Article International Airline Alliance Network Design with Uncertainty

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Abstract: This paper addresses the alliance route network design problem considering uncertainty of unit transportation cost. An alliance route network is constructed based on the hub-and-spoke (HS) network, in which airlines can achieve inter-area passenger transport through their international gateways. The design problem is formulated with a robust model containing a set of uncertain cost parameters. The model is established based on the three-subscript model of the HS network. A case study collected from real-world data is used to test the proposed model. The results show that the robust solution can reduce the impact of cost uncertainty.

Keywords: alliance route network; network design; hub-and-spoke network; robust model.

1. Introduction

The international airline alliance composed of airlines is the joint collaboration of airline companies aiming at establishing a union of global route system for improving competitiveness through code sharing or the joint operations. Different from typical airline operation modes, the airlines in the alliances are coordinated and treated equally to maximize the profits. For the implementation of the international airline alliance, diverse airports provide services to support the alliance although with multiple restrictions in practice. By having the international alliance, i.e. a collaboration between airline companies between the countries, some benefits are highlighted. For instance, limitations in the regulatory or legal barriers between countries are released [1]; passenger capacity for airlines can be increased; and the chance of improving profits is improved by having larger number of airline destinations. Moreover, the international airline alliance benefits the passenger experience by ways of having access to larger networks, having convenient services, having more choices of carriers, and being connected to more flyer points [2]. International airline alliance enables an effective coordination of flight schedules so as to minimize the travelers' waiting time and provide a sufficient time usage between flights [3]. The handover of passenger loads among airports can be simplified, as well as sharing the same aircraft maintenance services to reduce the maintenance time within an airline alliance network. The international airline alliance also facilitates the quality of service (QoS), for instance establishing a flexible price negotiation model, optimizing the airline routes, and the reduction of airlines' running expense.

For the international airline alliance, three typical categories are grouped in accordance with the topology among airline members, i.e., Star Alliance, Oneworld, and SkyTeam. with initial partners located in the major geographic regions and often involved in bilateral partnerships among other founders. The scheduled passenger volume of the three major alliances accounts for more than 50% of the global civil aviation industry [4].

Distinct from the domestic airline alliance, the international alliance is more challenging given some practical considerations, such as the large number of uncertain factors, docking through separate gateways, the uncertainties in the agile policy, changeable economic and social factors, and Snyder effect in uncertainties [5].

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To address the airline alliance problem, the problem is separated into two specific issues [6], i.e. the selection of alliance model and the route network design.

Some typical alliance models with the partial connection type are highlighted, such as competition model, strategic model. Authors in [7] evaluated and discussed the performance of applying strategic model for the alliance problem, where three alliance types are classified in accordance with the alliance strategy that are the complementary alliance (also named vertical competition model), parallel alliance (also named horizontal competition model), and hybrid alliance. Among them, the complementary model is widely used for the airline alliance owing to its elimination of the negative externalities between routes [8], and benefits of reducing the connection fare and improvement of social welfare. The connection time of the flight transfer is possible to be shortened significantly with the complementary alliance model leading to the possibility of having seamless transfer (one ticket to the end at the place of the departure with the direct luggage hanging service [9]) in potential. Consequently, this paper implements the complementary model as the prototype for addressing the model selection issue.

The route network is the foundation of airline operation. By re-designing and adjusting the route network, the transportation cost of airlines is reduced along with having a risk resistant model to provide reliable services, and provide promising strategies for the airline expansion in the future. In specific, the hub-and-spoke (HS) network is a promising network type to formulate the route network due to its feasibility and efficiency in practice. Several researches have been done for addressing the airline network optimization problem with the HS network. Authors in relative articles (Alumur & Kara, [10]; Cambell & O'Kelly, [11]) examine single allocation, multiple allocation, capacitated, uncapacitated, strict, non-strict with extensions.

However, above presented works only work for the common alliance problem and assume the deterministic alliance models with few constraints introduced in the models[12], which requires a proposition to exploit uncertain models with complex factors for designing the international alliance networks. The common alliance model neglects the uncertainty representing the unknown situations may be encountered in the practical operations, and the flexible parameters that are challenging to be estimated. For those flexible parameters, a reasonable interval range or possible probability distribution is obtained by statistic, such as, the airport capacity distribution, airline capacity, demands among airports, and unit transportation costs. Furthermore, finding the worst case scenario with respect to a determined uncertainty set (Bertsimas, Brown & Caramanis [13]) is another gap which has not sufficiently investigated in the previous researches.

