

# *Irreversibility analysis in Darcy-Forchheimer flow of nanofluid by a stretched surface*

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**Abstract:** The aim of this articles is to investigate the entropy optimization in unsteady MHD flow Darcy-Forchheimer nanofluids towards a stretchable sheet. The surface we tend to think about is porous and stretchy under acceleration. Flow occurs due to the stretching of the surface. Four distinct types of aqueous nanostructures are taken in this examination where copper oxide ( $CuO$ ), copper ( $Cu$ ), titanium dioxide ( $TiO_2$ ) and aluminum oxide ( $Al_2O_3$ ) are the nanoparticles. Irreversibility analysis are discussed through second law of thermodynamics. The expression of energy is mathematically designed and discussed according to heat generation / absorption, dissipation, thermal radiation, and joule heating. The nonlinear PDE (partial differential conditions) is first changed to ODE (normal differential conditions) through the use of appropriate similarity variables. Here we used the numerically embedded solution technique to develop a numerical result for the obtained nonlinear flow expression. Influence of various flow parameter velocity temperature distribution and entropy generation are discussed. Reduction occurs in velocity profile for larger porosity and magnetic parameters. An enhancement in entropy generation and temperature distribution is seen for Brinkman number. An opposite effect is noticed in velocity and temperature through solid volume friction.

**Keywords:** Darcy-Forchheimer; Surface stretch ; Unsteady boundary layer; MHD flow; Nanofluid; Heat generation / absorption and entropy analysis.

## **1: Introduction**

Nanomaterials can be described as a colloidal suspension of nano-size solid particles (1-100 nm) scattered uniformly in a base liquid. Nanoparticles are made from materials that are chemically stable like metals, oxides ceramics, carbides and non-metals such as graphite and CNTs. The commonly used base fluids are ethylene, water, oils and lubricants. Choi [1] worked on the enhancement of thermal conductivity of conservatively utilizable material (base liquid) through the insertion

of nanoparticles into it. Nanofluids enhance the thermophysical properties of the base fluids used. Stability of nano liquid is more than compared to other fluids. Nguyen et al. [2] discussed thermal conduction augmentation of water ( $H_2O$ ) base liquid with presence of Aluminium oxide. Thermal conductivity performance of nanomaterials depends on number of various parameter and properties scattered stages, molecule concentration, molecule measure, morphology and surfactants or dispersion. Nanofluids are produced in industries by several methods. One of the processes is where the nanoparticles are produced through gas condensation and then they are inserted into the base liquid. Ultrasound is used in process in order to make sufficient amalgamation of the particles and the base fluids [3-4]. Moreover, there is a relationship among the thermal conductivity and molecule size of nano particles' just as the temperature of the based liquid decreasing the particle size. There is an increment in the thermal transportation of nanofluid mixtures [5-6]. Irreversibility analysis in Darcy-Forchheimer CNT based nanoliquid flow with dissipation effect between rotating stretchable disk is explored by Khan et. al. [7]. Thermal transfer analysis in transient unsteady MHD flow of hybrid nanomaterials subject to porous stretching surface is illustrated by Nabwey and Mahdy [8]. Murshed et al. [9] explored the heat conductivity and consistency of nano-liquids both tentatively and hypothetically. Heat transfer analysis in magnetohydrodynamic reactive of viscous liquid with convective condition past a vertical plate is discussed by Makinde and Aziz [10]. Thermal and solutal transfer effect in ferro-nanoliquid flow with Lorentz and Soret effect is highlighted by Sabu. et al. [11]. Some relevant studies about nanomaterials flow are presented in Refs [13-20]. By keeping the origin of sheet fixed the two forces are applied, which are opposite in direction but equal in magnitude. Then such type of sheet is known as stretching sheet. In fluid dynamics the flow of fluid can be carried out by moving the boundaries of plates, applying the pressure gradient and the buoyancy forces. Many researchers have worked on the problems in which the flow is induced by stretching the surface and moving the plates due to its wide application in industry and engineering. Turkyilmazoglu [21] reported the magnetohydrodynamic flow of nanoliquids through an unstable dilation/compression surface. He demonstrated an accurate and analytical solutions. Time dependent MHD flow of pseudo-plastic nanoliquid with heat source is reported by Lin et al. [22]. Crane [23] revealed an MHD stream incited by a twisting/extending surface. Rasool et al. [24] announced a Darcy connection in nanofluid stream over nonlinearly extending sheet/surface. In many traditional industrial and mechanical fields, entropy generation minimization (EGM) is applied whenever fluid flow and heat transport is involved, enhancing flow engineering along with working conditions to improve hydroelectric performance. Focused. all practical and real life applications involved the study of EG in liquids, for example generators, Air conditioners heat pumps, Heat engines and power plants. The entropy generation

(EG) depends mainly on the following factors: heat transfer, chemical reactions, mass transfer, power transmission, viscosity losses and simultaneous effects of thermal and solutal. Bianco et al. [25] investigated the entropy generation of turbulent conductive flow water nanofluids in a tube in which constant wall heat flow conditions, in which they reported that for low concentrations of nanoparticles, the total entropy generation could be minimized. The stagnation point flow of MHD nanomaterials with minimal entropy generation on a stretched surface is explored by Rashidi and Bhatti [26]. Some modern irreversibility problem are mentioned in Refs. [27-37].

The theme of this article is to investigate the entropy generation in time dependent Darcy-Forchheimer flow towards a stretch surface. Energy expression are modeled through joule heating, heat source/sink and dissipation effects. Physical features of irreversibility analysis are examined thermodynamic second law. Ordinary system are obtained through similarity transformation. The obtained system are numerically solved through bvp4c Matlab software. Influence of various flow variables on velocity field temperature distribution and entropy generation rate. Thermo physical behaviors of nanofluid and base fluid are highlighted in Tables 1 and 2.

## 2: Modeling

Consider magnetohydrodynamics time dependent Darcy-Forchheimer flow towards a stretchable surface. Here water is considered as base liquid and four different type of nano particles (i.e. copper oxide ( $CuO$ ), copper ( $Cu$ ), titanium dioxide ( $TiO_2$ ) and aluminum oxide ( $Al_2O_3$ )) are considered. Energy expression are developed through joule heating, heat generation/absorption and dissipation effect. Physical description of irreversibility analysis are discusses through second law of thermodynamics. Consider  $U_w = (\frac{ax}{1-ct})$  is the stretching velocity with positive rate constant ( $a > 0$ ). Magnetic field of constant strength ( $B_0$ ) is applied. Flow sketch is highlighted in Fig. 1.

