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Comparison of Forecasting Ability for Energy Consumption in BRICS: ARIMA (1,1,1) and FGM (1, 1) Models

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Abstract: Brazil, Russia, China, India, and the Republic of South Africa (BRICS) represent developing economies facing different energy and economic development challenges. The current study aims to forecast energy consumption in BRICS at aggregate and disaggregate levels using the annual time series data set from 1992 to 2019 and to compare results obtained from a set of models. The time-series data are from the British Petroleum (BP-2019) Statistical Review of World Energy. The forecasting methodology bases on a novel Fractional-order Grey Model (FGM) with different order parameters. This study contributes to the literature by comparing the forecasting accuracy and the forecasting ability of the $FGM(1,1)$ with traditional ones, like standard $GM(1,1)$ and $ARIMA(1,1,1)$ models. Also, it illustrates the view of BRICS's nexus of energy consumption at aggregate and disaggregates levels using the latest available data set, which will provide a reliable and broader perspective. The Diebold-Mariano test results confirmed the equal predictive ability of $FGM(1,1)$ for a specific range of order parameters and the $ARIMA(1,1,1)$ model and the usefulness of both approaches for energy consumption efficient forecasting.

Keywords: Energy consumption, BRICS, GM (1, 1), Fractional-order, GREY, Forecasting accuracy

1. Introduction

Brazil, Russia, India, China, and South Africa (BRICS countries) belong to the most prominent and fastest developing economies. Although the dynamics of their growth differ across countries, they consume more and more energy. The study aims to forecast energy consumption in Brazil, China, India, Russia, and South Africa at both aggregates and disaggregate levels basing on the time series observed in the years 1992-2019.

Energy plays the most crucial role in the development and achieving the sustainable economic growth of any country. The significance of energy is more critical in countries with less reserve or domestic energy sources (oil, gas, coal, hydro, etc.). BRICS is falling in the list of countries spending many energy resources to fulfill their domestic needs in residential, agricultural, and industrial requirements. The financial spending on the import of crude oil is an extra burden on the economy. Therefore, there is a need for correct forecasting about energy consumption.

In the modern era, due to globalization, the relationship among different countries are more tied up with each other in terms of social, political, and economy-wise. There is fierce competition among the developed as well as developing countries. For fulfilling the economic challenges of the 21st century, every nation is trying to achieve a sustainable level of economic growth, so countries need a sustainable supply of energy to run their economies properly. Ultimately, energy requirements lead the energy consumption in the

country. However, there is vast potential to address this hot issue because massive flaws have been observed due to the traditional techniques.

The global energy consumption in 2019 amounted to 173340 tera-watt hours, while BRICS participated in this consumption in 35.79%. Particularly China is the leading energy consumer globally, consuming up to 22.71% of the global magnitude. It can also be observed that global energy consumption tends to decrease annually by 1-2%. However, in Brazil, China, India, and South Africa, energy consumption exhibits positive growth rates. On the other hand, Russia is reducing its energy consumption, and it follows the global decreasing trend[1].

When looking at the particular energy sources, the global energy consumption consisted of 30.93% oil, 25.30% coal, 22.67% natural gas, 8.00% biofuels and waste, 6.03% hydro, 4.00% nuclear, and 3.07% others in 2019[2]. Taking into account global energy consumption structure, in the paper, the focus is put on the aggregate energy consumptions and traditional energy sources, which are to be limited over time but still play a crucial role in energy consumption and keeps particular countries far from sustainable development goals. Thus the following disaggregates are included: oil, coal, natural gas, and hydro energy.

The paper's novelty lies in applying the fractional-order $GM(1,1)$ model ($FGM(1,1)$, hereafter) proposed by [1] to forecasting energy consumption in BRICS countries at both aggregates and disaggregates levels. This is the first application of this model in the empirical analysis to the authors' best knowledge. That is why the model needs to compare to well-known forecasting techniques based on the time series analysis, such as a standard grey model $GM(1,1)$ proposed by [2] and Auto Regressive, Integrated, Moving Average ($ARIMA(1,1,1)$), which is initially proposed by [3]. The model comparison is two-fold. In the first step, standard measures of forecasting accuracy such as mean square error (MSE) and mean absolute percentage error (MAPE). In contrast, in the second one, the models are compared for equal forecasting ability using the Diebold-Mariano test [4].

The rest of the paper has organized as follows. Subsection 1.1 provides an energy profile of BRICS countries, and subsection 1.2 reports the relevant literature review. Section 2 provides materials and methods. Section 3 presents the empirical results. Section 4 provides the discussion of results. The final section 5 concludes the paper and discusses policy implications.

1.1 Energy Profile of BRICS Countries

In this section, we briefly present the energy profile of BRICS. There is enormous potential in the energy sector of BRICS. The facts and figures of the following energy for BRICS have been taken from the BRICS energy report, 2020[3].

Brazil generated 306.8 million tonnes of oil equivalent (mtoe) of primary energy in 2018, with 14 mtoe of unutilized energy and natural gas reinjection (in 2019: 327 and 17 mtoe, respectively). Production of oil surpassed demand by 52.5 percent, accounting for most of the Brazilian surplus (in 2019: 64 percent).

After China and the United States (US), Russia is the world's third-largest producer and user of energy resources, accounting for 10% of global production and 5% of global consumption. The Russian energy complex, which includes the oil, gas, coal, electricity, and heat supply industries, is a significant source of revenue for the Russian Federation's budget.

After the US and China, India is the world's third-largest energy user, producing around 6% of global demand. Between 2010 and 2019, the country's energy consumption increased by 50%. At the same period, coal accounts for 56% of global primary energy

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[1] https://ourworldindata.org/energy -production-consumption (accessed 18.03.2021).

[2] https://ourworldindata. org/energy-production-consumption (accessed 18.03.2021).

[3] https://eng.brics-russia2020.ru/images/114/89/1148985.pdf (accessed 18.03.2021).

output. India produces just over half of its oil. The level and structure of energy production have changed significantly between 2010 and 2019: the volume of energy production has increased by 40%. The share of conventional biomass replaced by coal in the energy mix has decreased significantly.

China's energy output grew steadily in 2018, reaching 3.77 billion tons of coal equivalent, up 5.0 percent year on year and the highest amount in the last six years, accounting for 18.7% of global production. In 2018, fossil fuels accounted for 81.8 percent of China's energy output, with coal accounting for 69.1% and non-fossil accounting for 18.2%. China has surpassed the US as the world's largest hydropower, wind power, and solar power installed capacity nation. China's overall energy consumption in 2018 was 4.64 billion tons of coal equivalent, up 3.3 percent year on year. China's low rate of energy consumption growth helps to sustain the country's medium-high-speed economic growth.

The Republic of South African (RAS) is the continent's second-largest energy user. South Africa's total primary energy consumption in 2019 was 135 mtoe, down 5.6 percent from 2010. Coal dominates the energy demand structure, accounting for about 75% of total consumption. South Africa is a net energy exporter, exporting more than 45 mtoe of coal to global markets each year, while having minimal domestic oil and natural gas output and relying on imports for most of these fuels. The structure of energy production has remained nearly unchanged since 2010, but overall production has decreased slightly.

1.2 Literature review

There is a large plethora of literature available on the issue of energy consumption forecasting. Many studies used $ARIMA$ methods for forecasting energy consumption, e.g., [5-15], and some studies were forecasted by comparing the $ARIMA$ approach with some other methods. On the other hand, some studies used the grey methods for energy consumption forecasting. Referring only to the BRICS group of countries, there is numerous literature on energy consumption in China. Besides, Brazil and India are sometimes represented; however, Russia and South Africa are rarely the analysis subjects.

In past research, [10] investigated energy demand in the transport sector using $ARIMA$, exponential smoothing, and multi regression models. On the other hand, the study of [12] forecasted China's primary energy consumption by comparing the $ARIMA$ and grey models. In [13], the authors estimated electricity consumption for Brazil by applying the Spatial $ARIMA$ model. There are very few studies that evaluated energy consumption for BRICS by using the $ARIMA$ model. Some studies forecasted energy consumption by using the $ARIMA$ forecasting method like [5-10, 12] for China, [13] for Brazil, and [15] for South Africa. On the other hand, some studies used the grey Markov method with rolling mechanism and singular spectrum analysis for energy consumption forecasting like [16] for India. Similarly countrywide studies are [12, 17-74] for China; [75] for the US; [76] for China and India; [77] for BRICS; [78] for China and US, [79-80] for Brazil, and [81] for Asian countries.