This paper explores the methods to narrow down gaps for the airline network optimization with uncertainties. We apply the uncertainty optimization method to optimize the design of alliance route network, and present an international route network with both economy and risk resistance for airlines. Specifically, we propose an approach enabled with the complementary alliance model with the HS route method for the optimization purpose. An uncapacitated multiple p-hub median problem (UMpHMP) based alliance model is proposed with the integration of uncertain factors. The network optimization problem is addressed by developing a three-subscript model and measured on the worst case scenario. Compared with typical network model structures [12], the proposed three-subscript optimization model is applicable for more than two airline alliances and multiple gateway numbers, where the uncertain factors can be integrated to meet the practical operation requirements.

The rest of this paper is organized as follows. Section 2 provides a brief literature review of airline alliance and HS network. Section 3 constructs an alliance route network based on HS network and proposes a mathematical model to formulate the alliance route network design problem with uncertain cost. A case is provided in Section 4 to illustrate the proposed model. Section 5 presents the conclusion and suggestion for future research.

2. Literature review

Regarding the multiple airline alliance models, several works have been done. Oum, Park and Zhang [14] investigate the alliance as the complementary and the parallel model with the conclusion of better economic performance with the complementary alliance model. Zhang [7] examines the hybrid alliances whose members have complementary and overlapping routes, and

discusses the implications of hybrid alliances for the international airline alliances. From the alliance cooperation perspective, Oum, Park, Kim and Yu [15] group alliances into high-level and low-level categories according to the participation degree in the cooperation. A high-level alliance involves network-level collaboration representing that the allied airlines connect their route networks, while the low-level collaboration considers the route-level cooperation neglecting the network topology considerations.

Some researches have been done when designing and optimizing the airline alliance network. Wen and Hsu [16] take factors of the flight frequency and cost into consideration for the alliance network design, where a multi-objective function is presented based on the code sharing among member airlines for the purpose of maximizing the overall profits. Adler and Smilowitz [12] investigate the international alliances and mergers under the competitive environment. The result reveals that the optimal international gateway choices vary depending on the number of the remaining competitors in the market. Lordan, Sallan and Simo [17] focus on the reliability analysis with the three typical network models for the alliance network, and propose an alternative node selection strategy to evaluate their robustness and vulnerability. Lordan and Klophaus [18] analyze the vulnerability of member airlines for exiting to the alliance. Their results suggest that the Oneworld is the most vulnerable alliance with SkyTeam ranked the second and Star Alliance the third.

For designing the optimized networks, some works have been done. O'Kelly [19] firstly applies HS network to address the minimization of the cost in designing hub networks. Campbell ([20], [21]) proposes an integer linear program based method with the consideration of single and multiple allocation schemes. Ernst and Krishnamoorthy ([22], [23]) propose variants oriented from the hub location-allocation problems with fewer variables. The above typical linear formulations are adjusted according to unique specifications along with processing some distinguished features. Among them, the parameter uncertainty has been studied in addressing HS network design problems. After investigations, Fageda and Flores-Fillol obtain the conclusion that the hub-and-spoke network structure remains advantages at the higher congestion costs in the hubs [24]. For building route networks with the consideration of the market competition, Jiang and Zhang consider the long-term impact resulted from the high-speed rail competition on airlines, and develop an analytical model to investigate the airline impact on networks as well as the impacts on markets when the high-speed rail competition in trunk lines exists [25]. Babić and Kalić discuss the airline models operating in a competitive environment for selecting a network structure. In order to capture the interaction between competing airlines in the selection, the impacts of price, flight frequency, seat accessibility and route length on product differentiation are thoroughly studied [26].

For the network optimization with uncertainties, some studies are implemented. Averbakh [27] formulates the deviation robust optimization problem, and designs an optimization algorithm to convert the original problem whose objective function is MinMax into a deterministic objective function. Wang and He [28] present a robust optimization model with the regret model format for the logistics central localization purposed in an uncertain environment, where the proposed model outperforms the stochastic optimization model by simulations. Szucs [29] takes the costs of network elements as the uncertain factors, and proposes the solution enabled by the Dempster-Shafer theory and Dijkstra's algorithm for planning routes in a transport network. Shahabi and Unnikrishnan [30] formulate a robust model for the hub localization problem with features of considering incapacitated single and multiple allocation schemes and the uncertain demands in prior. The robust model is transformed from a mixed integer nonlinear program into a mixed integer conic quadratic program in the propositions.

3. Airline Alliance Network

In this section, we present mathematical formulations for the alliance route network design problem considering uncertainty of cost parameters. Three objectives are considered in modeling: (1) determine the optimal location of international gateways; (2) determine network configuration including transport paths for demands and flow volume on each path and (3) make the international gateway location decisions risk-resistant in terms of unit transportation cost.

3.1. Problem formulation

One demonstration of alliance route network is displayed in Figure 1. The nodes of the network include spokes and hubs. As one of the complex HS route networks, such alliance route network architecture has capabilities of connecting network of different airlines. The hubs are separated into regional hubs (R) and international gateways (K). Regional hubs connect local airports (i) within an area, while international gateways have connection with different areas. Alliance airlines may choose one or more international gateways from regional hubs to transfer passengers from different areas.