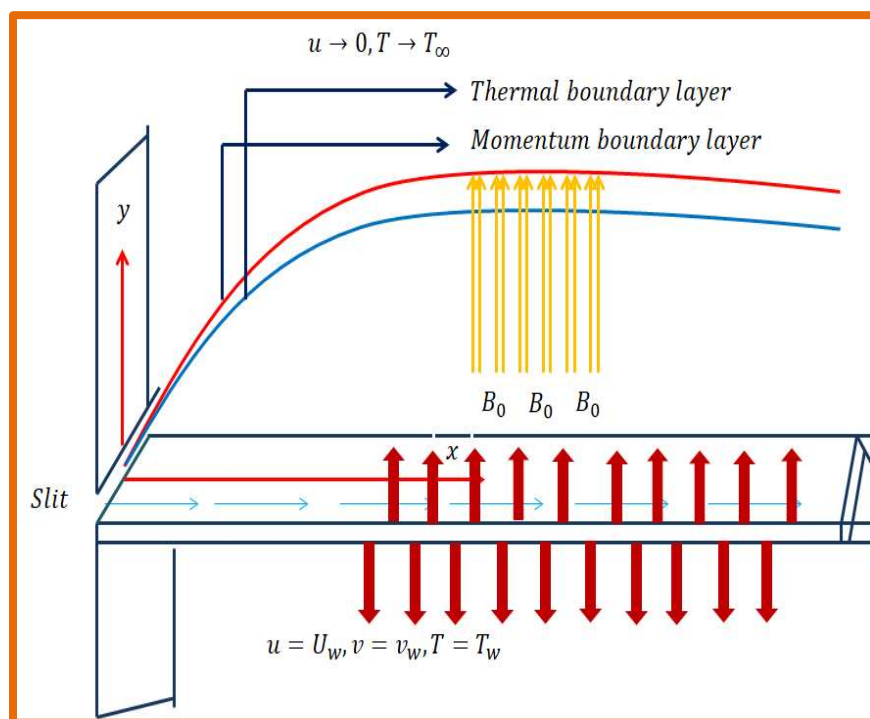


Fig. 1: Flow sketch.

Governing expression satisfy

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$	(1)
$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{nf} B_0^2}{\rho_{nf}} u - \frac{\nu_{nf}}{\kappa^*} u - F u^2,$	(2)
$\left. \begin{aligned} \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \frac{\kappa_{nf}}{(\rho C_p)_{nf}} \frac{\partial^2 T}{\partial y^2} + \frac{\mu_{nf}}{(\rho C_p)_{nf}} \left( \frac{\partial u}{\partial y} \right)^2 \\ &+ \frac{\sigma_{nf}}{(C_p \rho)_{nf}} B_0^2 u^2 + \frac{Q_0}{(C_p \rho)_{nf}} (T - T_\infty) \end{aligned} \right\},$	(3)

With

$\left. \begin{aligned} &\text{for } t \leq 0, u = 0 = v, T = T_\infty \text{ for all } (x, y) \\ &t > 0; u = U_w = \frac{ax}{1-ct}, v = v_w(t) = \frac{v_0}{\sqrt{1-ct}}, T = T_w(x, t) = T_\infty + \frac{bx}{(1-ct)^2}, y = 0 \\ &u \rightarrow 0, T \rightarrow T_\infty, \text{ as } y \rightarrow \infty. \end{aligned} \right\}$	(4)
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Here  $(x, y)$  denotes the Cartesian coordinates,  $(u, v)$  the velocity components,  $\sigma_{nf}$  the electrical conductivity,  $\rho_{nf}$  the density,  $\mu_{nf}$  the dynamic viscosity,  $\nu_{nf}$  kinematic viscosity,  $F(= \frac{C_b}{x\sqrt{\kappa^*}})$  non-uniform inertia coefficient of porous medium,  $k_{nf}$  the thermal conductivity,  $T_\infty$  the ambient temperature,  $(\rho C_p)_{nf}$  the heat

capacitance,  $T_w$  the wall temperature and  $Q_0$  coefficient of heat generation/absorption.

here subscript  $nf$  stand for nano fluid,  $s$  for solid particles.

Table 1: Mathematical expression of thermophysical properties of nanofluids [38-39]:

Characteristic of Nanofluid	Mathematical Form
Viscosity	$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}$
Viscosity	$\rho_{nf} = \rho_f \left(1 - \phi + \phi \frac{\rho_s}{\rho_f}\right),$
Heat Capacity	$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_s + \phi(\rho C_p)_f,$
Thermal Conductivity	$\frac{\kappa_{nf}}{\kappa_f} = \frac{(\kappa_s + 2\kappa_f) - 2\phi(\kappa_f - \kappa_s)}{(\kappa_s + \kappa_f) + \phi(\kappa_f - \kappa_s)},$
Electrical Conductivity	$\frac{\sigma_{nf}}{\sigma_f} = [1 + \frac{3(\sigma-1)\phi}{(\sigma+2) - (\sigma-1)\phi}], \sigma = \frac{\sigma_s}{\sigma_f},$

Letting

$$\left. \begin{aligned} u = \frac{\partial \psi}{\partial y} = \frac{ax}{1-ct} f'(\eta), v = -\frac{\partial \psi}{\partial x} = -\left(\frac{av_f}{1-ct}\right)^{\frac{1}{2}} f(\eta) \\ \eta = \left(\frac{a}{(1-ct)v_f}\right)^{\frac{1}{2}} y, \psi(x, y) = \left(\frac{av_f}{(1-ct)}\right)^{\frac{1}{2}} xf(\eta), \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty} \end{aligned} \right\}. \quad (5)$$

we have

$$\left. \begin{aligned} \frac{1}{(1-\phi)^{2.5}} f''' - \phi_1 (f'^2 - ff'' + A(f' + \frac{1}{2}\eta f'')) - Mf' \\ - \frac{\beta}{(1-\phi)^{2.5}} f' - \phi_1 F_r \cdot f'^2 = 0 \end{aligned} \right\}, \quad (6)$$

$$\left. \begin{aligned} \frac{1}{Pr} \frac{\kappa_{nf}}{\kappa_f} \theta'' + \phi_2 (f\theta' - \theta f' - A(\frac{1}{2}\eta\theta' + 2\theta)) + \frac{Ec}{(1-\phi)^{2.5}} f''^2 \\ + \phi_2 MEcf'^2 + \phi_2 Q_0 \theta = 0 \end{aligned} \right\}, \quad (7)$$

where

$$\phi_1 = (1 - \phi + \phi \frac{\rho_s}{\rho_f}), \phi_2 = (1 - \phi + \phi \frac{(\rho C_p)_s}{(\rho C_p)_f}), \quad (8)$$

