

Black Holes or an objects without event horizon?

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Abstract

The paper substantiates the possibility that objects that we usually identify with black holes are self-gravitating, fully or partially degenerate Fermi gas. This follows from the modification of Einstein's equations, which is based on a mathematical fact that the author of the GR could not have known in his time.

Keywords: Galactic center; Black Holes; Theory of gravitation.

1 Introduction

Establishing the existence of black holes and singularities is such an important task that it would be unforgivable to explore all possible interpretations of the results that the EHT project opens. Two such alternative possibilities [1, 2] are well known. In the present paper, one more possibility is considered.

It is caused by the mathematically justified need to modify the classical Einstein equations. Briefly, the reason for the modification of the classical Einstein equations is as follows. In classical electrodynamics, the equations of motion of a charge contain a 4-potential, which is determined up to arbitrary gradient. However, Maxwell's equations contain gradient-invariant quantities that have a direct physical meaning. In Einstein's theory of gravitation, the equations contain Christoffel symbols, which are also defined, which the creator of general relativity could not know, up to an arbitrary geodesic (projective) transformation. However, the classical Einstein equations are not invariant under these transformations. This necessitates the formulation of the theory of gravitation in terms of gauge-invariant objects.

The possibility of the existence of supermassive objects arising from the equations of gravitation was considered even before such an object was discovered in the center of our galaxy [14]. In paper [3] it was proposed as an alternative to black holes. Later, this possibility was studied in more detail in [12, 11a, 5, 14, 13]. Here new result is presented in viewed on EHT collaboration [5] results.

2 Projectively invariance

Let's start with Einstein's excellent hypothesis about the geodesic motion of a free particle under the influence of gravity.

The geodesic equations are invariant not only under an arbitrary transformation of coordinates, which is obvious, but also under geodesic (projective) mappings $\Gamma_{\beta\gamma}^{\alpha}(x) \rightarrow \bar{\Gamma}_{\beta\gamma}^{\alpha}(x)$ of the Christoffel symbols in any given coordinate system .

For example, if we use coordinate time $t = x^0/c$ (c is speed of light) as a parameter along geodesic lines, then the differential equations of a geodesic line are of the form

$$\ddot{x}^{\alpha} + (\Gamma_{\beta\gamma}^{\alpha} - c^{-1}\Gamma_{\beta\gamma}^0 \dot{x}^{\alpha}) \dot{x}^{\beta} \dot{x}^{\gamma} = 0, \quad (1)$$

where $\dot{x}^{\alpha} = dx/dt$, $\ddot{x}^{\alpha} = d\dot{x}^{\alpha}/dt$.

It easy to verify that these equations are invariant under the mapping

$$\bar{\Gamma}_{\beta\gamma}^{\alpha}(x) = \Gamma_{\beta\gamma}^{\alpha}(x) + \delta_{\beta}^{\alpha} \phi_{\gamma}(x) + \delta_{\gamma}^{\alpha} \phi_{\beta}(x), \quad (2)$$

where $\phi_{\alpha}(x)$ is any continuously differentiable vector field.

Such mappings of the Christoffel symbols induce some transformations $g_{\alpha\beta} \rightarrow \bar{g}_{\alpha\beta}$ of the metric tensor which are obtained by solving of the partial differential equation

$$\bar{g}_{\beta\gamma;\alpha}(x) = 2\phi_{\alpha}(x)\bar{g}_{\beta\gamma}(x) + \phi_{\beta}(x)\bar{g}_{\gamma\alpha}(x) + \phi_{\gamma}(x)\bar{g}_{\alpha\beta}(x), \quad (3)$$

where the semicolon denotes a covariant derivative.

It follows from (2) that the components $\bar{\Gamma}_{00}^i = \Gamma_{00}^i$. Therefore, in Newtonian limit, geodesic-invariance is not an essential fact. However this cannot be said about the relativistic case.

The property of invariance of geodesic equations with respect to geodesic (projective) mappings, leads to new physical consequences.

The properties of the classical field are determined in the features of the motion of test particles. These features are unique, inherent only in a single field. Identical and only identical fields have the same features of the motion of test particles.