Compared with HS network, the constructed alliance route network is characterized below:

1. The cooperation between alliance partners should meet the limitation of traffic rights.

2. The round-trip passenger flow of international routes usually differs greatly with asymmetry.

3. Usually, the domestic network of an airline is relatively complete before the airline joining an alliance, and the regional hubs can be given. However, airlines need to choose their international gateways from the set of regional hubs through optimization.

4. One or more international gateways coexist with each area for each airline, and all gateways are interconnected across international areas.





In order to reasonably reflect the practice and simplify the problem, the proposed model is subject to the following assumptions:

1. All international gateways are interconnected across international areas. Due to the scale economy that passengers converge at gateway airports, a discount factor α is incorporated into the cost between international gateways.

2. According to the actual transportation situation, international transportation usually does not exceed two transits, so airports other than the international gateways are regarded as "spokes" of international routes. Multi-allocation connection between "spokes" and international gateways is adopted. And χ is the discount factor from "spokes" to international gateways, δ is the discount factor from international gateways to "spokes". Generally, there is $0 \le \alpha < \chi, \delta \le 1$.

3. The regional hubs within each area are given and airlines choose their international gateways from this subset of regional hubs.

4. Inter-area journeys are limited to three legs, that is, if both the origin and destination nodes are "spokes", traveling across international areas will necessarily involve a three-leg journey. For example, to travel from i5 to i7 would involve one leg from i5-K1, a second leg from K1-K2 and finally a third leg from K2-i7. On the other hand, inter-area journeys contain at least one leg, and this happens when both the originating and destination nodes are the international gateways.

5. Inter-area traffic must all be transported from the originating node to the destination node.
6. The round-trip passenger demand of international routes usually are asymmetric, W_{ij} ≠ W_{ii}.

3.2. Alliance Route Network Model

Sets, parameters, and decision variables are introduced before describing the mathematical formulation.

Sets and Parameters:

A, set of international areas that need to establish alliances by airlines, $a \in A$.

N, set of all nodes in the network, $i \in N, j \in N$.

 N^a , set of nodes in the network of area $a \in A$, $N^a \subseteq N$.

H, set of all regional hubs in the network, $r \in H$, $k \in H$, $m \in H$, $t \in H$, $H \subseteq N$.

 H^a , set of regional hubs within area $a \in A$, $H^a \subseteq H$.

S, set of scenarios for the uncertain transportation cost, $s \in S$.

 W_{ij} , passenger demand from origin $i \in N^a$ to destination $j \in N \setminus N^a$.

 O_i , total traffic flow starting from node $i \in N^a$, $O_i = \sum_{i \in N \setminus N^a} W_{ii}$.

 $C_{ik}(s)$, unit transportation cost from "spoke" $i \in N^a$ to international gateway $k \in H^a$ under scenario $s \in S$;

 $C_{km}(s)$, unit transportation cost from international gateway $k \in H^a$ to international gateway $m \in H \setminus H^a$ under scenario $s \in S$;

 $C_{mj}(s)$, unit transportation cost from international gateway $m \in H \setminus H^a$ to "spoke" $j \in N^{(m)}$ under scenario $s \in S$, where $N^{(m)}$ denotes the set of nodes on the area containing gateway $m \in H \setminus H^a$.

Decision variables:

 h_k , 1 if an international gateway is located at node $k \in H^a$; 0 otherwise.

 Z_{ik} , total amount of flow from regional hub $i \in N^a$ to international gateway $k \in H^a$.

 Y_{km}^i , amount of flow from regional hub $i \in N^a$ to international gateway $k \in H^a$ that arrives at the gateway $m \in H \setminus H^a$.

 X_{mj}^i , amount of flow from regional hub $i \in N^a$ to node $j \in N \setminus N^a$ that through the international gateway $m \in H \setminus H^a$;

 $Z^*(s)$, the minimum total transportation cost of an alliance network constructed under scenario *s*.

Given the uncertain transportation effects, an alliance route network model is proposed as: $min \lambda$

subject to:

$$\sum_{\alpha \in A} \left[\sum_{i \in N^{\alpha}} \left(\sum_{k \in H^{\alpha}} \chi C_{ik}(s) Z_{ik} + \sum_{k \in H^{\alpha}} \sum_{m \in H \setminus H^{\alpha}} \alpha C_{km}(s) Y_{km}^{i} + \sum_{m \in H \setminus H^{\alpha}} \sum_{j \in N^{(m)}} \delta C_{mj}(s) X_{mj}^{i} \right) \right] \le (1 + \lambda) Z^{*}(s), \forall s \in S$$

