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Volatility modeling: an overview of equity markets in the euro area during COVID-19 Pandemic

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Abstract: Volatility is the most widespread measure of risk. Volatility modeling allows investors to capture potential losses and investment opportunities. This work aims to examine the impact of the two waves of COVID-19 infections on the return and volatility of the stock market indices of the euro area countries. The study also focuses on other important aspects such as time-varying risk premium and leverage effect. Thus, this investigation employed the Threshold GARCH(1,1)-in-Mean model with exogenous dummy variables. Daily returns of ten euro area stock indices from 4th January 2016 to 31th December 2020 has been used for the analysis. The results reveal that euro area stock markets respond differently to the COVID-19 pandemic. Specifically, the first wave of COVID-19 infections had a notable impact on stock market volatility of euro area countries with large and middle financial centres while the second wave had a significant impact only on stock market volatility of Belgium.

Keywords: Volatility; COVID-19 Pandemic; GARCH models; Euro area stock indices.

1. Introduction

The first COVID-19 infections were identified in Wuhan city, Hubei province of China in December 2019. On 21th February 2020, Northern Italy registered a significant increase of COVID-19 cases, while other European states started to report cases of infected people [1]. Later, on 11th March 2020, the World Health Organization (WHO) declared COVID-19 a pandemic [2]. The continuous spread of the novel coronavirus has wide effects on national economies and financial markets across the world. **Figure 1** shows the Gross Domestic Product (GDP) growth rates compared to the same quarter of previous year in the euro area (EA), European Union (EU) and North American Free Trade Agreement (NAFTA) from 1st quarter 2016 to 4th quarter 2020. During the last quarter of 2020, GDP decreased by 5.01% in the EA, 4.75% in the EU and 2.80% in the NAFTA compared with the last quarter of 2019. Probably due to containment measures of Covid-19, the worst peak occurred in the 2nd quarter 2020 (-14.71% in the EA, -13.89% in the EU and -10.26% in the NAFTA) [3]. According to International Monetary Fund's January 2021 World Economic Outlook (WEO), the euro area growth contraction for 2020 is estimated at -7.2% [4]. These poorly economics performance have led to a "bearish stock market" [5] and "financial market turmoil" [6]. For instance, the closing prices of Euro Stoxx 50 Index and S&P 500 Index show a sharp drop in the first quarter of 2020 (**Figure 2**, red area). Additionally, **Figure 2** highlights a rapid recovery of S&P 500 and a slow recovery of Euro Stoxx 50 during other quarters. Equally important is the high level of volatility, on 16th March 2020, CBOE Volatility Index¹ (VIX) closed at 82.69, surpassing the peak level of 80.86 on 20th November 2008 [8]. Volatility is a key factor of financial markets, it is a measure of financial risk or uncertainty of financial assets [9].

¹ CBOE Volatility Index is a volatility measure based on S&P 500 Index options. It was developed by Cboe Global Markets [7]

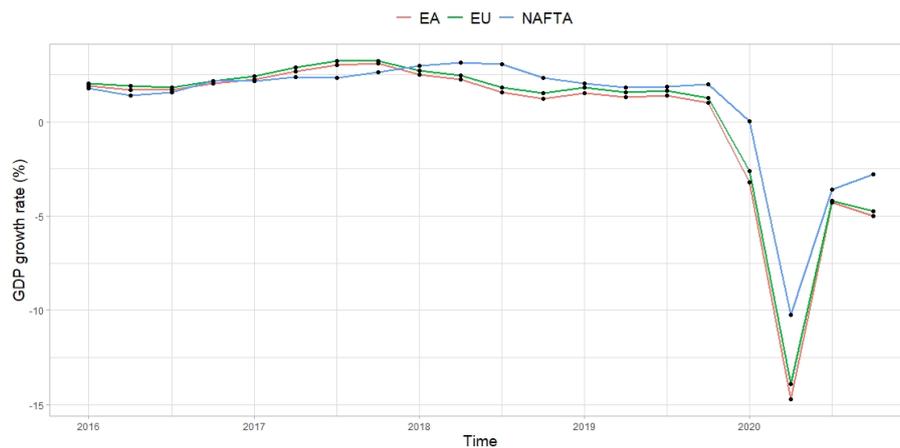


Figure 1. Growth Rates of Gross Domestic Product compared to the same quarter of previous year, seasonally adjusted data (1st quarter 2016 - 4th quarter 2020).



Figure 2. Time plot of daily closing prices (4th January 2016 - 31th December 2020). Panel (a) contains the daily closing prices of Euro Stoxx 50 (STOXX50E). Panel (b) contains the daily closing prices of S&P 500 (SPX).

To better understand the impact of coronavirus health crisis on European stock markets, we study the impact of COVID-19 pandemic on the daily returns of euro area stock market indices over the period from January 4, 2016 to December 31, 2020. In particular, the effect of COVID-19 pandemic is captured through two exogenous dummy variables. These variables reflect the two main waves of COVID-19 infections. We particularly focus on the euro area financial markets for at least two reasons. First, we believe that few studies have focused on euro area stock markets, especially on coronavirus pandemic period. Second, the size of equity markets varies significantly across the euro area member countries. It is possible to identify small, medium and large financial centers looking the relationship

between real GDP per capita and the stock market capitalization as a percentage of GDP across euro area countries (Figure 3) [10]. The large financial centers have a high real GDP per capita and a high stock market capitalization to GDP (%) such as Netherlands, Finland and France. On the contrary, small financial centers have a low real GDP per capita and a low stock market capitalization to GDP (%) such as the Baltic states of Estonia, Latvia, and Lithuania. Whereas mid-sized financial centers are located among large and small centers, for instance Belgium, Spain and Germany. Depending on the real GDP per capita and stock market capitalization to GDP i.e. the sizes of financial centers, we believe that euro area stock markets could respond differently to the COVID-19 pandemic.

Hence, using the GARCH modeling approach, the purpose of this paper is to analyse the impact of the COVID-19 pandemic on returns and volatility of the stock market indices of the euro area countries with different sizes of financial centers. These indices are AEX (the Netherlands), OMX Helsinki 25 (Finland), CAC 40 (France), DAX (Germany), BEL 20 (Belgium), IBEX 35 (Spain), FTSE MIB (Italy), FTSE/ATHEX 20 (Greece), Blue-Chip SBITOP (Slovenia) and OMX Baltic Benchmark Price index (the Baltic states of Estonia, Latvia, and Lithuania). They are summarized in Table 1.