With

$$\left. \begin{aligned} f(\eta) = f_w, f'(\eta) = 1, \theta(\eta) = 1, \text{ at } \eta = 0, \\ f'(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0, \text{ as } \eta \rightarrow \infty \end{aligned} \right\} \quad (9)$$

here  $A(=\frac{c}{a})$  denotes the unsteadiness parameter  $M(=\frac{\sigma_f B_0^2}{a\rho_f})$  the magnetic

parameter,  $\beta \left( = \frac{\nu_f}{\kappa^* \frac{a}{1-ct}} \right)$  the porosity parameter,  $F_r \left( = \frac{C_b x}{\sqrt{\kappa^*}} \right)$  the Forchheimer number,  $Pr \left( = \frac{\nu}{\alpha} \right)$  the Prandtl number,  $Ec \left( = \frac{x^2 a^2}{(1-ct)^2 C_{pf} (T - T_\infty)} \right)$  the Eckert number,  $Br \left( = Ec Pr \right)$  the Brinkman number and  $Q \left( = \frac{Q_0}{(\rho C_p)_f} \right)$  the heat generation/absorption variable.

Table 2: Thermo physical behaviors of base liquid and different nano particles [38-40]:

Physical properties	H <sub>2</sub> O	Cu	CuO	Al <sub>2</sub> O <sub>3</sub>	TiO <sub>2</sub>
$\rho (kgm^{-3})$	997.1	8933	6500	3970	4250
$C_p (J / KgK)$	4179	385	540	765	686.2
$\kappa (WmK)$	0.613	401	18	40	8.9538
$\sigma (S / m)$	$5.5 \times 10^{-6}$	$59.6 \times 10^6$	$2.7 \times 10^{-8}$	$35 \times 10^6$	$2.6 \times 10^6$

### 3: Engineering Quantities

Velocity and temperature gradient is given by

$$C_f = \frac{\tau_w}{\rho_f u_w^2} \text{ with } \tau_w = \mu_{nf} \left( \frac{\partial u}{\partial y} \right) \Big|_{y=0}, \quad (10)$$

$$Nu = \frac{x q_w}{k_f (T_w - T_\infty)} \text{ with } q_w = - (k_{nf}) \left( \frac{\partial T}{\partial y} \right) \Big|_{y=0}, \quad (11)$$

we have

$$C_f Re_x^{-1/2} = \frac{f''(0)}{(1-\phi)^{2.5}}, \quad (12)$$

$$Nu_x / Re_x^{-1/2} = - \frac{k_{nf}}{k_f} \theta'(0), \quad (13)$$

### 4: Entropy Modeling

Mathematically it is expressed as

$$E_G = \frac{k_f}{T_\infty^2} \left[ \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right] + \frac{\mu_{nf}}{T_\infty} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma_{nf} B_0^2}{T_\infty} u^2 + \frac{\mu_{nf}}{T_\infty \kappa^*} u^2, \quad (14)$$

we get

$$N_G(\eta) = \frac{E_G}{E_0} = \frac{\kappa_{nf}}{\kappa_f} \left[ \frac{1}{X^2} \theta(\eta)^2 + \text{Re}_L \theta(\eta)^2 \right] + \frac{Br \text{Re}_L}{\Omega(1-\phi)^{2.5}} f''(\eta)^2 + \frac{Br M^2}{\Omega} f'(\eta)^2 + \frac{\beta Br \text{Re}_L}{\Omega(1-\phi)^{2.5}} f'(\eta)^2, \quad (15)$$

where  $E_0 \left( = \frac{\kappa_f (\Delta T)^2}{L^2 T_\infty^2} \right)$  signify the entropy generation rate,  $X \left( = \frac{x}{L} \right)$  dimensionless parameter,  $N_G \left( = \frac{E_G}{E_0} \right)$  Entropy generation  $\text{Re} \left( = \frac{U_w L}{\nu_f} \right)$  the Reynolds number,  $Br \left( = \frac{\mu_f U_w^2}{\kappa_f \Delta T} \right)$  the Brinkman number and  $\Omega \left( = \frac{\Delta T}{T_\infty} \right)$  temperature difference parameter.

## 5: Graphical results and analysis

Non-linear system are solved through bvp4c Matlab software. Performance of various flow parameters on velocity field, temperature distribution and entropy rate are discussed. Surface drag force and Nusselt number are graphically examined. Here we studied four different class of nano particles ( $\text{Cu}$ ,  $\text{CuO}$ ,  $\text{Al}_2\text{O}_3$  and  $\text{TiO}_2$ ). Here our main focus is on copper ( $\text{Cu}$ ) nanoparticles.

### 5.1: Velocity field

Performance of velocity versus volume friction is revealed in Fig.2 Large volume friction reduces the velocity profile. Large magnetic parameter correspond to improve Lorentz force which augments the resistance to the liquid flow. Therefore velocity is diminished (see Fig.3). Fig.4 elucidates the variation of Forchheimer number on velocity. Velocity is reduced against Forchheimer number. Porosity parameter impact on velocity is illustrated in Fig.5. An increment in porosity variable corresponds to decays velocity field.

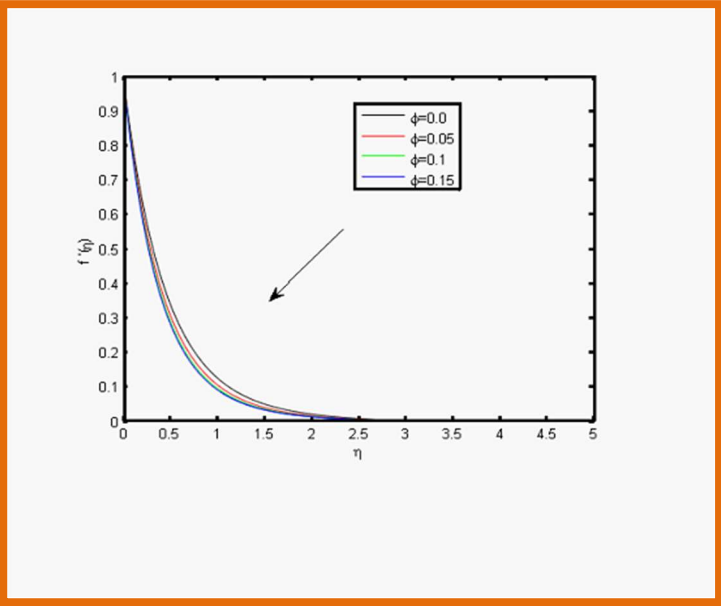


Fig. 2:  $\phi$  versus  $f'(\eta)$ .

