time and the costs for toll are assumed to be the only factors in the general disutility function that influence the total disutility of an individual traveler.
- No juridical, social or economical effects will be investigated.
- Toll is paid fully electronically.
- Vehicles on the toll lane do not have to stop or decelerate to pay toll.
- Traffic participants are fully informed about the downstream traffic conditions and the amounts of toll.
- Demand higher than the capacity of the road is not investigated in the simulations and analytical calculations.
- No research is done on the weaving effects when splitting the road in the toll road and the free road.
- The toll charge is a fixed amount: it is independent of the demand and it does nor vary within a simulation run.

3. Numerical Solution to Analytical Formulation

The following section provides a summary of the analytical formulation to the toll lane problem in three parts. First, the analytical equations are developed for one path, either with or without toll. Subsequently, the conditions that are required to achieve network equilibrium are stated. Next, the numerical solution for a specific demand level of 600 veh/h is presented. Finally, the sensitivity of this numerical solution to different variations of the basic four cases is illustrated.

a. Analytical Formulation for a Single Path

The toll lane problem is essentially a route choice problem in which individual vehicles have to decide whether to take the toll road or another route. At the disaggregate level, the route choice for a given individual can be solved by developing, for each choice alternative i, its disutility DU_i. This disutility can be expressed as a linear function, as shown in Equation 4.

$$DU_i = \alpha c_i + \beta t_i + \gamma T_i + \varepsilon_i \quad (4)$$

Where DU_i is the disutility of using alternative i, c_i is the operating costs of alternative i, t_i is the travel time of alternative i, T_i is the toll of alternative i, ε_i is the influence of unobserved factors affecting utility of alternative i, and α, β, γ is the set of coefficients that are driver dependent.

The first term in the disutility expression represents operating costs. The operating costs are typically a function of fuel costs and depreciation, and are commonly dependent upon trip distance. However, given that both alternative paths involve traveling the same distance, they are considered to be equal, and cancel each other out, and are therefore eliminated from further consideration in this paper.

The second term represents the disutility associated with travel time. This travel time is determined using the flow-dependent speeds computed in Section 2 of this paper. This is the only attribute that

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reflects the interactions that drivers have with other drivers on the same network. This flow dependency therefore makes the disutility of one driver dependent upon the path decisions and actions of the others. This second term is retained during subsequent parts of this analysis, as without it the solution to the path choice becomes rather trivial.

The third term represents the disutility associated with the toll charge. This toll is constant for all drivers in this example, but only applies to that route which uses to toll lane. Finally, the last term in the function, namely ε_i, explains factors that could not be observed or captured, but which do have an effect on the total disutility of a route. In this paper the problem is considered to be deterministic and this term is set to 0. It is therefore removed from further consideration.

As a result, Equation 5 simplifies for this paper to become Equation 5. In addition, if for the purposes of the subsequent calculations, disutility will be measured in units of travel time, coefficient $\beta = 1$ and is dimensionless. In addition, γ will become the inverse of the value of time, and is expressed in hours per dollar, as indicated in Equation 6.

$$DU_i = \beta t_i + \gamma T_i \quad (5)$$

$$\gamma = VT^{-1} \quad (6)$$

Where VT is the value of time ($/h).

b. Analytical Formulation for Equilibrium

To find the equilibrium flows on each route the first principle of Wardrop is applied. It requires one of two conditions to exist for every sub-population of drivers traveling from A to B with a common value of time, where travel impedance is used as a synonym for disutility or generalized cost:

All utilized paths should have the same travel impedance, or

Any unused path should have an equal or greater travel impedance.

The application of these two general principles to the toll lane network results in the need to demonstrate that the specifics of either Equation 7 or 8 apply.

$$\text{If } DU_i = DU_f \text{ then } q_t > 0 \text{ and } q_f > 0 \quad (7)$$

$$\text{If } DU_t > DU_f \text{ then } q_t = 0 \text{ and } q_f > 0 \quad (8)$$

It is important to note that, in the event that multiple sub-populations are present on the network concurrently, that the above conditions need to apply for each sub-population. Furthermore, if there is variability in the value of time within a sub-population, then for each percentile within each sub-population, these conditions must be satisfied as well

c. Ranking of Drivers by Value of Time

In order to find the equilibrium solutions for Cases I, II, III and IV it is important to compute the dis-utilities on routes 1 and 2 for every potential split of traffic between these 2 routes. This computation requires one to know the value of time of last marginal drivers who use the toll lane. These marginal values of time are illustrated in Figures 3a, b and c, as discussed next. Specifically, each Figure illustrates the marginal value of time for any potential fraction of demand on the toll lanes, where this fraction ranges from 0.0 to 1.0. This plot is the mirror image of the traditional way in which cumulative probabilities are plotted.

It is important to plot this reverse cumulative probability as that those drivers with the highest value of time will perceive the toll lane as having the least disutility. They will therefore be the first ones to use the toll lane. Consequently, Figure 3a illustrates a sorting of drivers from highest to lowest value

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of time for Cases I, II and III. In Case I, all drivers have the same value of time, but in Case II trucks have a higher value of time than business travelers, and they have a higher value of time than commuters. Consequently, trucks can be expected to use the toll lane first, and if needed only then will business travelers use the toll lane. In Case III, the value of time changes continuously, according to a normal distribution, and those at the high end of the distribution can be expected to us the toll lane first.

Figure 3b illustrates the ranking of drivers by value of time for both a normal distribution and a log normal distribution. It can be noted that for most of the range, both curves are virtually identical. However, very low values of time are much less likely for the log-normal distribution. Furthermore, for the log-normal distribution, negative values are also impossible.

Figure 3c illustrates one flavor of Case IV, namely one in which values of time are sorted within each sub-population. However, this ignores that some drivers, at the low end of the distribution with a high mean, may have a lower value of time than some drivers who are at the high end of a distribution with a lower mean. Consequently, Figure 3d illustrates what happens when all the drives in Case IV are sorted by value of time, independent of driver type.

d. Analytical Solutions: Demand of 6000 veh/h

Having sorted out the sequence in which drivers will switch from the free lanes to the toll lanes, it is now possible to determine the equilibrium for each of Cases I to IV. This is illustrated in Figures 4 a, b, c and d for a demand of 6000 vehicles per hour.

In all cases, a solution is found where the disutility on route 1 is equal to the sum of the travel time on that link, plus the toll divided by the relevant value of time. For route 2, there is no toll, and consequently, the disutility is simply equal to the travel time at the relevant traffic flow. This is relatively easy for Case I, as all drivers have the same value of time and therefore the same disutility for a given toll. However, for Cases II, III and IV the value of time is not the same, and consequently, one needs to keep track of the value of time of the specific sub-population the driver belongs to and the percentile of the driver within that sub-population, as is shown next.

In case I, all drivers have a value of time of \$ 10/h. Consequently, a toll of \$ 0.10 is equivalent to a travel time of 1/100th of an hour, or 36 seconds. This is illustrated in Figure 4a. As a result, it can be noted that a traffic equilibrium will now be reached when approximately 275 drivers are traveling on route 1, rather than 1500 vehicles per hour, which would have been the case if there had not been any tolls.

In Case II, not all drivers have the same value of time. Consequently, vehicles with the highest value of time will elect to use the toll road first, as they have the least aversion to tolls. If, after all vehicles with the highest value of time are on road 1, the disutility on road 1 is still lower than the disutility on road 2 for the second sub-population, they will also start to use the toll road. Figure 4b illustrates that for a demand of 6000 veh/h only the drivers with a value of time of \$15/h will use the toll lane, and that of these drivers only about 650 will use the toll lane. It can be noted that the solution is independent of the value of time of business travelers or commuters, or even the percentage of business travelers to commuters.