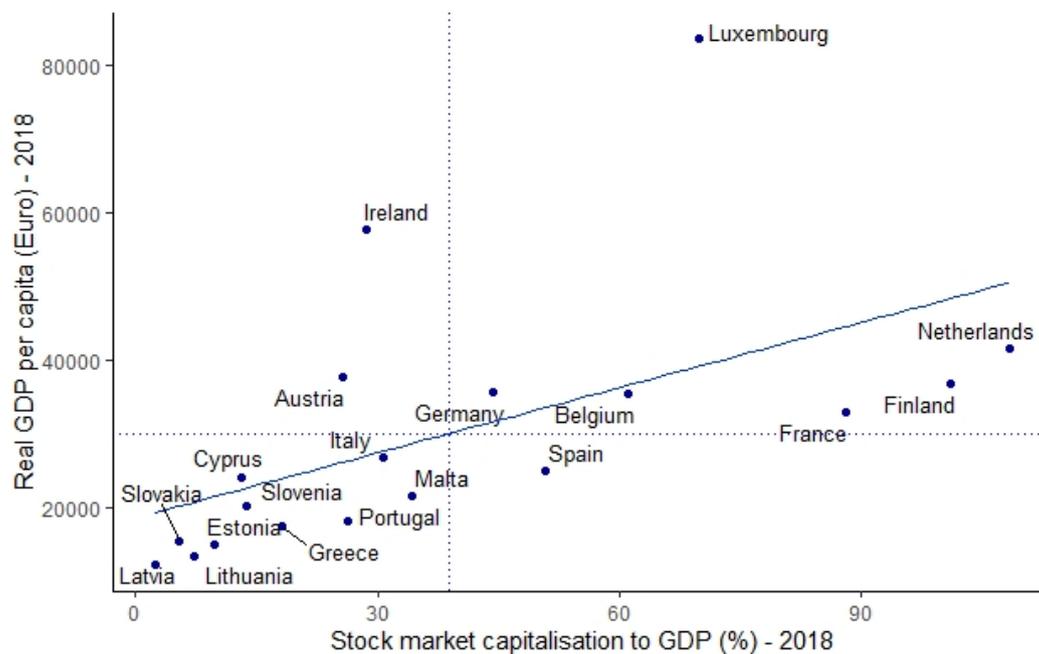


Figure 3. Relationship between real GDP per capita (Euro) and stock market capitalisation to GDP (%), 2018. Euro area financial centers.

Table 1. Analysed indices.

Country	Index	Financial center's size
Netherlands	AEX	Large
Finland	OMX Helsinki 25 (OMXH 25)	Large
France	CAC 40	Large
Belgium	BEL 20	Middle
Germany	DAX	Middle
Spain	IBEX 35	Middle
Italy	FTSE MIB	Middle
Greece	FTSE/ATHEX 20 (ATF)	Small
Slovenia	SBITOP	Small
Baltic states	OMXBBPI	Small

Firstly, this study employed the following statistical hypothesis tests to verify specific assumptions about financial time series under study: Jarque-Bera Test [11], Augmented Dickey-Fuller Test [12], Phillips-Perron Test [13], Ljung-Box Test [14], and Lagrange-Multiplier Test [15]. Secondly, we applied the Threshold GARCH(1,1)-in-Mean model with exogenous dummy variables to capture returns, volatility, COVID-19 impact, leverage effect and risk premium. This model belongs to the large family of generalized autoregressive conditional heteroscedasticity (GARCH) models. The rest of the paper is organized as follows. Section 2 includes the literature review. Section 3 describes the methodology and the data used. Section 4 illustrates the results of the analysis. Finally, Section 5 provides the results discussion.

2. Literature Review

Historically, the Autoregressive Conditionally Heteroscedastic (ARCH) model was introduced by Engle in 1982 through a UK inflation study [15], who gave rise to a vast literature and variety of models [16]. Generalised Autoregressive conditionally heteroscedastic (GARCH) model is an extension of ARCH model, it was developed by Bollerslev in 1986 [17]. These models are useful to capture the volatility clustering phenomenon, according to Mandelbrot (1963): *“large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes”* [18]. In 1987 Engle et al. [19] introduced the ARCH-in-Mean (ARCH-M) model to capture the risk premium, where the conditional variance enters in the mean equation as determinant of current risk premium. Thus, the risk premium *“rewards”* investors who buy risky assets i.e. assets with greater volatility. Asymmetric GARCH models captures the leverage effect, another stylised fact of financial time series. There is a leverage effect when *“negative returns (corresponding to price decreases) tend to increase volatility by a larger amount than positive returns (price increases) of the same magnitude”* [20]. In 1995 Hentschel [21] developed the family GARCH model which *“nests the most popular symmetric and asymmetric GARCH models”* [21]. Notable among these is the Threshold GARCH (TGARCH) model proposed by Zakoian [22]. In this study we used the TGARCH model nested in family GARCH model to specify the Conditional Volatility Equation.

The impact of the novel coronavirus on the financial market and stock market has been the topic of several empirical studies. Part of existing literature analyses COVID-19 effects on the stock market volatility and returns through GARCH modeling approach. Yousef (2020) [23] is the first who analysed the impact of the COVID-19 pandemic on the stock market volatility in G7 countries using GARCH and GJR-GARCH² models with dummy and control variables (COVID-19, oil price, gold price, and the EPU index). The results revealed that the COVID-19 pandemic increased stock market volatility in G7 countries. Similarly, Osagie et al. (2020) [24] examined the COVID-19 effects on the Nigeria stock exchange (NSE) using quadratic GARCH and exponential GARCH models with dummy variables. Their results showed that the COVID-19 conditioned negatively Nigeria stock returns. Equally important is the contribute of Shehzad et al. (2020) [25], they applied the Asymmetric Power GARCH model with dummy variables to investigate the impact of the Global Financial Crisis and COVID-19 on the following stock markets indices: S&P 500 (US), Nasdaq Composite Index (US), DAX 30 (Germany), FTSE MIB (Italy), Nikkei 225 Index (Japan), and SSEC (China). Furthermore, the study analysed the impact of the trade war between China and the United States on their stock markets. Shehzad et al.(2020) concluded that *“the European and the US markets are more affected by COVID-19 as compared to Asian markets”* [25]. Chaudhary et al. (2020) [5] employed the standard GARCH model to analyze the impact of the COVID-19 on the stock market indices of the top 10 countries based on GDP, and their results showed that, the COVID-19 pandemic increased the volatility of these indices. Using the GJR GARCH model, Bora and Basistha (2021) [26] investigated the impact of COVID-19 on the volatility of the two important stock market of India: Bombay Stock Exchange (BSE Sensex) and

² Named for Glosten, Jagannathan and Runkle 1993, it is a variant of Threshold GARCH Model.