$$(1)$$

$$\sum_{\alpha \in A} \sum_{k \in H^{\alpha}} h_k = p \tag{2}$$

$$\sum_{k \in H^{\alpha}} Z_{ik} = O_i; \forall i \in N^{\alpha}, \alpha \in A$$
(3)

$$\sum_{m \in H \setminus H^{\alpha}} X^{i}_{mj} = W_{ij}; \forall i \in N^{\alpha}, j \in N \setminus N^{\alpha}, \alpha \in A$$
(4)

$$\sum_{m \in H \setminus H^{\alpha}} Y_{km}^{i} = Z_{ik}; \forall i \in N^{\alpha}, k \in H^{\alpha}, \alpha \in A$$
(5)

$$\sum_{j\in N}^{(m)} X^{i}_{mj} = \sum_{k\in H^{\alpha}} Y^{i}_{km}; \forall i \in N^{\alpha}, m \in H \backslash H^{\alpha}, \alpha \in A$$
(6)

$$Z_{ik} \le O_i h_k; \forall i \in N^{\alpha}, k \in H^{\alpha}, \alpha \in A$$
(7)

$$X_{mj}^{i} \le W_{ij}h_{m}; \forall i \in N^{\alpha}, m \in H \setminus H^{\alpha}, j \in N^{(m)}, \alpha \in A$$
(8)

$$Z_{ik}, Y^{i}_{km}, X^{i}_{mj} \ge 0;$$

$$\forall i \in N^{\alpha}, k \in H^{\alpha}, m \in H \backslash H^{\alpha}, j \in N^{(m)}, \alpha \in A$$
(9)

$$h_k \in \{0, 1\}, \forall k \in H^{\alpha}, \alpha \in A \tag{10}$$

Formula (1) is the requirement of relative robust optimization, that is, for each design of alliance route network, the relative deviation between the total transportation cost and the optimal transportation cost under each scenario is calculated and the maximum relative deviation is required to be minimized. The total transportation cost in parentheses include collection cost, transfer cost and distribution cost. Constraints (2) indicate that the number of international gateways for an alliance network must be exactly p. Constraints (3) guarantee that all the traffic flow should be shipped out from the originating city. Constraints (4) guarantee that all the traffic flow should be delivered to the destination city. Constraints (5) and Constraints (6) are passenger flow balance constraints. Constraints (7) and Constraints (8) ensure that the flow transport through the international gateway is possible only if that gateway is open. Constraints (9) require that all the flow variables be non-negative, and Constraints (10) specify that the international gateway selection variables are binary.

3.3. Optimal Solution

The robust optimization of the alliance route network design model with uncertainty is to find minimum λ satisfying the constraint condition and route network design plan (selection of gateway and route, arrangement of OD flow) of minimum λ avoiding risks to the maximum extent. The solution principle is to give a relatively small constant value of λ (λ can be adjusted.), calculate the model (11) of each scenario with different gateway combination, and update λ to approach the minimum value of λ which the network design is continuously improved [31].

The model of each scenario with different gateway combination can be formulated.

$$Z(s) = \min \sum_{\alpha \in A} \left[\sum_{i \in N^{\alpha}} \left(\sum_{k \in H^{\alpha}} \chi(s) C_{ik} Z_{ik}(s) + \sum_{k \in H^{\alpha}} \sum_{m \in H \setminus H^{\alpha}} \alpha(s) C_{km} Y_{km}^{i}(s) + \sum_{m \in H \setminus H^{\alpha}} \sum_{j \in N^{(m)}} \delta(s) C_{mj} X_{mj}^{i}(s) \right) \right]$$
(11)

s.t.

$$\sum_{\alpha \in A} \sum_{k \in H^{\alpha}} h_k(s) = p \tag{12}$$

$$\sum_{k \in H^{\alpha}} Z_{ik}(s) = O_i(s); \forall i \in N^{\alpha}, \alpha \in A$$
(13)

$$\sum_{m \in H \setminus H^{\alpha}} X^{i}_{mj}(s) = W_{ij}(s); \forall i \in N^{\alpha}, j \in N \setminus N^{\alpha}, \alpha \in A$$
(14)

$$\sum_{m \in H \setminus H^{\alpha}} Y_{km}^{i}(s) = Z_{ik}(s); \forall i \in N^{\alpha}, k \in H^{\alpha}, \alpha \in A$$
(15)

$$\sum_{j\in N^{(m)}} X^{i}_{mj}(s) = \sum_{k\in H^{\alpha}} Y^{i}_{km}(s); \forall i\in N^{\alpha}, m\in H\backslash H^{\alpha}, \alpha\in A$$
(16)

$$Z_{ik}(s) \le O_i(s)h_k(s); \forall i \in N^{\alpha}, k \in H^{\alpha}, \alpha \in A$$
(17)

$$X_{mj}^{i}(s) \le W_{ij}(s)h_{m}(s); \forall i \in N^{\alpha}, m \in H \setminus H^{\alpha}, j \in N^{(m)}, \alpha \in A$$
(18)

$$Z_{ik}(s), Y_{km}^{i}(s), X_{mj}^{i}(s) \ge 0;$$

$$\forall i \in N^{\alpha}, k \in H^{\alpha}, m \in H \setminus H^{\alpha}, j \in N^{(m)}, \alpha \in A$$
(19)

$$h_k(s) \in \{0,1\}, \forall k \in H^{\alpha}, \alpha \in A$$

$$\tag{20}$$

The proposed solver is presented as follows in the Algorithm 1.

