Modeling and Control of Energy Produced by a Synchronous Generator Using Polynomial Fuzzy Systems and Sum-of-Squares Approach

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Abstract: The synchronous generator, as the main component of power systems, plays a key role in these system's stability. Therefore, utilizing the most effective control strategy for modeling and control the synchronous generator results in the best outcomes in power systems' performances. The advantage of using a powerful controller is to have the synchronous generator modeled and controlled as well as its main task i.e. stabilizing power systems. Since the synchronous generator is known as a complicated nonlinear system, modeling and control of it is a difficult task. This paper presents a sum of squares (SOS) approach to modeling and control the synchronous generator using polynomial fuzzy systems. This method as an efficacious control strategy has numerous superiorities to the well-known T–S fuzzy controller, due to the control framework is a polynomial fuzzy model, which is more general and effectual than the well-known T–S fuzzy model. In this case, a polynomial Lyapunov function is used for analyzing the stability of the polynomial fuzzy system. Then, the number of rules in a polynomial fuzzy model is less than in a T-S fuzzy model. Besides, derived stability conditions are represented in terms of the SOS approach, which can be numerically solved via the recently developed SOSTOOLS. This approach avoids the difficulty of solving LMI (Linear Matrix Inequality). The Effectiveness of the proposed control strategy is verified by using the third-part Matlab toolbox, SOSTOOLS.

Keywords: Synchronous generator; Polynomial fuzzy controller; Polynomial fuzzy system; Polynomial Lyapunov function; Stability; Sum of squares (SOS)

1. Introduction

Fuzzy logic, which has been considered as one of the most important parts of artificial intelligence either in the past or in the current time, was introduced by Professor Lotfizadeh in the form of Fuzzy sets [1]. The fuzzy set theory has been developed by Lotfizadeh to control plants. The fuzzy logic controller (FLC) is known as the most efficacious solution for a number of control issues. FLCs efficiency relates to the fact that they are less sensitive to parametric variations; therefore, they are more robust than the conventional classical controllers (such as PI, PD, and PID) in controlling system output [2]. FLC has been used as an effective control process because of several remarkable reasons such as quick decision-making capability, usability in nonlinear systems, and intuitive definition of controller behavior [2-6].

In the last two decades, the Takagi-Sugeno (T-S) fuzzy model based control methodology has received much attention as a powerful tool to deal with complex nonlinear control systems. The main contribution of T-S fuzzy model in representing nonlinear systems is that nonlinear systems are shown as a combination of local linear subsystems weighted by membership functions [7]. In addition, this fuzzy modeling method offers another excellent approach for describing higher order nonlinear systems, and then reduces the number of rules in their modeling [8].

The T-S fuzzy model can represent any smooth nonlinear systems by fuzzily blending linear sub-systems; moreover, their stabilization conditions based on Lyapunov stability theory can be represented in terms of linear matrix inequalities (LMIs) [7,8,9]. By the same token, designs have been carried out using LMI optimization techniques. In the T-S fuzzy model based control, for designing a fuzzy controller for the system the parallel distributed compensation (PDC) concept based on a common quadratic Lyapunov function has the main contribution [7], [9].

It is worth mentioning that, nowadays, numerous researches [10-14] try to utilize the T-S fuzzy model as the stabilization conditions of nonlinear systems because of the above mentioned reasons. Recently, in [15] a more general version of T-S fuzzy model has been introduced, it is named the polynomial fuzzy model. The main distinction between the T-S fuzzy model and the polynomial fuzzy model is that the former method only deals with constants in the system matrices; however, the latter one provides a perfect opportunity to deal with the polynomials in the system matrices. This great advantage of polynomial fuzzy model results in remarkable applications in nonlinear systems. Therefore, representation of the nonlinear systems with a number of polynomial terms can be controlled more efficiently [15,16]. There is a problem in handling the polynomial fuzzy model. To put it in other words, it is clear that T-S fuzzy model utilizes LMI optimization techniques, a numerical solution is obtained by convex optimization methods such as the interior point method. However, it cannot be used to solve stability analysis and control design problems directly in the polynomial fuzzy model [15, 16]. Despite the great success and popularity of LMI-based approaches, still there exists a large number of design problems that either cannot be represented in terms of LMIs, or the results obtained through LMIs are too conservative and the polynomial fuzzy model is one of that problems. Hence, the paper [15] introduced a sum-of-squares (SOS) optimization technique to perform stability analysis and control design for the polynomial fuzzy model. The problems represented in terms of SOS can be numerically solved by free third-party MATLAB toolboxes such as SOSTOOLS [17] and SOSOPT [18].

In this paper SOS approach for modeling and control of the synchronous generator using polynomial fuzzy systems is presented. The proposed SOS-based approach was selected for modeling and control of this vital system since this method has proved its high efficiency and obvious superiority over T-S fuzzy model. One of the advantages as discussed above is that the polynomial fuzzy model framework is a general version of T-S fuzzy model, hence is more effective in representing nonlinear control systems. The second one is that, one polynomial Lyapunov function that contains quadratic Lyapunov function was employed to stabilize the fuzzy polynomial system and its stability conditions. Hence, the obtained stability conditions from proposed SOS-based approach are more general than those based on the existing LMI-based approaches to T–S fuzzy model and control. The derived stability conditions were represented in terms of SOS can be numerically solved via the recently developed SOSTOOLS [19]. These SOS conditions cannot be generally solved via convex optimization methods. SOSTOOLS [19] is a free, third-party MATLAB toolbox that solves SOS problems. The techniques behind it are based on the SOS decomposition for multivariate polynomials, which can be efficiently computed using semidefinite programming. SOSTOOLS is developed as a consequence of the recent interest [15].

Synchronous generators are the most important parts and electrical energy suppliers of all power systems. They usually operate together (or in parallel), forming a large power system supplying electrical energy to the loads or consumers. Synchronous generators are built in large units, their rating ranging from tens to hundreds of megawatts. They convert mechanical power to ac electric power. The source of mechanical power, the prime mover, may be a diesel engine, a steam turbine, a water turbine, or any similar device.

One stable model of synchronous generator improves the performance and the stability of nonlinear power systems, and provides several benefits such as saving time, energy, and money. Therefore, a helpful and powerful control strategy such as SOS-based polynomial fuzzy control strategy for modeling and control of synchronous generator will be a cost effective, time and energy saving strategy to improve the performance and the stability of nonlinear power systems, as well as enhances the dynamic response of the operating system.

The rest of the paper is organized as follows: In section 2 dynamic model of the synchronous generator is presented. Next, a general form of the fuzzy logic controller is introduced. In the section 4 the polynomial fuzzy model and the polynomial Lyapunov function are described, precisely. Then, the stability analysis via SOS are explained. In section 6, designing the polynomial fuzzy controller is shown. Finally, in section7, the synchronous generator behavior in presence of the introduced polynomial fuzzy controller and without it is analyzed carefully. In the last section, conclusion explains the whole paper briefly.

2. Dynamic Model of the Synchronous Generator

The detailed nonlinear model of a synchronous generator is a sixth-order model. However, the third-order model is of crucial interest for studying control systems of the generator as well as their synthesis [20]. Therefore, the detailed nonlinear model is usually reduced to a generalized one-axis nonlinear third-order model. Generator structure diagram and the simplified model of the synchronous generator are shown in figure1. and figure2. respectively.

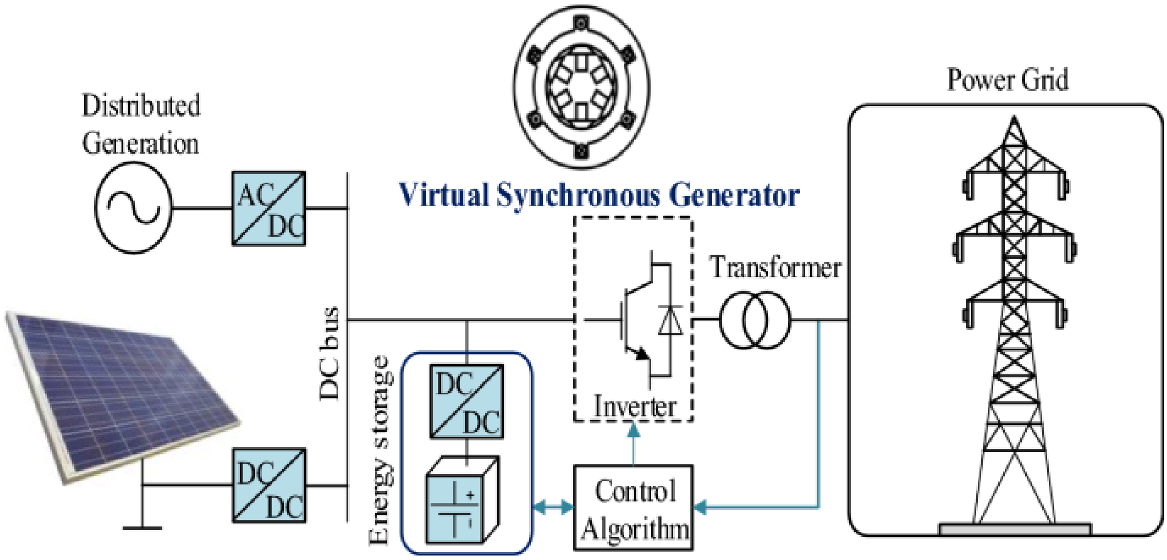

Figure 1. Synchronous Generator Diagram

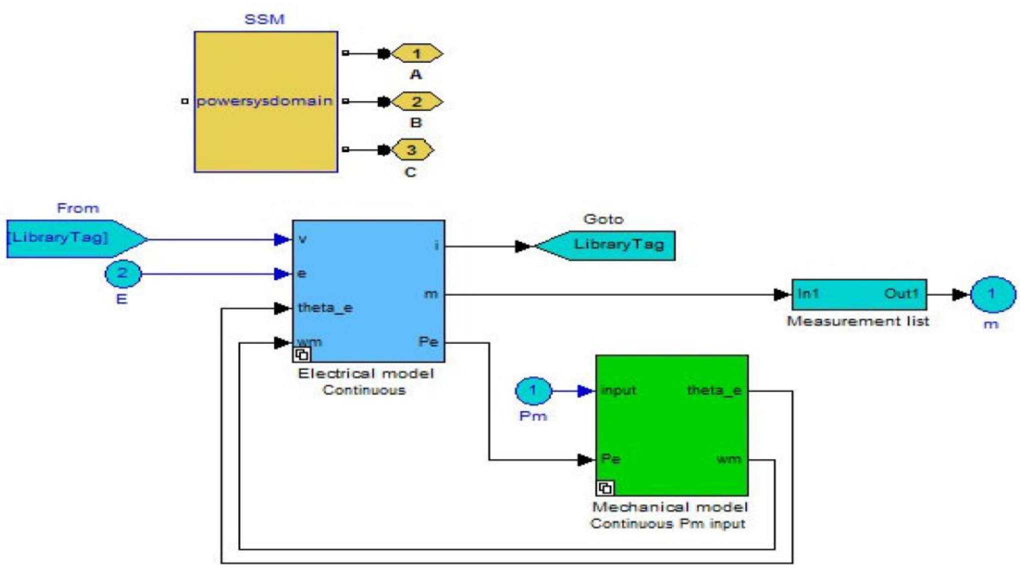

Figure 2. Simplified model of third-order of the synchronous generator in Simulink MATLAB