National Stock Exchange of India (NSE Nifty). Their findings revealed that the BSE Sensex became volatile during COVID-19 period. About international stock markets, Bhunia and Ganguly (2020) [27] focused on the volatility and leverage effect before and during the period of Covid-19. The GARCH modeling approach is also applied to exchange rate volatility [28], gold price volatility [29] as well as the volatility of other financial assets.

This study contributes to the existing literature about reactions of equity markets to COVID-19 pandemic. Moreover, it provides an overview of equity markets in the euro area during coronavirus pandemic. The study also identifies the differences and characteristics among euro area stock markets, such as: returns, volatility, COVID-19 impact, leverage effect and risk premium. Hence, these last aspects could help portfolio managers to optimize their decisions.

3. Data and Methodology

3.1. Data

In this study, data on daily closing prices of euro area indices (**Table1**) have been collected from the Investing.com website [30]. The time period taken for the study is divided into two parts: pre-COVID-19 period from 4th January 2016 to 31th December 2019 and COVID-19 period from 2nd January 2020 to 31th December 2020. To underline the COVID-19 pandemic scenario, the considered period does not include the periods of the global financial crisis (2007-2009) and the sovereign debt crisis (2010-2011) [31]. The daily returns of the ten euro area market indices (**Table1**) were calculated with “the natural log difference approach” [5]

$$r_{i,t} = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right), \quad (1)$$

where:

- $r_{i,t}$ is the daily return on index i at time t ;
- $P_{i,t}$ is the daily closing price of index i at time t ;
- $P_{i,t-1}$ is the daily closing price of index i at time $t - 1$.

3.2. Descriptive Statistics

Descriptive statistics calculated on the daily returns for all indices provide fundamental information on the characteristics of the data generating process. Thus, this study employed the following Descriptive Statistics: mean, median, standard deviation, skewness and kurtosis.

3.3. Jarque–Bera Test

The Jarque–Bera Test was applied to verify if daily returns were normally distributed. Let N be the number of observations, this statistical test compares the observed skewness (sk) and kurtosis (ku) to their values under normality, i.e. 0 and 3 [16]:

$$JB = \frac{N}{6} \left(sk^2 + \frac{1}{4}(ku - 3)^2 \right). \quad (2)$$

The null hypothesis (H_0) is that the daily returns are normally distributed and the alternative (H_a) is that the daily returns are not normally distributed; H_0 is rejected if P-value ≤ 0.05 .

3.4. Unit Root Tests

The unit root tests are used to check whether a time series is stationary or nonstationary. In this study, the Augmented Dickey–Fuller (ADF) Test and Phillips-Perron (PP) Test were applied. The ADF test is based on the estimate of the following regression:

$$\Delta r_t = \alpha_0 + \delta_1 t + \gamma_1 r_{t-1} + \sum_{i=1}^p \beta_i \Delta r_{t-i} + \epsilon_t. \quad (3)$$

Here, Δ symbolizes the first difference operator, r represents the time series, t is the time period, p is the optimum number of lags, α_0 is the constant value, δ_1 is the time trend coefficient, γ_1 and β_i are parameters, and ϵ is a stochastic error term. The null hypothesis is that there is a unit root ($H_0 : \gamma_1 = 0$), and the alternative (H_a) is stationarity. Whereas, Phillips-Perron test “use nonparametric serial correlation method to take care of serial correlation in the error term without adding lagged difference term” [26]. In both tests, H_0 is rejected if P-value ≤ 0.05 .

3.5. Ljung–Box Test

The Ljung-Box Test was employed to check for the absence of serial autocorrelation for a set of lags “ p ”. Specifically, the Weighted Ljung-Box Test of Fisher and Gallagher (2012) [32,33] was applied on the standardized residuals of the main model. The Ljung-Box statistical test is equal to:

$$Q_{LB}(p) = N(N+2) \sum_{i=1}^p \frac{\hat{\rho}_i^2}{N-i}. \quad (4)$$

Where, N is the number of observations, ρ represents the autocorrelation, and p is the number of lags. The null hypothesis is $H_0 : \rho_1 = \rho_2 = \dots = \rho_p = 0$. The alternative (H_a) is that at least one ρ_i is not equal to 0. H_0 is rejected if P-value ≤ 0.05 .

3.6. Lagrange-Multiplier (ARCH-LM) Test

The Lagrange-Multiplier (ARCH-LM) Test was used to check for “ARCH effect” [15] and heteroscedasticity. Besides, the Weighted ARCH-LM Test of Fisher and Gallagher (2012) [32,33] was applied on the standardized residuals of the main model. The ARCH-LM Test is based on the estimate of the following *auxiliary regression*:

$$\epsilon_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i \epsilon_{t-i}^2 + u_t. \quad (5)$$

Here, ϵ^2 is the square residual measured by the primary regression model, α_0 is the constant value, α_i is a parameter and u is a noise term. The null hypothesis is that there is no ARCH effect i.e. $H_0 : \alpha_1 = \alpha_2 = \dots = \alpha_p = 0$. The alternative (H_a) is that at least one α_i is not equal to 0. H_0 is rejected if P-value ≤ 0.05 .

3.7. Threshold GARCH(1,1)-in-Mean Model with Exogenous Dummy Variables

To capture the COVID-19 impact we introduced two exogenous dummy variables in the conditional mean (6) and conditional volatility (7) equations of the model. They correspond to the two waves of COVID-19 infections defined as follows:

- $WAVE1_{i,t}$ assumes the value of 1 during the 1st wave of COVID-19 infections, i.e. for the period from 1st January 2020 to 31th July 2020, otherwise it is equal to 0;
- $WAVE2_{i,t}$ assumes the value of 1 during the 2nd wave of COVID-19 infections, i.e. for the period from 1st August 2020 to 31th December 2020, otherwise it is equal to 0.