However, the Ricci tensor does not remain invariant in the geodesic transformation of the Christoffel symbols [17] :

$$\bar{R}_{\alpha\beta} = R_{\alpha\beta} - \phi_{\alpha\beta}, \quad (4)$$

where $\phi_{\alpha\beta} = \phi_{\alpha;\beta} - \phi_{\alpha}\phi_{\beta}$, and $\phi_{\alpha;\beta}$ is a covariant derivative with respect to x^{α} .

This fact means that the connection between the behavior of test particles and their description by means of the characteristics of Riemannian space-time is ambiguous. Equation $R_{\alpha\beta} = 0$, generally speaking, does not give all physically possible properties of space-time which are defined by properties of test points.

A similar situation exists in electrodynamics, where the equations of motion of the charge are invariant with respect to gradient transformations of 4-potential. However, in this case, the field equations contain quantities that

are invariant under the gradient transformations of the potential, and only this quantities have a direct physical meaning.

Therefore, it should be assumed that in the case of gravitation, the equations for finding physically admissible characteristics of gravitation must contain only projectively invariant quantities. Geodesic (projective) mappings are natural gauge transformations in any theory of gravitation, based on the hypothesis that test particles move along the geodesic.

3 Projectively invariant equation of gravitation

The only projectively invariant object that can be created by the Christoffel symbols is the Thomas symbols [17]

$$\Pi_{\alpha\beta}^{\gamma} = \Gamma_{\alpha\beta}^{\gamma} - (n+1)^{-1} \left[\delta_{\alpha}^{\gamma} \Gamma_{\beta} + \delta_{\beta}^{\gamma} \Gamma_{\alpha} \right] \quad (5)$$

where $\Gamma_{\alpha} = \Gamma_{\alpha\beta}^{\beta}$. However they are not a tensor. Tensor object can be created only if Minkowski space has also some physical sense, in other words, only in a bimetric theory of gravitation. The simplest geodesic-invariant tensor $B_{\beta\gamma}^{\alpha}$ can be formed as follows [5]:

$$B_{\beta\gamma}^{\alpha} = \Pi_{\beta\gamma}^{\alpha} - \overset{\circ}{\Pi}_{\beta\gamma}^{\alpha} \quad (6)$$

where

$$\overset{\circ}{\Pi}_{\alpha\beta}^{\gamma} = \overset{\circ}{\Gamma}_{\alpha\beta}^{\gamma} - (n+1)^{-1} \left[\delta_{\alpha}^{\gamma} \overset{\circ}{\Gamma}_{\beta} + \delta_{\beta}^{\gamma} \overset{\circ}{\Gamma}_{\alpha} \right] \quad (7)$$

and $\overset{\circ}{\Gamma}_{\alpha} = \overset{\circ}{\Gamma}_{\alpha\beta}^{\beta}$ are the Thomas symbols for the Minkowski space-time in a used coordinate system.

In paper [5] a theory in which geodesic mappings play a role of gauge transformations is considered. The investigation of the problem leads to the conclusion that the simplest bimetric geodesic-invariant generalization of Einstein's vacuum equations are:

$$\nabla_{\alpha} B_{\beta\gamma}^{\alpha} = B_{\beta\delta}^{\epsilon} B_{\epsilon\gamma}^{\delta}. \quad (8)$$

The symbol ∇_{α} denotes a covariant derivative in Minkowski space with respect to x^{α} .

These equations have similarities to vacuum equations of classical electrodynamics in Minkowski space:

$$\mathcal{F}_{\alpha;\gamma}^{\gamma} = 0, \quad \mathcal{F}_{\alpha\beta} = \partial_{\alpha} A_{\beta}(x) - \partial_{\beta} A_{\alpha}(x),$$

which are invariant relative to mappings of 4-potentials: $A_{\alpha} \rightarrow A_{\alpha} + \phi(x)_{,\alpha}$. However, eqs. (8) contains the right-hand side which expresses self-interaction of gravitation.

When we select the covariant gauge conditions in the form $Q_{\alpha} = \Gamma_{\alpha\beta}^{\beta} - \overset{\circ}{\Gamma}_{\alpha\beta}^{\beta} = 0$, these equations coincide with the classical vacuum Einstein equations.

In other words, the classic vacuum Einstein's equations $R_{\alpha\beta} = 0$ are equations (8) at the gauge conditions $Q_{\alpha} = 0$.