Algorithm 1 An iterative optimization algorithm for the alliance route network.

- 1: Initialization $Z^*(s) = +\infty$, s = 1, ..., S, $\lambda = T$, T is a small positive number.
- 2: Select p regional hubs as international gateways. The possible combinations correspond-
- ing to p international gateways are $C_{H^a}^{a'} \times C_{H^b}^{b'} \times C_{H^c}^{c'} \times ...$, one of the combination is represented by H, H^a represents the set of optional hubs in area a. $\{a, b, c, ...\} = A$, $a' + b' + c' + ... = p, a', b', c' \ge 1$.
- 3: For the combination *H*, use the model (11) to find the solution under scenario s, s = 1, ..., S, obtain the total transportation cost Z(H, s).
- 4: For scenario s, if $Z(H,s) \ge Z^*(s)$, turn to Step 5, else $Z(H,s) < Z^*(s)$, renew $Z^*(s) = Z(H,s)$, turn to Step 5.
- 5: If $\frac{Z(H,s)-Z^*(s)}{Z^*(s)} \leq T$ for all s = 1, ..., S, output $\lambda = T$, $H^* = H$, H^* is the optimal international gateway combination, else turn to Step 6.
- 6: if $\frac{Z(H,s)-Z^*(s)}{Z^*(s)} \ge T$ for some s, turn to Step 2.

4. Case Study

4.1. Data settings

In this section, we select a Chinese-based and US-based airline alliance network as the general example to examine the performance of the proposed models, where the use case can be easily extended to other scenarios. We select 10 cities in China and 8 cities in US to form the route network and implement the airline alliance. The distribution of airports and relevant parameter configurations are presented in Table 1.

Node	Airport	Node	Airport	Node	Airport
1	PEK*\Beijing	7	CKG\Chongqing	13	DTW*\Detroit
2	CAN\Guangzhou	8	XIY*\Xi'an	14	LAX*\Los Angeles
3	PVG*\Shanghai	9	WUH*\Wuhan	15	MSP*\Minneapolis
4	CTU\Chengdu	10	NKG\Nanjing	16	SFO\San Francisco
5	SZX\Shenzhen	11	ATL*\Atlanta	17	SEA\Seattle
6	KMG*\Kunming	12	JFK*\New York	18	ORD\Chicago

Table 1: City airport

The city airports marked with * are the regional hubs among corresponding airlines. Transportation between these regional hubs is assumed to meet the traffic regulations. The crucial data such as the passenger demands are sourced from the Market Information Data Transfer Database and segment distance data [32].

We apply the unit transportation cost function to measure the distance between the nodes (with $C_{rr} = 0$). We use the cost per available seat kilometer (CASK) with the great circle distance between the relevant nodes for measuring the cost parameters [33] due to the varying CASK values affected by the distance. Moreover, the uncertain factors can be conveniently integrated in CASK as the main indicator. Given the published three aircraft categories, i.e. wide-body, narrow-body and regional jets, the average CASK is computed accordingly. The wide-body aircraft serve long distance markets (distance > 5000 km), narrow-bodies serve distances between 1000 and 5000 km and regional jets are utilized in the short distance markets (distance < 1000 km). Similar to Adler et al. (2018) [33], the CASK values used in this research are presented in Table 2.

Table 2: Costs per available seat kilometer

	Short Haul	Medium Haul	Long Haul
CNY per ASK	0.3724	0.2894	0.2941

To assess the impact of uncertainty in transportation cost on the design of alliance network, we assume that the values of the CASK fluctuate within intervals [-20%, +20%]. We generate several scenarios by changing the CASK magnitudes, as shown in Table 3. As risk control is critical in the robust optimization, boundary scenario should be considered in the robust optimization. There is no special requirement on the number of scenarios.

Scenario	CASK-SH	CASK-MH	CASK-LH
number	deviation (%)	deviation (%)	deviation (%)
1	0	0	0
2	20	20	20
3	-20	-20	-20
4	20	20	-20
5	20	-20	20
6	-20	20	20
7	20	-20	-20
8	-20	20	-20
9	-20	-20	20

Table 3: Scenarios

As shown in Table 3, we generate nine different scenarios for a combination of the three CASK, among which scenario 1 is called the base case. In each scenario $s \in S$, we can get a set of values of unit transportation cost.