The following equations describe a third-order dynamic model of the synchronous generator :

$$\begin{cases} \dot{\delta}(t) = \omega(t) - \omega_0 \\ \dot{\omega}(t) = \frac{K_D}{2H}\left(\omega(t) - \omega_0\right) + \frac{\omega_0}{2H}\left(P_m - P_e(t)\right) \\ \dot{v_q}(t) = \frac{1}{T_{do}^{'}}\left(v_f(t) - v_q(t)\right) \end{cases} \tag{1}$$

Where

$$\begin{cases} v_q(t) = \frac{X_d}{X_{ds}^{'}} v_q^{'}(t) - \frac{X_d - X_d^{'}}{X_{ds}^{'}} V_s \cos\left(\delta(t)\right) \\ v_f(t) = K_E G_F(t) \\ P_e(t) = \frac{V_s v_q}{X_{ds}} \sin\left(\delta(t)\right) \end{cases} \tag{2}$$

And

$$X_{ds} = X_d + X_T + \frac{1}{2} X_L \tag{3}$$

$$X_{ds}^{'} = X_d^{'} + X_T + \frac{1}{2} X_L \tag{4}$$

Variables and parameters in the equations of the third-order model of the generator are introduced in the below table:

Table 1. Description of the system variables and parameters

Symbol	**Description**
$\delta(t)$	Rotor angle of the generator (radian)
$\omega(t)$	Speed of the rotor
$\omega_0(t)$	Synchronous machine speed of the generator (radian per second)
K_D	Damping constant of the generator (pu)
H	The inertia constant of the generator (sec)
P_m	Mechanical input power of the generator (pu)

P_e	Active electrical power delivered by the generator (pu)
$v_q(t)$	The EMF of the q-axis of the generator (pu)
$v_q^{'}(t)$	The transient EMF in the q-axis of the generator (pu)
$v_f(t)$	The equivalent EMF in the excitation winding of the generator
$T_{d0}^{'}$	d-axis transient short circuit time constant of the generator (sec)
K_E	The gain of the excitation amplifier of generator
G_F	Control input of the excitation amplifier with gain K_E
X_{ds}	The total direct reactance of the system (pu)
$X_{ds}^{'}$	Total transient reactance of the system (pu)
X_d $X_d^{'}$	The d-axis reactance of the generator (pu) The d-axis transient reactance of the generator (pu)
V_s	Infinite bus voltage (pu)

The state variables of the generator are defined as follow:

$$x_1(t) = \delta(t) \;\; , \;\; x_2(t) = \omega(t) - \omega_0 \, , \;\; x_3(t) = v_q(t) \tag{5}$$

Hence, state variables vector for the generator will be:

$$x(t) = [x_1 \quad x_2 \quad x_3]^T \tag{6}$$

The control input $u(t)$ also considered as follows:

$$u(t) = \frac{K_E}{T_{do}^{'}} G_F(t) \tag{7}$$

The nonlinear equations of the system, define the following constants for the generator:

$$\begin{cases} \alpha_1 = -\dfrac{K_D}{2H} \\ \alpha_2 = -\dfrac{\omega_0}{2HX'_{ds}} V_s \\ \alpha_3 = -\dfrac{\omega_0 (X_d - X'_d)}{4HX_{ds} X'_{ds}} V^2_s \\ \alpha_4 = \dfrac{\omega_0}{2H} P_m \\ \alpha_5 = -\dfrac{1}{T'_{d0}} \dfrac{X_{ds}}{X'_{ds}} \\ \alpha_6 = \dfrac{X_d - X'_d}{T'_{d0} X'_{ds}} V_s \end{cases} \tag{8}$$

The equations (1) and (2) can be rewritten by using (8) as follows:

$$\begin{cases} \dot{x}_1 = x_2(t) \\ \dot{x}_2 = \alpha_1 x_2(t) + \alpha_2 x_3(t) \sin(x_1(t)) + \alpha_3 \sin(2x_1(t)) + \alpha_4 \\ \dot{x}_3 = \alpha_5 x_3(t) + \alpha_6 \cos(x_1(t)) + \alpha_2 \sin x_1 u(t) \end{cases} \tag{9}$$

In these equations the rotor angle is the first state variable, the second one is the rotor speed deviation, and the last state variable describes the voltage. Considering the above described equations, it is clear that the systems is complex and difficult to control because of nonlinear terms. Therefore, employing a powerful approach to control it is a significant issue.

3. Fuzzy Logic Control

In modeling and control of systems with no accurate mathematical model, fuzzy Logic Controls (FLCs) can be of great help. The fuzzy-model-based control methodology provides a natural, simple, and effective design approach to complement other nonlinear control techniques that require special and rather involved knowledge [21, 22].

The main part of the fuzzy logic controller is a set of linguistic control rules related to fuzzy implication and compositional rule of inference. The fuzzy logic controller is the most rapid methodology, and it is simple to design. It requires no precise system mathematical model and can deal with the nonlinearity of haphazard complications. Representing the local dynamics of each fuzzy implication (rule) by a linear system model is the main feature of this model. It is done by linguistic rules with an IF-THEN general structure, which is the origin of human logic. The following figure (Figure3.) and example clearly represent the above-mentioned features of fuzzy logic systems and control.

The structure of a fuzzy controller is shown in Figure3. It consists of fuzzification inference engine and defuzzification blocks:

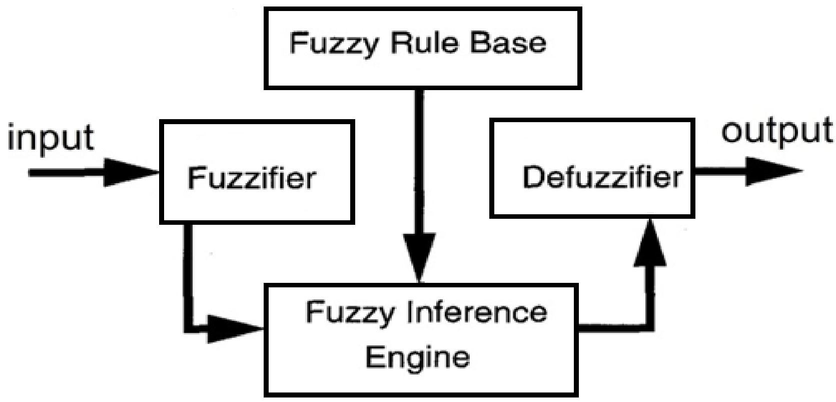

Figure 3. Basic configuration of fuzzy systems with fuzzifier and defuzzifier

Example 1: Consider the following nonlinear system:

$$\begin{cases} \dot{x}_1 = -x_1 + x_1 x_2^3 + u \\ \dot{x}_2 = -x_2 + (3 + x_2) x_1^3 \end{cases} ;\ \boldsymbol{x}_1, x_2 \in [-1,1] \tag{10}$$

$$\dot{x} = \begin{bmatrix} -1 & x_2^2 x_1 \\ (x_2+3)x_1^2 & -1 \end{bmatrix} x \ ; x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{11}$$

If the nonlinear items ${x_2}^2 x_1$ and $(x_2+3){x_1}^2$ in equation (11) are replaced by z_1 and z_2 respectively, the following equation is obtained:

$$\dot{x} = \begin{bmatrix} -1 & z_1 \\ z_2 & -1 \end{bmatrix} x \tag{12}$$

For $x_1, x_2 \in [-1,1]$:

$$\begin{cases} max_{x_1,x_2}\ z_1 = 1,\ min_{x_1,x_2}\ z_1 = -1 \\ max_{x_1,x_2}\ z_2 = 4,\ min_{x_1,x_2}\ z_2 = 0 \end{cases} \tag{13}$$

Therefore:

$$\begin{cases} z_1 = x_1 x_2^2 = M_1(z_1).1 + M_2(z_1).(-1) \\ z_2 = (3+x_2)x_1^2 = N_1(z_2).1 + N_2(z_2).(-1) \end{cases} \tag{14}$$

Where

$$\begin{cases} M_1(Z_1) + M_2(Z_1) = 1 \\ N_1(Z_2) + N\ (Z_2) = 1 \end{cases} \tag{15}$$

The membership functions are given as follows:

$$\begin{cases} M_1(z_1)=\dfrac{z_1-z_{1min}}{z_{1max}-z_{1min}}, M_2(z_1)=\dfrac{z_{1max}-z_1}{z_{1max}-z_{1min}} \\ N_1(z_2)=\dfrac{z_2-z_{2min}}{z_{2max}-z_{2min}}, N_2(z_2)=\dfrac{z_{2max}-z_2}{z_{2max}-z_{2min}} \end{cases} \tag{16}$$

$$\begin{cases} M_1(Z_1)=\dfrac{Z_1+1}{2}, M_2(Z_1)=\dfrac{1-Z_1}{2} \\ N_1(z_2)=\dfrac{z_2}{2}, N_2(z_2)=\dfrac{4-z_2}{4} \end{cases} \tag{17}$$

$$\begin{aligned} &IF\ z_1\ is\ M_1\ AND\ z_2\ is\ N_1\ THEN\ \dot{x}=A_1x \\ &IF\ z_1\ is\ M_1\ AND\ z_2\ is\ N_2\ THEN\ \dot{x}=A_2x \\ &IF\ z_1\ is\ M_2\ AND\ z_2\ is\ N_1\ THEN\ \dot{x}=A_3x \\ &IF\ z_1\ is\ M_2\ AND\ z_2\ is\ N_2\ THEN\ \dot{x}=A_4x \end{aligned} \tag{18}$$

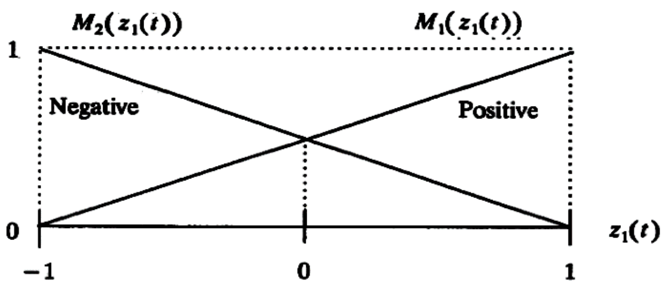

Figure 4. Membership functions for $M(z_1(t))$

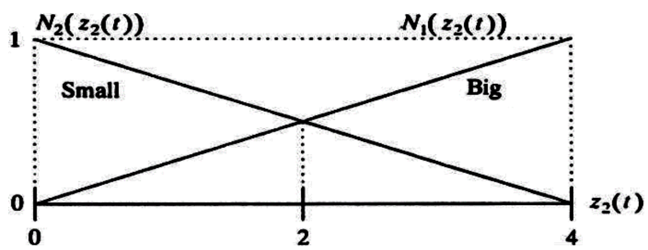

Figure 5. Membership functions for $N(z_1(t))$

Figures 4. and 5. show membership functions.

$$A_1=\begin{bmatrix}-1 & 1\\ 4 & -1\end{bmatrix}, A_2=\begin{bmatrix}-1 & 1\\ 0 & -1\end{bmatrix}, A_3=\begin{bmatrix}-1 & -1\\ 4 & -1\end{bmatrix}, A_4=\begin{bmatrix}-1 & -1\\ 0 & -1\end{bmatrix}$$

$$B_1=B_2=B_3=B_4=\begin{bmatrix}1\\0\end{bmatrix} \tag{19}$$

With defuzzification process:

$$\dot{x} = \sum_{i=1}^{4} h_i(z) A_i x \quad (20)$$

As

$$\begin{aligned} h_1(z) &= M_1(z_1) \times N_1(z_2) \\ h_2(z) &= M_1(z_1) \times N_2(z_2) \\ h_3(z) &= M_2(z_1) \times N_1(z_2) \\ h_4(z) &= M_2(z_1) \times N_2(z_2) \end{aligned} \quad (21)$$

This model exactly shows nonlinear system in area $[-1,1] \times [-1,1]$ of space $x_1 - x_2$.