In Case III, there is only a single mean value of time, but the value of time of individual drivers fluctuates about this mean in accordance with an externally specified coefficient of variation. This requires one to track, for each increase in volume of on the toll lane, the specific percentile that this volume represents on the value of time distribution of Figure 3b. For example, the 300th driver out of a total of 6000 translates into the 95th percentile on the cumulative distribution of value of time, and it is this value that must be at equilibrium.

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It can be noted in Figure 4b that a traffic equilibrium develops when approximately 625 vehicles use the toll lanes. This number is very similar to the solution for Case II. Figure 4c also illustrates that this solution is also very similar independent of whether the value of time follows a normal distribution (Case 3a) or a log-normal distribution (Case 3b).

Case IV typically requires a solution that is a hybrid between Case II and III. Specifically, as in Case II one needs to first find which sub-population will be traveling both on the free lanes and on the toll lanes. Subsequently, as was the required in Case III, one needs to determine what percentile of drivers need to switch to the toll road within that sub-population in order for its perceived toll to be equal to the difference in path travel times. This approach is valid if the mean values of time for each sub-population are very different or if the coefficients of variation are very small. However, when this is not the case, the lower percentile values of time for the sub-population with the higher mean become less than the higher percentile values for the sub-populations with a lower value of time.

Figure 4d illustrates that the solutions for Case 4a and 4b are very similar. Specifically, it matters little if vehicles are loaded following a sorting only within a given type or if the whole population is sorted. This is, however, not a general finding. For example, one can note that near a demand on the toll lane of about 1100 veh/h, Cases 4a and 4b deviate from each other considerably.

e. Summary of Cases for a Demand Level of 6000 veh/h

Figure 4e provides a summary of all 4 cases. It can be noted that Case I provides a significantly different estimate of the level of use of the toll lane compared to either Cases II, III or IV. Specifically, while Case I would suggest that only about 275 vehicles would use the toll lane, Cases II, III and IV suggest toll lane usage rates closer to 625-675 vehicles per hour. Clearly, this difference has significant revenue implications.

4. Simulation Modeling of Tolls in Large Networks

The following section of the paper first provides a summary of the features of the INTEGRATION traffic simulation model (Van Aerde and Rakha, 2021) that are relevant to the toll problem being examined in this paper. Subsequently, the results of applying two different traffic assignment techniques within INTEGRATION are presented and compared to the analytical solutions that were demonstrated earlier.

a. Modeling of Toll Roads within INTEGRATION

The INTEGRATION model provides a unique integrated combination of a microscopic traffic simulation model and a dynamic traffic assignment model. In the context of this toll problem, it implies that the traffic simulation part of the model automatically captures the dynamic dependencies of travel time/speed on traffic flow on either the tolled lanes or the free lanes. Concurrently, the dynamic traffic assignment portion of the model ensures that drives will select paths through the network that minimize their perceived total disutility at the time they traverse the network.

A further attribute of INTEGRATION is its ability to represent time-dependent tolls as being levied on each link, where these tolls can be different for different driver types. In order to capture the response of drivers to these tolls, the user can specify different mean values of toll for each driver subpopulation and different coefficients of variation about this mean. When values of time are distributed about a sub population mean, a normal or log-normal distribution can be considered. A normal distribution is usually the easiest to work with, however, the log-normal distribution provides

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the added realism that values of time are typically skewed and that for high coefficients of variation, low percentiles of value of time will never become negative.

In summary, INTEGRATION provides the capability to model the impact of tolls on networks much larger than the one examined in this paper, multiple tolls can be levied. Furthermore, tolls levied at a particular site can be different for different driver sub-populations, and can be time-varying. However, before tackling these more complex/realistic problems, it is important to first demonstrate that the solutions produced by INTEGRATION are consistent with those that can be derived independently through a process that does not involve the model. This is the limited objective of this paper.

b. Comparison of Numerical Solution to Simulation Results

In order to compare the numerical solutions to the analytical formulation against the results from the assignment/simulation model, the expected traffic volumes on the toll lane were estimated for Cases I to IV for a range of traffic demands. These results are illustrated in Figures 5a and 5b for the analytical formulation and the simulation, respectively.

Figure 5a indicates that without a toll, the traffic flow on the toll lane would be roughly equal to 25% of the total demand, as the toll lane also provides 25% of the available capacity. However, with the toll, the level of toll lane use is expected to be much less for all representations of the value of time. Specifically, Case I results in the lowest use of the toll road given that no driver is has value of toll greater than $10/h. In Case II, the percent of drivers using the toll road includes, as some drivers are will to pay as much as $15/h, while the 50% of those with a value of time of only $7/h stay on the free lanes as before. Finally, for Cases III and IV a percentage of the drivers have values of time in excess of $15/h. Consequently, they are willing to start to use the toll road sooner, even compared to Case II. However, at higher traffic demands the number of drivers desiring to use the toll lanes increases, causing the marginal value of time to drop to a level that is more consistent with the top 30% in Case II.

Figures 5b illustrates the split of traffic flow between the toll lane and the free lane for Cases I to IV when analyzed using the Dynamic Traffic Assignment capability with the INTEGRATION assignment/simulation model. It can be noted that Figure 5b is very similar to Figure 5a, in that the non-toll scenario stands out from being very different from any of the toll Cases. Furthermore, within the toll alternatives, the number of toll lane users is much lower for Case I compared to II, which in turn is somewhat lower than either Cases III or IV.

The similarity between Figures 5a and b first illustrates that the simulation model provides estimates of toll lane flow that are generally consistent with analytical estimates. However, as important is the fact that the simulation model is even sensitive to the different assumptions that can be made with respect to how the value of time is represented. Clearly, this similarity is not a proof that the simulation model is accurate for all large network situations, but it does provide the users with considerable confidence that the solutions are reasonable for those cases where a comparison to an analytical formulation is possible.

5. Conclusions and Recommendations

From the above analysis, the following conclusions can be made:

1. There are a number of different ways in which the value of time can be represented within an analysis of toll lane usage. Some of these representations are simple, require less field data collection and are mathematically easy to analyze,

2. Other representations are more realistic but require more field data, and require the application of numerical techniques to be solved,
3. The estimated levels of usage of the toll lane are heavily dependent upon the level of approximation of the value of time of drivers; therefore if the value of time does vary by sub-population and within sub-populations, these variations should be explicitly considered in the analysis.

Finally, when problems are of such complexity to preclude a numerical solution to analytical formulations, this paper has shown that combined assignment/simulation models can provide results consistent with even the most sophisticated analytical formulations.

The analysis of this paper is but a stepping stone along a longer road towards solving all toll lane related issues. The following is a set of recommended next steps:

- The same network should be analyzed considering dynamic demands and the presence of queues,
- The same network should be analyzed considering tolls that vary by driver sub-population and by time-of-day,
- Larger networks should be analyzed to determine the impacts of weaving into and out of such toll lanes,
- Larger networks should be analyzed to determine the differential impact between a separate toll road versus simply a toll lane, and
- Surveys should be conducted to determine how many driver sub-populations can be realistically identified, and to establish how different their mean value of time and coefficient of variation might be.

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Table 1: Summary of Numerical Data for Toll Lane Cases I, II, III and IV.