Based on the model formulation in Section 3.3, we know that it is a linear program. The model is coded under AIMMS platform and solved by CPLEX12.5 with CPLEX options set to their default values. All tests are executed using a personal computer running the Microsoft Windows 7 operation system and equipped with Intel Core i5 CPU 6500 3.20 GHz and 4 GB RAM.

4.2. Computational results

In this section, we present computational analysis with the robust optimization model of alliance route network design problem, to assess the effects of uncertainty in the unit transportation cost on the resulting solutions. We assume that the number of international gateways selected from the two areas is three. The collection and distribution discount factors are taken equal to one; i.e., $\chi = \delta = 1$. For the alpha value, on the other hand, we let $\alpha \in 0.2, 0.4, 0.6, 0.8$, as it is customarily done in the literature.

We solve the problem separately under each scenario with the goal of minimizing the total transportation cost and also solve the robust model based on the nine scenarios. The results of optimal international gateway locations and the total transportation costs are presented in Table 4.

Note: s1...s9: scenario number; RM: robust optimization model; Intl. GL: optimal international gateway locations; Trans. costs: total transportation costs in the respective optimal solutions.

Note that in robust solutions total transportation costs are different under each scenario, hence, the value of the "Trans. Costs" are left empty. Observe from Table 4 that, for each scenario, the optimal international gateway locations are not always the same when the alpha value changes. For instance, the solution of the base case suggests locating international gateways at Node 3, 12, 14 for $\alpha = 0.2$, Node 1, 12, 14 for $\alpha = 0.4$ and $\alpha = 0.6$, and Node 1, 13, 14 for $\alpha = 0.8$. This proves that the optimal solutions are sensitive to the alpha value. In addition, the magnitude of the total transportation costs increases with the increase of the alpha value.

	lpha=0.2		lpha=0.4		lpha=0.6		lpha=0.8	
	Intl. GL	Trans. costs						
s1	3, 12, 14	223129610	1, 12, 14	365549030	1, 12, 14	506512065	1, 13, 14	647009456
s2	3, 12, 14	267755532	1, 12, 14	438658836	1, 12, 14	607814479	1, 13, 14	776411347
s3	3, 12, 14	178503688	1, 12, 14	292439224	1, 12, 14	405209652	1, 13, 14	517607565
s4	3, 12, 14	208191272	1, 12, 14	325888407	1, 12, 14	438658836	1, 12, 14	551429264
s5	1, 12, 14	241491590	1, 12, 14	410647234	1, 12, 14	579766838	1, 14, 15	743667894
s6	3, 13, 14	260392325	1, 13, 14	429436112	1, 13, 14	596487252	1, 13, 14	763538391
s7	3, 12, 14	183698164	3, 12, 14	297876805	1, 12, 14	410647234	1, 12, 14	523417662
s8	3, 13, 14	201692264	1, 13, 14	318068686	1, 13, 14	429436112	1, 13, 14	540803538
s9	1, 12, 14	236054009	1, 12, 14	405209652	1, 14, 15	572475719	1, 14, 15	736129225
RM	3, 12, 14		1, 12, 14		1, 12, 14		1, 12, 14	

Table 4: Results with uncertain unit transportation cos	sts
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For each alpha value, the optimal international gateway locations may be different in each of the scenarios. For example, when $\alpha = 0.8$, although Node 1 and Node 14 are selected as international gateways in all the scenarios, the third gateway is not always the same. Specifically, the optimal solution in Scenario 1, 2, 3, 6, 8 is $\{1, 13, 14\}$, while in Scenario 4, 7 is $\{1, 12, 14\}$, and in Scenario 5, 9 is $\{1, 14, 15\}$. If an alliance route network is constructed based on the optimal solution in the base case, the international gateways are PEK, DTW and LAX, which will be different from the optimal gateways PEK, JFK and LAX in Scenario 4, 7 and the optimal gateways PEK, LAX and MSP in Scenario 5, 9. This suggests that the selected international gateways for the base case will no longer be optimal in some other scenarios. In general, only when the cost data in a certain scenario is optimal, the solutions might not be optimal in other scenarios and may even deviate significantly from the original optimal solutions.

We can see from Table 4 that the optimal solutions of the robust model are different than the solutions obtained under some scenarios. In order to illustrate that the robust solution can adapt to many possible scenarios with different cost parameters, we take $\alpha = 0.8$ as an example and calculate the relative deviation of the robust solution in each scenario. The results are shown in Figure 2.