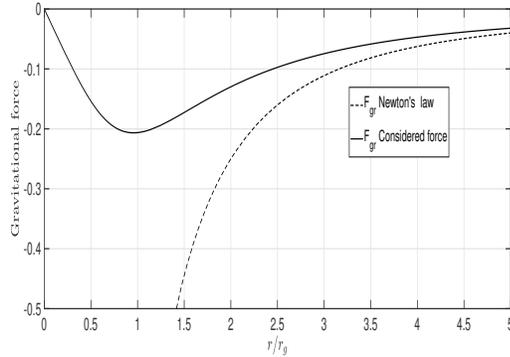


Figure 1: Gravitational force

4 Physical peculiarity of the equations

Using the fact that the theory is bimetric¹, we find the equations of motion of particles in a spherically symmetric field from the Lagrangian in Ninkowski space

$$L = mc[A(r)\dot{r}^2 + B(r)(\dot{\theta}^2 + \sin^2 \theta \dot{\phi}^2) - c^2 C(r)]^{1/2} \quad (9)$$

that gives from the eqs. (8) the following solution

$$A = \frac{r^4}{f^4(1 - r_g/f)}, \quad B = f^2, \quad C = 1 - \frac{r_g}{f}, \quad (10)$$

where $f = (r^3 + r_g^3)^{1/3}$ and $r_g = 2MG/r^2$.

These equations lead to the same results that Einstein's equations at the distances from the gravity center which are much larger of the Schwarzschild radius r_g of the central mass. However, gravity properties become quite different when the distance approaches r_g . The classical newtonian expression for the force $m\ddot{r}$ acting on a mass m as should be change to follows:

$$F = \frac{mMG}{r^2} \left(1 - \frac{r_g}{f}\right) \quad (11)$$

where $r_g = 2GM/c^2$ is the Schwarzschild radius of a mass M .

Singularity is missing in this model. The gravitational force decreases within the Schwarzschild radius, and eventually tends to zero along with the distance from the center.

Typically, the Schwarzschild radius is small compared with the dimensions of celestial objects. That is, almost always the condition $r \gg r_g$ is executed. Therefore, for comparison with observations in most cases only the expansion

¹The physical meaning of the coexistence in the used theory of gravitation of the Riemann and Minkowski spaces is considered in the Appendix.

of functions $A(x)$, $f(x)$, and $C(x)$ in a Taylor series in powers of $\phi = r_g/r$ are of interest.

Accurate to terms of second order with respect to ϕ these functions are of the form:

$$C = 1 - \phi, \quad B = r^2, \quad A = 1 + \phi + \phi^2 \quad (12)$$

Therefore, in a first approximation, the line element ds^2 of the field in the rest frame coincides with the commonly used expression in general relativity:

$$ds^2 = (1 - \phi) dx^{02} - (1 + \phi) dr^2 - r^2[d\theta^2 + \sin^2 \theta d\varphi].$$

And to this accuracy all observed effects, of course, will be the same as in the general theory of relativity.

This shows that the observed effects differ from the results that follow from the Einstein equations only when the Schwarzschild radius is approached.

5 Energy of gravitation

A correct expression for the density of the gravitational field in this model can be obtained directly from the expression for the force (11) that does not has singularity.

We define the potential of the force F as $U(r) = \int_0^r \frac{F(r')}{m} dr'$.

Double differentiation of function $U(r)$ gives

$$U'' = -\frac{2U'}{r} - \frac{2G^2 M^2}{c^2 f^4}.$$

Now denote

$$t_{00} = \frac{GM^2}{2\pi f^4}.$$

As a result, we obtain that function $U(r)$ satisfies the differential equation

$$\nabla^2 U = -4\pi\bar{\rho} \quad (13)$$

where the mass density is $\bar{\rho} = t_{00}/c^2$.

This equation has a clear physical meaning. This is the Poisson equation for a spherically symmetric field, in which $\bar{\rho}$ is the mass density of the gravitational field created by the mass M . In the absence of another matter, only the density of the gravitational field can lead to the fact that the potential will differ from zero.

This result shows clearly that t_{00} is the density of the gravitational field in the considered theory where the gravitational force is given by Eq. (11).

