Type of the Paper (Article.)

Analytical and Semi-Analytical Solutions for the Self and Mutual Inductances of Concentric Coplanar Ordinary and Bitter Disk Coils

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Abstract: In this paper, closed and semi-closed form solutions are presented for the self - and mutual inductance of ordinary and Better disk coils which lie concentrically in a plane. The solutions are given as the combination of the elliptic integral of the second kind and a simple integral or only as the elliptic integral of the second kind. All formulas or obtained in remarkably simple form and give extremely accurate results with significantly neglectable computational time. All cases either regular or singular (disks in contact or overlap) are covered. The formulas for the mutual inductances can be directly used to calculating the self-inductance of the ordinary disk coil or the Bitter disk coil, respectively. Many presented examples show the excellent numerical agreement with previous publish methods.

Keywords: Ordinary disk coils; Bitter disk coils; mutual inductance, self-inductance.

1. Introduction

Several monographs and papers are devoted to calculating the self and the mutual inductance for the ordinary circular coils (massive coils of the rectangular cross-section, thin wall solenoids, disk coils) with the azimuthal current density [1–12]. The conventional coils used in many applications such as all ranges of transformers, generators, motors, current reactors, magnetic resonance applications, antennas, coil guns, medical electronic devices, superconducting magnets, tokamaks, electronic and printed circuit board design, plasma science, etc., are very well-known. Also, there are circular coils with the radial current density (massive coils of the rectangular cross-section, disk coils) which are interesting from an engineering aspect. These coils are the well-known Bitter coils [13–18] which supply extremely high magnetic fields up to 45 T. There are other applications such as the homopolar motors, homopolar generators, and Faraday's disk. The thin disk coils (pancakes) are of the particular interest in previously mentioned electromagnetic applications. In [8] J.T. Conway propose the analytical solutions for the self- and mutual Inductances of ordinary concentric coplanar disk coils. He gives the excellent solution as generalized hypergeometric functions which are closely related to elliptic integrals. The method used is a Legendre polynomial expansion of the inductance integral, which renders all integrations straightforward. In this paper we give remarkably simple solutions for calculating the self- and mutual inductance of the ordinary concentric coplanar and the Bitter disk coils. In the case of the ordinary concentric coplanar disks the solutions are obtained in semi-analytical and analytical form. The solutions are obtained over the elliptic integral of the second kind $E(k)$ and one simple integral whose kernel function is integrable over all interval of integration. In the case of the concentric coplanar Bitter disk coils the solutions are obtained in the analytical form over the elliptic integral of the second kind $E(k)$. All cases given in [8] are verified and confirmed by the presented approach. Also, the self- and the mutual inductance of the concentric coplanar disk coils are verified by the filament method [15–18]. All obtained results are in an excellent agreement.

2. Basic Expressions

2.1 Ordinary concentric coplanar disks

The mutual inductance between two ordinary coplanar disks is given by [8],

$$M = \frac{\mu_0 N_1 N_2}{(R_2 - R_1)(R_4 - R_3)} \int_0^\pi \int_{R_1}^{R_2} \int_{R_3}^{R_4} \frac{r_1 r_2 \cos(\theta) dr_1 dr_2 d\theta}{\sqrt{r_1^2 - 2r_1 r_2 \cos(\theta) + r_2^2}} \quad (1)$$

where,

N_1 and N_2 are equally spaced turns of disks,

R_1 and R_3 are inner radii of disks (m),

R_2 and R_4 are outer radii of disks(m),

$\mu_0 = 4\pi \cdot 10^{-7}(\text{H/m})$

The self-inductance of a disk coil with equally spaced turns N and R_1 and R_2 inner and outer radii and, respectively, is given by [8],

$$L = \frac{\mu_0 N^2}{(R_2 - R_1)^2} \int_0^\pi \int_{R_1}^{R_2} \int_{R_1}^{R_2} \frac{r_1 r_2 \cos(\theta) dr_1 dr_2 d\theta}{\sqrt{r_1^2 - 2r_1 r_2 \cos(\theta) + r_2^2}} \quad (2)$$

In (1) and (2) r_1 and r_2 are the cylindrical coordinates.