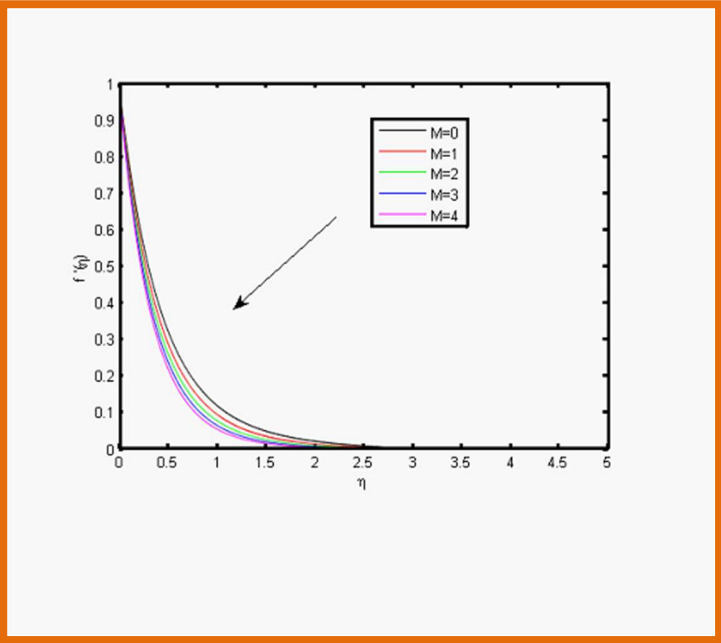
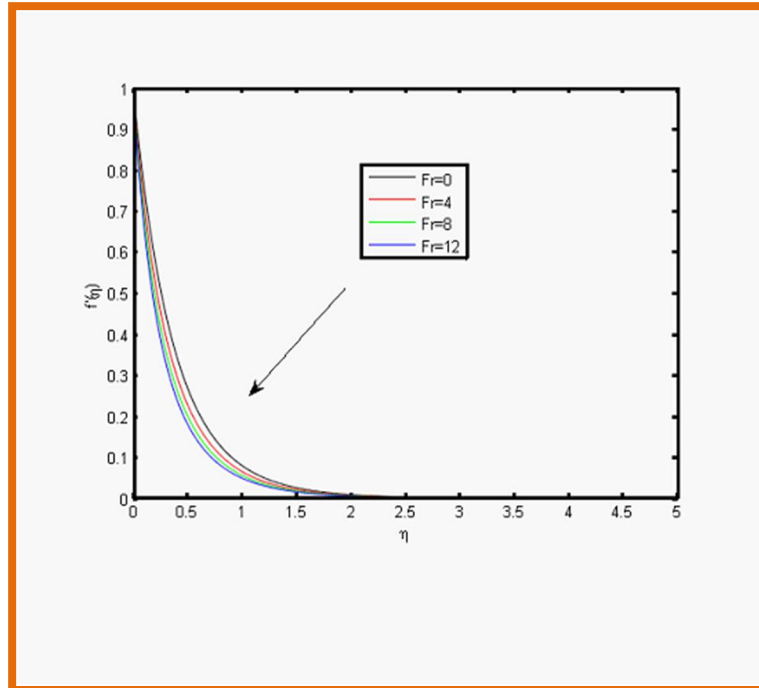
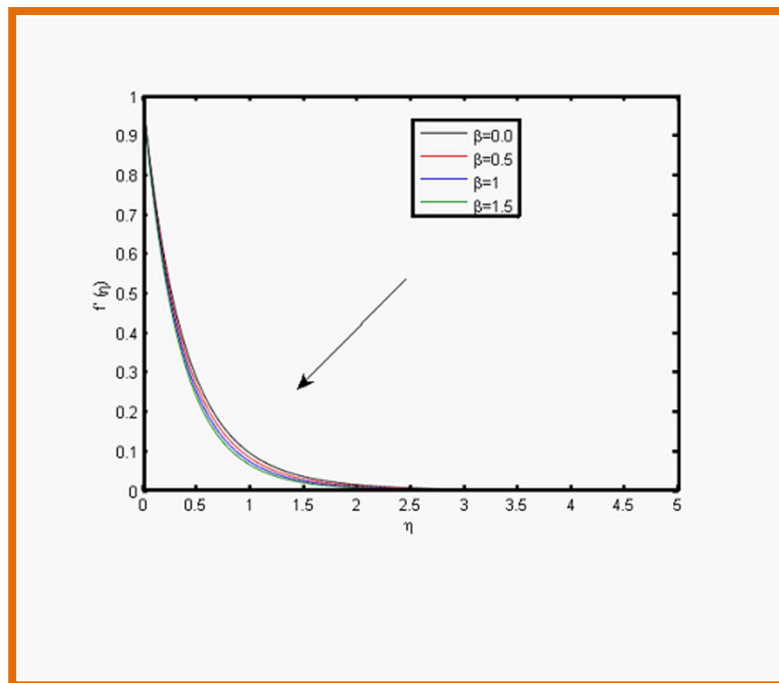


Fig. 3:  $M$  versus  $f'(\eta)$ .



Fig. 4:  $Fr$  versus  $f'(\eta)$ .Fig. 5:  $\beta$  versus  $f'(\eta)$ .

## 5.2: Temperature

Physical description of temperature versus volume friction variable ( $\phi$ ) is illustrated in Fig 6. An augmentation in temperature is enhanced for volume friction variable. Unsteadiness variable impact on temperature is portrayed in Fig.7. A reduction is

seen in temperature distribution with variation of unsteadiness parameter  $(A)$  . Significant effect of  $(M)$  on temperature is displayed in Fig.8. An improvement in temperature is noted for magnetic parameter. Physically larger magnetic variable augments the disturbance in liquid particles which improve collision among the liquid particles. Thus temperature is boosted. Fig.9 sketch to see the Brinkman number effect on temperature. Larger Brinkman number enhance the kinetic energy and as a result temperature is augmented.