We can also write

$$t_{00} = \varkappa^{-1} \frac{r_g^2}{2f^4} \quad (14)$$

which gives, when integrating over entire space with $\varkappa = 4\pi G/c^4$

$$\int t_{00}dV = Mc^2$$

Thus the energy of a point mass is finite and equal to the expected value. In classical electrodynamics, a similar integral for the Coulomb field diverges.

Besides, it should be noted also that the integration of the density t_{00} over the volume of the visible Universe (radius $R \sim 10^{28}cm$, $\rho \sim 6 \cdot 10^{-30}g/cm^3$) gives the correct value of its mass $\sim 10^{56}g$.

For the above reasons, we can consider the expression (14) as the energy density of the gravitational field of a mass M .

6 Possible nature of the supermassive objects

The following simple considerations are based only on the assumption of the existence of objects with a Schwarzschild radius, and do not depend on the model of the theory used.

The radius of a spherically symmetric homogeneous mass equal to the Schwarzschild radius of the matter contained in it is

$$R_{cr} = c \left(\frac{3}{8\pi G\rho} \right)^{1/2} \quad (15)$$

When the average density ρ is equal to $10^4g/cm^3$, the radius R_{cr} is equal to $\sim 10^{11}cm$, which is an order of magnitude larger than the radius of the Sun. The mass of such object is of the five order the the Sun mass. If such an object is only partially degenerate, then they mass can be much more.

To take a deeper look at the situation, consider a graph of the total energy of self-gravitating spherically symmetric degenerate matter. Using the average values for the characteristics of the quantities used, it can be written in the following form In that equation

$$E = E_k - Vt_{00}. \quad (16)$$

where M in t_{00} , is mass of the configuration under consideration in volume $V = 4/3\pi R^3$.

These graph are shown in fig. 2 for the several value of a spherically configuration of mass

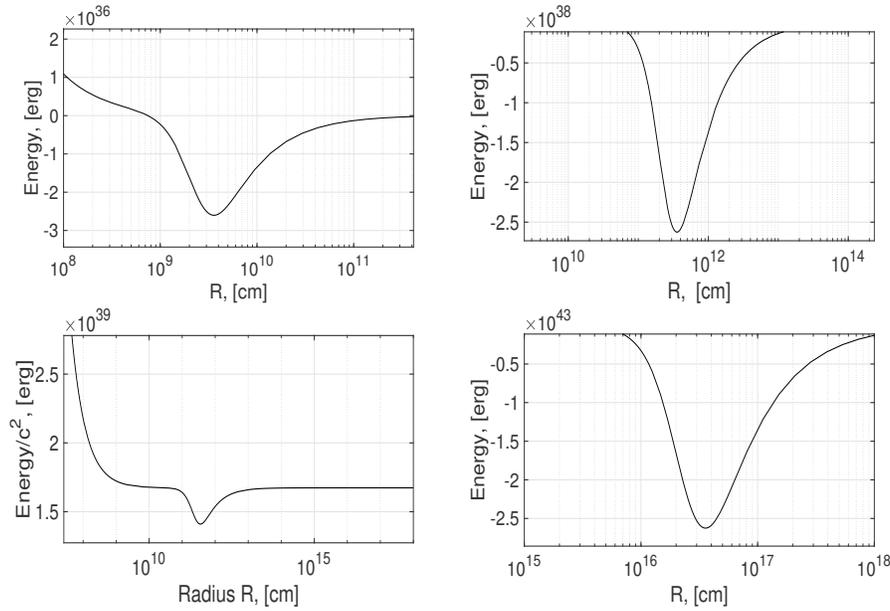


Figure 2: The mass $M = \mathcal{E}/c^2$ of the configuration as a function of total number of particles .

In equation 16, E_k is the internal energy of a relativistic gas of electrons in volume V [12]

$$E_k = \frac{cV(m_e c)^4}{8\pi^2 \hbar^3} (x_0(2x_0^2 + 1)(x_0^2 + 1)^{1/2} - \operatorname{asinh}(x_0));$$

$$p_0 = \hbar(3\pi^2 * n)^{1/3}$$

$$x_0 = \frac{p_0}{m_e c}, \quad n = N/V$$

N is the particles number in volume V .

$M_0 = Nm_n$, m_e and m_n are mass of electron and neutron, respectively.

Vt_{00} is the gravitational binding energy in volume V .