In [14] the authors analyzed several versions of grey models (e.g., grey model including $GM(1,1)$; $GM(1,n)$; Rolling $GM(1,1)$ Rolling $GM(1,1,X_n)$ and Rolling $NOGMN(1,1)$, and forecasted electricity consumption from 2015 to 2020. Most of the studies used the standard $GM(1,1)$. There is some technical problem in the grey prediction model's methodology, as most of the studies fail to fulfill the principle of "new information priority" proposed by [2]. The grey forecasting method proposed by [2] has gained popularity among researchers because it is efficient in a small number of observations [17]. To tackle this problem, the study of [23] developed the new grey model based on the initial condition by considering the last data point of the one accumulated generating operation (AGO) sequence as the initial condition. [25] analyzed electricity consumption for China by using the continuous fractional-order grey model and forecasted from 2010 to 2014. [27] analyzed grey $GM(1,1)$, Gross weight grey model, and $GVGM(1,1)$ for China and forecasted from 2010 to 2020. [28] analyzed energy consumption for China by using the improved hybrid grey model (INHGM-Markov) and forecast from 2018 to 2022. Similarly, the grey method is suitable to tackle forecasting in the case of inaccuracy of data.

Some researchers developed the model's extended versions using the standard $GM(1,1)$ grey forecasting model, as [18] proposed an improved version of the seasonal rolling grey prediction model to estimate the accurate forecasting for traffic flow problems. Moreover, [19] proposed fractional-order accumulation techniques and forecasted from 1999 to 2007 for China and from 1999 to 2008 for the US. [20] developed a new time-delayed polynomial grey model, which has shown outstanding results when forecasting China's natural gas consumption and forecasted from 2005 to 2013 and 2014 to 2020. [21] predicted China's energy consumption by incorporating genetic programming in the grey prediction approach and forecasted from 2004 to 2007. Similarly, [22] developed a generalized fractional-order grey model using the fractional calculus and forecasted it from 2010 to 2014. [39] forecast coal stockpiles for China using grey spontaneous combustion forecasting models and forecasted from day 11 to 20. [40] analyze the electricity consumption for China by using grey prediction with the nonlinear optimization method and forecasted from 2014 to 2020. [41] examined electricity consumption by using grey $GM(1,1)$ and combined improved grey ($DCOGM(1,1)$) prediction models and forecasted from 2017 to 2021. The results show that $DCOGM(1,1)$ shows better results than the traditional grey $GM(1,1)$ model. [42] analyzed electricity consumption for China by using the grey polynomial prediction model and forecasted from 2011 to 2015. [43] analyzed the energy vehicle industry for China by using grouping approach-based nonlinear grey Bernoulli model (DGA-based $NGBM(1,1)$) and $GM(1,1)$ forecasting from 2018Q1 to 2020Q4 [50] forecast for by using Self-adaptive intelligence grey predictive model and forecasted for 2014. The results indicate that the Self-adaptive grey model shows better results than $GM(1,1)$ and discrete grey ($DGM(1,1)$) models. [81] used hybrid dynamic grey model for forecasted from 1999 to 2007 for China, from 1999 to 2008 for the US.

2. Materials and Methods
This section is divided into three subsections related to data sources description, forecasting models, and forecasting accuracy.

2.1 . Data Sources

The present study is based on the secondary data source consisting of annual observations on the BRICS economy for 1992-2019. The starting date is limited by the case of Russia being founded in 1991 after the dissolution of the Soviet Union. The currents study uses the energy consumption in BRICS and for empirical analysis. The data on energy consumption (EC) at aggregate and disaggregate energy consumption components (oil, gas, coal, and hydroelectric) are taken from British Petroleum (BP-2019) Statistical Review of World Energy. All variables are measured in mtoe units, and the description of variables is as: (1) Aggregate energy consumption (agg) (2) Oil consumption (oil) (3) Gas consumption (gas) (4) Coal consumption (coal) (5) Hydroelectric consumption (hydro).

2.2. Methodology

The current study is based on $FGM(1,1)$ grey model. The model was introduced to the literature in 2019. Its application to energy consumption forecasting is still not recognized by the authors of the paper [1] based on simulation results. In the current study, the $FGM(1,1)$model is compared to a standard $GM(1,1)$ model, as well as the $ARIMA(p,d,q)$ widely recognized in the forecasting literature. We focus on model comparison in terms of forecasting ability.

2.2.1 *Unit Root Testing and ARIMA (p,d,q) Model*

The most recognized representation for nonstationary time series is the $ARIMA$ model that can be written in the form:

$$\emptyset(L)(1-L)^d(Y_t - \mu_t) = \theta(L)\varepsilon_t \qquad (1)$$

where $\emptyset(L)$, and $\theta(L)$ are polynomials in the lag operator, L, defined such that $L^n x_t = x_{t-n}$, μ_t, is the unconditional mean, d is the order of integer differencing, and ε_t is a white noise process (i.i.d. normally distributed) (for further details, see [3, 82]). This model is termed an $ARIMA(p,d,q)$ to indicate p lags in the AR and q lags in the MA terms, and d is an integer differencing. To estimate the parameters of the $ARIMA$ model, the maximum likelihood method is recommended. The model selection procedure, related to lag parameters p and q, is based on the Akaike information criterion (AIC) and Bayesian information criterion (BIC) [83].

$ARIMA$ model refers to integrated time series, which become stationary after d-th times differencing. To determine whether a time series is stationary or not, the Augmented Dickey and Fuller test is typically used [84]. It is essential to check the time series stationarity at the beginning of the analysis to fit the correct type of model. The graphical analysis of all series suggests that there would be a need to take a trend in the unit root testing procedure. Hence, the study used the intercept and trend in all cases. The null hypothesis assumes that the time series is nonstationary of order 1, i.e., $I(1)$, and the alternative hypothesis assumes stationarity (i.e., $I(0)$). If the time series is stationary, the $ARIMA(p,q)$ model is applied.

2.2.2. *Fractional-order GM (1,1) Model*

The construction of the fractional-order $GM(1,1)$ grey model methodology is explained by [1]. As it is quite a new approach, it is presented in this section.

Definition1: The sequence of raw data series is $X^{(0)} = x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))$, where $r \in R^+$, which is known as $X^{(r)} = (x^{(r)}(1), x^{(r)}(2), x^{(r)}(1), \ldots, x^{(r)}(n))$ is the r th-order accumulating sequence of $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))$ [48] where $\Gamma(n)$ is denoting the gamma function,

$$x^{(r)}(k) = \sum_{i=1}^{k} \frac{\Gamma(r+k-i)}{\Gamma(k-i+1)\Gamma(r)} x^{(0)}(i), k = 1,2,\cdots,n \qquad (2)$$

Definition 2: Assume that $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots\ldots, x^{(0)}(n))$ is the sequence of raw data, where$r \in R^+$, which is known as $X^{(r)} = (x^{(-r)}(1), x^{(-r)}(2), \ldots\ldots, x^{(-r)}(n))$ is the rth order reducing generation sequence of $X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \ldots\ldots\ldots, x^{(0)}(n)\right)$. [48] where,

$$x^{(-r)}(k) = \sum_{i=0}^{k-1} (-1)^i \frac{\Gamma(r+1)}{\Gamma(i+1)\Gamma(r-i+1)} x^{(0)}(k-i) \qquad (3)$$

Theorem 1: Assume that $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))$ is the sequence of raw data, $X^{(p)}$ is the p th-order accumulating generation sequence of $X^{(0)}$, where $p \in R^+$, and $X^{(-q)}$ is known as q th-order reducing generation sequence of $X^{(0)}$, where $q \in R^+$. It implies that $\left(X^{(p)}\right)^{(-q)}$ is the q th-order reducing generation sequence of $\left(X^{(p)}\right)$, and $\left(X^{(q)}\right)^{(-p)}$ is the p th-order accumulating generation sequence of $X^{(-q)}$. The following conditions will exist if:

If $p-q > 0, X^{(p-q)}$ is the $(p-q)$ th-order accumulating generation sequence of $X^{(0)}$.