The Threshold GARCH(1,1)-in-Mean model with exogenous dummy variables is specified as follow:

Conditional Mean Equation

$$r_{i,t} = \mu + mx_1 WAVE1_{i,t} + mx_2 WAVE2_{i,t} + \lambda_1 \sigma_{i,t}^2 + \epsilon_{i,t} \quad (6)$$

Conditional Volatility Equation

$$\begin{aligned} \sigma_{i,t} &= \omega + vx_1 WAVE1_{i,t} + vx_2 WAVE2_{i,t} + \alpha_1 \sigma_{i,t-1} (|z_{i,t-1}| - \gamma_1 z_{i,t-1}) + \beta_1 \sigma_{i,t-1} \\ \text{with } \epsilon_{i,t} &= \sigma_{i,t} z_{i,t}, \quad z_{i,t} \sim sstd(0, 1, \nu). \end{aligned} \quad (7)$$

In the Conditional Mean Equation (6), $r_{i,t}$ and $\epsilon_{i,t}$ indicate the returns and error terms of stock index i at time t , respectively. Besides, μ is the constant term. The coefficients mx_1 and mx_2 determine the impact of $WAVE1_{i,t}$ and $WAVE2_{i,t}$ on the conditional mean, respectively. If mx_1 and mx_2 are negative and statistically significant, the waves of Covid-19 infections caused a reduction in the mean returns of the euro area stock markets. Considering Engle et al. (1987) [19], the Conditional Mean Equation (6) also includes the risk premium term λ_1 . If λ_1 is different from 0 and statistically significant there is a “correlation between risk and expected return” [34]. Specifically, with $\lambda_1 > 0$, returns are positively related to their conditional variance ($\sigma_{i,t}^2$).

As already mentioned above in Section 2, the Conditional Volatility Equation (7) is based on the TGARCH model [22] nested in family GARCH model of Hentschel (1995) [21]. TGARCH model returns “the conditional standard deviation as a linear function of shocks and lagged standard deviations” [21]. Thus, $\sigma_{i,t}$ denotes the conditional standard deviation of stock index i at time t . ω is the constant term, while α_1 and β_1 are non-negative parameters that capture the ARCH effect and the GARCH effect, respectively. Besides, γ_1 captures the leverage effect. A positive γ_1 indicates that negative shocks increase volatility by a larger amount than positive shocks, while if $\gamma_1 = 0$ the shocks effect is symmetric. The terms vx_1 and vx_2 embody the impact of $WAVE1_{i,t}$ and $WAVE2_{i,t}$ on the conditional standard deviation, respectively. If vx_1 and vx_2 are positive and statistically significant, the waves of Covid-19 infections increase the volatility of the euro area equity markets. $z_{i,t}$ is the standardized residual of market i at time t . To capture the leptokurtosis and asymmetry of daily returns, $z_{i,t}$ were modeled using the skewed student-t distribution (*sstd*) with mean 0, variance 1 and ν degrees of freedom.

To optimize the serial correlation, the Conditional Mean Equation of BEL 20 (Belgium) and FTSE/ATHEX 20 (Greece) includes an autoregressive process of order one. Thus, Equation (6) gets transformed into Equation:

$$r_{i,t} = \mu + mx_1 WAVE1_{i,t} + mx_2 WAVE2_{i,t} + \phi_1 r_{i,t-1} + \lambda_1 \sigma_{i,t}^2 + \epsilon_{i,t} \quad (8)$$

here, ϕ_1 measures the time link between time t and time $t - 1$ (the amount of feedback or memory). The Conditional Volatility Equation (7) is unchanged.

Threshold GARCH(1,1)-in-Mean with exogenous dummy variables was estimated for all indices under study (Table 1) through the R’s **rugarch** package [35].

4. Results

4.1. Descriptive Statistics Summary and Statistical Tests Results

Table 2 shows the summary of the Descriptive Statistics of daily returns for all indices under two different periods:

- (A) pre-COVID-19 period from 4th January 2016 to 31th December 2019;
- (B) COVID-19 period from 2nd January 2020 to 31th December 2020.

Looking at the Descriptive Statistics of the Table 2, a few interesting considerations arise. Whereas the daily returns of all indices during the pre-COVID-19 period show positive mean returns, the COVID-19 period has negative daily mean returns. The minimum daily return of all indices occurred during the COVID-19 period (first quarter of 2020) with the exception of the ATF index (Greece), its minimum daily return occurred during the pre-COVID-19 period. In the same way the maximum daily return of all indices occurred during the COVID-19 period, this is probably due to the “market

rebound". Additionally, the standard deviation is large for all indices during the COVID-19 period. The standard deviation of SBITOP (Slovenia) and OMXBBPI (the Baltic States) is quite low compared to other indices during the COVID-19 period. Besides, all indices present negative skewness and high kurtosis during COVID-19 period, this aspect "indicates chances of high losses" [26]. On the other hand, the pre-COVID-19 period is characterized by moderate asymmetry and kurtosis values. Consequently, daily returns are not normally distributed as demonstrated by the Jarque-Bera Test (JB) in Table 3. Furthermore, Table 3 shows the results of other statistical hypothesis tests employed in this study. Considering ADF Test and PP Test, all the indices have a higher test statistic than critical value and a low P-value, hence the null hypothesis of unit root can be rejected, and all indices are stationary in their first difference. The small P-values of the ARCH-LM Test confirm the presence of ARCH effect and heteroscedasticity, thus the null hypothesis of no ARCH effect is rejected.

Table 2. Descriptive Statistics Summary.

Indices	Descriptive Statistics						
	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis
(A) pre-COVID-19 period							
AEX	0.000330	0.000801	0.034328	-0.058731	0.008640	-0.510444	3.789948
OMXH 25	0.000257	0.000500	0.032634	-0.087509	0.009758	-0.748260	6.923950
CAC 40	0.000273	0.000420	0.040604	-0.083844	0.009521	-0.779379	7.398629
BEL 20	0.000081	0.000397	0.030667	-0.066132	0.008819	-0.679289	4.437738
DAX	0.000251	0.000773	0.034457	-0.070673	0.009823	-0.525730	3.499413
IBEX 35	0.000024	0.000283	0.036876	-0.131853	0.010767	-1.890304	22.596608
FTSE MIB	0.000124	0.000716	0.049111	-0.133314	0.012957	-1.086358	12.191209
ATF	0.000251	0.001253	0.096824	-0.172561	0.016334	-1.383679	16.606192
SBITOP	0.000276	0.000202	0.034698	-0.027937	0.005904	0.003892	2.167711
OMXBBPI	0.000227	0.000317	0.019922	-0.029506	0.004266	-0.751921	5.360405
(B) COVID-19 period							
AEX	0.000127	0.000636	0.085907	-0.113758	0.017946	-1.093380	8.663198
OMXH 25	0.000330	0.000599	0.066647	-0.106786	0.017979	-1.089623	6.534863
CAC 40	-0.000288	0.000321	0.080561	-0.130983	0.020580	-1.112610	8.224969
BEL 20	-0.000344	0.000507	0.073606	-0.153275	0.021392	-1.649272	10.860476
DAX	0.000137	-0.000009	0.104143	-0.130549	0.020921	-0.869483	8.480384
IBEX 35	-0.000653	0.000374	0.082253	-0.151512	0.021521	-1.325902	10.829625
FTSE MIB	-0.000218	0.001241	0.085495	-0.185411	0.022613	-2.589287	19.980233
ATF	-0.000691	0.000286	0.116725	-0.149518	0.027465	-1.030415	7.873790
SBITOP	-0.000109	0.000853	0.059589	-0.093825	0.013507	-1.966691	13.901583
OMXBBPI	0.000300	0.000795	0.046703	-0.107607	0.013174	-3.799412	30.244574

Table 3. Statistical Tests Results.