4. Polynomial Fuzzy Model and Polynomial Lyapunov Function

4.1. Polynomial fuzzy model

In this section, before introducing the polynomial fuzzy model, the T–S fuzzy-model-based control is explained. It provides an opportunity to compare the two mentioned models' performances and proving the advantages of the polynomial fuzzy model. The main application of T–S fuzzy-model-based control is that it is an effective strategy to represent any smooth nonlinear control systems by the T–S fuzzy models (with liner model consequence), and the system stability is analyzed based on quadratic Lyapunov functions. The T–S fuzzy model is described by fuzzy IF-THEN rules that represent local linear input-output relations of a nonlinear system. In this model, the dynamics of each fuzzy implication (rule) are shown as a linear system model, and this is the main feature of the T–S fuzzy model. The overall fuzzy model of the system is achieved by the fuzzy blending of the linear system models. After modeling systems, analyzing the stability should be considered. In this case, the quadratic Lyapunov function should be defined, and the stability conditions result in solving LMIs. Finally, the stability conditions can be efficiently solved numerically by interior point algorithms such as the LMI toolbox of MATLAB, which is hard and timewasting sometimes.

To solve the above-mentioned problems and prepare further beneficial results, a simpler and more general method is introduced as the polynomial fuzzy model [15]. For proving the stability of this model a polynomial Lyapunov function should be defined. The stability conditions for polynomial fuzzy systems based on polynomial Lyapunov functions could be reduced to SOS problems, which avoids the difficulty of the LMIs and could be solved readily. Therefore, instead of the LMI toolbox, these problems can be solved via SOSTOOLS. In the following, the polynomial fuzzy model of a general nonlinear system is described.

Suppose a nonlinear system as follows:

$$\dot{x}(t) = f(x(t), u(t)) \quad (22)$$

As discussed above, a so called polynomial fuzzy model is introduced to represent the nonlinear system (22).

The main difference between T-S fuzzy model and a polynomial fuzzy model lies in the consequent part representation. The T-S fuzzy model features linear model consequence, however, the introduced polynomial fuzzy model has polynomial model consequence as below:

Model rule i:

$$If\ z_1(t)\ is\ M_{1i}\ and \ldots and\ z_p(t)\ is\ M_{pi}$$

$$Then\ \dot{x}(t) = A_i(x(t))\hat{x}(x(t)) + B_i(x(t))u(t), i = 1,2,\ldots,r \qquad (23)$$

Where $A_i(x(t)) \in R^{n\times n}$ and $B_i(x(t)) \in R^{n\times m}$ are polynomial matrices in $x(t)$. $\hat{x}(x(t))$ is a column vector whose entries are all monomials in $x(t)$.

The overall polynomial fuzzy model is obtained by fuzzy blending of each polynomial model equation in the consequent part. By using the weighted average of each rule's output, the defuzzification process of model (23) can be represented as:

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} w_i(z(t))\{A_i(x(t))\hat{x}(x(t)) + B_i(x(t))u(t)\}}{\sum_{i=1}^{r} w_i(z(t))} = \sum_{i=1}^{r} h_i(z(t))\{A_i(x(t))\hat{x}(x(t)) + B_i(x(t))u(t)\} \qquad (24)$$

If $\hat{x}(x(t)) = x(t)$, $A_i(x(t))$ and $B_i(x(t))$ are constant matrices for all i, then $A_i(x(t))\hat{x}(x(t)) + B_i(x(t))u(t)$ reduces to $A_i x(t) + B_i u(t)$. Then (24) reduces to (25):

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\{A_i x(t) + B_i u(t)\} \qquad (25)$$

Where, (25) shows the T-S fuzzy model of above nonlinear system. Therefore, (24) or polynomial fuzzy model is a more general representation compared to T-S fuzzy model (25).

4.2. Polynomial Lyapunov function

Analyzing the stability of mentioned polynomial system could be simple in using a polynomial Lyapunov function, in this case the stability results and conditions could be relaxed. The proposed polynomial Lyapunov function is defined as below:

$$\hat{x}^T(x(t))P(x(t))\hat{x}(x(t)) \qquad (26)$$

Where $P(x(t))$ is a polynomial matrix in $x(t)$. If $\hat{x}(x(t)) = x(t)$ and $P(x(t))$ is a constant matrix, then (26) reduces to the quadratic Lyapunov function $x^T(t)Px(t)$. Therefore, it is clear that (26) is a more general representation.

5. Sum of Squares for Stability Analysis

5.1.Sum of squares

One of the most important objectives of this paper is utilizing the SOS method as the computational method to provides significantly more relaxed stability results than the existing LMI approaches to T–S fuzzy models and avoid the difficulty of solving the LMI. A multivariate $f(x(t))$ where $x(t) \in R^n$ is an SOS if there exist polynomials $f_1(x(t)),\dots, f_m(x(t))$ such that $f(x(t)) = \sum_{i=1}^{m} f_i^2(x(t))$. It is clear that $f(x(t)) > 0$ for all $x(t) \in R^n$ [23].

5.2.Stability conditions

In this section, the stability of system (24) is analyzed. The zero equilibrium of the system (24) with $u = 0$ is stable if there exists a symmetric polynomial matrix $P(x) \in R^{(n\times n)}$ such that (27) and (28) are satisfied, where $\sigma_1(x)$ and $\sigma_{2i}(x)$ are nonnegative polynomials for all x: (In this section, we drop the notation with respect to time t)

$$\hat{x}^T(x)(P(x) - \sigma_1(x)I)\hat{x}(x) isSOS \qquad (27)$$

$$-\hat{x}^T(x)(P(x)T(x)A_i(x) + A_i^T(x)T^T(x)P(x) + \sum_{k=1}^{n} \frac{\partial P}{\partial x_k}(x)A_i^k(x)\hat{x}(x) + \sigma_{2i}(x)I)\hat{x}(x) isSOS \quad \forall i \qquad (28)$$

Where $T(x)$ is a polynomial matrix whose $(i, j)th$ entry is given by

$$T^{ij}(x) = \frac{\partial \hat{x}_i}{\partial x_j}(x) \qquad (29)$$

If $P(x)$ is a constant matrix, then the stability holds globally.

Remark1: When $A_i(x), B_i(x)$, and $P(x)$ are constant matrices and $\hat{x}(x) = x$. The system (24) and the polynomial Lyapunov function (26) are the same as the T–S fuzzy model and the quadratic Lyapunov function. Thus, the proposed SOS approach to polynomial fuzzy models contains the existing LMI approaches to T–S fuzzy models as a special case. Therefore, the SOS-based polynomial fuzzy models provide significantly more relaxed stability results than the existing LMI approaches to T–S fuzzy models.

6. Designing Polynomial Fuzzy Controller

A fuzzy controller with polynomial rule consequence can be constructed from the given polynomial fuzzy model (23).

The ith rule of polynomial fuzzy controller is as follows:
Control rule i:

$$\begin{aligned} &If\ z_1(t) is\, M_{1i}\, and \ldots and\ z_p(t) is\, M_{pi} \\ &Then\, u(t) = -F_i\left(x(t)\right)\hat{x}(t), i = 1, 2, \ldots, r \end{aligned} \tag{30}$$

Where $F_i \in R^{m\times n}$ is the polynomial feedback gain in rule j. Thus, the following polynomial fuzzy controller is applied to the nonlinear plant represented by the polynomial fuzzy model:

$$u(t) = -\sum_{i=1}^{r} h_i\left(z(t)\right) F_i\left(x(t)\right)\hat{x}\left(x(t)\right) \tag{31}$$

Therefore, from (24) and (31) the closed-loop system can be represented as:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(z(t)\right) h_j\left(z(t)\right) \\ &\times\left\{A_i\left(x(t)\right) - B_i\left(x(t)\right) F_j\left(x(t)\right)\right\}\hat{x}\left(x(t)\right) \end{aligned} \tag{32}$$

Where $A_i\left(x(t)\right) \in R^{n\times n}$ and $B_i\left(x(t)\right) \in R^{n\times m}$ are polynomial matrices in $x(t)$. If $\hat{x}\left(x(t)\right) = x(t)$, $A_i\left(x(t)\right)$, $B_i\left(x(t)\right)$ and $F_j\left(x(t)\right)$ are constant matrices for all i and j then the above equation can be summarized to Takagi-Sugeno equation. Therefore, (32) are more general representations compared to Takagi-Sugeno equation.

In the next step the stability of the closed-loop control system (32) should be considered.

6.1.SOS design conditions

Theorem1: *To provide required conditions for stability of closed-loop system (32) one polynomial matrix* $S(\tilde{x})$ *and the polynomial matrix* M_i *should be defined to satisfy the following conditions, where* $\sigma_1(x)$ *and* $\sigma_{2ij}(x)$ *are nonnegative polynomials such that* $\sigma_1(x) > 0$ *for* $x \neq 0$ *and* $\sigma_{2ij}(x) \geq 0$ *for all* x:

($A_i^k(x)$ denotes the kth row of $A_i(x)$, $K=\{k_1,k_2,\dots,k_m\}$ denotes the row indices of $B_i(x)$ whose corresponding row is equal to zero, and define $\hat{x}=(x_{k1},x_{k2},\dots,x_{km})$).

$$v^{T}\left(S(\tilde{x})-\sigma_1(x)I\right)v \text{ is SOS} \tag{33}$$

$$\begin{aligned}
&-v^T(T(x)A_i(x)S(\tilde{x})-T(x)B_i(x)M_j(x)+\\
&S(\tilde{x})A_i^T(x)T^T(x)-M_j^T(x)B_i^T(x)T^T(x)+\\
&T(x)A_j(x)S(\tilde{x})-T(x)B_j(x)M_i(x)+\\
&S(\tilde{x})A_j^T(x)T^T(x)-M_i^T(x)B_j^T(x)T^T(x)-\\
&\sum_{k\in K}\frac{\partial S}{\partial x_k}(\tilde{x})A_i^k(x)\hat{x}(x)-\sum_{k\in K}\frac{\partial S}{\partial x_k}(\tilde{x})A_j^k(x)\hat{x}(x)+\\
&\sigma_{2ij}(x)I)v \text{ is } SOS, i\le j
\end{aligned} \tag{34}$$

Where v is independent of x. $T(x)$ is a polynomial matrix and i, j are entries that are given by:

$$T^{ij}(x)=\frac{\partial \hat{x}_i}{\partial x_j}(x) \tag{35}$$

If (34) holds with $\sigma_{2ij}(x)>0$ for $x\neq 0$ then the zero equilibrium is asymptotically stable. If $S(\tilde{x})$ is a constant matrix then the stability holds globally. A stabilizing feedback gain $F_i(x)$ can be obtained from $S(\tilde{x})$ and $M_i(x)$ as:

$$F_i(x)=M_i(x)S^{-1}(\tilde{x}) \tag{36}$$

Algorithm1: Application of Sum of Square Programming

1: Initialize the sum of squares program

2: Define x_i

3: Define M_i

4: Define V as an independent vector of x

5: System characteristics in state space

6: σ_i values

7: Equation (35)
8: Equation (33)
9: Equation (34)
10: Define SOSP constraints
11: Calling solver
12: Get solution
13: Equation (36)

14: Obtained $F_i(x)$ from SOSTOOLS

Positive polynomials are optimized by SOSTOOLS Algorithm1.