If $p-q < 0, X^{(p-q)}$ is the $(q-p)$ th-order reducing generation sequence of $X^{(0)}$.

The fractional-order accumulating generation operator and the reducing generation operator satisfy the commutative and exponential laws.

$$X^{(p-q)} = \left(X^{(p)}\right)^{(-q)} = \left(X^{(-q)}\right)^{(p)} \quad (4)$$

Definition 3: Assume that $X^{(r)} = (x^{(r)}(1), x^{(r)}(2),\dots, x^{(r)}(n))$ is defined as definition 1, and $X^{(-r)} = (x^{(-r)}(1), x^{(-r)}(2), \dots\dots, x^{(-r)}(n))$ is follow the definition 2.

Thus, $Z^{(-r)} = (z^{(r)}(2), z^{(r)}(3), \dots\dots, z^{(r)}(n)$, where

$$Z^{(r)}(k) = \frac{x^{(r)}(k)+x^{(r)}(k-1)}{2} \ , \ k = 2,3,4,\dots\dots\dots,n, \quad (5)$$

The model formula

$$x^{(r-1)}(k) + az^{(r)}(k) = b \quad (6)$$

is representing th $FGM(1,1)$. The following conditions are distinguished:

if r=1, Eq (6) is representing the $GM(1,1)$

$$x^{(0)}(k) + az^{(1)}(k) = b,$$

Which is called as standard grey $GM(1,1)$ model described in [2]

if r=0, Eq (6) is representing the direct $GM(1,1)$ modeling.

$$x^{(-1)}(k) + az^{(0)}(k) = b.$$

It is expected that the development coefficient a to be negative and the intension parameter b to be positive (See, [85] chapter 7, p.149).

Theorem 2: We are following definition 3 for $FGM(1,1)$, and the parameter vector is explained as:

$x^{(r-1)}(k)+az^{(r)}(k)$=b, $\hat{a} = [a,b]^T$ is the $GM(1,1)$ parameter, and the least-square estimate of the parameters satisfies the following equations.

$$\hat{a} = (B^T B)^{-1} B^T Y \quad (7)$$

The construction of observation matrices is described in [1].

It follows,

$$\mathbf{Y} = \begin{bmatrix} x^{(r-1)}(2) \\ x^{(r-1)}(3) \\ \vdots \\ x^{(r-1)}(n) \end{bmatrix}, \mathbf{B} = \begin{bmatrix} -z^{(r)}(2) & 1 \\ -z^{(r)}(3) & 1 \\ \vdots & \vdots \\ -z^{(r)}(n) & 1 \end{bmatrix} \quad (8)$$

where,

$$x^{(r-1)}(k) = ((x^{(r)})^{-1}(k) - x^{(r)}(k-1)$$

$$= \sum_{i=1}^{k} \frac{\Gamma(r+k-i)}{\Gamma(k-i+1)\Gamma(r)} x^{(0)}(i) - \sum_{i=1}^{k-1} \frac{\Gamma(r+k-1-i)}{\Gamma(k-i)\Gamma(r)} x^{(0)}(i), k = 2,3,\cdots,n, k-i \geq 1$$

$$z^{(r)}(k) = \frac{x^{(r)}(k) + x^{(r)}(k-1)}{2}$$

$$= \frac{\sum_{i=1}^{k} \frac{\Gamma(r+k-i)}{\Gamma(k-i+1)\Gamma(r)} x^{(0)}(i) + \sum_{i=1}^{k-1} \frac{\Gamma(r+k-i)}{\Gamma(k-i+1)\Gamma(r)} x^{(0)}(i)}{2}, k = 2,3,\cdots,n$$

Therefore, it follows

$$\mathbf{Y} = \begin{bmatrix} \sum_{i=1}^{2} \frac{\Gamma(r+2-i)}{\Gamma(2-i+1)\Gamma(r)} x^{(0)}(i) - \sum_{i=1}^{1} \frac{\Gamma(r+2-1-i)}{\Gamma(2-i)\Gamma(r)} x^{(0)}(i) \\ \sum_{i=1}^{3} \frac{\Gamma(r+3-i)}{\Gamma(3-i+1)\Gamma(r)} x^{(0)}(i) - \sum_{i=1}^{2} \frac{\Gamma(r+3-1-i)}{\Gamma(3-i)\Gamma(r)} x^{(0)}(i) \\ \vdots \\ \sum_{i=1}^{n} \frac{\Gamma(r+n-i)}{\Gamma(n-i+1)\Gamma(r)} x^{(0)}(i) - \sum_{i=1}^{n-1} \frac{\Gamma(r+n-1-i)}{\Gamma(n-i)\Gamma(r)} x^{(0)}(i) \end{bmatrix}$$

$$= \begin{bmatrix} (r-1)x^{(0)}(1) + x^{(0)}(2) \\ \frac{r(r-1)}{2} x^{(0)}(1) + (r-1)x^{(0)}(2) + x^{(0)}(3) + \\ \vdots \\ \sum_{i=1}^{n} \frac{\Gamma(r+n-i)}{\Gamma(n-i+1)\Gamma(r)} x^{(0)}(i) - \sum_{i=1}^{n-1} \frac{\Gamma(r+n-1-i)}{\Gamma(n-i)\Gamma(r)} x^{(0)}(i) \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} -\dfrac{x^{(r)}(1) + x^{(r)}(2)}{2} & 1 \\ -\dfrac{x^{(r)}(2) + x^{(r)}(3)}{2} & 1 \\ \vdots & \vdots \\ -\dfrac{x^{(r)}(n-1) + x^{(r)}(n)}{2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\dfrac{1}{2}\left[(r+1)x^{(0)}(1) + x^{(0)}(2)\right] & 1 \\ -\dfrac{1}{2}\left[\dfrac{r(r+3)}{2}x^{(0)}(1) + (r+1)x^{(0)}(2) + x^{(0)}(3)\right] & 1 \\ \vdots & \vdots \\ -\dfrac{1}{2}\left[\displaystyle\sum_{i=1}^{n} \dfrac{\Gamma(r+n-i)}{\Gamma(n-i+1)\Gamma(r)}x^{(0)}(i) + \sum_{i=1}^{n-1} \dfrac{\Gamma(r+n-i)}{\Gamma(n-i+1)\Gamma(r)}x^{(0)}(i)\right] & 1 \end{bmatrix}$$

The model properties are defined by Definition 4 and Theorem 3.

Definition 4: Assume that $x^{(r-1)}(k)$ and $Z^{(r)}(k)$ as defined as in Theorem 1:

$$\frac{dx^{(r)}}{dt} + ax^{(r)} = b \qquad (9)$$

is known as the whitenization function of the $GM(1,1)$ differential equation.

$$x^{(r-1)}(k) + az^{(r)}(k) = b.$$

Theorem 3: Assume that **B**, **Y** and $\widehat{a}$ are based on the Theorem 2, as if the following $GM(1,1)$ parameter and least square equation,

$$\hat{a} = [a, b]^T = (B^T B)^{-1} B^T Y,$$

then, the solution (i.e., time response function) of the whitenization function of the $FGM(1,1)$,

$$\frac{dx^{(r)}}{dt} + ax^{(r)} = b \qquad \text{is given by}$$

$$\hat{x}^{(r)}(t) = \left(x^{(1)}(1) - \frac{b}{a}\right)e^{-at} + \frac{b}{a} \qquad (10)$$

The time response sequence of $FGM(1,1)$,
$x^{(r-1)}(k) + az^{(r)}(k) = b$, is given in equation (10),

$$\hat{x}^{(r)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-a(k-1)} + \frac{b}{a}, \quad k=1,2,\ldots\ldots,n \qquad (11)$$

Let $x^{(1)}(0) = x^{(0)}(1)$, then the restored value of $x^{(0)}(k)$ is given as in equation (12)

$$\hat{x}^{(0)}(k) = \left(\hat{x}^{(r)}\right)^{(-r)}(k) = \begin{cases} x^{(0)}(1) & k = 1 \\ \sum_{i=0}^{k-1} (-1)^i \dfrac{\Gamma(r+1)}{\Gamma(i+1)\Gamma(r-i+1)}\hat{x}^{(r)}(k-i) & k = 2,3,\cdots,n \end{cases} \qquad (12)$$

2.2.3 *Forecasting accuracy measures*

The most popular measures of forecast accuracy concentrate on computing forecast errors. There are plenty of such measures, such as MSE and MAPE (See, [86] (p.309). Their usefulness consists of showing the differences in accuracy of computed forecasts but say nothing about the method of forecasting.