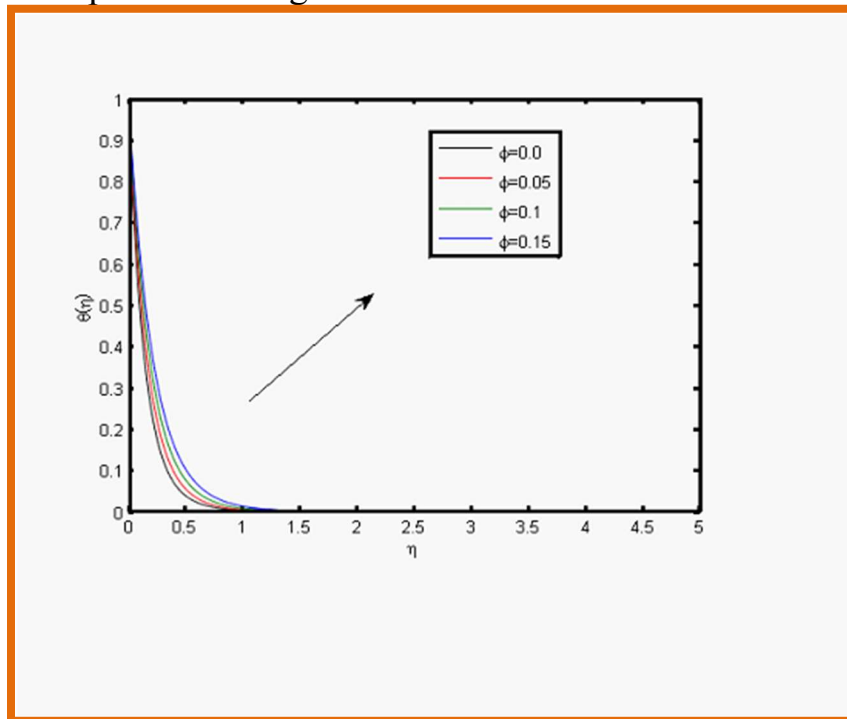


Fig. 6:  $\phi$  versus  $\theta(\eta)$ .

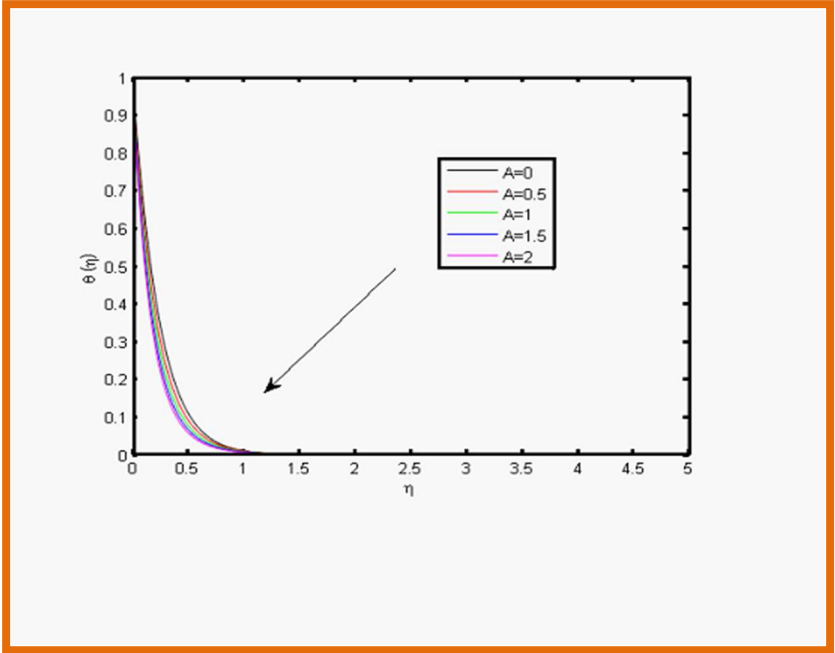


Fig. 7:  $A$  versus  $\theta(\eta)$ .

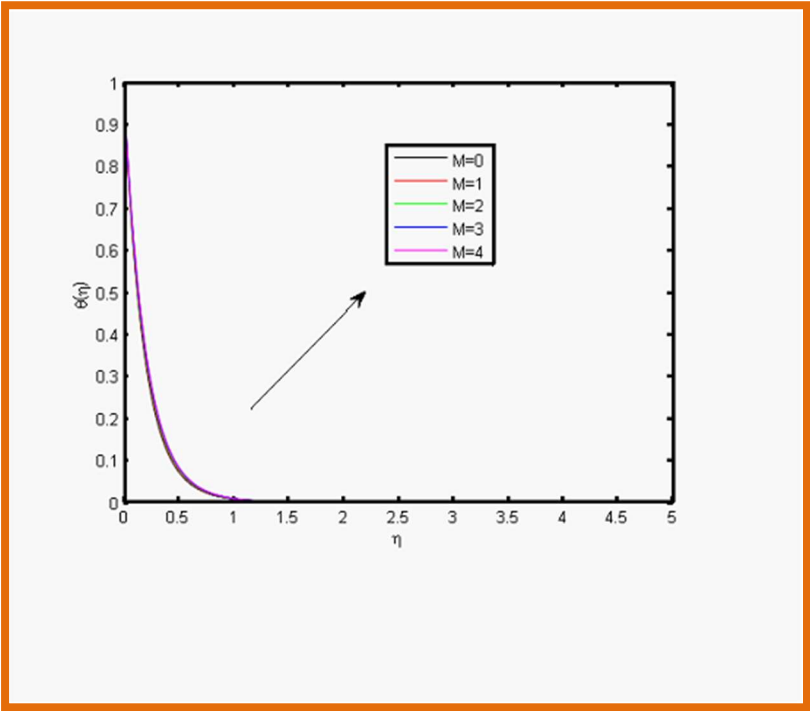


Fig. 8:  $M$  versus  $\theta(\eta)$ .

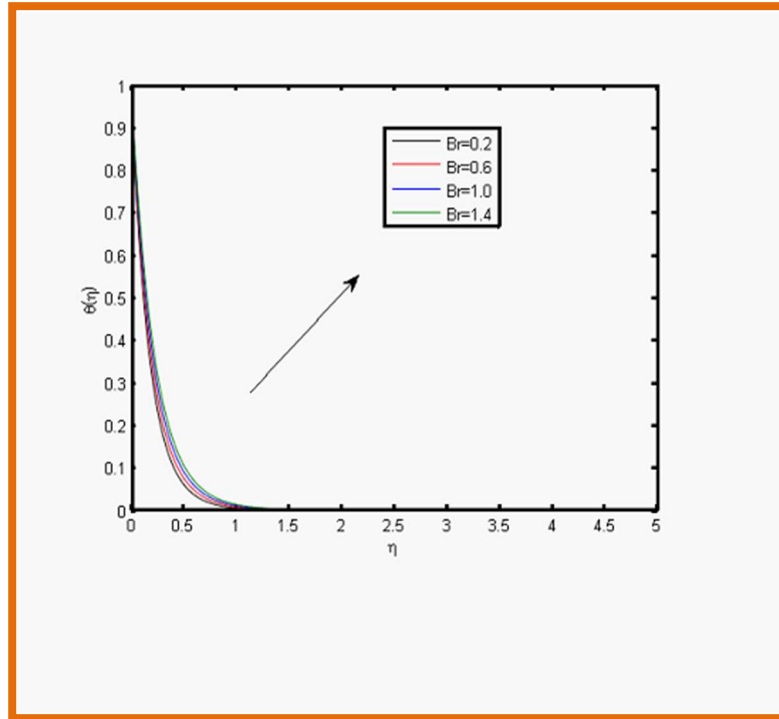


Fig. 9:  $Br$  versus  $\theta(\eta)$ .