7. Modeling and Stability Analysis of the Synchronous Generator

7.1. Nonlinear synchronous generator system

The model of simple transmission system containing power plant is shown in figure 6. The polynomial fuzzy system is used to improve transient stability and power system oscillations damping.

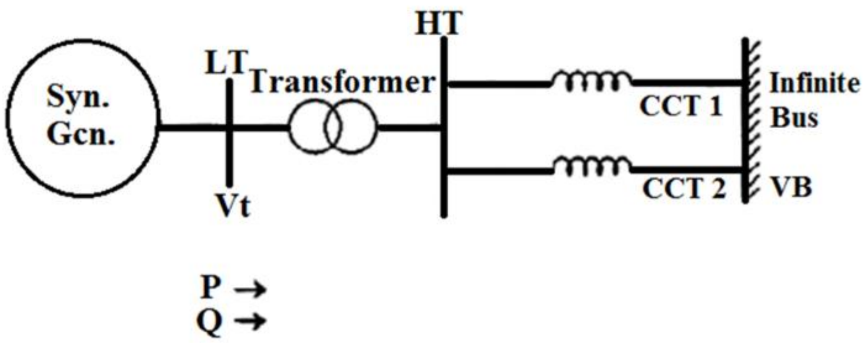

Figure 6. Bus power system

The state space equations for the system discussed above (figure 6) is given as follow: (assuming $K_D = 0$: damping constant of generator)

$$\begin{cases} \dot{x}_1 = x_2(t) \\ \dot{x}_2 = \alpha_2 x_3(t)\sin(x_1(t)) + \alpha_3 \sin(2x_1(t)) + \alpha_4 \\ \dot{x}_3 = \alpha_5 x_3(t) + \alpha_6 \cos(x_1(t)) + \alpha_2 \sin x_1 u(t) \end{cases} \quad (37)$$

The nominal values of system parameters are shown in Table 2:

Table 2. System parameters

x_d	1.863
x'_d	0.257
x_T	0.127
T'_{do}	6.9
x_L	0.4853
H	4
W_0	314.159
P_m	0.9
V_s	0.9552

By applying values of Table 2, the following are obtained:

$$\begin{cases} \dot{x}_1 = x_2(t) \\ \dot{x}_2 = -146x_3 \sin x_1 + 60\sin\left(2x_1(t)\right) + 35 \\ \dot{x}_3 = -x_3(t) + 0.86\cos\left(x_1(t)\right) - 146\sin x_1 u(t) \end{cases} \tag{38}$$

Figures 7-9 show the time responses of the system behavior:

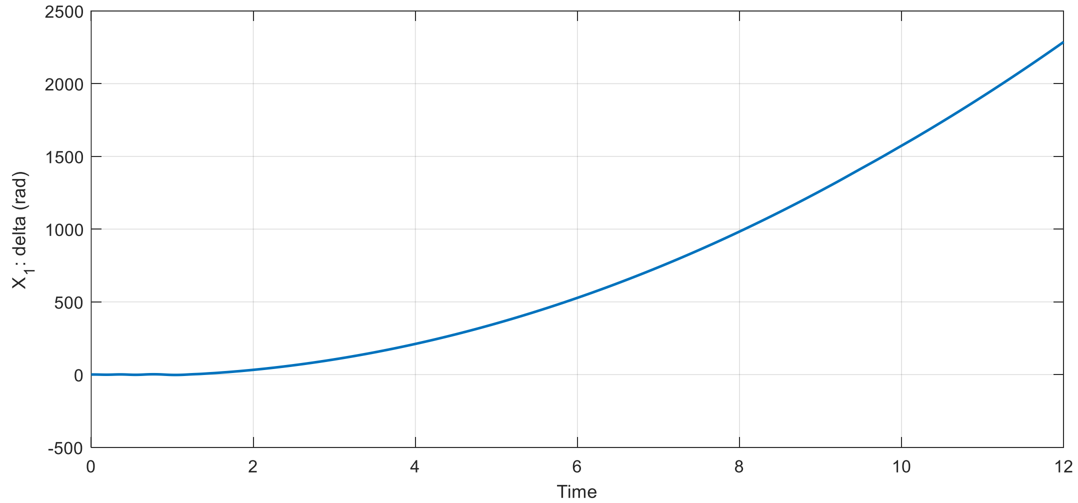

Figure 7. Rotor angle (radian)

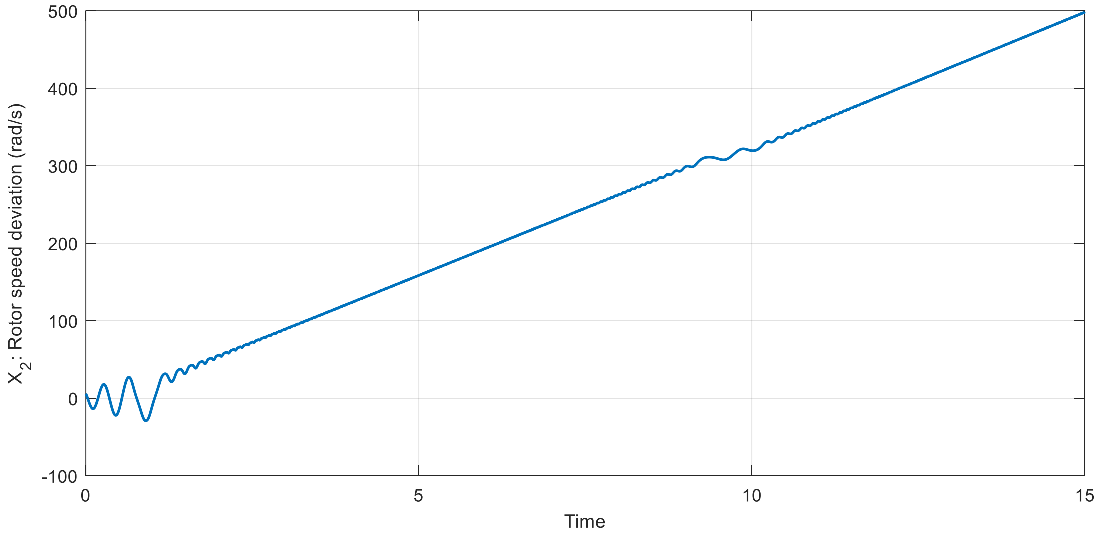

Figure 8. Rotor speed deviation (radian per second)

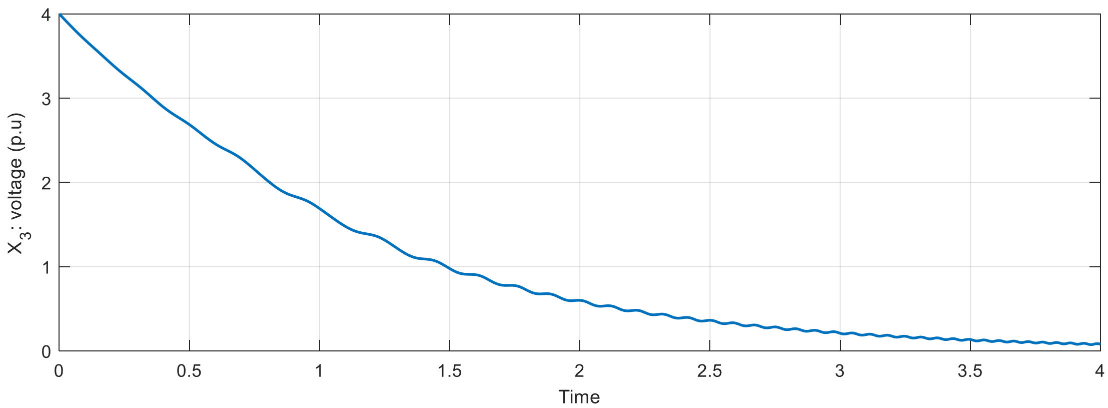

Figure 9. Terminal voltage (P.U.)

As shown in Figures 7 and 8, the angle and the speed deviation of the rotor goes to infinity with respect to time, which exhibit unstable behavior. Also in Figure 9, the variable x_3 goes to zero with an uncontrolled initial value. Hence, we should improve and control it.

Since phase plot is a useful graphical tool to understand the stable or unstable behavior of equilibrium points of nonlinear systems, figures 10, 11, 12 are prepared to show the behavior of a nonlinear system with $u=0$. As shown, x_1 and x_2 exhibit unstable behavior, therefore, nonlinear system is unstable.