In 1995, [4] derived a testing procedure of equal predictive accuracy. The hypothesis to be tested says that the alternative methods are equally accurate on average. The general idea of Diebold-Mariano's test relies on two-time series, including actual values and forecasts of a predicted variable, say y_t and $\hat{y}_{it}$, as well as on the loss function depending on the forecast and actual values only through the forecast error, defined as: $g(y_t, \hat{y}_{it}) = g(\hat{y}_{it} - y_t) = g(e_{it})$. The loss function may take many different forms, which is discussed further in this part. What we compare is a loss differential between the two forecasts, coming from two competing models of the form: $d(t) = g(e_{1t}) - g(e_{2t})$. The forecasting methods are equally accurate if $E(d(t)) = 0$, which is assumed under the null hypothesis.

This paper applied the Diebold-Mariano test to compare a standard $GM(1,1)$ model, $ARIMA(1,1,1)$, and $FGM(1,1)$ model estimated for different r values. The loss function based on the mean square error was selected for comparison because the differences between forecast errors were relatively low. It is worth emphasizing that such a comparison is possible only if a sufficient number of observations are necessary to estimate the $ARIMA$ model.

3. Results

In this section, the data used in the analysis is presented. To begin the analysis, the time series are presented in figure A1 in the appendix. The descriptive statistics of the selected aggregate and disaggregate energy consumption series of BRICS are provided in table A1 in the appendix.

The mean value of "agg" ranges from 111.94 mtoe in South Africa to 1909.42 mtoe in China in BRICS countries. However, the mean value of "coal" ranges from 13.71 mtoe in Brazil to 1298.20 mtoe in China. Similarly, the average "gas" is lowest in South Africa with 2.42 mtoe, while it is highest in Russia with 346.38 mtoe. On the other hand, average "hydro" is lowest in South Africa with 0.30 mtoe and highest in China with 123.78 mtoe. Finally, average "oil" is lowest in South Africa with 24.55 mtoe, while it is highest in China with 360.67 mtoe. The distribution of energy series for each country is skewed in nature (as the mean is different from the median), so the median is a better measure of the center instead of the mean, and interquartile range (IQR) is a better measure of a spread than standard deviation (SD). It can be seen that most of them represent a growing trend, so they are assumed to be nonstationary.

The opposite conclusion can be derived when hydro energy consumption in India, Russia, and South Africa is observed. As far as variation is concerned, South Africa has the least variation (13.51 mtoe), while China has the highest variation (919.14 mtoe) for "agg". Brazil has the least variation (2.09 mtoe), while China has the highest variation (557.56 mtoe) among the "coal" series. South Africa has the least variation (1.25 mtoe), while China has the highest variation (76.87 mtoe) among the "gas" series. South Africa has the least variation (0.17 mtoe) while China has the highest variation (81.93 mtoe) among the "hydro" series. Finally, South Africa has the least variation (3.05 mtoe) while China has the highest variation (167.76 mtoe) among the "oil" series.

In most cases, the unit root hypothesis was confirmed apart from four series, i.e., Russia's "agg", "coal" and "hydro" energy, and South Africa's "hydro" energy. As concerns Russia, decreasing trends is observed in coal energy consumption. Total (aggregate) energy consumption was decreasing in 1992-1998. Oil and gas consumption decreased in 1992-1996; since that time, a growing tendency has been observed. Testing for normality using Jarque and Bera test [87] indicates that most of the time series satisfied the normal distribution. Only aggregate energy consumption and coal energy consumption in Russia do not satisfy this condition.

The results of forecasting energy consumption using the $FGM(1,1)$ model are presented in table 1 for aggregate energy consumption and in tables A3-A6 for disaggregates energy consumption. All results of aggregate and disaggregate energy consumption is reported with the values of MAPE, MSE, development coefficient (a), grey input (b) with different orders (r) varying from $r = 0$ to $r = -1.5$. In table 1, the results of aggregate energy consumption are reported; for Brazil, the order (r) is r=0.9 with the minimum MAPE= 3.43 with the appropriate sign of grey parameters, $a = -0.021$, and $b = 125.064$. For China, the order (r) is r=0.5 with the minimum MAPE= 10.030 with the parameters $a = -0.021$, and $b = 125.064$. For India, the order (r) is r=1 with the minimum MAPE= 2.163 with the parameters $a = -0.051$, and $b = 204.588$. For Russia, the order (r) is r=1 with the minimum MAPE= 3.632 with the parameters $a = -0.003$, and $b = 632.45$. Finally, South Africa has the least MAPE= 21.565 with the parameters $a = -0.001$, and $b = 1.376$, where the value of r=0.

Table 1. Mean absolute percentage error (MAPE) and Mean Square Error (MSE) for Aggregate Energy Consumption of BRICS

	r=0	r=0.01	r=0.05	r=0.1	r=0.5	r=0.9	r=1	r=1.5	r=-0.01	r=-0.05	r=-0.1	r=-0.5	r=-0.9	r=-1.5	ARIMA(1,1,1)
FGM (1,1) for Brazil															
MSE	115.761	114.048	113.330	121.564	183.547	104.364	121.952	2365.346	118.233	137.248	194.597	120.572	153.293	71846.171	34.952
MAPE	3.437	3.467	3.843	4.299	5.652	3.430	3.887	15.476	3.464	3.894	5.659	4.182	5.294	114.571	1.689
a	0.016	0.018	0.024	0.028	0.009	-0.021	-0.027	-0.054	0.014	0.004	-0.012	0.771	1.475	2.876	---
b	9.516	10.424	14.011	18.400	57.183	125.064	150.880	381.851	8.604	4.943	0.586	36.399	13.890	-7.802	---
FGM (1,1) for China															
MSE	39766.001	38854.842	35862.192	33235.534	30342.711	38666.280	44284.071	192619.194	40755.833	45700.224	55315.626	934292.685	192632.280	2535304.295	1455.956
MAPE	11.649	11.426	10.681	10.145	10.030	10.567	11.432	17.974	11.882	12.919	14.596	62.912	29.165	96.097	1.889
a	-0.021	-0.021	-0.020	-0.020	-0.033	-0.052	-0.056	-0.077	-0.021	-0.022	-0.025	0.019	0.842	2.014	---
b	57.208	60.395	73.612	91.213	288.417	665.437	808.421	2071.191	54.069	41.990	28.009	7.239	106.628	-4.462	---
FGM (1,1) for India															
MSE	305.050	264.233	181.024	169.925	458.723	179.881	125.988	4767.086	358.196	775.327	2398.166	27525.071	7235.069	231705.058	75.811
MAPE	3.463	3.065	2.463	2.918	5.302	3.199	2.163	10.946	3.921	6.467	11.662	41.465	20.290	106.842	1.514
a	-0.042	-0.041	-0.038	-0.035	-0.034	-0.047	-0.051	-0.070	-0.043	-0.048	-0.054	0.131	1.543	2.960	---
b	2.785	3.621	7.118	11.814	64.392	165.641	204.588	556.371	1.965	-1.147	-4.601	13.565	48.380	-1.380	---
FGM (1,1) for Russia															
MSE	2557.965	3397.955	10340.732	889.763	2634.297	2009.022	1210.561	13152.140	1928.408	758.559	309.188	594.381	16285.304	3961075.140	NA
MAPE	6.689	7.659	15.072	3.786	5.938	4.659	3.632	11.189	5.825	3.619	2.245	3.203	18.722	293.588	NA
a	0.230	0.161	-0.038	-0.042	0.044	0.005	-0.003	-0.037	0.294	0.456	0.516	0.916	1.861	2.592	---
b	149.005	107.228	-27.610	-29.666	237.035	525.883	632.457	1579.718	185.251	252.662	241.844	94.585	12.318	-61.080	---
FGM 1,1) for South Africa															
MSE	314.658	313.896	314.996	321.056	365.344	322.015	312.152	809.537	316.329	347.810	469.286	497.888	1777.788	166344.037	11.193
MAPE	21.565	21.750	22.218	22.419	23.788	22.161	21.932	31.946	21.368	22.720	28.334	29.611	52.934	444.068	2.294
a	-0.001	0.003	0.018	0.033	0.027	-0.008	-0.015	-0.046	-0.005	-0.013	-0.002	0.336	0.587	0.604	---
b	1.376	1.962	4.654	8.259	33.693	72.221	86.774	216.973	0.845	-0.527	-0.133	5.297	-0.143	-3.801	---