These graphs show that all configurations of self-gravitating objects are stable. The graphs also show that the mass of the completely degenerate Fermi gas not exceeding $10^{39}g$. The mass corresponding to the object in M78 does not have sufficient density. This suggests that objects with masses are more complex objects, the upper layers of which will not radiate for another reasons. It is also impossible to exclude gross errors in determining the mass of such objects[21]. However, in any case, one might think that objects with masses less than $10^{39}g$ are a viable alternative to black holes.

7 Can we observe features in the deflection of light rays?

To answer this question, let's find the radius of the ring of light from the object in M87. This can be done in two ways. First, the deflection angle, defined as the angle between the asymptotic incoming and outgoing trajectories, obtained in [6] for an arbitrary Lagrangian like () based on a Weinberg method.

$$\Theta = \arcsin \left(u \frac{C(r_m)}{B(r_m)} \right)^{1/2} \quad (17)$$

where where u is the impact parameter, defined as the distance between the black hole and each of the asymptotic photon trajectories, related to the closest approach distance r_m by

$$u = \left(\frac{B(r_m)}{C(r_m)} \right)^{1/2} .$$

Second, we can use Nemirov's simple and elegant method [7], suitable for flat space. This gives

$$\Theta = \arcsin \left(\frac{R}{D} \left(\frac{1 - r_g/D}{1 - r_g/R} \right)^{1/2} \right) \quad (18)$$

In this equation R is the radius of the object, r_g is its Schwarzschild radius, D is the distance from an observer.

The both method gives the same result - $0.22 \mu\text{uas}$. that is consistent with observation, and consequently, this result does not allows us test absence of singularity at the current level of measurement accuracy .

However, there is another method of testing the theory, based on the possible existence of the Schwarzschild gravitational radius of the Universe.

8 Schwarzschild radius of the Universe

8.1 Motion of a fluid

The expanding Universe can be viewed as an isentropic fluid. It is shown in [38] that the macoscopically small elements ("particles") of such a spherically symmetric fluid are described in exact accordance with the relativistic Euler equations by the following Lagrangian

$$L = -mc \left(G_{\alpha\beta} \frac{dx^\alpha}{dt} \frac{dx^\beta}{dt} \right)^{1/2} \quad (19)$$

where $G_{\alpha\beta} = \chi^2 g_{\alpha\beta}$, $\eta_{\alpha\beta}$ is a solution of our vacuum equations of gravitation in Minkowski space, $\chi = w/\rho c^2$ where w is the volume density of the enthalpy where

$$\chi = \frac{w}{\rho c^2} = 1 - \frac{\varepsilon}{\rho c^2} + \frac{p}{\rho c^2},$$

where w is the enthalpy per unit volume, ρ is the density, ε is the density of gravitational binding energy, p is the pressure.

The integral of motion is

$$\frac{mc^2\chi}{[1 - (\dot{r}/c)]^{1/2}} = Const.$$

This equation is the relativistic Bernoulli equation in Minkowski space.

The constant has a meaning of the energy of the particle and , therefore, this equation can be read as

$$\frac{\chi}{(1 - v^2/c^2)^{1/2}} = \bar{E} \quad (20)$$

where the constant $\bar{E} = E/mc^2$ is the dimensionless energy E of the “particles” of the fluid and $v = \dot{x}$.

At present, the pressure is negligible and therefore $\chi = 1 + \varepsilon/\rho c^2$, and for small distance from an observer and at $\bar{E} = 1$, the velocity $v(r)$ is proportional to the distance r from the observer:

$$v = \frac{4}{3}\sqrt{G\pi\rho}r,$$

We obtain the Hubble law holds and, at the density $\rho = 6 \cdot 10^{-30}g\,cm^{-3}$, the Hubble constant is

$$H_0 = \frac{4}{3}(G\pi\rho)^{1/2} = 1.5 \cdot 10^{-18}s^{-1}$$

If \bar{E} is not equal to 1, the equations of fluid motion contradict the Hubble law.