### 5.3: Entropy optimization

Fig.10 sketch to show the effect of porosity variable on entropy rate. An enhancement in entropy rate is seen for porosity variable ( $\beta$ ). Fig.11 exhibit outcome of magnetic variable on entropy optimization. An augmentation in magnetic variable improve Lorentz force, which augments the collision amongst the nano particles. Therefore entropy optimization is boosted. An increment in Brinkman number leads to augments heat generation which is created by fluid friction and as a result entropy rate is boosted (see Fig 12). Influence of Reynold number ( $Re$ ) on entropy generation is revealed in Fig.13 Higher Reynold number ( $Re$ ) improve the entropy rate.

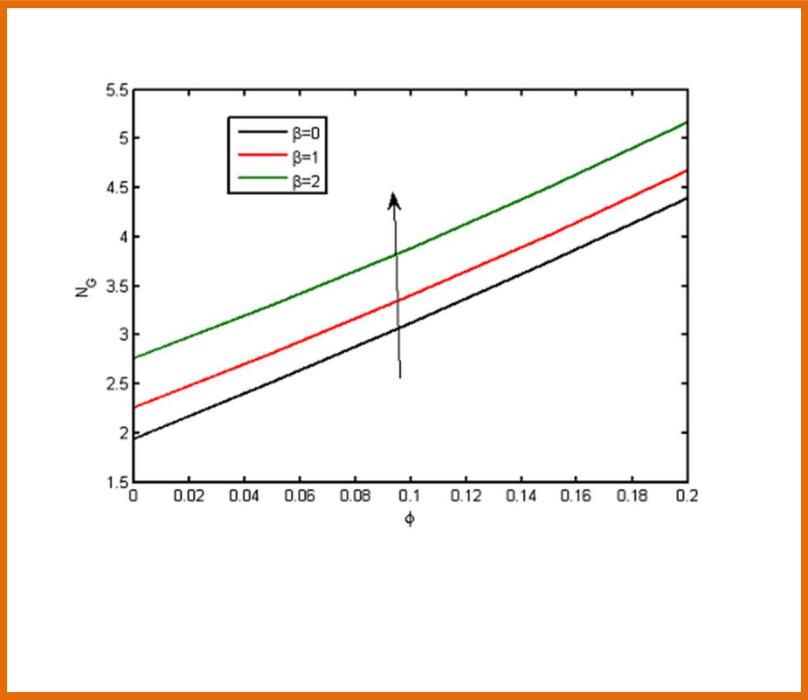


Fig. 10:  $\beta$  versus  $N_G$ .

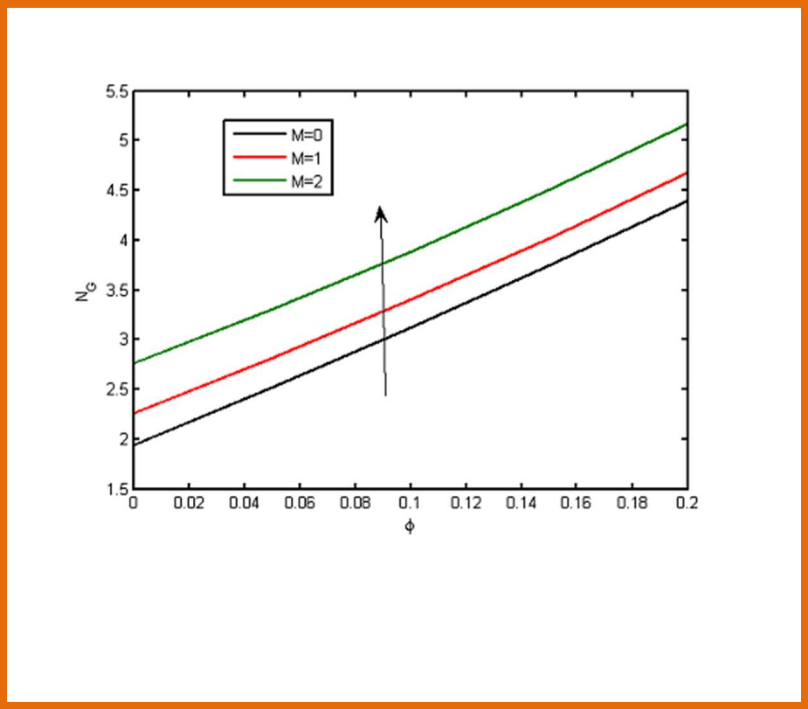


Fig. 11:  $M$  versus  $N_G$ .

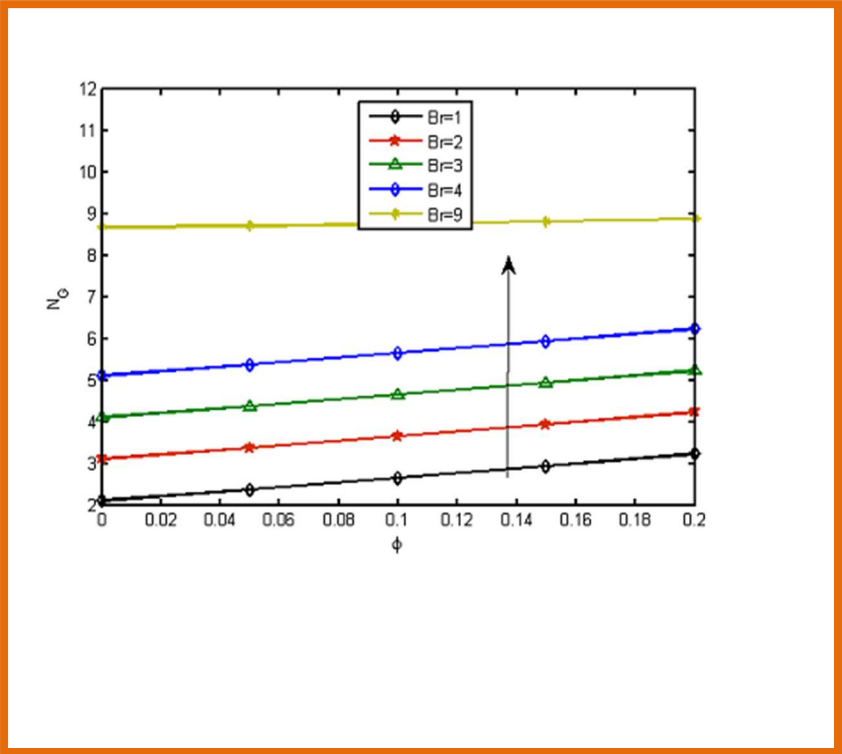


Fig. 12:  $Br$  versus  $N_G$ .