In table A3, the results of oil consumption are reported. For Brazil, the order (r) is r=0.9 with the minimum MAPE= 5.030 with the appropriate sign of grey parameters, $a = -0.013$, and $b = 60.198$. For China, the order (r) is r=0.1 with the minimum MAPE= 2.795 with the parameters $a = -0.029$, and $b = 14.555$. For India, the order (r) is r=0.9 with the minimum MAPE= 2.457 with the parameters $a = -0.042$, and $b = 56.271$. For Russia, the order (r) is r=1.5 with the minimum MAPE= 5.566 with the parameters $a = -0.034$, and $b = 361.965$. Finally, South Africa has the least MAPE= 2.151 with the parameters $a = -0.005$, and $b = 17.103$, where the value of r=0.9.

In table A4, the results of gas consumption are reported. For Brazil, the order (r) is r=0.1 with the minimum MAPE= 22.262 with the appropriate sign of grey parameters, $a = -0.001$, and $b = 1.441$. For China, the order (r) is r=1 with the minimum MAPE= 18.884 with the parameters $a = -0.124$, and $b = 9.974$. For India, the order (r) is r=0.1 with the minimum MAPE= 7.842 with the parameters $a = -0.001$, and $b = 11.814$. For Russia, the order (r) is r=1 with the minimum MAPE= 3.408 with the parameters $a = -0.008$, and $b = 309.562$. Finally, South Africa has the least MAPE= 30.042 with the parameters $a = -0.050$, and $b = 0.853$, where the value of r=0.9.

In table A5, the results of coal consumption are reported. For Brazil, the order (r) is r=1 with the minimum MAPE= 8.040 with the appropriate sign of grey parameters, $a = -0.013$, and $b = 11.142$. For China, the order (r) is r=0.5 with the minimum MAPE= 13.971 with the parameters $a = -0.016$, and $b = 246.587$. For India, the order (r) is r=1 with the minimum MAPE= 4.489 with the parameters $a = -0.054$, and $b = 105.423$. For Russia, the order (r) is r=1.5 with the minimum MAPE= 12.986 with the parameters $a = -0.027$, and $b = 249.931$. Finally, South Africa has the least MAPE= 3.335 with the parameters $a = -0.003$, and $b = 57.857$, where the value of r=0.9.

In table A6, the results of hydro consumption are reported. For Brazil, the order (r) is r=0.9 with the minimum MAPE= 5.552 with the appropriate sign of grey parameters, $a = -0.006$, and $b = 52.191$. For China, the order (r) is r=1 with the minimum MAPE= 11.418 with the parameters $a = -0.081$, and $b = 34.593$. For India, the order (r) is r=1 with the minimum MAPE= 8.642 with the parameters $a = -0.027$, and $b = 15.549$. For Russia, the order (r) is r=1.5 with the minimum MAPE= 14.648 with the parameters $a = -0.036$, and $b = 96.269$. Finally, South Africa has the least MAPE= 122.067 with the parameters $a = -0.040$, and $b = .729$, where the value of r=1.5.

The conclusion of model comparison using the Diebold-Mariano test given in table 2 is that the $GM(1,1)$ model and $ARIMA(1,1,1)$ possess an equal predictive ability for energy consumption, so the model choice must be based on additional measures for forecasts accuracy, as well as on the data availability. Since the observation number is large, one can rely on the stochastic time series model. On the other hand, grey models are beneficial. As concerns $FGM(1,1)$, the conclusion is quite similar, but for small negative r values like -1.5, the results are much worse. This conclusion is useful because one can limit the range of possible r values between (-1.00; +1.00]. The two results (China, hydro r=0.00 and India coal, r=-0.05), when the null hypothesis of equal predictive ability was rejected, can be considered random.

Table 2. Diebold-Mariano test results (p-values) for comparison of GM(1,1) and FGM(1,1)

GM(1,1)	Brazil					China					India					Russia					South Africa				
FGM(1,1)	agg	oil	gas	coal	hydro	agg	oil	gas	coal	hydro	agg	oil	gas	coal	hydro	agg	oil	gas	coal	hydro	agg	oil	gas	coal	hydro
r=0	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	0.044	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1
r=0,01	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1
r=0,05	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1
r=0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1
r=0,5	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1
r=0,9	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1
r=1,5	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1
r=-0,01	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1
r=-0,05	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	0.040	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1
r=-0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1
r=-0,5	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1
r=-0,9	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1	>0,1
r=-1,5	0.020	0.022	>0.1	0.024	0.021	0.016	0.019	>0,1	0.048	0.040	0.027	0.038	0.024	0.024	0.026	0.015	0.012	0.015	0.015	0.024	0.023	0.029	0.039	0.020	0.000
ARIMA(1,1,1)	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1

Note: Rejecting the null hypothesis is highlighted with the shaded area; agg=aggregate.

4. Discussion

BRICS countries belong to the developing economies group, although China has become a global competitor in some areas [88]. Developing countries desire precise forecasts of both scales of growth and energy consumption. As the economy grows, energy consumption increases as well. However, there are strict limitations to energy consumption. They divide into global and country-specific limitations. Global recommendations origin is in the United Nations' 2030 Agenda for the Sustainable Development Goals (SDGs), which all 193 UN member states accepted. Among 17 SDGs, goal 7 assumes access to affordable, reliable, sustainable, and modern energy for all, and goal 13 covers urgent action to combat climate change and its impacts. Country-specific limitations consist of natural resource exploitation, technology for energy production, and transformation and consciousness of the necessity of rationalization. Considering the above, the energy policy of BRICS countries needs continuous monitoring for forecasting accuracy and its structure according to the SDGs requirements.

The results of the grey and $ARIMA$ models' comparison presented in the current paper revealed as follows:

1. Fractional Grey Model $FGM(1,1)$ allows a broad spectrum of parameters that adjust to the empirical data. FGM-based approach is more comprehensive than the standard $GM(1,1)$ model, which is "a special case" of $FGM(1,1)$ for $r=1$.

2. According to the Diebold-Mariano test results, the estimated $FGM(1,1)$ models - taking parameters' range [-1; 1] confirmed equal predictive ability with $GM(1,1)$ model as well as $ARIMA(1,1,1)$ model.

3. Although grey-type models are mostly recommended for short time series, their predictive ability is equal to $ARIMA$ models designed for long time series. However, taking values of MSE and MAPE in empirical study, $ARIMA(1,1,1)$ model highly outperformed $FGM(1,1)$ in 19 cases on 25. Only in China's case of oil consumption, the $FGM(1,1)$ model has the minimum MAPE and MSE values. The remained five series were stationary, so $ARIMA(1,1,1)$ model was not estimated.

4. For some parameter "r" values, empirical $FGM(1,1)$ models do not satisfy the grey model assumption, i.e., $a < 0$ and $b > 0$. In such circumstances, it is recommended to estimate the model for another "r" parameter value.

5. Grey-type models are helpful for forecasting in the case when only a few observations are available. Still, for long and nonstationary time series, standard time series models perform better.

In the paper [89], the authors provided a methodological comparison of probability models, fuzzy math, grey systems, and rough sets. It appears that grey models are evidently preferred in the case of small samples and incomplete information sets. They concentrate on the law of reality. On the other hand, stochastic models, such as $ARIMA$ are designed for large samples and follow historical law. The general conclusion that both types of models possess equal predictive ability indicated by the Diebold-Mariano test allows selecting the proper procedure for a given data set and forecasting perspective. The exact values of MAPE and MSE are less informative because they are valid only for a given sample. Therefore, the presented results are in line with both theory and expectations.