8.2 Acceleration of Universe as a test gravity properties in vicinity Schwarzschild radius

The Lagrange equation obtained from (19) is

$$\ddot{r} = -c^2 \frac{\chi'(r)}{\chi(r)} \left(1 - \frac{\dot{r}^2}{c^2}\right)$$

which with the equation $v^2/c^2 = 1 - \chi$ leads to the following equation for acceleration of a fluid in the expanding Universe.

$$\ddot{r} = -c^2 \frac{d\chi}{dr} \chi$$

A graph of $\ddot{r}(r)$ is shown on the fig.3

First, this graph shows that the acceleration of the expansion of the Universe is due to the properties of gravity and this is verifiable through observations[11a].

Secondly, this graph shows that the gravitational force of the substance of the Universe is limited by the radius This graph shows that the gravitational

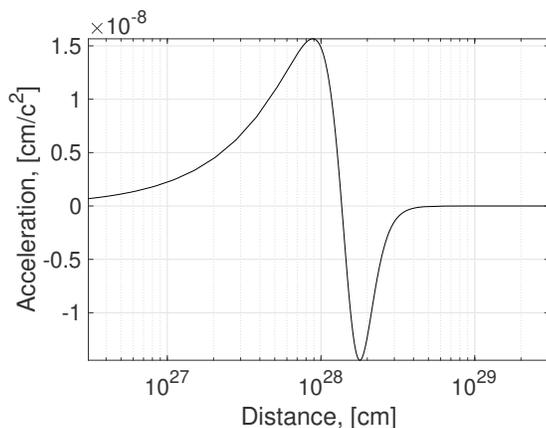


Figure 3: Distance from an observer vs. the Universe acceleration

force of the substance of the Universe is limited by the radius R_{cr} (15), and then rapidly decreases. Therefore R_{cr} can be called the gravitational radius of the Universe.

The calculation of the deceleration parameter :

$$q = -\frac{\ddot{r}r}{\dot{r}^2}.$$

gives some confidence in the above result.

The observation gives the present-day value of this parameter q_0 as approximately -0.55 . Graph 4 shows q as a function of distance from the observer in the theory used here.

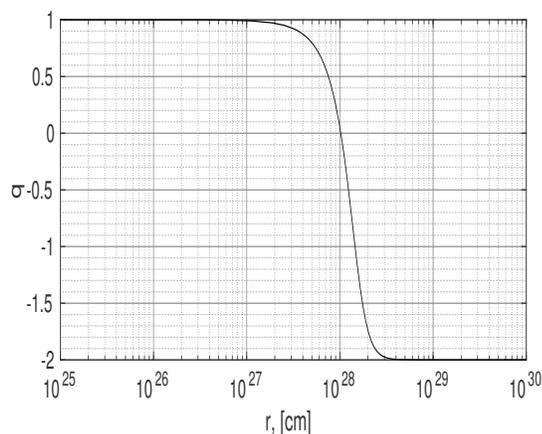


Figure 4: Parameter q vs. distance to the object.

Graph 4 shows that the observed value of pre sent-day $q_0 \approx -0.55$ corresponds to the geometrical distance of the remote objects of about 10^{28} cm . It is consistent with fig. 3 that shows the acceleration of distance object.

Appendix

Physical meaning of the bimetricity

In Einstein's classical theory, coordinates play a twofold and difficultly compatible role. On the one hand, coordinates are only a way to parameterize events, that is, points in space-time. From this point of view, they are completely arbitrary. On the other hand, they play the role of gauge transformations.

However, the equations of motion of test particles admit invariance with respect to projectively (geodesic) metrics (3), and due to this fact, the equations defining a space-time metric must also be invariant with respect to these transformations.

A projectively invariant generalization of Einstein's equations seems to be possible only within the framework of some bimetric theory of gravity.

Rosen [8] was the first to recognize the need for introducing Minkowski space into theory. The possibility of considering Einstein's equations in flat space was also considered by some authors after the paper of Tiring[9].

The physical meaning of bimetricism used in the present paper is based on ideas going back to Poincaré, who realized that there is a strange situation: In order to characterize the properties of the geometry of space, we must know the properties of measuring instruments, and in order to characterize the properties of instruments, we must know the geometric space properties. In the modern interpretation, this can be summarized as follows: The physical, operational sense has only the aggregate "space-time geometry + properties of measuring instruments" [6]. In this form, this fact has never been realized in physics.

However, a step in its implementation can be made if we notice that it is the reference frame used by the observer is the measuring tool that is necessary to establish the geometric properties of space-time. Therefore, it should be assumed that the following statement is true: Only the combination "space-time geometry + properties of the reference frame used" has physical meaning.