Indices	JB Test		ADF Test		PP Test		ARCH-LM Test	
	Statistic	P-value	Statistic	P-value	Statistic	P-value	Statistic	P-value
AEX	11959	0	-10.695	0.01	-1312.0	0.01	81.2	1.99e-08
OMXH 25	6268.2	0	-10.807	0.01	-1232.8	0.01	149	0
CAC 40	14480.48	0	-11.004	0.01	-1312.7	0.01	127	2.22e-16
BEL 20	29957.67	0	-10.339	0.01	-1246.7	0.01	239	0
DAX	12209.41	0	-10.608	0.01	-1338.5	0.01	108	5.98e-13
IBEX 35	27741.8	0	-10.545	0.01	-1382.8	0.01	283	0
FTSE MIB	33638.95	0	-10.091	0.01	-1462.8	0.01	275	0
ATF	11855.24	0	-9.8266	0.01	-1177.6	0.01	155	0
SBITOP	35174.43	0	-8.6752	0.01	-1481.4	0.01	94.4	1.30e-10
OMXBBPI	349400.4	0	-9.0819	0.01	-1426.8	0.01	90.8	5.22e-10

4.2. Results of Conditional Mean Equation

Table 4 shows the results of the Conditional Mean Equation (6). The coefficient μ is not statistically significant for all indices. The impact of the two waves of COVID-19 infections on the conditional mean is not statistically significant for all indices. However, there are two notable exceptions on the mx_1 term: it is negative and statistically significant for ATF index (Greece) whereas it is positive and statistically significant for OMXBBPI index (the Baltic States). Moreover the results reveal a statistically significant coefficient of the risk premium for AEX (the Netherlands), OMXH 25 (Finland), BEL 20 (Belgium) and DAX (Germany) indices. BEL 20 index has the highest λ_1 coefficient. **Figure 4** highlights the estimated “time-varying risk premium” [21] as the product $\lambda_1 \cdot \sigma_{i,t}^2$, the maximum peaks occurred during the late 1st quarter of 2020. To conclude, ϕ_1 coefficient is statistically significant for ATF (Greece) and BEL 20 (Belgium) indices, implying a time link between $r_{i,t}$ and $r_{i,t-1}$.

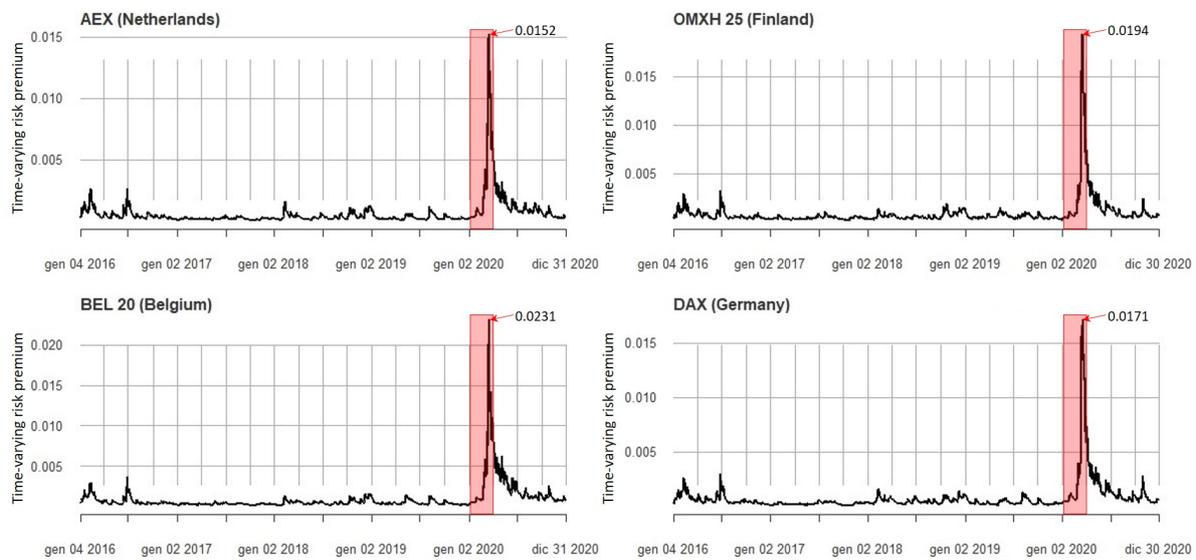


Figure 4. Estimated time-varying risk premium for the entire study period (4th January 2016 - 31th December 2020).

Table 4. Results of Conditional Mean Equation.

Variable	Estimate	Std. Error	t-value	P-value
AEX (Netherlands)				
μ	-0.000108	0.000216	-0.49946	0.617454
mx_1	-0.001357	0.001159	-1.17124	0.241503
mx_2	0.000584	0.000923	0.63290	0.526799
λ_1	4.988566	2.079694	2.39870	0.016453
OMXH 25 (Finland)				
μ	-0.000457	0.000309	-1.480611	0.138710
mx_1	0.000066	0.001036	0.063984	0.948983
mx_2	0.000404	0.000958	0.421910	0.673091
λ_1	5.932241	2.237929	2.650772	0.008031
CAC 40 (France)				
μ	0.000012	0.000234	0.051865	0.958637
mx_1	-0.001349	0.001129	-1.194617	0.232237
mx_2	-0.000362	0.000986	-0.367391	0.713328
λ_1	3.010943	1.726031	1.744432	0.081084
BEL 20 (Belgium)				
μ	-0.000431	0.000255	-1.6934	0.090381
mx_1	-0.001990	0.001395	-1.4263	0.153780
mx_2	-0.001310	0.001034	-1.2660	0.205516
ϕ_1	0.070139	0.028625	2.4502	0.014277
λ_1	6.286127	1.732860	3.6276	0.000286
DAX (Germany)				
μ	-0.000262	0.000246	-1.06567	0.28658
mx_1	-0.001217	0.001318	-0.92314	0.35594
mx_2	-0.000263	0.001077	-0.24369	0.80747
λ_1	4.367039	0.658676	6.63003	0.00000
IBEX 35 (Spain)				
μ	-0.000557	0.000355	-1.56960	0.116509
mx_1	-0.002069	0.001456	-1.42113	0.155279
mx_2	-0.000439	0.001268	-0.34634	0.729084
λ_1	4.664612	3.171524	1.47078	0.141351
FTSE MIB (Italy)				
μ	-0.000224	0.000567	-0.39567	0.692345
mx_1	-0.000414	0.001449	-0.28573	0.775083
mx_2	0.000257	0.001373	0.18737	0.851371
λ_1	2.140860	2.707931	0.79059	0.429184
ATF (Greece)				
μ	-0.000048	0.000543	-0.089063	0.929032
mx_1	-0.001972	0.000947	-2.082157	0.037328
mx_2	0.000252	0.001503	0.167480	0.866993
ϕ_1	0.078743	0.026921	2.924908	0.003446
λ_1	1.574275	2.188090	0.719474	0.471849
SBITOP (Slovenia)				
μ	0.000077	0.000290	0.26660	0.789775
mx_1	0.000102	0.000512	0.19923	0.842079
mx_2	0.000153	0.000738	0.20799	0.835238
λ_1	3.681973	7.172696	0.51333	0.607719
OMXBBPI (Baltic States)				
μ	0.000125	0.000141	0.880294	0.378700
mx_1	0.001084	0.000498	2.177435	0.029448
mx_2	0.000021	0.000471	0.044939	0.964156
λ_1	5.170328	5.238970	0.986898	0.323693