Case	Commuters			Business			Freight		
	%	Mean VT	COV VT	%	Mean VT	COV VT	%	Mean VT	COV VT
I				100	10	0.0			
II	50	8	0.0	20	10	0.0	30	15	0.0
III				100	10	0.3			
IV	50	8	0.2	20	10	0.2	30	15	0.2

Table 2: Network parameters

Lane Capacity	2000 veh/h/lane
Free-flow Speed	60 km/h
Speed-at-capacity	30 km/h
Jam Density	133.33 veh/km/lane

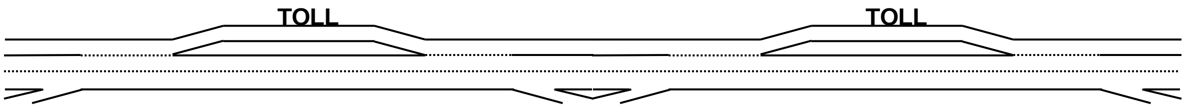

Figure 1a: Configuration of Freeway with Multiple Toll Lane Diversion Opportunities

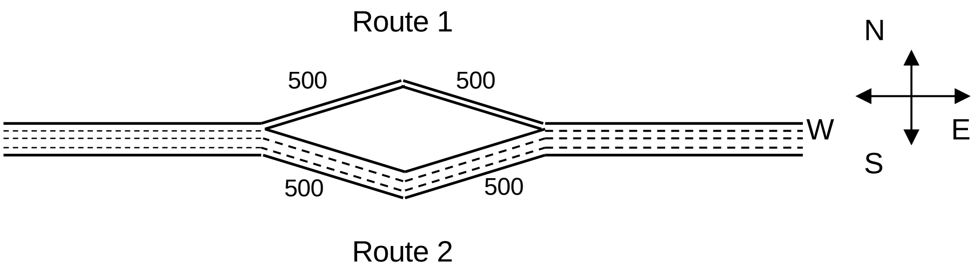

Figure 1b: Configuration of Single Toll Lane Diversion

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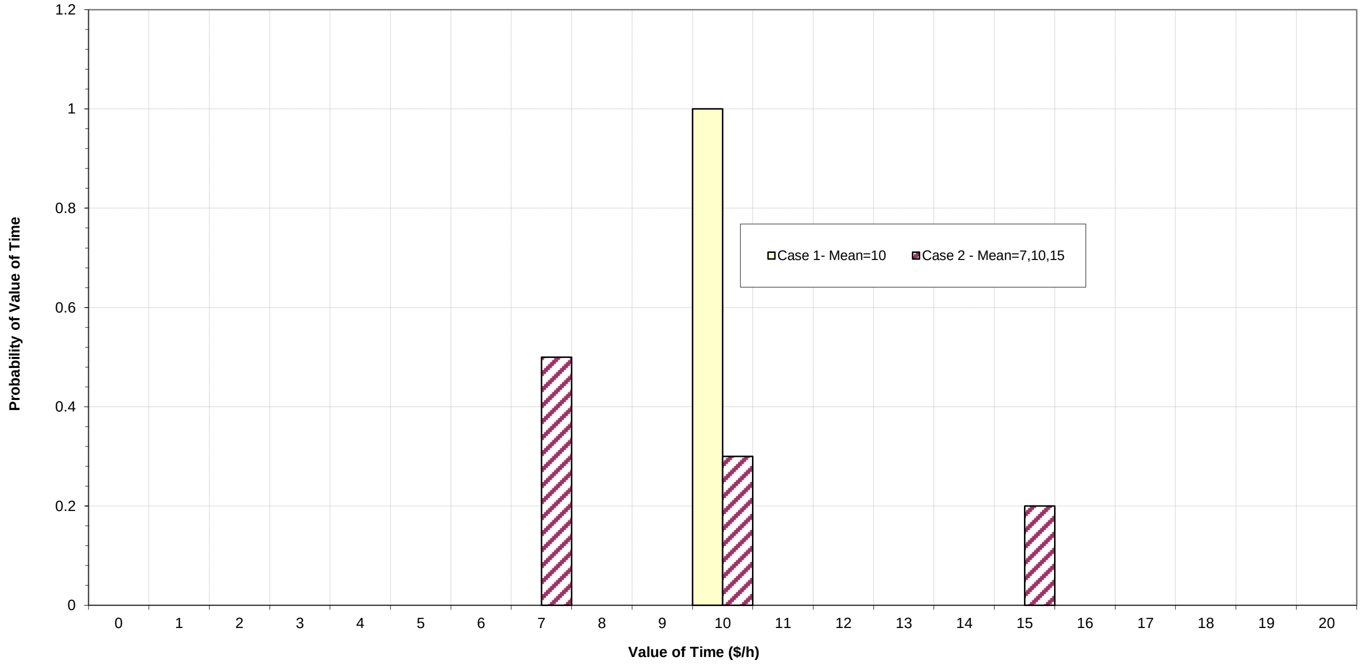