Of course, by reference frame we mean here not a coordinate system, but a physical device consisting of a reference body and a clock attached to it.

Remembering all the above, we now consider a classical field \mathcal{F} in an inertial reference system (*IRF*), where space-time according to experience is Minkowski space. The world lines of the particles of mass m moving under the action of the field \mathcal{F} form the reference body of a non-inertial reference which can be named the proper reference frame (*PRF*) of the field \mathcal{F} .

If an observer in a *PRF* of the field \mathcal{F} is at rest, his world line coincides with the world line of some point of the reference body. It is obvious for the observer that the accelerations of the point masses forming his reference body are equal to zero in non-relativistic and relativistic meaning. That is, if the line element

of space-time in an inertial reference frame is denoted by $d\sigma$ and $u^\alpha = dx^\alpha/d\sigma$ is the field of 4- velocities of the point masses forming the reference body, then the absolute derivative of u^α is equal to zero: ²

$$Du^\alpha/d\sigma = 0. \quad (21)$$

(We mean that an arbitrary coordinate system is used.)

The same should occur in the *PRF* used. That is, if the line element of space-time in the *PRF* is denoted by ds , the 4-velocity vector $\zeta^\alpha = dx^\alpha/ds$ of the point-masses forming the reference body of the *PRF* should satisfy the equation

$$D\zeta^\alpha/ds = 0 \quad (22)$$

The equation (22) uniquely determines the fundamental metric form in *PRFs*.

Indeed, the differential equations of these world lines are at the same time the Lagrange equations describing, in Minkowski space, the motion of the point masses forming the reference bodies of the *PRF*. The eq. (22) can be derived from a Lagrange action S by the principle of the least action. Therefore, the equations of the geodesic lines can be obtained from a line element $ds = k dS$, where k is a constant, $dS = \mathcal{L}(x, \dot{x})dt$, and $\mathcal{L}(x, \dot{x})$ is a Lagrange function describing, in Minkowski space, the motion of identical point masses m forming the body reference of the *PRF*. The constant k is equal to $-(mc)^{-1}$, as follows from the analysis of the case when the frame of reference is inertial, when $\mathcal{L}(x, \dot{x}) = -mc ds$ at the signature (+ - - -).

Thus, if we proceed from relativity of space and time in the Berkeley-Leibniz-Mach-Poincaré (BLMP) sense, then the line element of space-time in *PRFs* can be expected to have the following form [5]

$$ds = -(mc)^{-1} dS(x, dx). \quad (23)$$

For example, if the field \mathcal{F} is electromagnetic, then the space-time in such reference frames is Finslerian[5, 6]. And the space-time in the reference frames comoving to an isentropic ideal fluid is conformal to the Minkowski space.

In the case of gravity, we proceed from Thirring's assumption that gravity is described by a tensor field $\psi_{\alpha\beta}(x)$ and the Lagrangian describing the motion of test particles has the form

$$L = -mc[g_{\alpha\beta}(\psi) \dot{x}^\alpha \dot{x}^\beta]^{1/2}.$$

Then it is obvious from () that space-time in the *PRF* is Riemannian with a linear element of the form

$$ds = (g_{\alpha\beta}(x)dx^\alpha dx^\beta)^{1/2}$$

²WE USE NOTATIONS AND DEFINITIONS, FOLLOWING THE LANDAU AND LIFSHITZ BOOK [47].

This conclusion leads to the possibility of a double interpretation of gravity. Gravity can be considered as physical field ψ in inertial reference frames and manifests itself as curvature in its own reference frames (*PRFs*) of this field.³

In the present paper, the description of gravity in Minkowski space is used since only in this case we can get a correct expression for the field energy.

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³1) The question: "Where is this inertial reference frame ?" does not make sense, since, as noted in the footnote on page 1, the properties of the reference system do not have a physical meaning in themselves. In the concrete, we can imagine that we are in an inertial reference frame and consider the gravity of the Universe as a physical field in Minkowski space, but we can also assume that we are in the reference frame comoving to the radial flow of galaxies and consider space-time as Riemannian.

2) Are both descriptions of gravity fully equivalent or just locally? This question remains open, but this plays no role for problem under consideration.

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