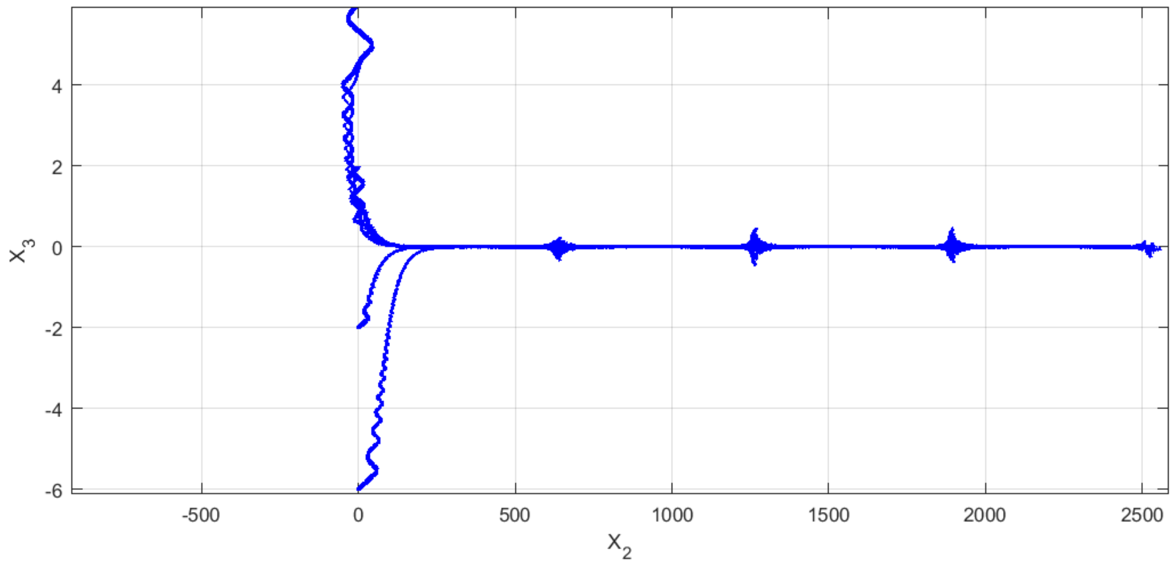

Figure 10. Behavior in $x_2 - x_3$ plane

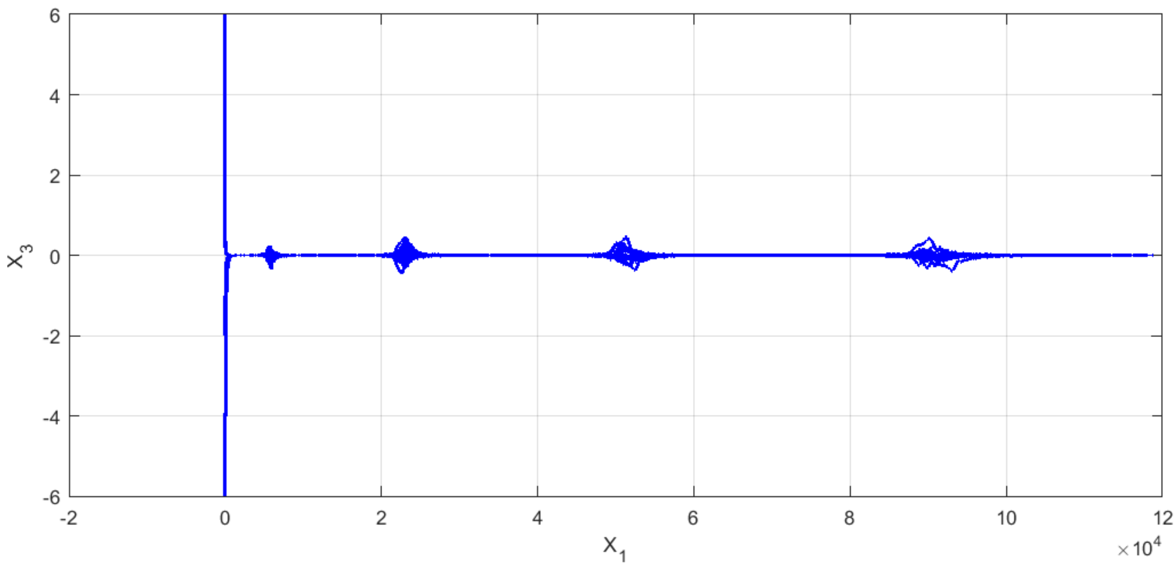

Figure 11. Behavior in $x_1 - x_3$ plane

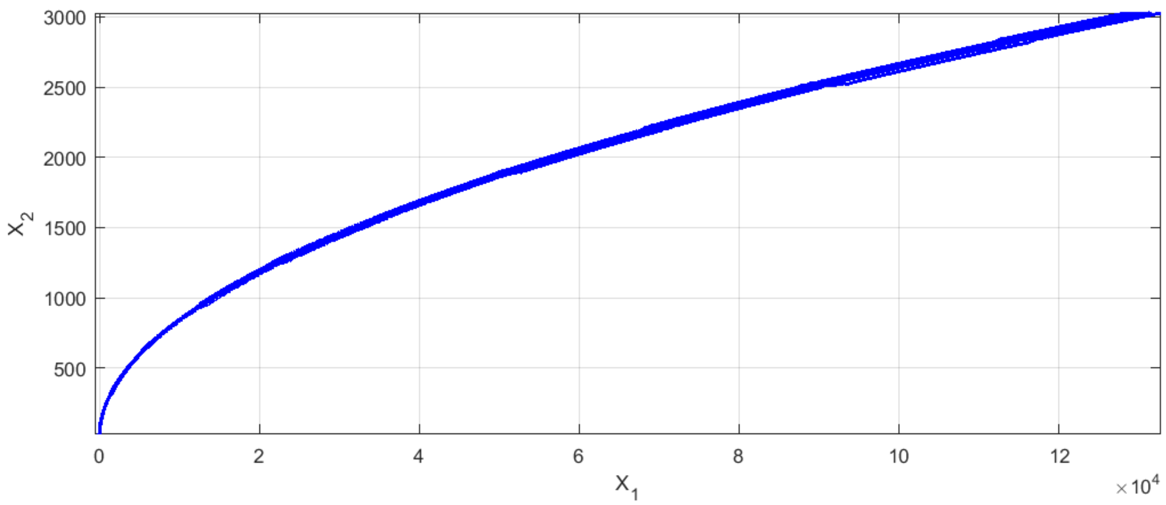

Figure 12. Behavior in $x_1 - x_2$ plane

7.2. Controller design

To meet the objective of control and stabilize the system, modeling and designing a polynomial fuzzy controller is necessary.

In this step one more variable is defined as $x_4 = \cos x_1(t)$. According to derivative of the chain rule:

$$\dot{x}_4 = -\dot{x}_1 \sin(x_1(t)) = -x_2 \sin(x_1(t)) \quad (39)$$

By replacing $\sin 2x = (2\sin x)(\cos x)$ the above equation could be written as follow:

$$\begin{cases} \dot{x}_1 = x_2(t) \\ \dot{x}_2 = -146x_3 \sin x_1 + 120x_4 \sin(x_1(t)) + 35 \\ \dot{x}_3 = -x_3(t) + 0.86x_4 - 146\sin x_1 u(t) \\ \dot{x}_4 = -x_2 \sin(x_1(t)) \end{cases} \quad (40)$$

It is clear that:

$$-0.2172 \le \frac{\sin x}{x} \le 1 \quad (41)$$

Membership functions are given as follow:

$$h_1(z) = \frac{z - z_{min}}{z_{max} - z_{min}}, h_2(z) = \frac{z_{max} - z}{z_{max} - z_{min}} \quad (42)$$

Therefore, the fuzzy model of the system is obtained as follows:

$$h_1(z)=\frac{\sin x_1+0.2172x_1}{1.2172x_1},\; h_2(z)=\frac{x_1-\sin x_1}{1.2172x_1} \tag{43}$$

$$A_1(x)=\begin{bmatrix} 0 & 1 & 0 & 0 \\ -146x_3+120x_4 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0.86 \\ -x_2 & 0 & 0 & 0 \end{bmatrix} \tag{44}$$

$$A_2(x)=\begin{bmatrix} 0 & 1 & 0 & 0 \\ -146\times(-0.2172)x_3+120\times(-0.2172)x_4 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0.86 \\ 0.2172x_2 & 0 & 0 & 0 \end{bmatrix} \tag{45}$$

$$B_1=\begin{bmatrix} 0 \\ 0 \\ -146 \\ 0 \end{bmatrix}, B_2=\begin{bmatrix} 0 \\ 0 \\ -146\times(-0.2172) \\ 0 \end{bmatrix} \tag{46}$$

By considering the values of σ as follows:

$$\begin{aligned} \sigma_1 &= 0.001 \\ \sigma_2 &= 0.001 \end{aligned} \tag{47}$$

The SOS design conditions in Theorem 1 are feasible. From (33), (34) and (36), and using SOSTOOLS in MATLAB $F_i(x)$ is obtained as follows:

$$\begin{cases} F_1 = M_1 \times S^{-1} \\ F_2 = M_2 \times S^{-1} \end{cases} \tag{48}$$

$$\begin{cases} F_1(1,1) = -2.3548e^{-18x_2} + 2.9111e^{-5x_3} + 0.0009443x_4 - 0.1023 \\ F_1(1,2) = 3.8025e^{-17x_2} - 2.5505e^{-6x_3} - 8.2738e^{-5x_4} + 0.00171 \\ F_1(1,3) = 2.4496e^{-15x_2} + 1.8947e^{-17x_3} + 6.146e^{-16x_4} - 0.00115 \\ F_1(1,4) = -3.6107e^{-15x_2} - 1.630e^{-17x_3} - 5.287e^{-16x_4} + 0.00099 \end{cases} \tag{49}$$

$$\begin{cases} F_2(1,1) = 4.191e^{-18x_2} + 2.9441e^{-5x_3} - 0.057827x_4 - 0.64291 \\ F_2(1,2) = 6.7675e^{-17x_2} - 2.5794e^{-6x_3} + 0.0050664x_4 + 0.01023 \\ F_2(1,3) = -4.3598e^{-15x_2} + 1.916e^{-17x_3} - 3.763e^{-14x_4} - 0.00762 \\ F_2(1,4) = 6.4262e^{-15x_2} - 1.6485e^{-17x_3} + 3.238e^{-14x_4} + 0.00655 \end{cases} \quad (50)$$

F can be written as (regardless of some small amounts):

$$\begin{cases} F_1(1,1) = 0.00094435x_4 - 0.10235 \\ F_1(1,2) = 0.0017107 \\ F_1(1,3) = -0.0011586 \\ F_1(1,4) = 0.00099656 \end{cases} \quad (51)$$

$$\begin{cases} F_2(1,1) = -0.057827x_4 - 0.64291 \\ F_2(1,2) = 0.0050664x_4 + 0.010235 \\ F_2(1,3) = -0.007623 \\ F_2(1,4) = 0.0065559 \end{cases} \quad (52)$$

Therefore, the next equation is obtained:

$$\begin{cases} F_1 = [0.00094435x_4 - 0.10235, 0.0017107, -0.0011586, 0.00099656] \\ F_2 = [-0.057827x_4 - 0.64291, 0.0050664x_4 + 0.010235, -0.007623, 0.0065559] \end{cases} \quad (53)$$

The fuzzy controller is obtained from the equation (31):

$$u(t) = -\sum_{i=1}^{r} h_i(z(t)) F_i(x(t)) \hat{x}(x(t))$$

Finally, the closed-loop system could be obtained from (32):

$$\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \times \\ \{A_i(x(t)) - B_i(x(t)) F_j(x(t))\} \hat{x}(x(t))$$

Figures 13-18 show the control result via the designed stabilizing controller.

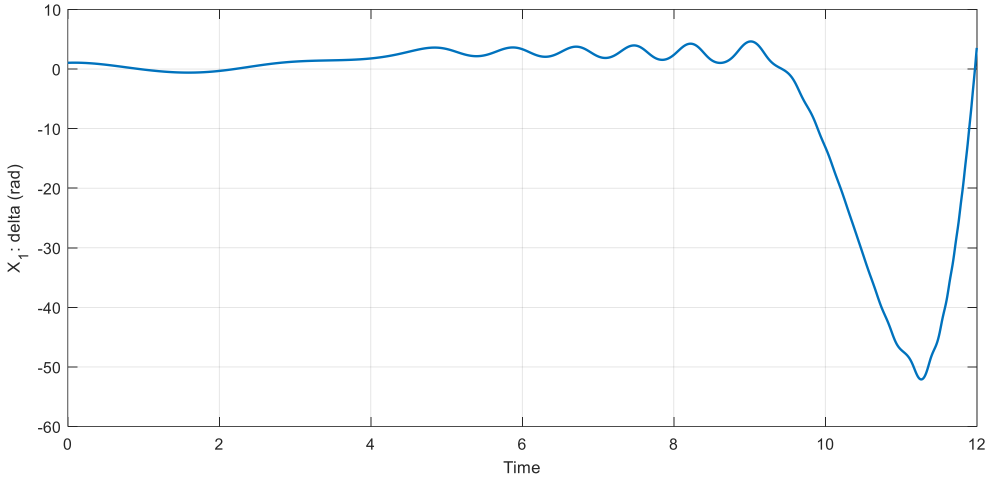

Figure 13. Rotor angle (radian)