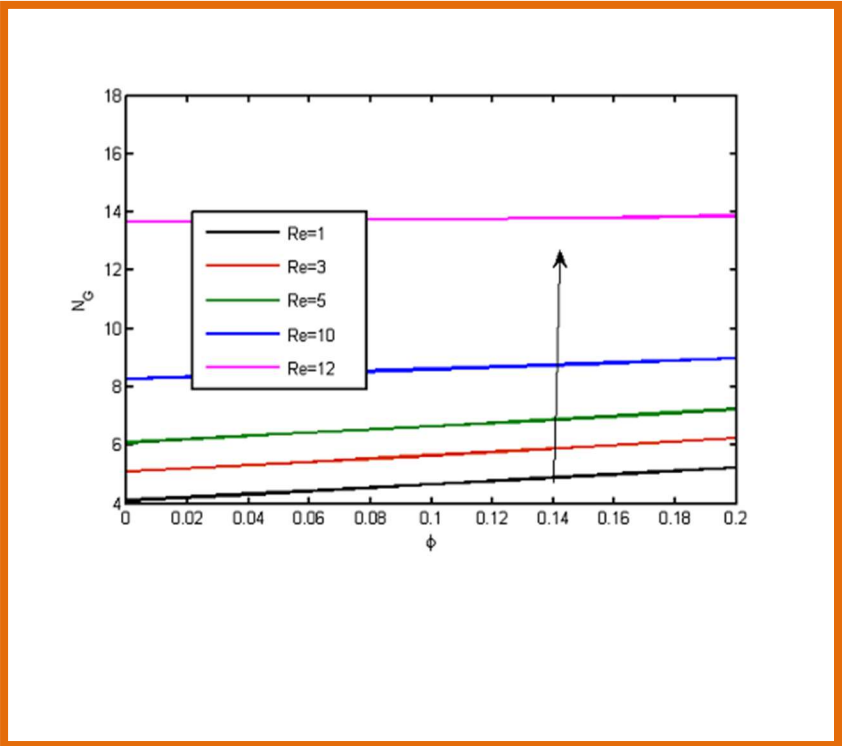


Fig. 13:  $Re$  versus  $N_G$ .

## 5.4: Physical quantities

Influence of various sundry variables on surface drag force and heat transfer rate are discussed in Figs. (14-17).

### 5.4.1: Skin friction coefficient

Fig. 14 sketch to show the variations of skin friction coefficient against  $A$  and  $\phi$ . Here clearly noted that velocity gradient has opposite effect for  $A$  and  $\phi$ . Skin friction coefficient reduces versus higher magnetic parameter.

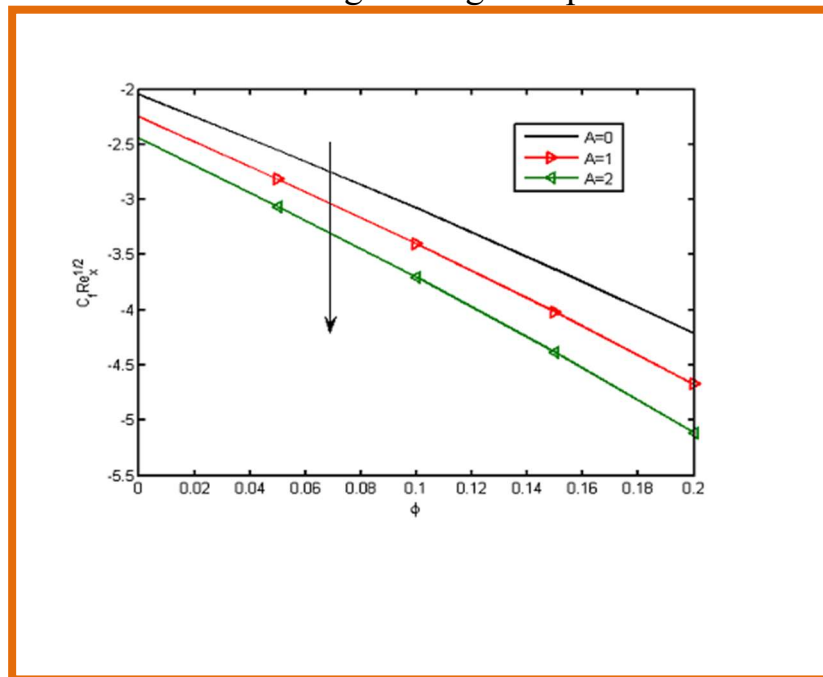


Fig. 14:  $A$  and  $\phi$  versus  $C_f$ .