4.3. Results of Conditional Volatility Equation

Table 6 shows the results of the Conditional Volatility Equation (7). The coefficient ω is positive and statistically significant for all indices. The results suggest a significant positive impact of the first wave of COVID-19 infections on the conditional standard deviation, the term vx_1 is statistically significant for AEX, OMXH 25, CAC 40, BEL 20, DAX, IBEX 35 and FTSE MIB indices. Thus, the first wave of COVID-19 infections increased the stock market volatility of euro area countries with large and middle financial centres: the Netherlands, Finland, France, Belgium, Germany, Spain and Italy. In contrast, vx_1 is not statistically significant for ATF, SBITOP and OMXBBPI indices, i.e. Greece, Slovenia and the Baltic States (with small financial centres). Considering the second wave, the term vx_2 is not statistically significant for all indices except for BEL 20 (Belgium). Specifically, $vx_1 > vx_2$ for BEL 20, this means that first wave had a greater impact than second wave on stock market volatility. The results highlight that the coefficients α_1 and β_1 are positive and statistically significant for all indices, underlining the presence of ARCH and GARCH effects. Additionally, the leverage effect term γ_1 is positive and statistically significant for all indices (except the OMXBBPI index of the Baltic States where P-value > 0.05), i.e. negative shocks have a greater impact than positive shocks on stock market volatility. The *Skew* and *Shape* parameters are statistically significant for all indices, showing that the skewed student-t distribution suitably captured the leptokurtosis and the asymmetry of daily returns. **Table 5** shows the estimated volatility peaks and the volatility persistence. During COVID-19 period the ATF index (Greece) exhibited the maximum peak of estimated volatility while the SBITOP index (the Baltic States) exhibited the lowest volatility peak compared to other indices. Besides, the volatility persistence [33] is about close to 1 for all euro area stock markets indices, this means that shocks persist for a long time period before to fade away i.e. they have “long memory” [23].

Table 5. Estimated volatility peaks and volatility persistence.

Indices	Volatility Peaks	Volatility Persistence
AEX	0.0551	0.9540
OMXH 25	0.0571	0.9415
CAC 40	0.0687	0.9562
BEL 20	0.0606	0.9507
DAX	0.0626	0.9543
IBEX 35	0.0621	0.9383
FTSE MIB	0.0757	0.9447
ATF	0.0840	0.9573
SBITOP	0.0313	0.9449
OMXBBPI	0.0415	0.9406

In conclusion, **Figures A1** and **A2** in the **Appendix A** show the estimated conditional standard deviation (volatility) versus the absolute daily returns of euro area stock indices.

Table 6. Results of Conditional Volatility Equation.

Variable	Estimate	Std. Error	t-value	P-value
AEX (Netherlands)				
ω	0.000374	0.000042	8.97047	0.000000
vx_1	0.000421	0.000155	2.72372	0.006455
vx_2	0.000097	0.000140	0.69280	0.488436
α_1	0.087282	0.007236	12.06273	0.000000
γ_1	1.000000	0.151637	6.59471	0.000000
β_1	0.887497	0.000413	2146.93433	0.000000
Skew	0.819234	0.033350	24.56479	0.000000
Shape	7.296351	1.356883	5.37729	0.000000
OMXH 25 (Finland)				
ω	0.000549	0.000059	9.237703	0.000000
vx_1	0.000350	0.000117	2.981309	0.002870
vx_2	0.000086	0.000133	0.645025	0.518911
α_1	0.081623	0.007851	10.397171	0.000000
γ_1	1.000000	0.172359	5.801828	0.000000
β_1	0.878117	0.000458	1916.490492	0.000000
Skew	0.858881	0.037724	22.767708	0.000000
Shape	10.676601	2.516136	4.243252	0.000022
CAC 40 (France)				
ω	0.000386	0.000096	4.023639	0.000057
vx_1	0.000452	0.000218	2.071096	0.038350
vx_2	0.000198	0.000172	1.154827	0.248161
α_1	0.103238	0.018929	5.454051	0.000000
γ_1	0.999999	0.172066	5.811726	0.000000
β_1	0.879291	0.020522	42.845497	0.000000
Skew	0.870862	0.035127	24.791904	0.000000
Shape	5.602945	0.814265	6.880987	0.000000
BEL 20 (Belgium)				
ω	0.000416	0.000055	7.5188	0.000000
vx_1	0.000495	0.000165	3.0012	0.002689
vx_2	0.000335	0.000148	2.2689	0.023275
α_1	0.079069	0.009437	8.3788	0.000000
γ_1	1.000000	0.157939	6.3316	0.000000
β_1	0.891050	0.013243	67.2850	0.000000
Skew	0.861401	0.033965	25.3618	0.000000
Shape	6.398950	1.041216	6.1457	0.000000
DAX (Germany)				
ω	0.000448	0.000029	15.20404	0.000000
vx_1	0.000502	0.000171	2.94691	0.00321
vx_2	0.000149	0.000158	0.94026	0.34708
α_1	0.085015	0.013466	6.31354	0.000000
γ_1	1.000000	0.162817	6.14187	0.000000
β_1	0.891599	0.016260	54.83440	0.000000
Skew	0.875440	0.033202	26.36728	0.000000
Shape	5.122348	0.745970	6.86670	0.000000

Table 6. Results of Conditional Volatility Equation (*continued*).