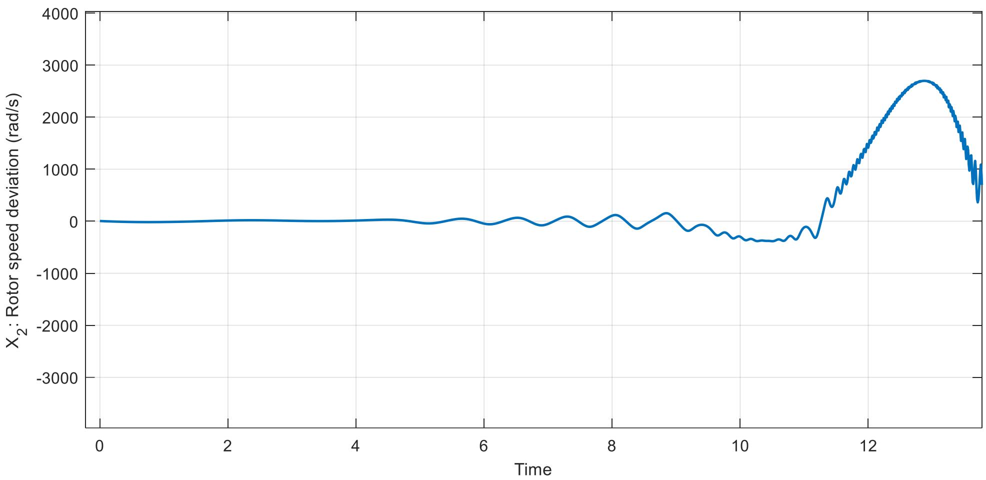

Figure 14. Rotor speed deviation (radian per second)

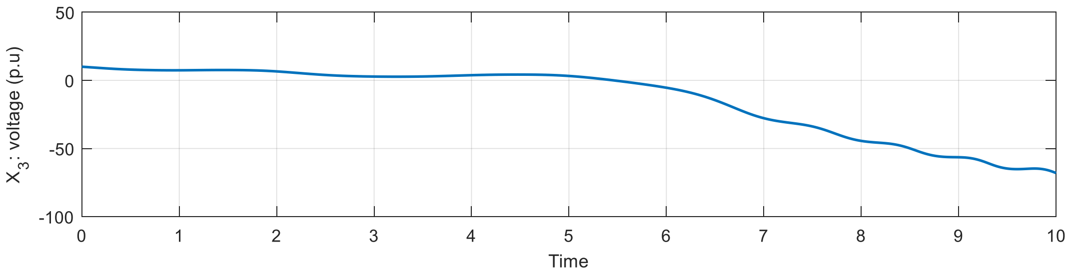

Figure 15. Terminal voltage (P.U.)

The explained unstable system in previous sections is stabilized via SOS designed controller. In the figures 13-15, it can be seen that the components of the rotor angle, its speed and the terminal voltage have reached a stable value, and by taking the advantage of the introduced method all of them could be controlled.

we considered the value of x_1 in Figure13. of $\left(-1\times10^{-4},5\times10^{-4}\right)$, In order to show the more details (including the initial value, etc.), so it is clear that x is stable.

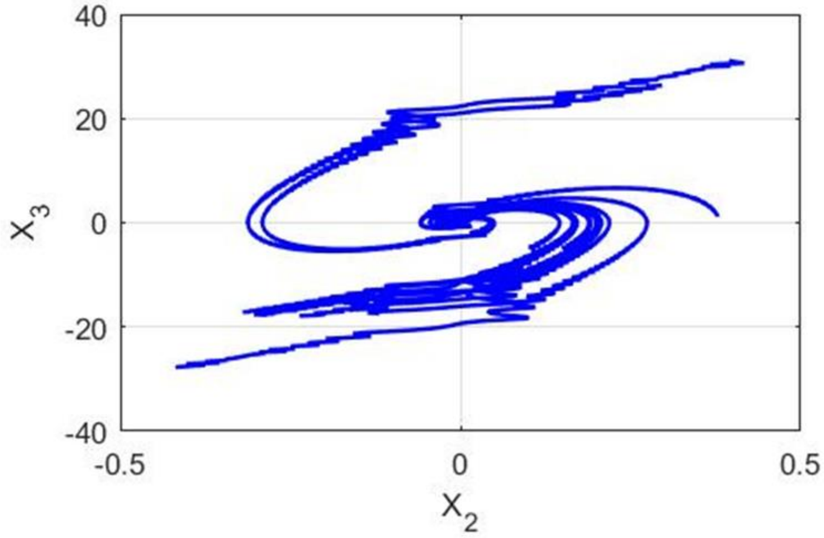

Figure 16. Behavior in $x_2 - x_3$ plane

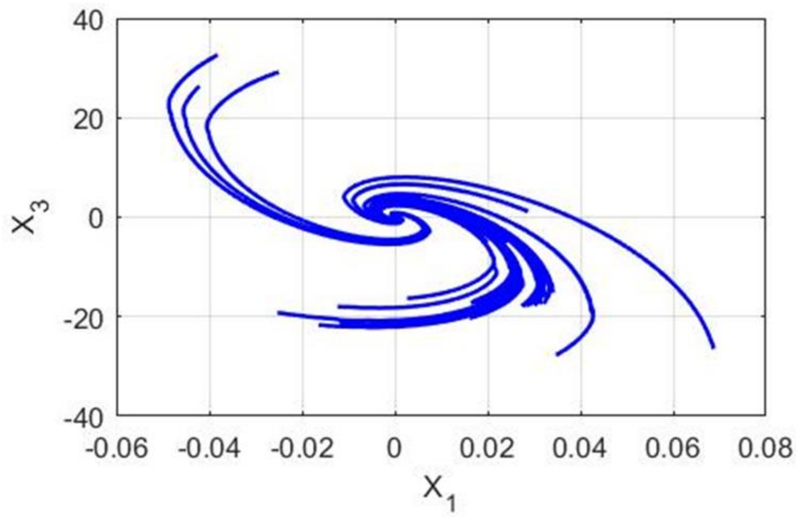

Figure 17. Behavior in $x_1 - x_3$ plane

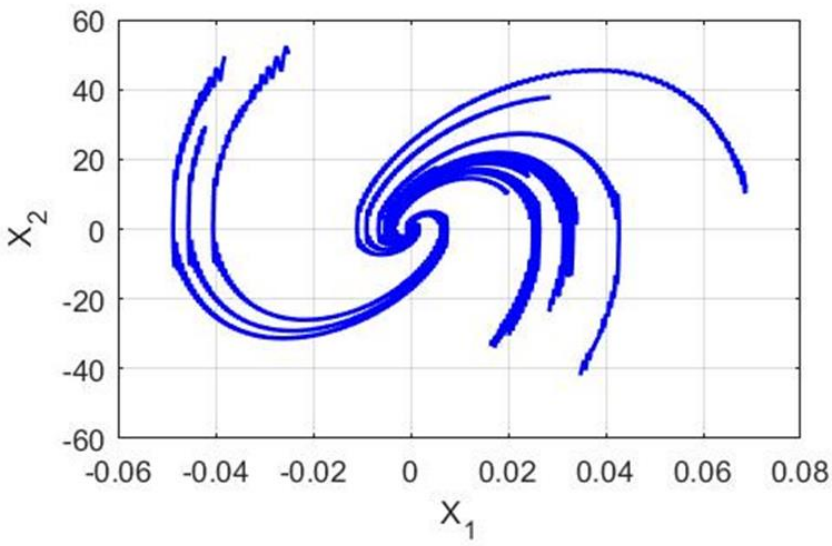

Figure 18. Behavior in $x_1 - x_2$ plane

Phase plot figures 16, 17, and 18 show the stability of the system behavior on the $x_1 - x_2$ plane using the obtained state feedback. In fact, the controller guarantees the global asymptotic stability of controlled system. It should be noted that the path of the states on the x_1 and x_2 plane approaches to the origin. In this case, the equilibrium point is called the stable node. Consequently, the paths are said to be stable (as t increases, the paths lead to the origin).

7.3. Design example

In this step, to show the results more accurately, it is assumed that the system's state equations are as follows:

$$\begin{cases} \dot{x}_1 = x_2(t) \\ \dot{x}_2 = 20.564\sin(2x_1(t)) + 35.342 \\ \dot{x}_3 = -0.516x_3(t) + 0.354\cos(x_1(t)) + u(t) \end{cases} \quad (54)$$

State equations of the system include nonlinear terms. By plotting the system time response and, and as shown in figures 19-25, x_1 and x_2 show an unstable behavior.

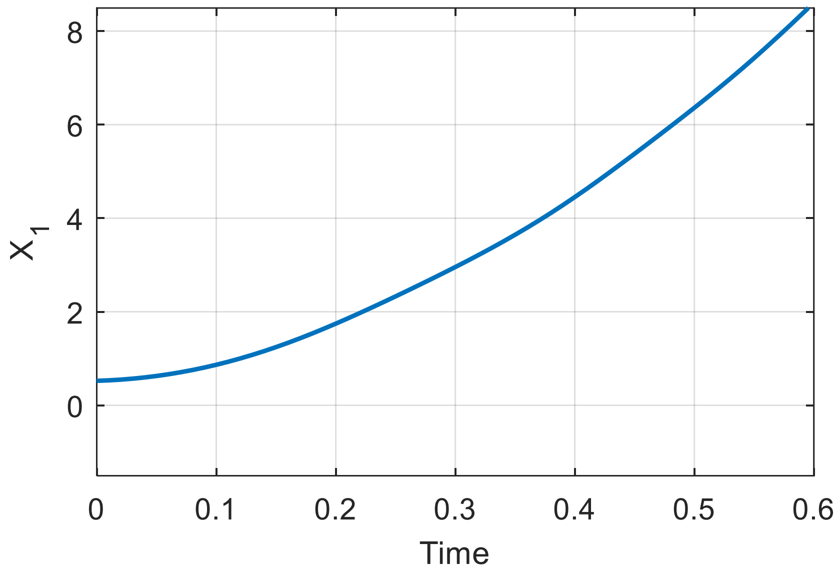

Figure 19. Rotor angle (radian)

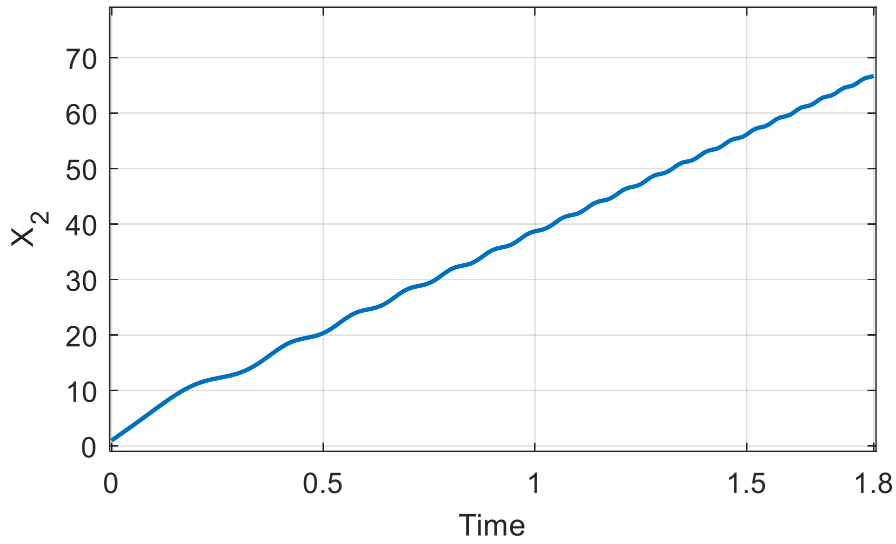

Figure 20. Rotor speed deviation (radian per second)

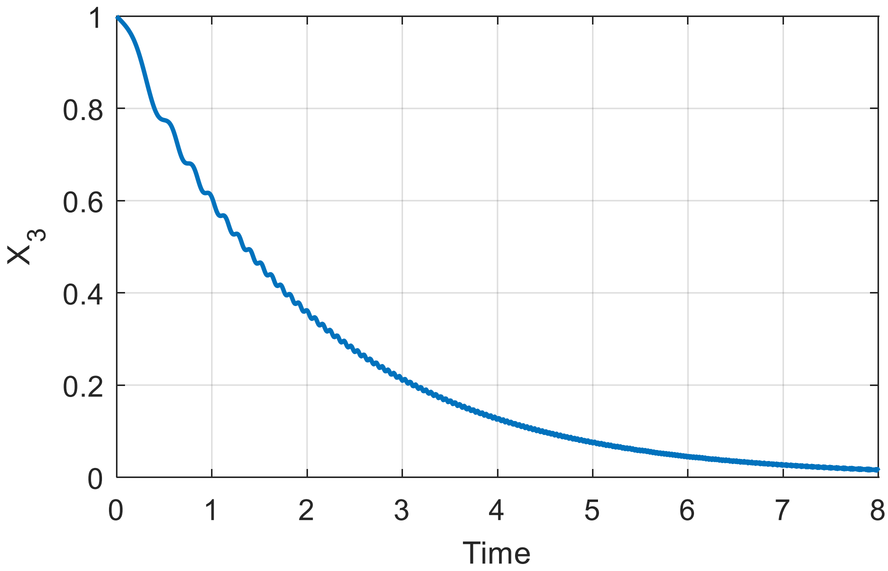

Figure 21. Terminal voltage (P.U.)