Figure 2a:Value of Time Probability Distribution for Cases I and II

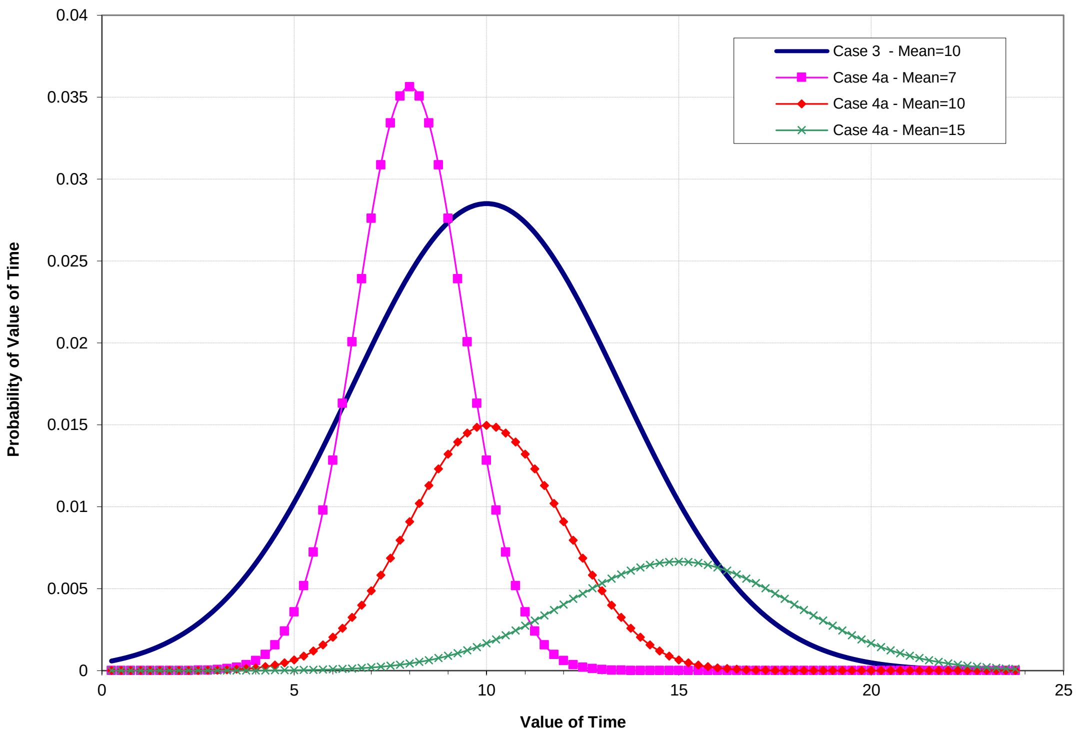

Figure 2b: Value of Time Probability Distribution for Case III and IV

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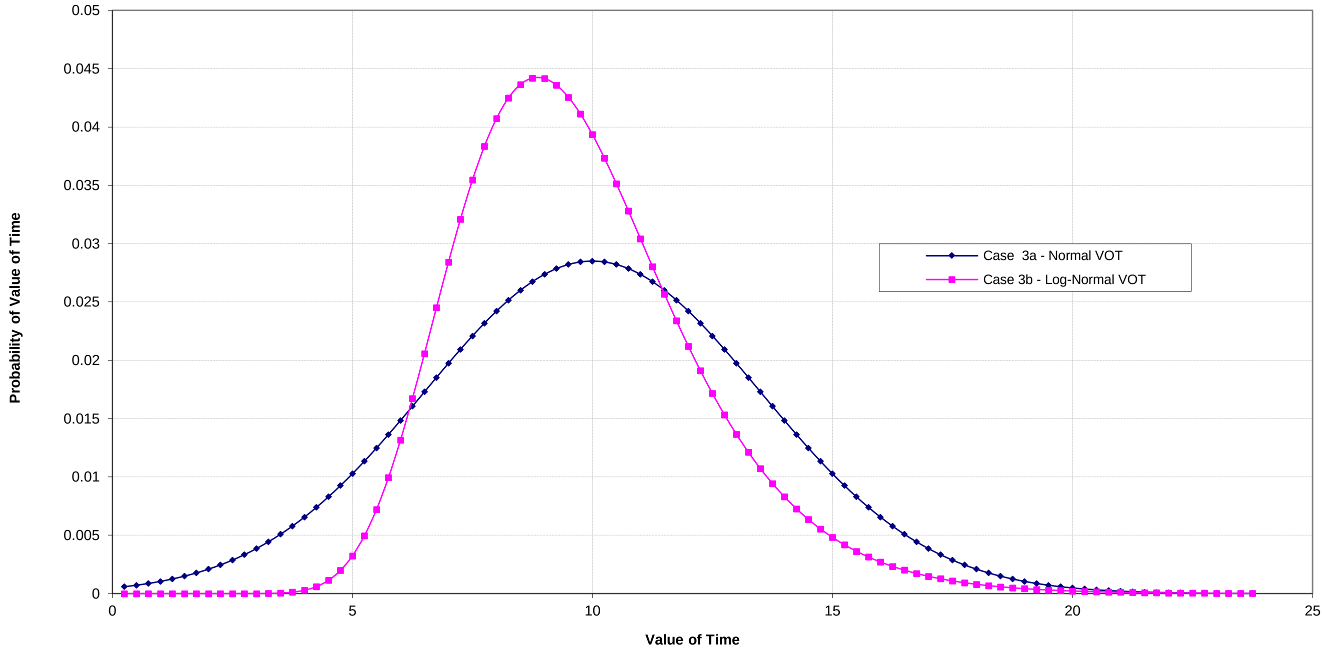

Figure 2c: Value of Time Distribution for Cases 3a and 3b

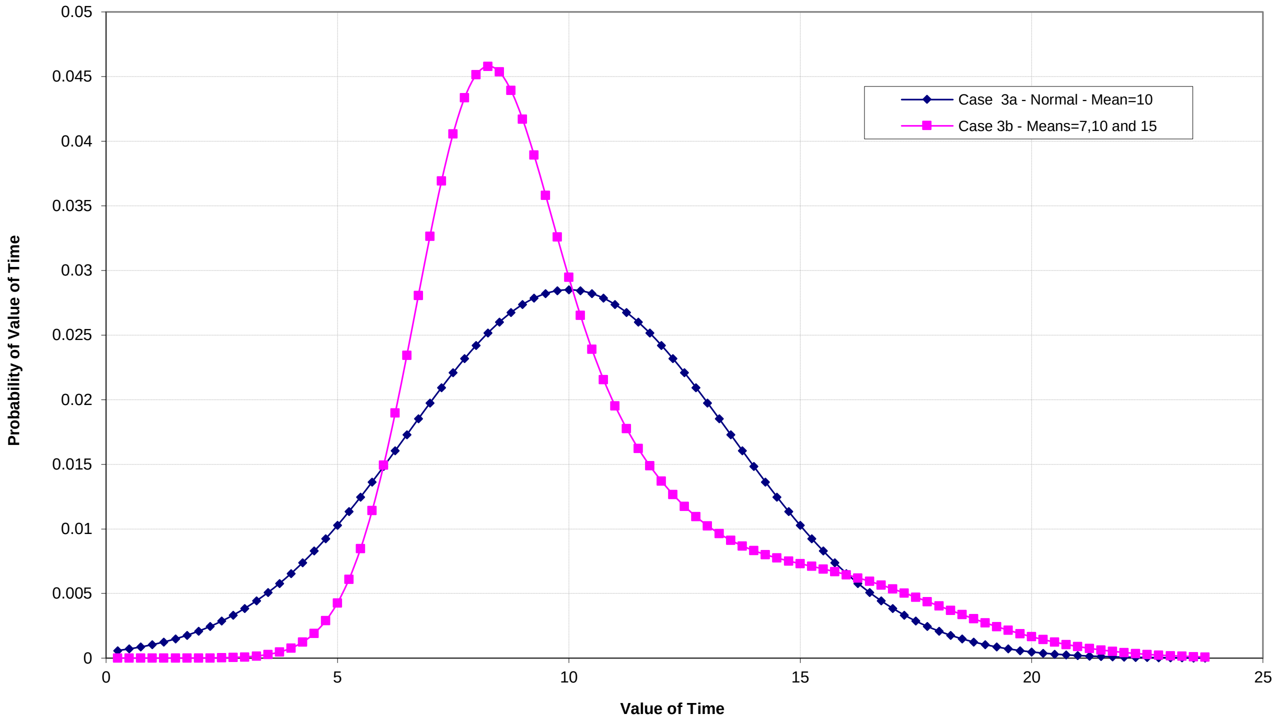

Figure 2d: Value of Time Distribution for Case 3a versus 4b

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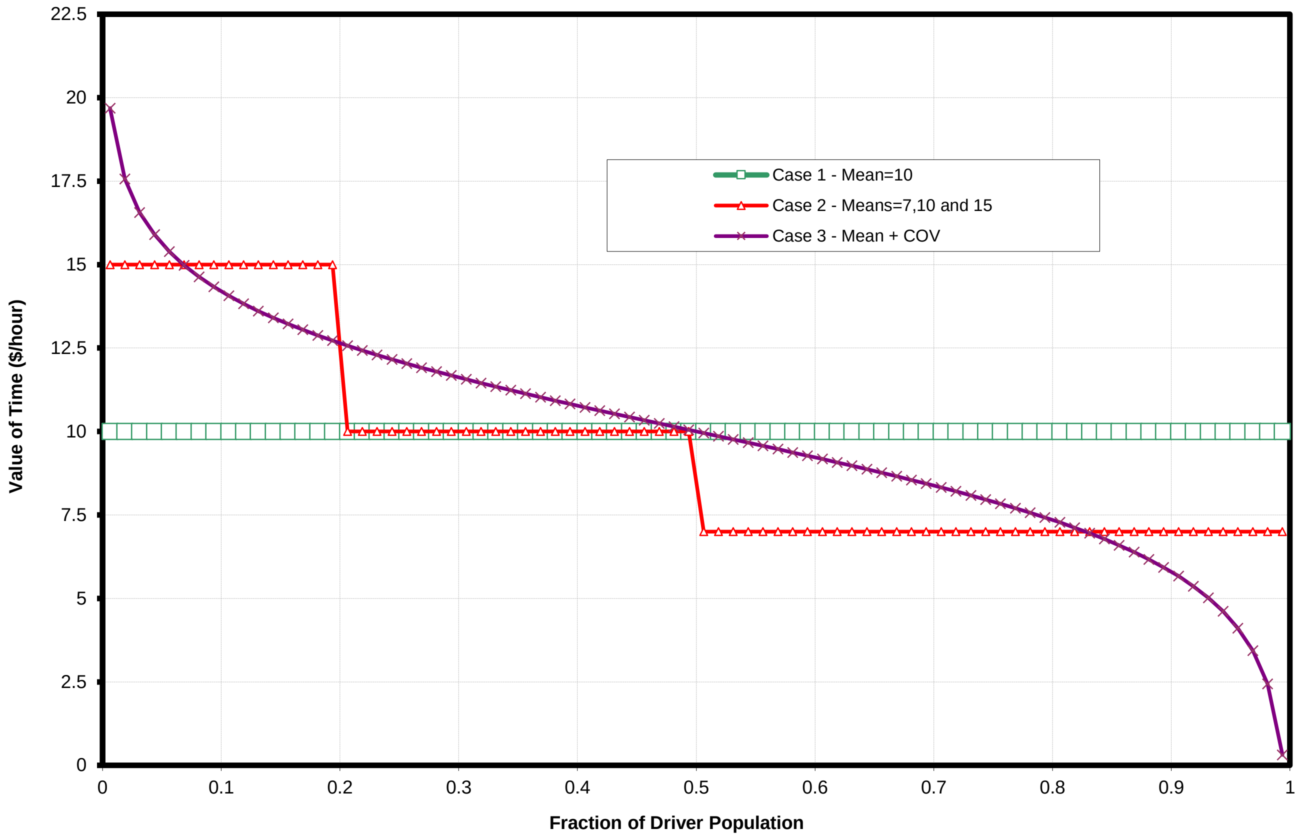

Figure 3a: Value of Time as Function of Toll Lane Use: Cases 1, 2 and 3.