Variable	Estimate	Std. Error	t-value	P-value
IBEX 35 (Spain)				
ω	0.000609	0.000206	2.95323	0.003145
vx_1	0.000580	0.000248	2.34141	0.019211
vx_2	0.000324	0.000205	1.57940	0.114245
α_1	0.094024	0.024977	3.76437	0.000167
γ_1	0.709623	0.186698	3.80092	0.000144
β_1	0.867895	0.035438	24.49044	0.000000
<i>Skew</i>	0.899439	0.035301	25.47919	0.000000
<i>Shape</i>	5.875950	0.913772	6.43043	0.000000
FTSE MIB (Italy)				
ω	0.000649	0.000215	3.01588	0.002562
vx_1	0.000483	0.000254	1.89779	0.057724
vx_2	0.000103	0.000198	0.51911	0.603686
α_1	0.087781	0.020942	4.19160	0.000028
γ_1	1.000000	0.208372	4.79910	0.000002
β_1	0.878881	0.029566	29.72578	0.000000
<i>Skew</i>	0.866659	0.036536	23.72050	0.000000
<i>Shape</i>	5.969577	0.934168	6.39026	0.000000
ATF (Greece)				
ω	0.000633	0.000180	3.520491	0.000431
vx_1	0.000196	0.000190	1.032927	0.301638
vx_2	0.000167	0.000212	0.783654	0.433243
α_1	0.103826	0.019568	5.305980	0.000000
γ_1	0.506735	0.142909	3.545854	0.000391
β_1	0.880972	0.020342	43.307602	0.000000
<i>Skew</i>	0.904579	0.034051	26.565801	0.000000
<i>Shape</i>	4.975379	0.716048	6.948385	0.000000
SBITOP (Slovenia)				
ω	0.000343	0.000124	2.75778	0.005820
vx_1	0.000163	0.000120	1.35870	0.174243
vx_2	0.000111	0.000100	1.10451	0.269372
α_1	0.087790	0.021489	4.08533	0.000044
γ_1	0.304170	0.152841	1.99011	0.046579
β_1	0.879952	0.030573	28.78228	0.000000
<i>Skew</i>	0.975162	0.037394	26.07834	0.000000
<i>Shape</i>	5.281005	0.776383	6.80206	0.000000
OMXBBPI (Baltic States)				
ω	0.000260	0.000094	2.751597	0.005931
vx_1	0.000172	0.000107	1.608322	0.107765
vx_2	0.000121	0.000093	1.293827	0.195725
α_1	0.137346	0.028555	4.809877	0.000002
γ_1	0.119587	0.096397	1.240569	0.214765
β_1	0.841993	0.036783	22.890757	0.000000
<i>Skew</i>	0.925409	0.035909	25.771170	0.000000
<i>Shape</i>	4.295693	0.525850	8.169049	0.000000

4.4. Diagnostic Tests Results

Table 7 shows the results of the Weighted Ljung-Box Test and the Weighted ARCH-LM Test. These diagnostic tests were carried out on standardized residuals to check for the absence of serial autocorrelation and heteroscedasticity. Regarding the Weighted Ljung-Box Test, the *null hypothesis of no serial correlation* cannot be rejected for all indices except for the OMXBBPI index of the Baltic States where its P-value < 0.05. The *null hypothesis of no heteroscedasticity* cannot be rejected for all indices. All the estimated models are correct but the OMXBBPI model could be improved to optimize serial autocorrelation.

Table 7. Diagnostic Tests Results.

Indices	Weighted Ljung-Box Test		Weighted ARCH-LM Tests	
	Statistic	P-value	Statistic	P-value
AEX	5.307	0.1303	4.7396	0.2522
OMXH 25	4.648	0.1836	2.702	0.5715
CAC 40	6.085	0.0859	2.8693	0.5386
BEL 20	4.052	0.2173	1.557	0.8099
DAX	4.206	0.2295	2.2706	0.6601
IBEX 35	4.375	0.2108	0.3051	0.9925
FTSE MIB	2.374	0.5330	0.4804	0.9801
ATF	2.603	0.5372	6.2319	0.1264
SBITOP	5.126	0.1434	1.8018	0.7592
OMXBBPI	21.169	9.235e-06	5.524	0.1767

5. Discussion

This empirical study applied the Threshold GARCH(1,1)-in-mean model with exogenous dummy variables to analyze the impact of the COVID-19 infections waves on daily returns of euro area stock indices. The euro area countries under study differ in the financial centre size, i.e. they have different levels of real GDP per capita and stock market capitalisation to GDP (%). The findings show that the first wave of COVID-19 infections had a notable impact on stock market volatility of euro area countries with large and middle financial centres. Furthermore the second wave of COVID-19 infections had a significant impact only on stock market volatility of Belgium (BEL 20). On the other hand the two waves did not have a significant impact on the conditional mean of daily returns, but there are two exceptions: the negative impact of the first wave of COVID-19 infections on the conditional mean of ATF index (Greece) and the positive impact of the first wave of COVID-19 infections on the conditional mean of OMXBBPI index (the Baltic States). The two waves of COVID-19 infections had a not meaningful impact on the stock market volatility of Greece, Slovenia and the Baltic States which present low levels of real GDP per capita and stock market capitalisation to GDP (see **Figure 3**). The results on volatility about Germany, France and Italy confirm those of Chaudhary et al. (2020) [5]. Moreover, findings support the leptokurtosis and asymmetry of stock daily returns. Another key aspect is the time-varying risk premium for Netherlands, Finland, Belgium and Germany stock markets. Belgium stock market has the highest risk premium compared to the other three countries. These risk premia compensate for risk changes (volatility changes) due to COVID-19 waves. Additionally, the results confirm the presence of the leverage effect for all stock markets except for the Baltic States stock market where the leverage effect coefficient (γ_1) is small and not significant. To conclude, these findings are useful to portfolio managers and investors to optimize their portfolio choices as well as risk managers, policymakers, academicians, and researchers.