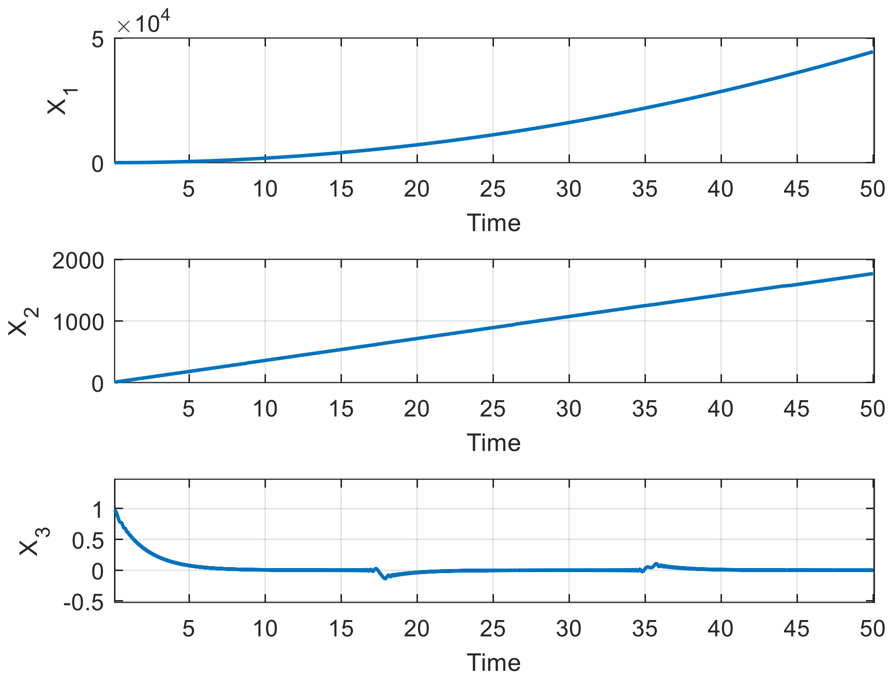

Figure 22. Power system responses

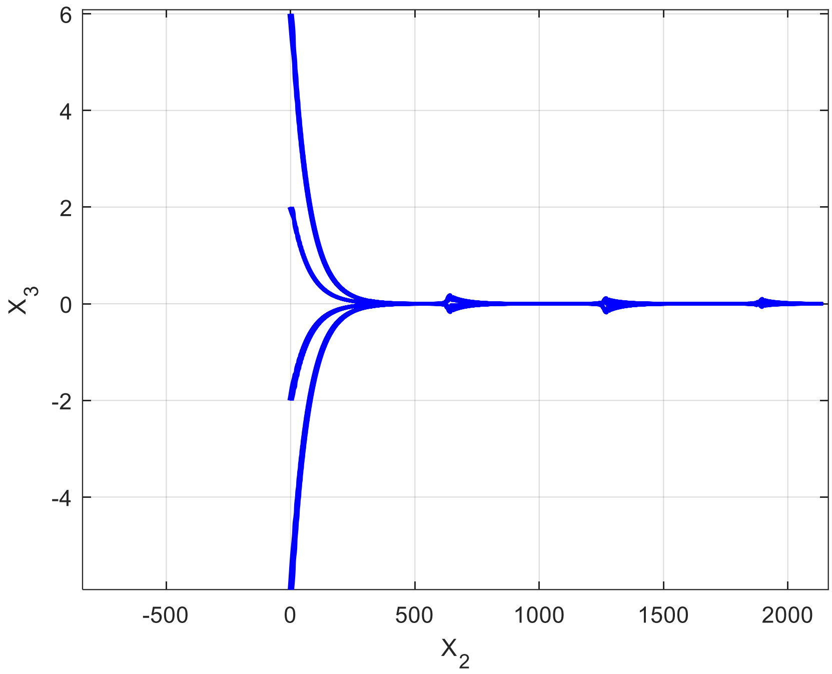

Figure 23. Behavior in $x_2 - x_3$ plane

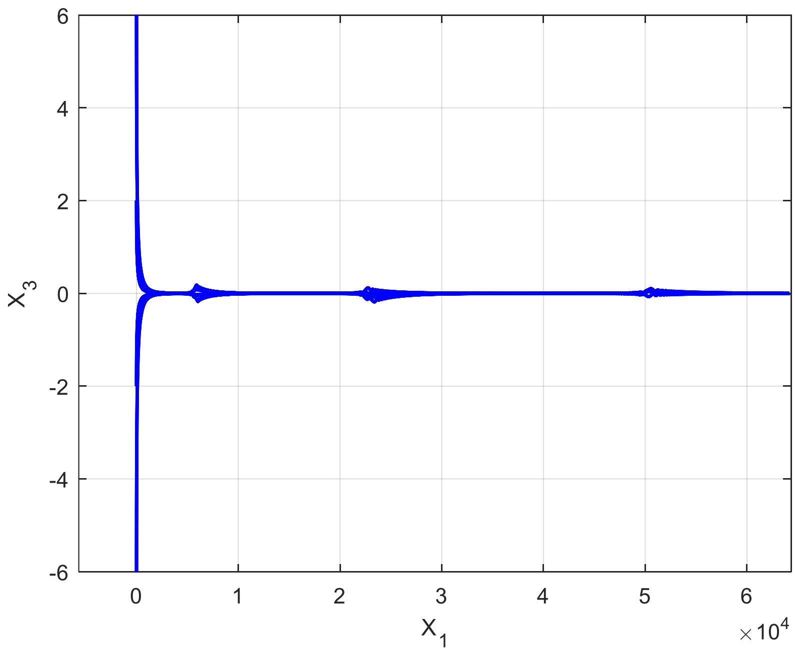

Figure 24. Behavior in $x_1 - x_3$ plane

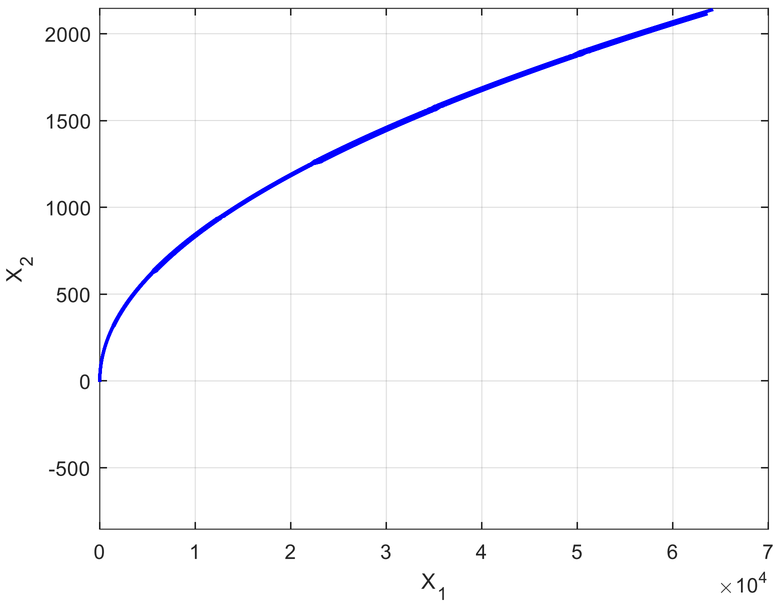

Figure 25. Behavior in $x_1 - x_2$ plane

Figures 19, 20, 21, and 22 show the time response of the system, which rotor angle increases slightly and the rotor speed increases to infinity respect to time. The variable (given the assumptions in the problem) in Figure 21 behaves more stable, so the goal is to control the variables. Also, Figures 23, 24, and 25 show the values of variables and go to infinity. Therefore, the nonlinear system is unstable. In this system using SOSTOOLS, the following are obtained:

$$\begin{cases} F_1 = [-0.0077146x_4 - 0.07597, -0.00091803x_4 + 0.042675, -0.33956, 0.34027] \\ F_2 = [0.0016756x_4 + 0.10108, 0.0001994x_4 - 0.0483, -0.33956, 0.34027] \end{cases} \quad (55)$$

Figures 26-29 show the result of the control by the stabilizer controller.