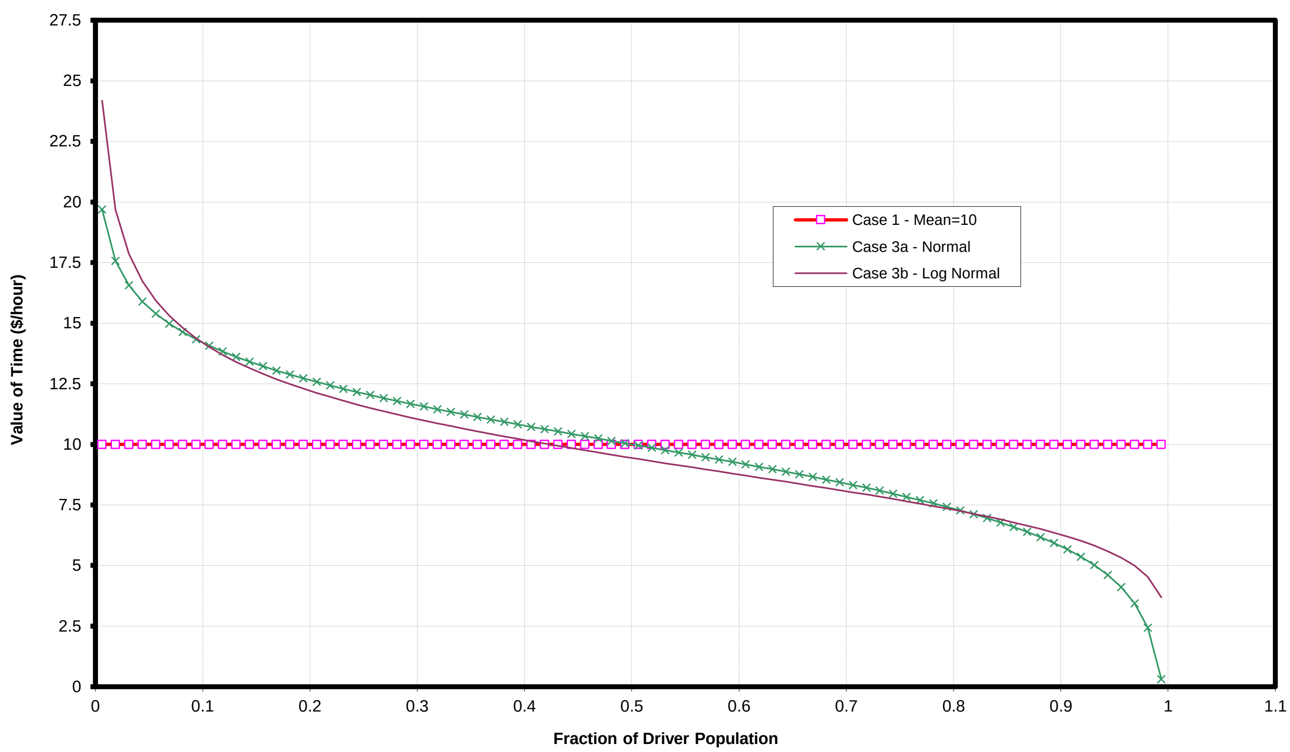

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Figure 3b: Value of Time as Function of Toll Lane Use: Cases 1 versus 3a and 3b.

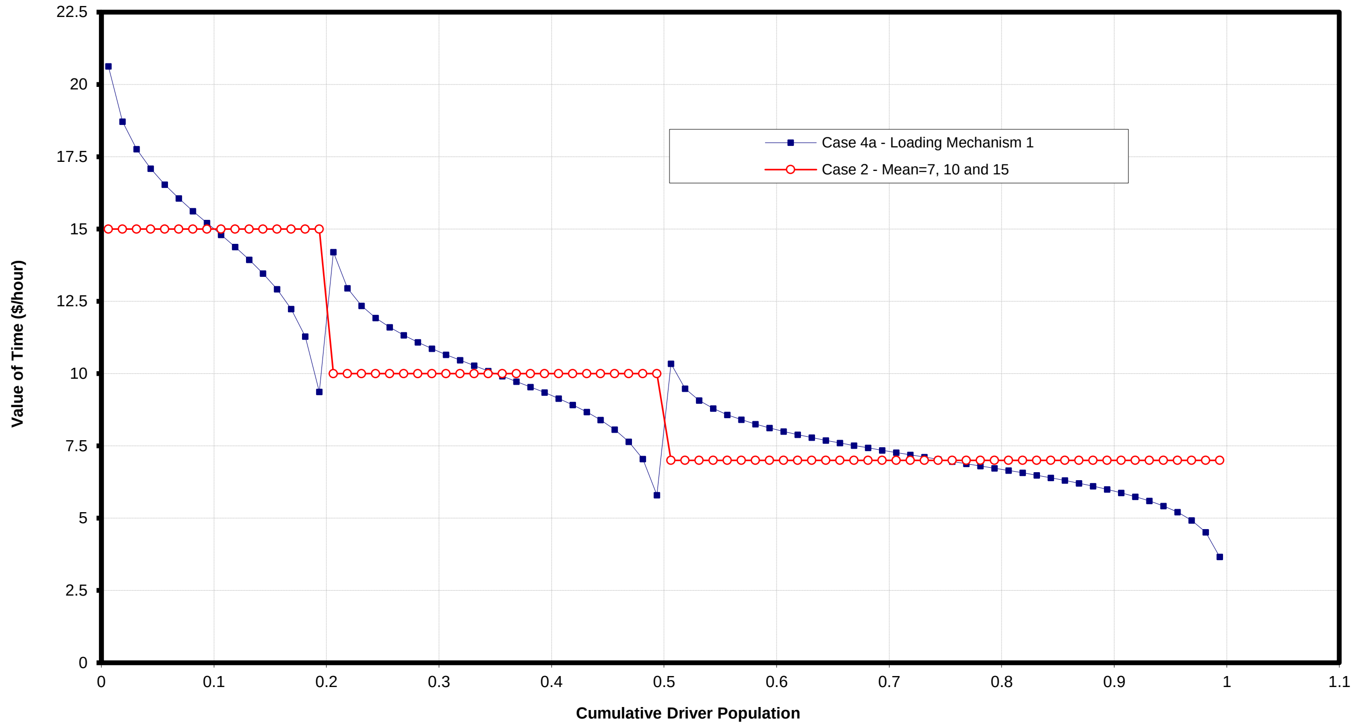

Figure 3c: Value of Time as Function of Toll Lane Use: Cases 2 versus 4a.