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Appendix A. Estimated Conditional Volatility Plots

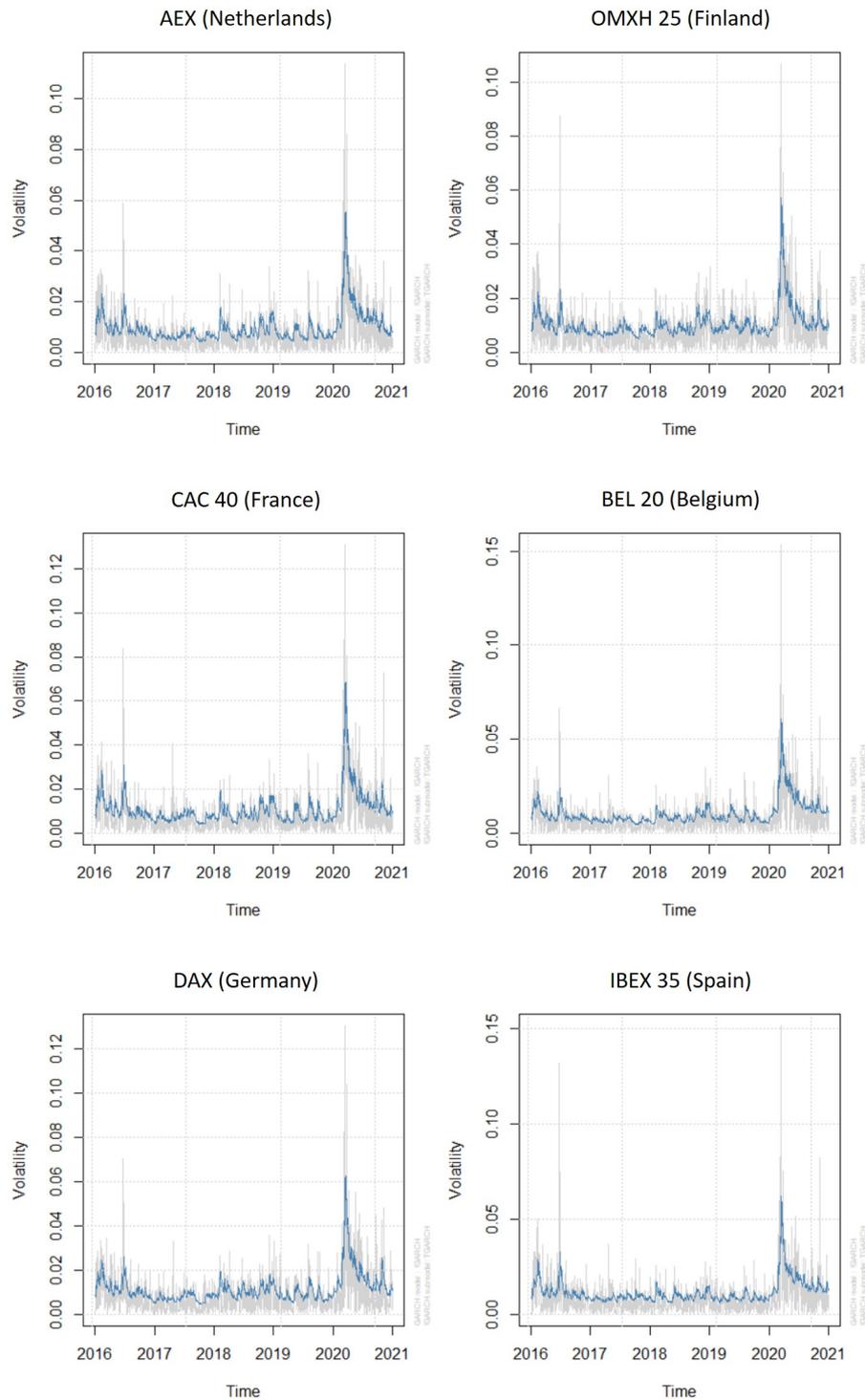


Figure A1. Estimated conditional standard deviation (volatility) versus absolute daily returns of euro area stock indices. Period from 4th January 2016 to 31th December 2020.

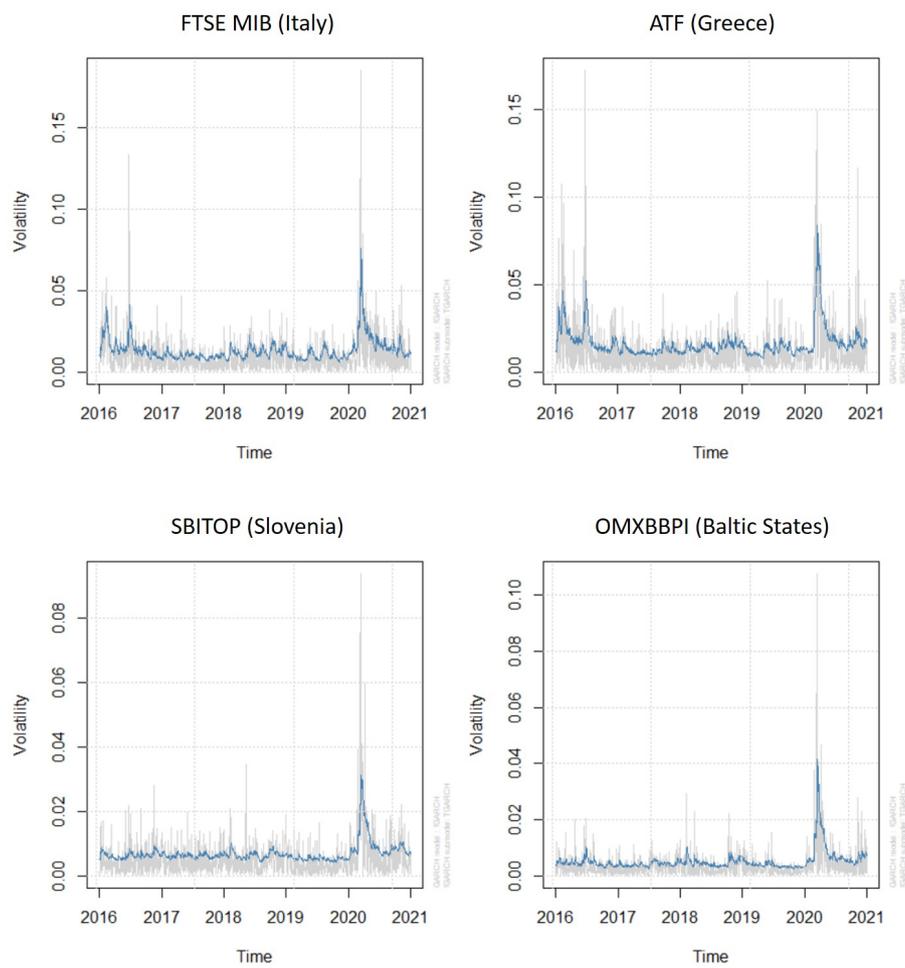


Figure A2. Estimated conditional standard deviation (volatility) versus absolute daily returns of euro area stock indices. Period from 4th January 2016 to 31th December 2020.

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