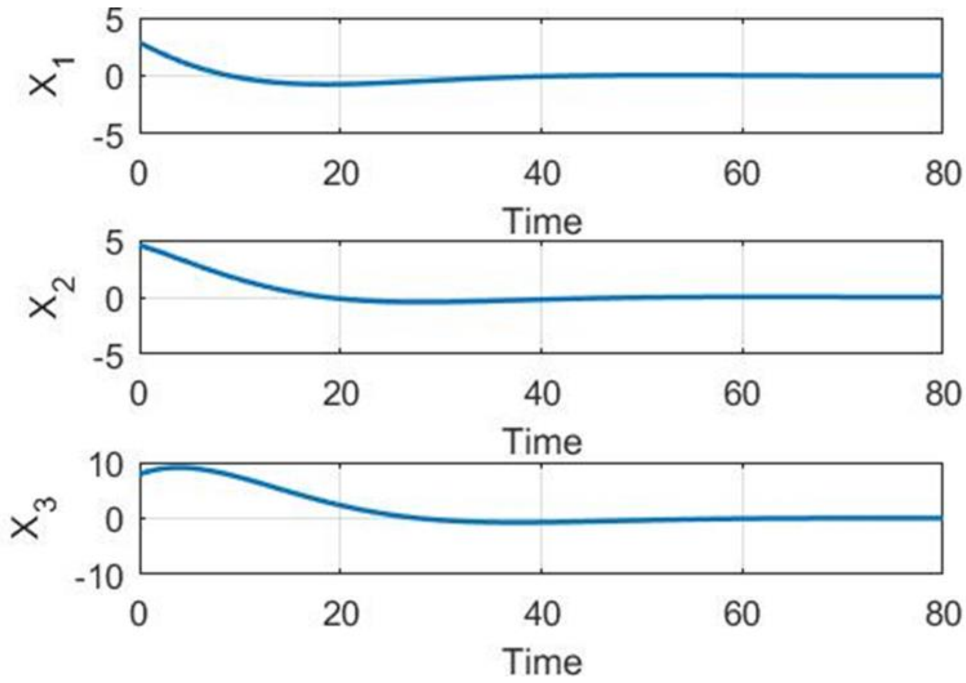

Figure 26. Power system responses

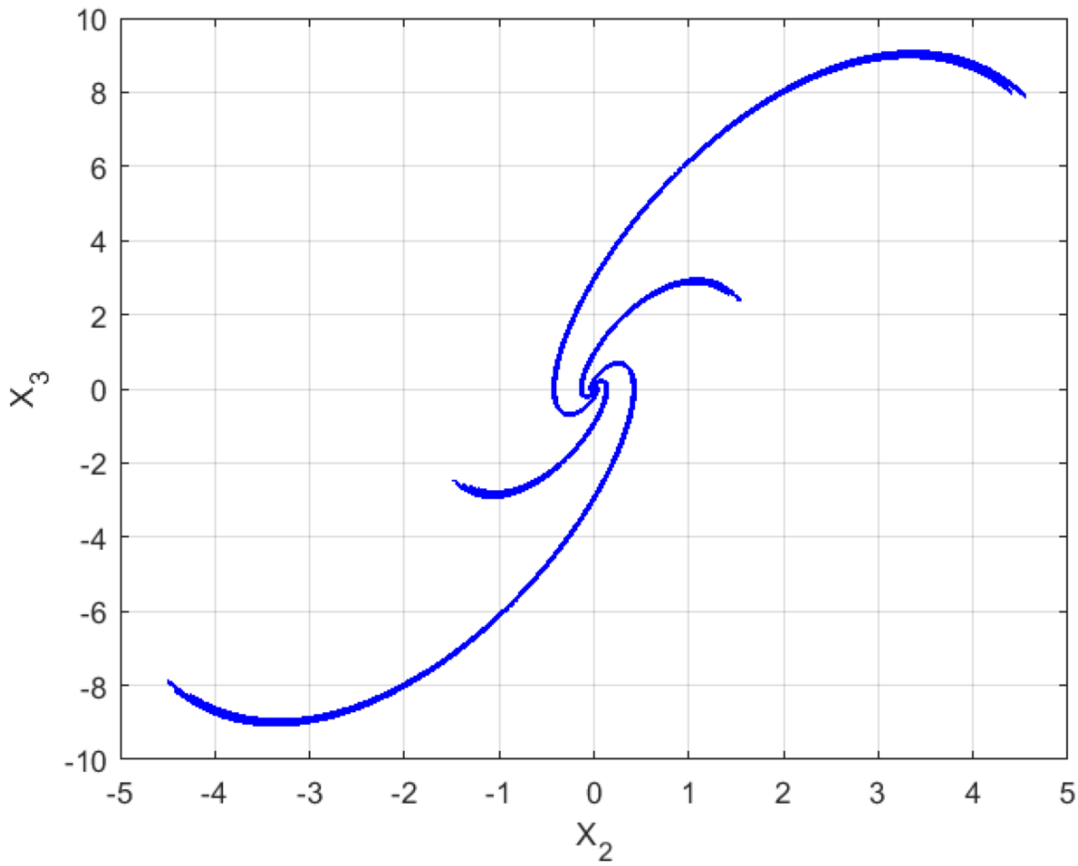

Figure 27. Behavior in $x_2 - x_3$ plane

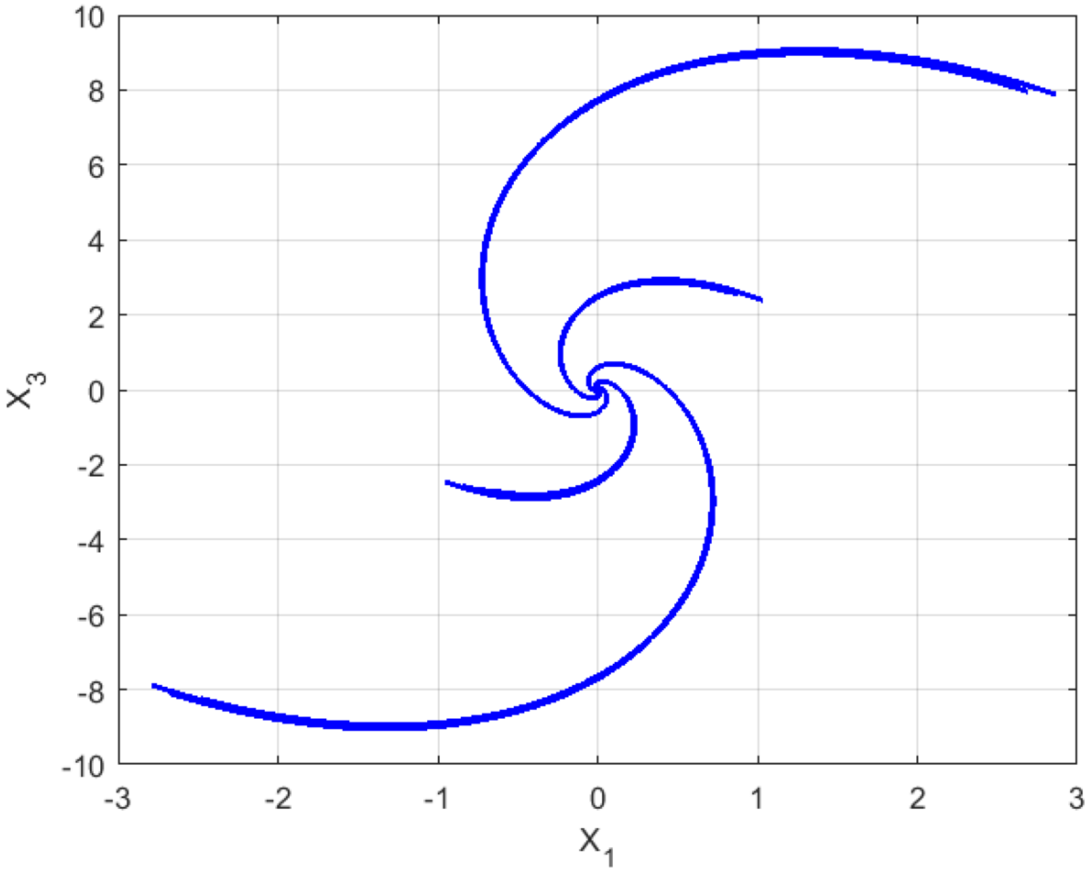

Figure 28. Behavior in $x_1 - x_3$ plane

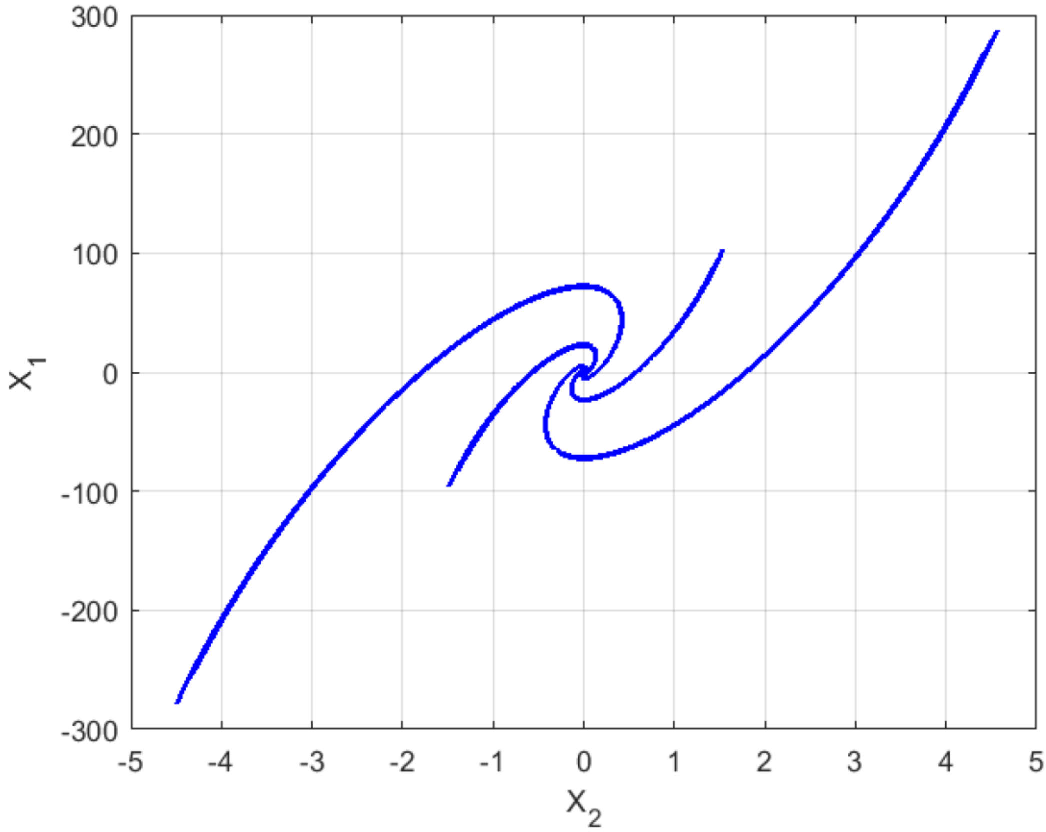

Figure 29. Behavior in $x_1 - x_2$ plane

Figure 26 shows the time response graph of the system, which indicates the stability of system, and variables reach their final value after a certain time. The system response after the design and implementation of the polynomial fuzzy controller has been improved and stabilized. The figures 27,28, and 29 show the stability of the system behavior in $x_1 - x_2$ plane using the obtained state feedback. As shown in figures, the unstable system is affected by the designed controller of the sum of square, and stabilize, finally. It should be noted that the path of the states in x_1 and x_2 plane approaches to the origin. In this case, the equilibrium point $x = 0$ is called the stable node. Consequently, the paths are said to be stable (as t increases, the paths lead to the origin).

8.Conclusion

This paper discussed the synchronous generators as a highly complicated system and its importance in power systems and their stability. Then, it presented SOS approach to modelling and control of the synchronous generator in terms of polynomial fuzzy systems as an efficacious method. First of all, the state equations of the synchronous generator were described. Secondly, a polynomial fuzzy modeling and control framework that is more general and effective than the T–S fuzzy model and control was introduced. Thirdly, stability of the fuzzy polynomial systems has been obtained based on polynomial Lyapunov functions that contain quadratic Lyapunov functions as a special case. The stability and stabilizability conditions presented in this paper are more general and relaxed than those of the existing LMI-based approaches to T–S fuzzy model and control. SOS-based approach offers less strict analysis and design conditions comparing to the current LMI approach. Stability conditions can be represented in terms of SOS and, then, can be numerically (partially symbolically) solved via the recently developed SOSTOOLS. The simulation results have been acquired for the generator system with Fuzzy Polynomial Controller and without it. Validity of the proposed approach was demonstrated using the third-part Matlab toolbox, SOSTOOLS.

Author Contributions: All the authors conceived the idea, developed the method, and contributed to the formulation of methodology and experiments. Moreover, all authors read and approved the final manuscript.

Conflicts of Interest: The authors declare no conflict of interest.

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