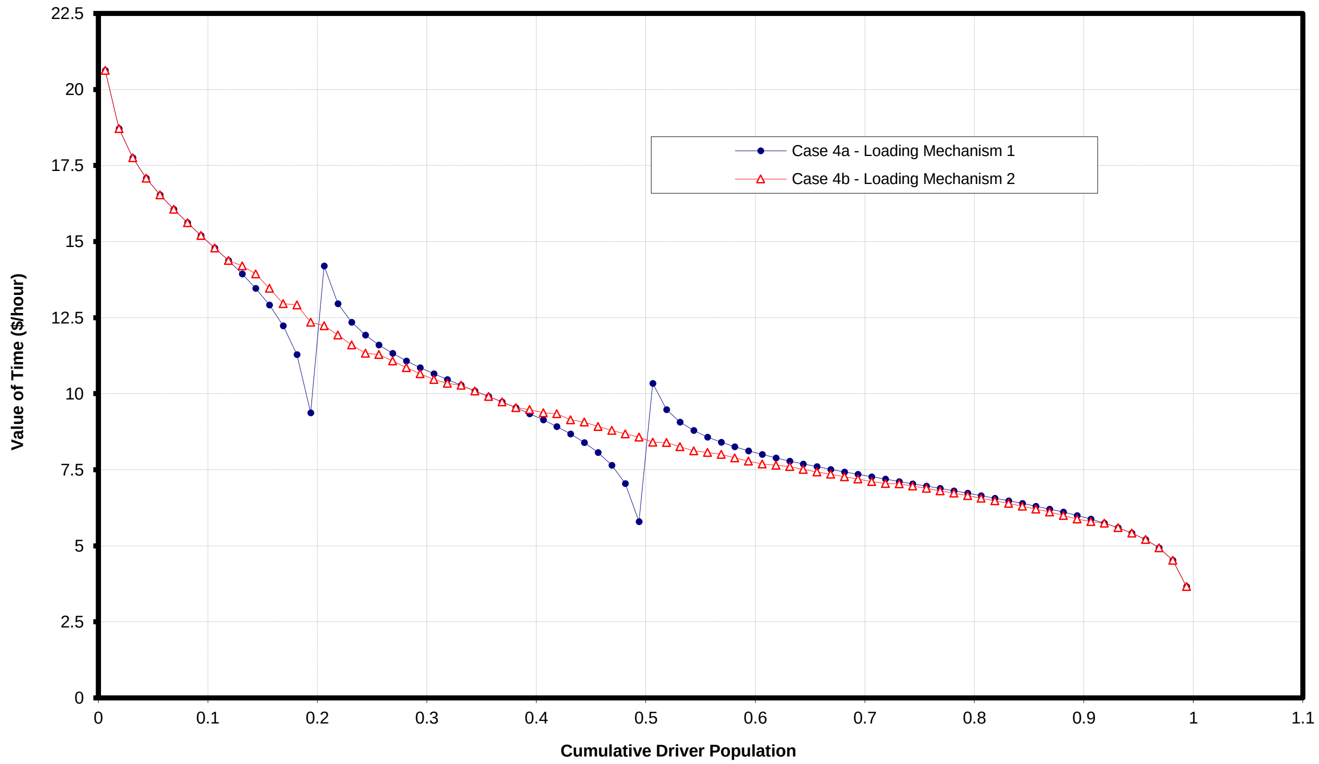

Figure 3d: Value of Time as Function of Toll Lane Use: Case 4a and 4b.

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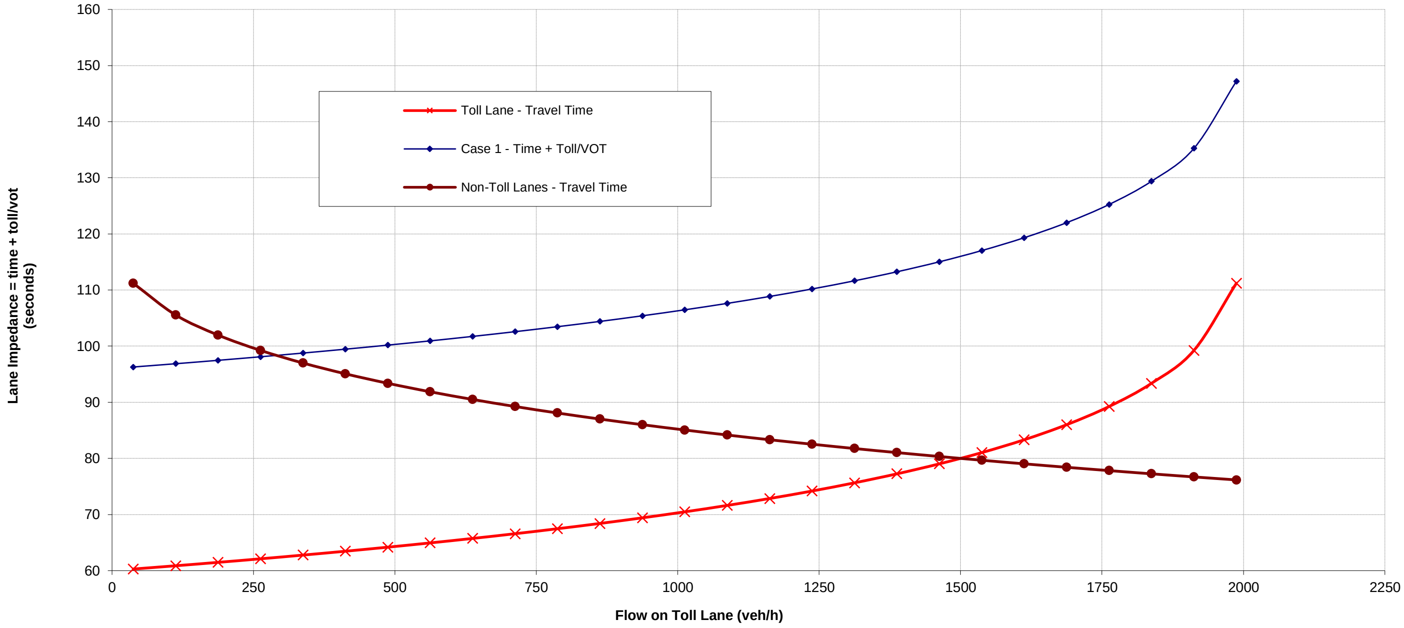