Figure 2. Relative deviations

The calculation results show that in each scenario, the relative deviation of the robust solution is small. Even in the worst case such as Scenario 8 (-20%, 20%, -20%), the relative deviation is 1.68%. In other words, even if the robust solution $\{1, 12, 14\}$ differs from the optimal solution under Scenario 8, the relative deviation from the total cost of the optimal solution $\{1, 13, 14\}$ does not exceed 2%. In addition, we can find that even though the robust solution is the same as the optimal solution of Scenario 4 and 7, there is a certain deviation in the total transportation cost. This is due to the difference in transport paths and flow volume on each path.

It concludes that since optimal solutions are sensitive to the unit transportation costs, with high amount of uncertainty in the unit transportation costs, it is better to adopt the solution obtained with the robust model instead of adopting the solution of a particular scenario. It proves that the model presented in Section 2.2 can realize the transportation of alliance routes and reduce the impact of cost uncertainty on the design of alliance route network.

Taking $\alpha = 0.8$ as an example, the international gateways constructed according to the robust solution are PEK, JFK and LAX. Connections between the city airports can be seen in Figure 3.

The dotted lines indicate the connections between "spokes" and international gateways, and the solid lines indicate the connections between the international gateways. PEK is selected as the gateways of China, JFK and LAX are selected as the gateways of the US. Thus, passengers across the two areas can be transported through the links between the international gateways.

To evaluate the impact of the number of international gateways on robust solutions, we calculate the optimal solutions for different ρ values according to different alpha values, as shown in Table 5.

	$\rho=2$	$\rho=3$	$\rho=4$	<i>ρ</i> =5	
$\alpha = 0.2$	3, 14	3, 12, 14	1, 3, 12, 14	1, 3, 12, 13, 14	
$\alpha = 0.4$	1, 14	1, 12, 14	1, 3, 12, 14	1, 3, 12, 13, 14	
$\alpha = 0.6$	1, 14	1, 12, 14	1, 3, 12, 14	1, 3, 12, 14, 15	
$\alpha = 0.8$	1, 14	1, 12, 14	1, 3, 12, 14	1, 3, 12, 14, 15	

Table 5: Optimal results for different numbers of international gateways

As can be seen from Table 5, for different alpha values, the frequency of each gateway selection is not same. In our case, for example, when $\alpha = 0.6$ or $\alpha = 0.8$, the frequency of gateway selection is 1, 14 > 12 > 3 > 15. With the number of international gateways increasing, the total transportation cost of the alliance network continues to decrease. The model proposed in this paper can select gateway location and network configuration, and can also provide more decision-making option for airline partner.

Through simulations, we obtain more results. The big number of gateways attribute to the excessive diversion and flow dispersion leading to the discount capacity of inter-hub transportation furthermore. The small number of gateways lead to the limited choices of gateways for the origin-destination (OD) flow transfer resulting in serious bypass transportation. Consequently, the gateway number should be seriously considered for the airline and airline partners when constructing the alliance route network. Specifically, when multiple regional hubs have better acknowledge of uncertain parameters, the gateway number can be increased so as to improve the coordination capability among airlines. Moreover, by having the robust optimization formulation, the deviation under uncertain cost can be reduced, which makes the optimized route network more resistant to risks and economic decisions.

5. Conclusions

The international alliances are built on the extensions of existing bilateral relationships and are implemented to allow the largest international carriers in the world to link their routes and frequent flyer programs into international networks. The rational construction of the alliance route network is important for the airlines to maximize the use of the alliance network resources to expand its network.

In this paper, we construct an alliance route network based on the HS airline network. Uncertainties are taken into account in designing the alliance network. We build an alliance route network optimization model considering uncertainty of unit transportation costs by modifying the three-subscript model of HS network. To date, this has not been proposed in the literature. The proposed model is tested on a real world dataset of a China-based airline and a US-based airline. The optimal alliance route network shows that two airlines have achieved inter-area transport through their international gateways. The calculation results indicate that the optimal solutions are sensitive to the unit transportation costs, and it is better to adopt the solution obtained with the



Figure 3. Alliance route network optimization design

robust model instead of adopting the solution of a particular scenario. By evaluating the impact of the number of gateways on robust solutions, we conclude that the model proposed in this paper can reduce the total transportation cost compared with previous studies.

The proposed model and method enable the route network design with the resistant capability to risks and economic decisions. The model is extended from the three-subscript route network model, which is promising for addressing the airline alliance problems.

The methodology developed in this paper shows potentials to other alliance network optimization problems with more complicated structures, such as the capacity limitation among the hub airports with the consideration of the passenger demands and cost uncertainties. The dynamic and flexible cost impacts on the optimization performance, along with the real-time estimation of costs also demand further investigations.