5. Conclusions

The BRICS are emerging economies concerning the production and management of resources and require a consistent supply of having energy resources. The BRICS countries should monitor energy consumption, focusing on the supply-demand gap of energy and its components and facilities provided to local and foreign investors. Therefore, forecasting is quite significant for energetic policy projection. Accurate forecasts of energy consumption are vital when demand grows faster. On the other hand, BRICS's energy consumption values can be offered as fluctuating and increasing.

This study aims to compare different decision-making types for energy demand forecasting in BRICS. First, this paper focused on forecasting the annual energy consumption for BRICS. Secondly, it compared $ARIMA$, and $FGM(1,1)$ models with actual data in 1992-

2019 using estimated errors (MAPE) and (MSE). Results have revealed that $ARIMA$ and $FGM(1,1)$ models perform close findings. Thirdly, model comparison using the Diebold-Mariano test confirmed the equal predictive ability of $ARIMA(1,1,1)$ and $FGM(1,1)$ unless the FGM parameter ranges [-1, 1].

The empirical findings allow formulating some recommendations. BRICS countries need to follow SDGs concerning energetic policy keeping their economic growth level increasing. It implies a gradual structural change from traditional towards renewable energy sources. A structural change always means a significant limitation of the observations; therefore, the $FGM(1,1)$ model is recommended for predicting energy consumption in aggregate and disaggregate levels.

Appendix A

Table A1. Summary statistics of energy consumption for BRICS

Variable	Brazil					Russia					India					China					South Africa				
	agg	oil	gas	coal	hydro	agg	oil	gas	coal	hydro	agg	oil	gas	coal	hydro	agg	oil	gas	coal	hydro	agg	oil	gas	coal	hydro
Mean	224.48	97.12	17.87	13.72	75.63	668.91	146.39	346.38	102.32	39.16	456.45	140.38	31.60	249.51	23.24	1909.42	360.69	81.16	1298.20	123.78	111.94	24.55	2.42	81.39	0.30
Med	214.21	91.52	17.54	13.00	78.15	670.93	137.99	352.03	99.16	39.55	405.72	129.09	30.18	215.24	23.71	1892.64	346.59	45.04	1389.66	95.99	115.44	25.47	2.91	82.57	0.26
Max	296.25	129.59	36.92	17.62	95.44	819.31	238.82	390.80	156.98	42.10	813.50	244.53	51.84	444.73	34.46	3384.43	666.52	264.26	1969.07	270.33	129.00	28.62	3.91	93.82	0.68
Min	139.15	64.37	3.19	10.68	53.34	598.74	125.21	297.00	83.93	36.20	218.18	63.79	12.41	123.67	16.16	758.40	134.64	13.68	578.80	31.21	88.12	17.92	0.80	66.35	0.03
S. D.	52.31	18.04	11.20	2.09	11.44	48.43	24.75	24.91	16.69	1.89	188.65	53.38	13.40	108.11	5.53	919.14	167.76	76.87	557.56	81.93	13.51	3.05	1.25	8.99	0.17
Ske	0.04	0.05	0.19	0.46	-0.27	1.05	2.24	-0.26	1.75	-0.08	0.47	0.39	0.12	0.53	0.16	0.16	0.30	0.98	-0.05	0.54	-0.33	-0.65	-0.14	-0.19	0.98
Kurt	1.61	2.07	1.66	1.98	2.09	4.77	8.47	2.01	5.97	1.71	1.90	2.15	1.57	1.83	1.72	1.44	1.77	2.73	1.24	1.78	1.63	2.31	1.20	1.59	3.19
J-B	2.25	1.03	2.28	2.19	1.29	8.78	58.27	1.45	24.65	1.98	2.44	1.53	2.45	2.91	2.03	2.96	2.16	4.59	3.62	3.09	2.72	2.50	3.86	2.50	4.53
Prob	0.32	0.60	0.32	0.33	0.52	0.01	0.00	0.48	0.00	0.37	0.29	0.46	0.29	0.23	0.36	0.23	0.34	0.10	0.16	0.21	0.26	0.29	0.15	0.29	0.10
Obs.	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28

Note: Med: Median; Max=Maximum; Min=Minimum; S.D.= Standard deviation; Ske= Skewness; Kurt= Kurtosis; J-B= Jarque-Berra; Prob=Probability; Obs= Observations.

Table A2. Unit Root (ADF) Testing for BRICS

		Brazil					Russia					India					China					South Africa				
	Variable	agg	oil	gas	coal	hydro	agg	oil	gas	coal	hydro	agg	oil	gas	coal	hydro	agg	oil	gas	coal	hydro	agg	oil	gas	coal	hydro
Level	**Statistic**	-0.992	0.101	-0.718	-1.575	-1.882	-3.761	-2.499	-0.859	-3.421	-3.403	4.640	2.436	-0.801	2.449	-0.598	0.227	2.157	3.298	-0.801	1.651	-1.282	-2.214	-0.539	-1.513	-3.760
	Prob.	0.741	0.958	0.826	0.481	0.335	0.009*	0.127	0.785	0.020*	0.020*	1.000	1.000	0.802	1.000	0.855	0.969	1.000	1.000	0.801	0.999	0.623	0.206	0.868	0.512	0.009*
First difference	**Statistic**	-4.052	-4.186	-3.516	-5.597	-4.586	-3.286	-4.259	-4.843	-3.707	-5.383	-1.456	-3.713	-3.653	-3.373	-5.749	-2.202	-4.722	0.605	-2.556	-6.243	-5.892	-5.038	-4.671	-5.908	-4.463
	Prob.	0.005*	0.004*	0.018*	0.000*	0.001*	0.026*	0.003*	0.001*	0.011*	0.000*	0.539	0.010*	0.012*	0.022*	0.000*	0.210*	0.001*	0.986	0.116	0.000*	0.000*	0.000*	0.001*	0.000*	0.002*

Note: * indicate the rejection of the null hypothesis of a unit root at the 1% significant levels, respectively; agg=aggregate.

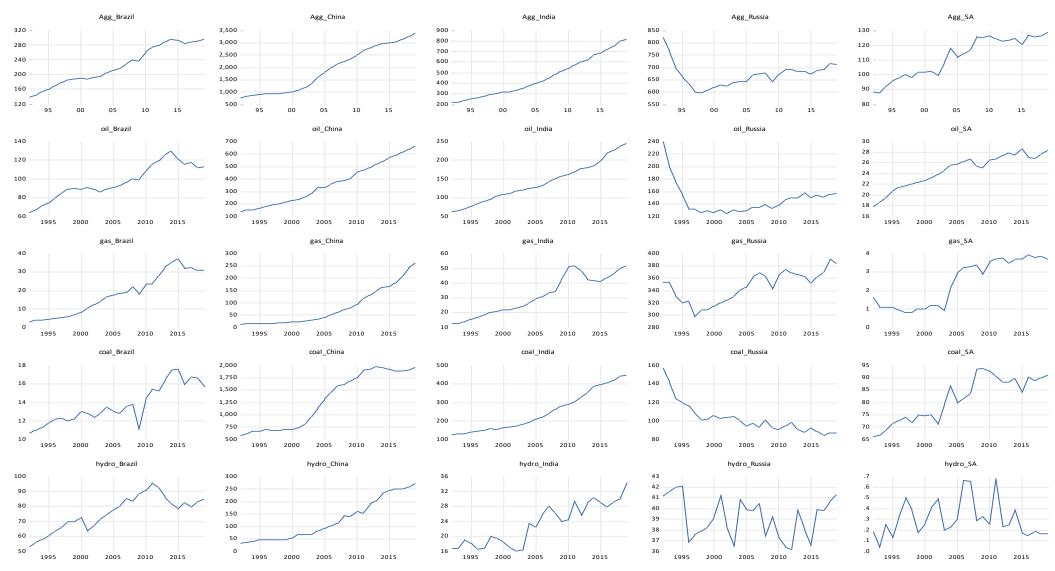**Figure A1.** Graphical trends of energy consumption for BRICS