Figure 4a: Case I Disutilities and Equilibrium

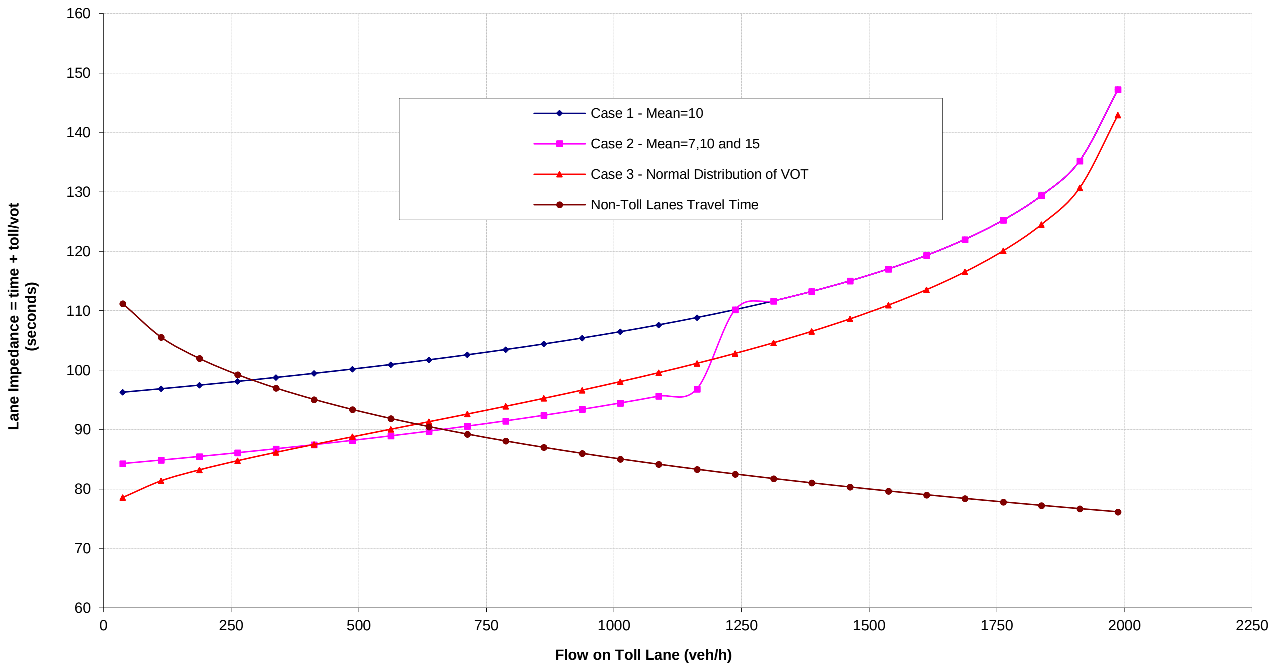

Figure 4b: Case I, II and III Disutilities and Equilibrium

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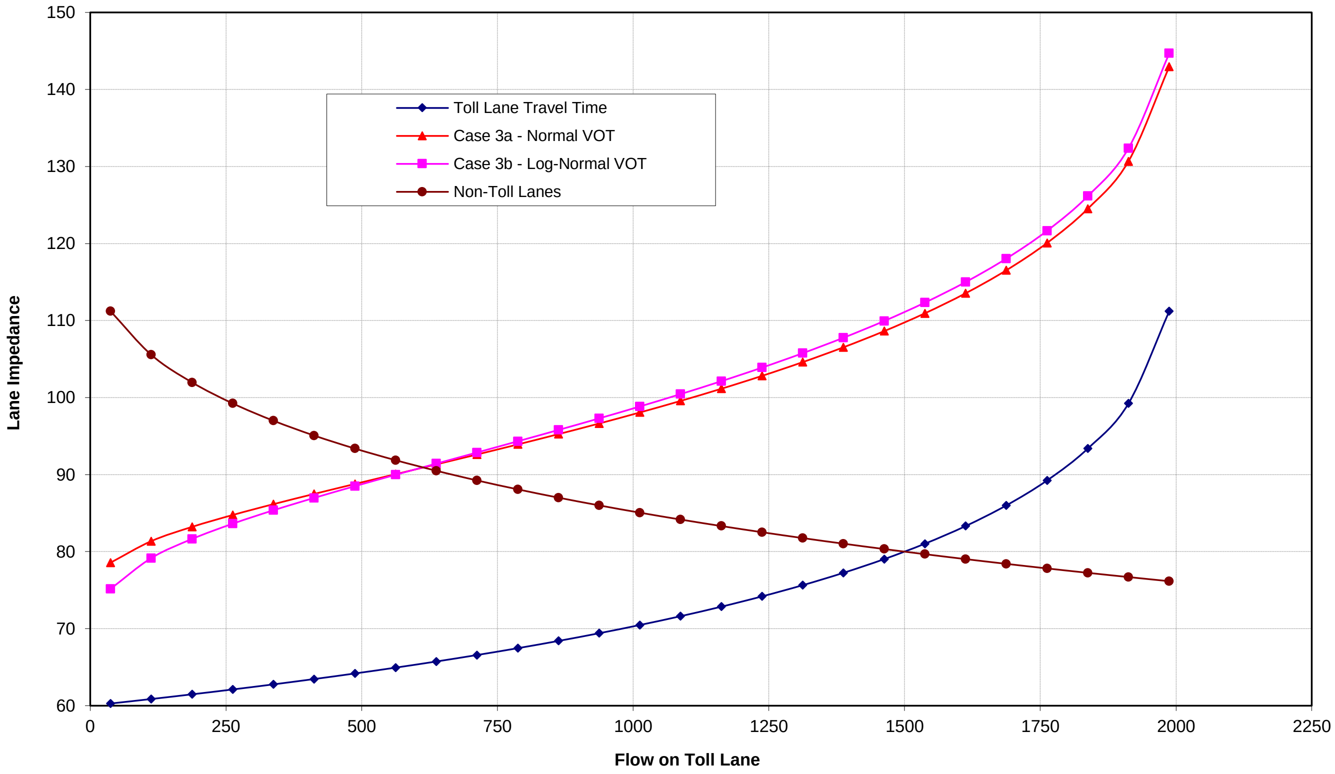