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References

- 1. Oum, T.H.; Park, J.H. Airline alliances: current status, policy issues, and future directions. J Air Transp Manag 1997, 3, 133–144.
- 2. Adler N, H.E. Regulating inter-firm agreements: The case of airline codesharing in parallel networks. *Transp Res B-Meth* 2016, 84, 31–54.
- 3. Park J H, Zhang A, Y. Analytical models of international alliances in the airline industry. *Transp Res B-Meth* 2001, 35, 865–886.
- 4. https://www.iata.org.
- 5. Snyder, L.V. Facility location under uncertainty: a review. *IIE trans* 2006, *38*, 547–564.
- 6. Brueckner, J.K. The economics of international codesharing: an analysis of airline alliances. Int J Ind Organ 2001, 19, 1475–1498.
- 7. Zhang, A. Competition models of strategic alliances. *Res Transp Econ* 2005, 13, 75–100.
- 8. Ji Jie, l.Y. Flight frequency, congestion cost and complementary aviation alliance: congestion analysis of hub and spoke network airports. *Manag Rev* **2012**, *24*, 164–170.
- 9. Jiayue, L. Optimization design of airline route network. *Electr Technol Softw Eng* 2019, 20, 4–5.
- 10. Alumur, S.; Kara, B.Y. Network hub location problems: The state of the art. Eur J Oper Res 2008, 190, 1–21.
- 11. Szűcs, G. Route planning based on uncertain information in transport networks. Transport 2012, 27, 79-85.

- 12. Adler, N.; Smilowitz, K. Hub-and-spoke network alliances and mergers: Price-location competition in the airline industry. *Transp Res B-Meth* **2007**, *41*, 394–409.
- 13. Bertsimas, D.; Brown, D.B.; Caramanis, C. Theory and applications of robust optimization. SIAM review 2011, 53, 464-501.
- 14. Oum, T.H.; Park, J.H.; Zhang, A. Globalization and strategic alliances: The case of the airline industry; 2000.
- 15. Oum, T.H.; Park, J.H.; Kim, K.; Yu, C. The effect of horizontal alliances on firm productivity and profitability: evidence from the global airline industry. *J Bus Res* **2004**, *57*, 844–853.
- 16. Wen, Y.H.; Hsu, C.I. Interactive multiobjective programming in airline network design for international airline code-share alliance. *Eur J Oper Res* **2006**, *174*, 404–426.
- 17. Lordan, O.; Sallan, J.M.; Simo, P.; Gonzalez-Prieto, D. Robustness of airline alliance route networks. *Commun Nonlinear Sci* 2015, 22, 587–595.
- Lordan, O.; Klophaus, R. Measuring the vulnerability of global airline alliances to member exits. *Transp Res Proc* 2017, 25, 7 16. World Conference on Transport Research - WCTR 2016 Shanghai. 10-15 July 2016.
- 19. O'kelly, M.E. A quadratic integer program for the location of interacting hub facilities. Eur J Oper Res 1987, 32, 393-404.
- 20. Campbell, J.F. Integer programming formulations of discrete hub location problems. Eur J Oper Res 1994, 72, 387–405.
- 21. Campbell, J.F. Hub location and the p-hub median problem. Oper Res 1996, 44, 923–935.
- 22. Ernst, A.T.; Krishnamoorthy, M. Efficient algorithms for the uncapacitated single allocation p-hub median problem. *Locat Sci* **1996**, *4*, 139–154.
- 23. Ernst, A.T.; Krishnamoorthy, M. Exact and heuristic algorithms for the uncapacitated multiple allocation p-hub median problem. *Eur J Oper Res* **1998**, *104*, 100–112.
- 24. Fageda X, F.F.R. A note on optimal airline networks under airport congestion. Econ Lett 2015, 128, 90–94.
- 25. Jiang C, Z.A. Airline network choice and market coverage under high-speed rail competition. Transp Res A-Pol 2016, 92, 248–260.
- 26. Babić D, K.M. Modeling the selection of airline network structure in a competitive environment. J Air Transp Manag 2018, pp. 42–52.
- 27. Averbakh, I. Minmax regret solutions for minimax optimization problems with uncertainty. Oper Res Lett 2000, 27, 57-65.
- 28. Baohua, W.; Shiwei, H. Robust optimization model and algorithm for logistics center location and allocation under uncertain environment. *J Transp Syst Eng Inform Technol* **2009**, *9*, 69–74.
- 29. Campbell, J.F.; O'Kelly, M.E. Twenty-five years of hub location research. Transp Sci 2012, 46, 153–169.
- 30. Shahabi, M.; Unnikrishnan, A. Robust hub network design problem. Transp Res E-Log 2014, 70, 356–373.
- 31. Jinfu, Z. Air transport planning; Northwest University of Technology Press: China, 2009.
- 32. http://www.gcmap.com.
- 33. Adler, N.; Volt. The multi-airline p-hub median problem applied to the African aviation market. Transp Res A-Pol 2018, 107, 187 202.