Table A3. Mean absolute percentage error (MAPE) and Mean Square Error (MSE) for Oil Consumption of BRICS

	r=0	r=0.01	r=0.05	r=0.1	r=0.5	r=0.9	r=1	r=1.5	r=-0.01	r=-0.05	r=-0.1	r=-0.5	r=-0.9	r=-1.5	ARIMA(1,1,1)
						FGM (1,1) for Brazil									
MSE	49.603	49.386	115.761	52.035	61.483	42.264	47.209	554.869	50.008	54.215	66.398	55.720	57.320	13798.910	12.622
MAPE	5.180	5.229	3.437	5.828	6.954	5.030	5.632	17.095	5.128	5.157	5.645	5.899	6.013	115.989	2.552
a	0.053	0.055	0.060	0.062	0.024	-0.013	-0.020	-0.050	0.051	0.038	0.014	0.525	1.234	2.701	---
b	6.992	7.501	9.428	11.640	29.519	60.981	73.023	181.164	6.471	4.276	1.513	9.415	3.614	-4.374	---
						FGM (1,1) for China									
	r=0	r=0.01	r=0.05	r=0.1	r=0.5	r=0.9	r=1	r=1.5	r=-0.01	r=-0.05	r=-0.1	r=-0.5	r=-0.9	r=-1.5	
MSE	173.090	163.078	135.180	118.389	145.729	231.141	346.868	4832.628	184.703	137.248	433.358	83239.662	4709.281	93920.959	141.491
MAPE	3.866	3.716	3.198	2.795	3.745	4.356	5.595	14.754	4.025	3.894	6.633	80.335	23.117	90.676	2.787
a	-0.032	-0.032	-0.030	-0.029	-0.036	-0.052	-0.056	-0.076	-0.033	0.004	-0.039	-0.028	1.073	1.877	---
b	8.143	8.747	11.243	14.555	51.763	124.539	152.515	403.135	7.548	4.943	2.562	-2.197	28.462	0.568	---
						FGM (1,1) for India									
	r=0	r=0.01	r=0.05	r=0.1	r=0.5	r=0.9	r=1	r=1.5	r=-0.01	r=-0.05	r=-0.1	r=-0.5	r=-0.9	r=-1.5	
MSE	24.484	26.001	32.126	39.551	53.420	17.790	21.358	653.221	22.992	18.043	26.741	1694.731	445.230	19010.504	14.203
MAPE	3.248	3.343	3.651	4.148	5.602	2.457	3.206	15.675	3.140	2.741	3.453	33.074	15.496	96.138	2.112
a	-0.038	-0.037	-0.032	-0.028	-0.027	-0.042	-0.046	-0.067	-0.039	-0.045	-0.054	0.147	1.464	2.613	---
b	1.392	1.704	2.987	4.670	22.514	56.272	69.239	185.918	1.084	-0.109	-1.493	4.815	13.837	-0.474	---
						FGM (1,1) for Russia									
	r=0	r=0.01	r=0.05	r=0.1	r=0.5	r=0.9	r=1	r=1.5	r=-0.01	r=-0.05	r=-0.1	r=-0.5	r=-0.9	r=-1.5	
MSE	188.868	212.817	371.977	751.580	235.376	339.298	282.710	189.970	168.724	112.864	73.178	53.418	3378.958	389209.753	58.974
MAPE	8.593	9.093	11.887	17.409	8.047	9.725	8.781	5.566	8.143	6.758	5.526	4.440	39.446	436.351	4.332
a	0.389	0.364	0.236	0.047	0.030	0.006	-0.001	-0.034	0.412	0.484	0.549	1.111	2.091	2.527	---
b	53.226	51.381	37.649	8.659	41.620	115.046	140.362	361.965	54.558	56.147	53.643	22.856	-0.617	-18.040	---
						FGM(1,1) for South Africa									
	r=0	r=0.01	r=0.05	r=0.1	r=0.5	r=0.9	r=1	r=1.5	r=-0.01	r=-0.05	r=-0.1	r=-0.5	r=-0.9	r=-1.5	
MSE	0.322	0.322	0.352	0.439	0.999	0.419	1.011	38.694	0.326	0.389	0.687	2.363	1.411	1311.244	0.223
MAPE	1.799	1.787	1.957	2.217	3.480	2.151	3.291	17.950	1.811	1.988	3.091	5.561	3.791	138.693	1.894
a	0.072	0.075	0.082	0.084	0.035	-0.005	-0.013	-0.045	0.068	0.040	-0.024	0.641	1.467	2.966	---
b	2.149	2.323	2.927	3.561	8.411	17.103	20.436	50.270	1.964	1.058	-0.383	2.878	1.055	-1.208	---

Note: MSE=Mean Standard Error; MAPE=Mean absolute percentage Error.

Table A4. Mean absolute percentage error (MAPE) and Mean Square Error (MSE) for Gas Consumption of BRICS

	r=0	r=0.01	r=0.05	r=0.1	r=0.5	r=0.9	r=1	r=1.5	r=-0.01	r=-0.05	r=-0.1	r=-0.5	r=-0.9	r=-1.5	ARIMA(1,1,1)
FGM (1,1) for Brazil															
MSE	13.971	13.862	13.461	13.031	11.422	15.033	17.749	60.883	14.085	14.589	15.360	42.822	58.652	2234.459	4.822
MAPE	22.107	22.113	22.170	22.262	24.350	29.581	31.343	43.135	22.101	22.173	22.340	31.303	66.971	278.951	9.528
a	0.008	0.007	0.004	-0.001	-0.035	-0.062	-0.068	-0.092	0.009	0.012	0.016	0.007	0.006	0.108	---
b	1.166	1.192	1.300	1.441	2.972	5.820	6.883	16.021	1.140	1.038	0.915	0.072	-0.104	-0.090	---
FGM (1,1) for China															
MSE	2386.569	2286.868	1933.065	1576.655	422.742	236.068	233.796	455.330	2491.238	2964.885	3703.170	22144.389	49126.530	63696.575	25.285
MAPE	60.179	58.943	54.298	49.114	26.881	19.728	18.884	19.937	61.446	66.841	74.376	175.801	310.118	473.308	7.415
a	-0.108	-0.108	-0.108	-0.109	-0.115	-0.122	-0.124	-0.133	-0.108	-0.107	-0.107	-0.098	-0.054	0.067	---
b	0.756	0.787	0.920	1.101	3.377	8.018	9.794	25.724	0.725	0.608	0.477	-0.092	-0.046	-0.445	---
FGM (1,1) for India															
MSE	18.215	18.125	17.824	17.565	18.099	22.225	24.800	82.087	18.312	18.766	19.530	158.415	176.382	2806.706	4.925
MAPE	8.607	8.498	8.143	7.842	8.005	10.882	12.105	22.167	8.727	9.325	10.433	45.288	49.903	178.481	4.435
a	-0.002	-0.002	-0.001	-0.001	-0.018	-0.040	-0.046	-0.070	-0.003	-0.005	-0.008	0.028	0.282	0.551	---
b	1.364	1.432	1.709	2.072	5.985	13.333	16.108	40.412	1.298	1.040	0.738	0.142	0.278	-0.408	---
FGM (1,1) for Russia															
MSE	1597.445	1070.673	199.004	149.132	594.394	338.965	211.700	4671.837	1708.874	804.480	274.693	262.158	1829.579	668162.502	133.125
MAPE	11.005	8.753	3.498	2.980	5.601	4.213	3.408	13.490	11.384	7.178	4.153	4.028	11.768	229.299	2.555
a	-0.042	-0.046	-0.019	0.021	0.039	0.001	-0.008	-0.041	-0.028	0.152	0.402	0.793	1.639	2.507	---
b	-13.414	-14.730	-3.854	16.874	120.034	257.855	309.562	770.353	-8.978	43.940	99.941	45.230	9.518	-25.102	---
FGM (1,1) for South Africa															
MSE	0.551	0.539	0.496	0.454	0.315	0.330	0.350	0.687	0.563	0.622	0.718	1.937	2.245	36.106	0.057
MAPE	48.536	47.852	45.358	42.625	31.557	30.042	30.360	34.505	49.252	52.327	56.664	93.233	100.429	353.199	13.235
a	-0.012	-0.012	-0.012	-0.013	-0.028	-0.050	-0.055	-0.078	-0.012	-0.011	-0.010	0.124	0.454	0.669	---
b	0.049	0.053	0.070	0.092	0.360	0.853	1.035	2.592	0.046	0.032	0.018	0.032	0.011	-0.073	---