Figure 4c: Case 3a and 3b Disutilities and Equilibrium

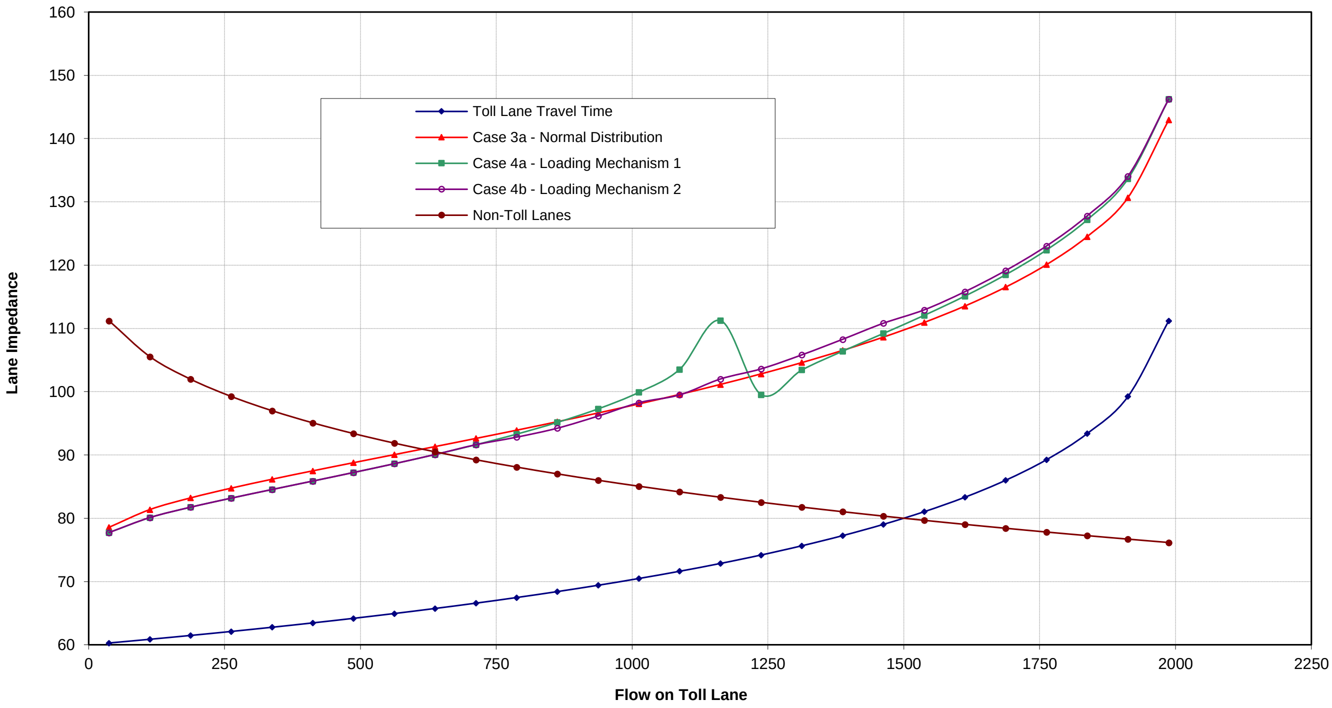

Figure 4d: Case 3a, 4a and 4b Disutilities and Equilibrium

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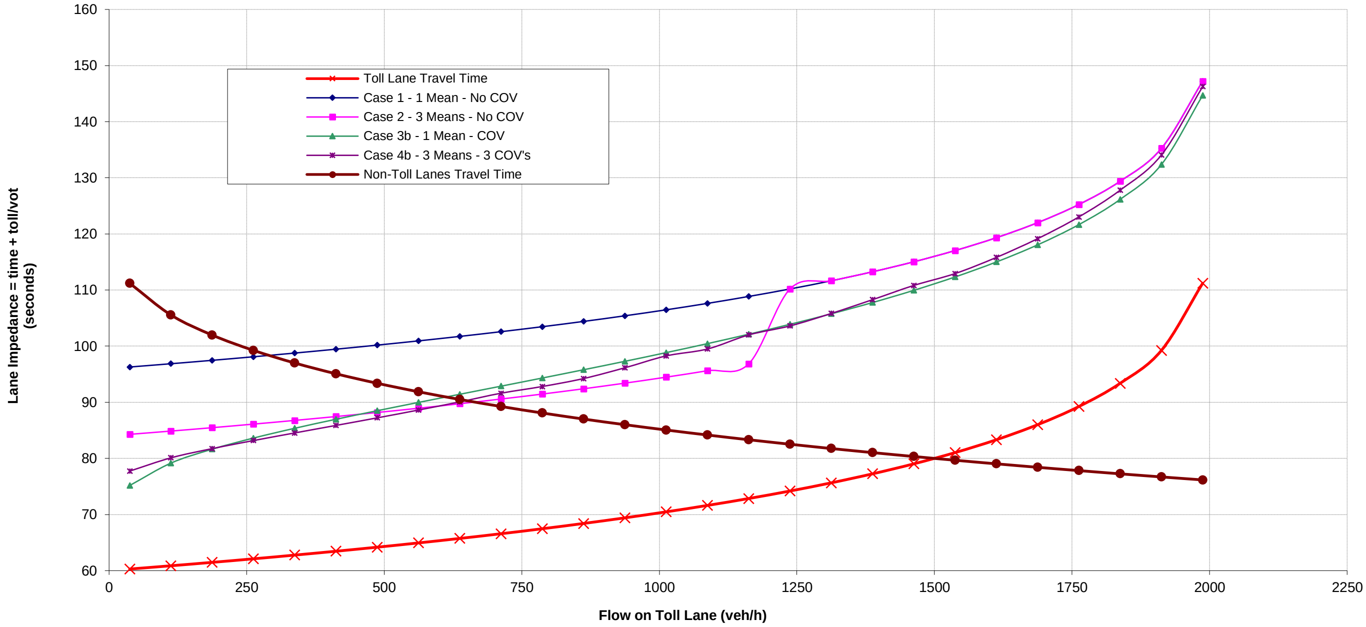

Figure 4e: Comparison of Cases I, II, III and IV

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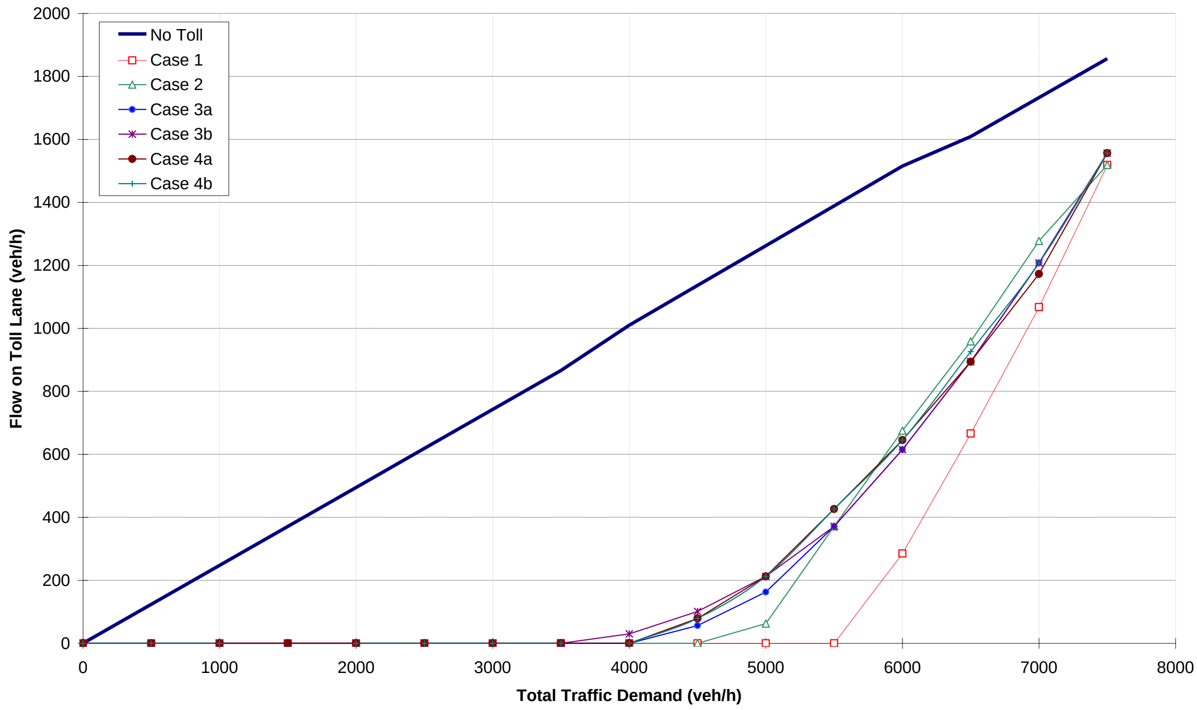

Figure 5a: Impact of Total Demand on Toll Lane use: Analytical Formulation

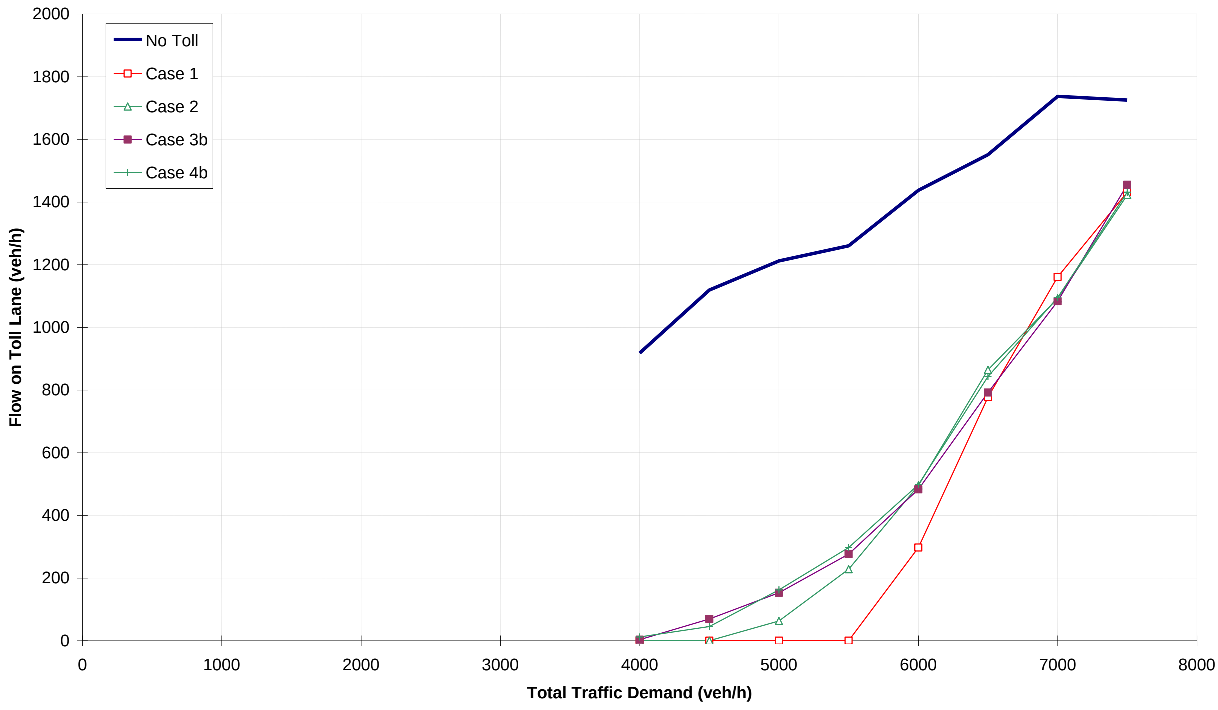

Figure 5b: Impact of Total Demand on Toll Lane use: Simulation Results