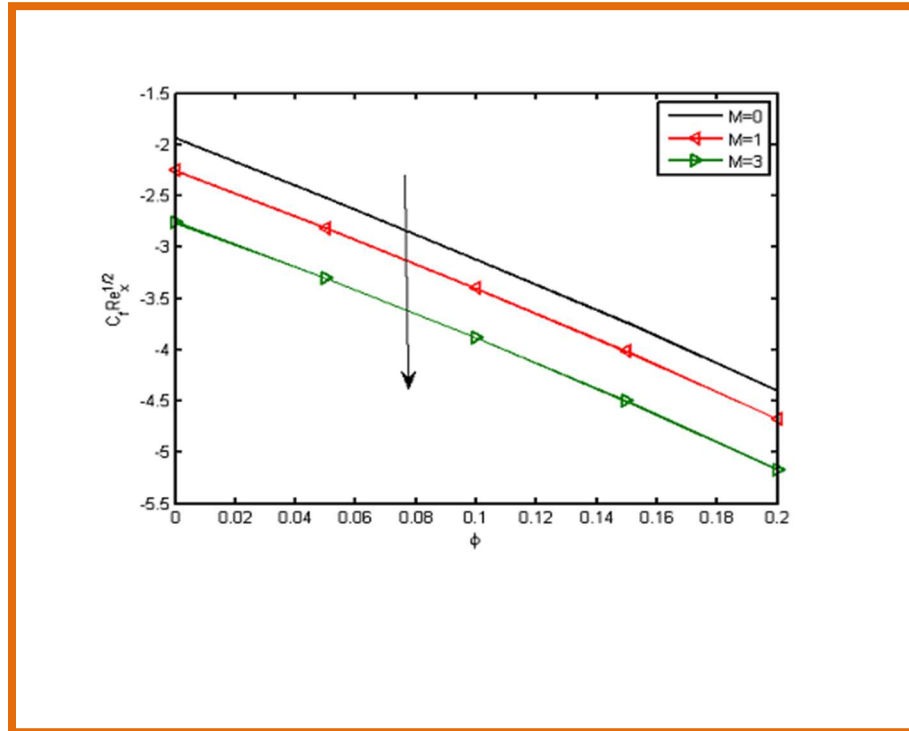


Fig. 15:  $M$  and  $\phi$  versus  $C_f$ .

#### 5.4.2: Nusselt number

An increment in magnetic parameter reduces the heat transfer rate (see Fig. 16). Influence of stretching parameter on Nusselt number is revealed in Fig. 17. An enhancement in stretching parameter improve the heat transfer rate.



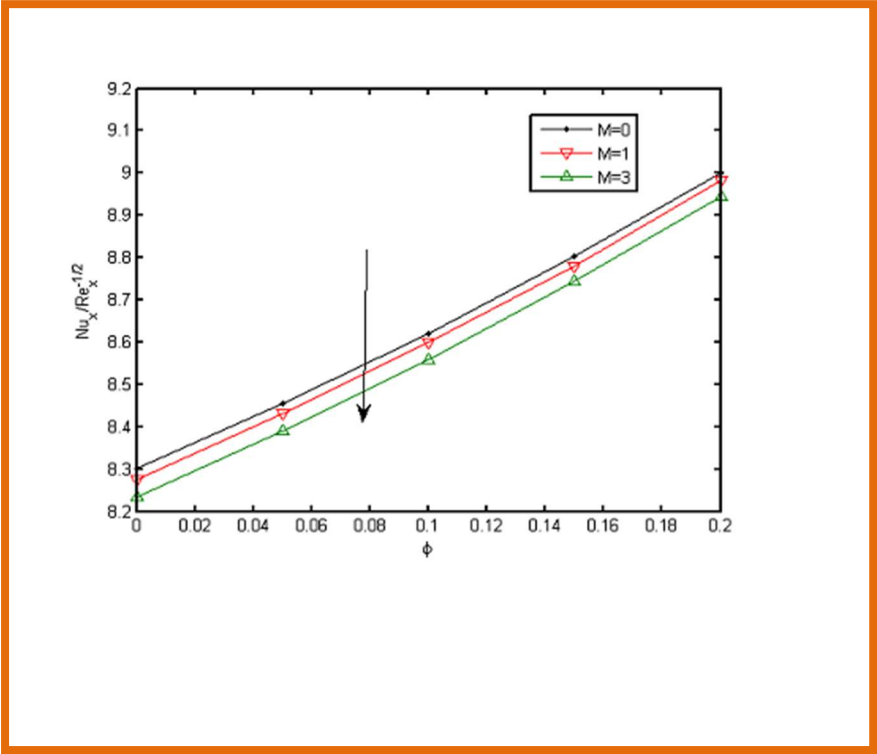


Fig. 16:  $M$  and  $\phi$  versus  $Nu$ .

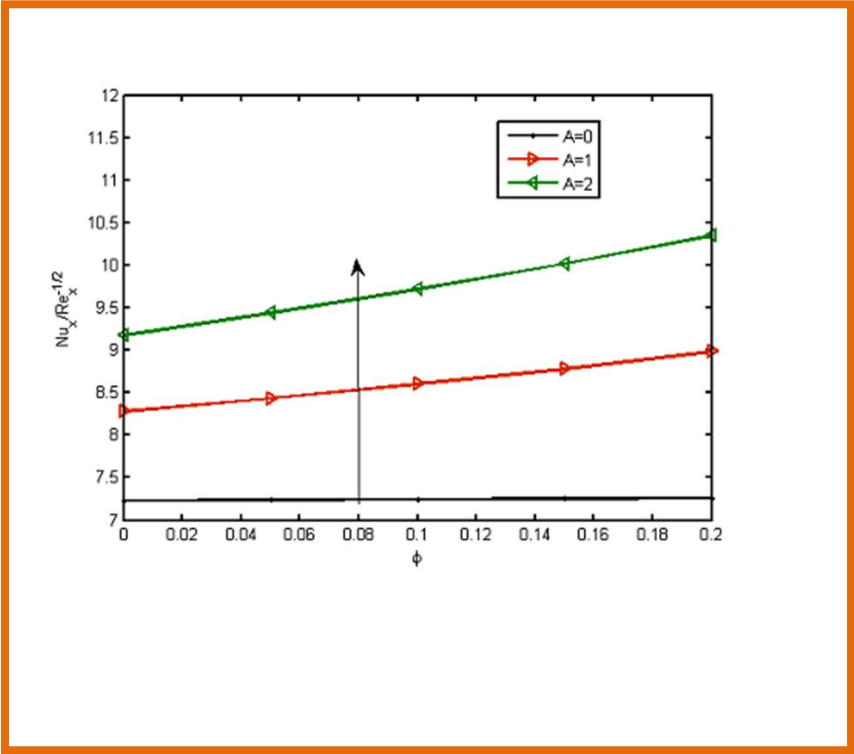


Fig. 17:  $A$  and  $\phi$  versus  $Nu$ .

## ***6: Conclusions***

Key observation of the present flow are given below

- Reduction in velocity field is noticed through magnetic and porosity parameters.
- Higher Forchheimer number and unsteadiness parameters leads to reduce the velocity profile.
- An augmentation in temperature distribution and entropy generation are observed with variation of Brinkman number.
- Larger Magnetic variable corresponds to augments the entropy generation rate and temperature distribution.
- Temperature distribution is reduced with variation of unsteadiness parameter.
- Similar characteristic for entropy generation rate is seen through porosity parameter.
- Magnetic parameter has reducing effect on both velocity gradient and heat transfer rate.
- An opposite effect is observed in temperature gradient and surface drag force for stretching parameter.

## ***Declaration of Competing Interest***

None.

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