Table A 5. Mean absolute percentage error (MAPE) and Mean Square Error (MSE) for Coal Consumption of BRICS

	r=0	r=0.01	r=0.05	r=0.1	r=0.5	r=0.9	r=1	r=1.5	r=-0.01	r=-0.05	r=-0.1	r=-0.5	r=-0.9	r=-1.5	ARIMA(1,1,1)
FGM (1,1) for Brazil															
MSE	3.022	3.040	3.128	3.245	3.726	3.068	2.980	11.746	3.007	3.095	4.226	4.271	16.223	1863.829	0.763
MAPE	8.267	8.410	8.823	9.278	11.457	8.704	8.040	19.536	8.106	8.484	12.653	13.056	28.945	319.597	4.321
a	0.012	0.018	0.038	0.054	0.033	-0.005	-0.013	-0.044	0.006	-0.012	-0.001	0.367	0.629	0.645	---
b	0.345	0.452	0.888	1.385	4.437	9.289	11.142	27.770	0.243	-0.058	-0.012	0.727	-0.016	-0.516	---
FGM (1,1) for China															
MSE	40148.271	39558.183	37475.784	35439.618	31570.931	37702.083	41820.481	133382.202	40767.469	43544.982	47717.402	175312.752	90654.063	1102950.215	1980.428
MAPE	15.554	15.364	14.691	14.165	13.971	14.936	15.868	22.104	15.761	16.716	18.127	39.026	28.406	94.467	2.691
a	0.006	0.006	0.006	0.005	-0.016	-0.041	-0.047	-0.072	0.006	0.005	0.003	0.097	0.706	1.851	---
b	58.656	61.412	72.762	87.674	246.587	538.901	648.459	1604.314	55.931	45.376	33.035	21.642	43.347	-19.890	---
FGM (1,1) for India															
MSE	413.770	375.372	273.808	216.300	257.278	177.617	143.883	1153.488	459.400	746.177	1537.331	9666.674	3228.482	73129.170	62.304
MAPE	8.160	7.632	6.039	5.535	7.020	5.436	4.489	8.385	8.732	11.729	17.242	45.917	25.595	115.115	2.511
a	-0.040	-0.040	-0.038	-0.036	-0.037	-0.050	-0.054	-0.072	-0.041	-0.043	-0.047	0.126	1.237	2.362	---
b	1.905	2.296	3.950	6.213	32.779	85.160	105.423	289.759	1.524	0.102	-1.420	6.772	20.189	-2.474	---
FGM (1,1) for Russia															
MSE	20.822	19.540	15.492	13.169	64.766	57.719	35.647	410.340	22.203	28.623	38.245	74.944	578.569	156299.051	NA
MAPE	3.764	3.646	3.250	2.842	6.480	5.514	4.183	12.986	3.880	4.453	5.221	7.576	23.016	398.090	NA
a	0.253	0.245	0.210	0.141	0.095	0.026	0.015	-0.027	0.261	0.293	0.334	0.866	1.898	2.453	---
b	23.099	23.192	22.979	18.459	56.429	104.897	123.841	294.931	22.971	22.239	21.062	10.176	-2.168	-12.251	---
FGM (1,1) for South Africa															
MSE	14.173	13.693	12.783	13.175	23.058	14.400	17.494	392.652	14.761	17.875	36.789	31.934	27.484	19716.100	11.116
MAPE	3.040	2.989	3.146	3.517	5.101	3.335	3.565	17.534	3.102	4.134	6.564	5.632	5.657	164.637	2.96
a	0.036	0.042	0.061	0.071	0.037	-0.003	-0.011	-0.044	0.028	-0.006	0.041	0.670	1.486	2.630	---
b	3.818	4.591	7.336	10.128	28.091	57.857	69.193	170.402	3.015	-0.081	2.354	9.554	2.946	-4.523	---

Table A 6. Mean absolute percentage error (MAPE) and Mean Square Error (MSE) for Hydroelectric Consumption of BRICS

	FGM (1,1) for Brazil														ARIMA(1,1,1)
	r=0	r=0.01	r=0.05	r=0.1	r=0.5	r=0.9	r=1	r=1.5	r=-0.01	r=-0.05	r=-0.1	r=-0.5	r=-0.9	r=-1.5	
MSE	26.193	25.638	24.188	23.534	26.283	29.985	39.439	422.779	26.844	30.436	33.004	49.510	43.965	11437.767	12.919
MAPE	4.903	4.871	4.786	4.766	5.250	5.552	6.888	20.785	4.951	5.228	5.451	7.822	7.371	134.446	3.7851
a	0.067	0.071	0.079	0.081	0.035	-0.006	-0.014	-0.047	0.064	0.041	0.006	0.509	1.221	2.431	---
b	6.286	6.809	8.695	10.716	25.763	52.191	62.277	152.041	5.741	3.346	0.511	6.738	2.384	-3.319	---
	FGM (1,1) for China														
	r=0	r=0.01	r=0.05	r=0.1	r=0.5	r=0.9	r=1	r=1.5	r=-0.01	r=-0.05	r=-0.1	r=-0.5	r=-0.9	r=-1.5	
MSE	1102950.215	344.024	316.631	289.252	199.725	215.014	240.196	832.985	360.018	397.743	458.891	4067.114	10319.203	36363.546	97.298
MAPE	94.467	18.552	17.499	16.520	13.636	11.436	11.418	16.457	19.201	20.629	22.744	66.711	113.149	193.771	8.923
a	1.851	-0.041	-0.042	-0.044	-0.061	-0.077	-0.081	-0.097	-0.040	-0.039	-0.038	-0.033	0.098	0.458	---
b	-19.890	3.993	4.499	5.175	13.017	28.589	34.593	88.877	3.751	3.289	2.750	-0.035	0.723	-0.147	---
	FGM (1,1) for India														
	r=0	r=0.01	r=0.05	r=0.1	r=0.5	r=0.9	r=1	r=1.5	r=-0.01	r=-0.05	r=-0.1	r=-0.5	r=-0.9	r=-1.5	
MSE	5.671	5.531	5.410	5.421	6.653	5.422	5.055	21.686	6.019	21.591	352.601	15.608	47.200	3145.920	4.534
MAPE	8.768	8.602	8.617	8.788	10.103	9.148	8.642	13.992	9.057	15.380	56.726	16.589	30.760	234.639	8.354
a	-0.062	-0.057	-0.039	-0.022	0.002	-0.021	-0.027	-0.053	-0.068	-0.090	-0.113	0.429	0.861	1.203	---
b	-0.786	-0.666	-0.173	0.449	5.315	12.772	15.549	40.178	-0.903	-1.325	-1.647	1.988	0.707	-0.538	---
	FGM (1,1) for Russia														
	r=0	r=0.01	r=0.05	r=0.1	r=0.5	r=0.9	r=1	r=1.5	r=-0.01	r=-0.05	r=-0.1	r=-0.5	r=-0.9	r=-1.5	
MSE	9.288	22.673	3.967	4.429	11.002	5.421	3.322	77.324	5.413	3.266	2.991	7.240	23.523	9454.585	NA
MAPE	6.744	10.793	4.074	4.359	7.240	5.056	4.248	14.648	4.782	4.036	3.762	5.829	11.862	242.602	NA
a	-0.006	-0.058	-0.018	0.103	0.062	0.010	0.001	-0.036	0.047	0.174	0.240	0.653	1.509	2.536	---
b	-0.221	-2.277	-0.493	6.097	17.020	33.298	39.591	96.269	1.718	5.609	6.485	3.787	0.662	-2.922	---
	FGM (1,1) for South Africa														
	r=0	r=0.01	r=0.05	r=0.1	r=0.5	r=0.9	r=1	r=1.5	r=-0.01	r=-0.05	r=-0.1	r=-0.5	r=-0.9	r=-1.5	
MSE	0.049	0.047	0.041	0.036	0.024	0.026	0.028	0.049	0.050	0.054	0.054	0.026	0.454	28.255	NA
MAPE	54.051	51.839	50.271	51.327	58.643	67.637	72.447	122.067	56.466	63.976	67.613	78.740	293.018	1968.815	NA
a	-0.005	-0.002	0.011	0.030	0.067	0.014	0.003	-0.040	-0.008	-0.016	-0.022	0.013	0.040	0.035	---
b	-0.002	-0.001	0.005	0.016	0.136	0.269	0.317	0.729	-0.003	-0.007	-0.009	-0.007	-0.007	-0.006	---

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