2.1. Concentric coplanar Bitter disks

The mutual inductance between two Bitter disks is given by [17],

$$M_B = \frac{\mu_0 N_1 N_2}{\ln \frac{R_2}{R_1} \ln \frac{R_4}{R_3}} \int_0^\pi \int_{R_1}^{R_2} \int_{R_3}^{R_4} \frac{\cos(\theta) dr_1 dr_2 d\theta}{\sqrt{r_1^2 - 2r_1 r_2 \cos(\theta) + r_2^2}} \quad (3)$$

and the self-inductance of the Bitter disk coil is given by [17],

$$L_B = \frac{\mu_0 N^2}{[\ln \frac{R_2}{R_1}]^2} \int_0^\pi \int_{R_1}^{R_2} \int_{R_3}^{R_4} \frac{\cos(\theta) dr_1 dr_2 d\theta}{\sqrt{r_1^2 - 2r_1 r_2 \cos(\theta) + r_2^2}} \quad (4)$$

3. Calculation Method

3.1. Ordinary concentric coplanar disks

Before the first integration let us take the following substitution $\theta = \pi - 2\beta$ so that (1) becomes,

$$M = \frac{-2\mu_0 N_1 N_2}{(R_2 - R_1)(R_4 - R_3)} \int_0^{\pi/2} \int_{R_1}^{R_2} \int_{R_3}^{R_4} \frac{r_1 r_2 \cos(2\beta) dr_1 dr_2 d\beta}{\sqrt{r_1^2 + 2r_1 r_2 \cos(2\beta) + r_2^2}} \quad (5)$$

The first integration in (5), [19-20] gives:

$$I_{r_1} = \int_{R_1}^{R_2} \frac{r_1 dr_1}{\sqrt{r_1^2 + 2r_1 r_2 \cos(2\beta) + r_2^2}} = \left\{ \sqrt{r_1^2 + 2r_1 r_2 \cos(2\beta) + r_2^2} - r_2 \cos(2\beta) \operatorname{asinh} \frac{r_1 + r_2 \cos(2\beta)}{r_2 \sin(2\beta)} \right\}_{R_1}^{R_2}$$

The second integration in (5), [19-20] gives:

$$I_{r_2} = \int_{R_3}^{R_4} r_2 I_{r_1} dr_2 = \frac{1}{3} \sum_{n=1}^{n=4} (-1)^{n-1} \{ (l_n^2 + \rho_n^2) \sqrt{l_n^2 + 2l_n \rho_n \cos(2\beta) + \rho_n^2} - l_n^3 \cos(2\beta) \operatorname{asinh} \frac{\rho_n + l_n \cos(2\beta)}{l_n \sin(2\beta)} - \rho_n^3 \cos(2\beta) \operatorname{asinh} \frac{l_n + \rho_n \cos(2\beta)}{\rho_n \sin(2\beta)} \}$$

where,

$$\rho_1 = \rho_2 = R_1, \quad \rho_3 = \rho_4 = R_2, \quad l_1 = l_4 = R_3, \quad l_2 = l_3 = R_4$$

The third integration in (5), [19-20] gives:

$$I_3 = \int_0^{\pi/2} \cos(2\beta) I_{r_2} d\beta = \frac{1}{6} \sum_{n=1}^{n=4} (-1)^{n-1} \{ l_n \rho_n (l_n + \rho_n) E(k_n) - \int_0^{\pi/2} l_n^3 \operatorname{asinh} \frac{\rho_n + l_n \cos(2\beta)}{l_n \sin(2\beta)} d\beta - \int_0^{\pi/2} \rho_n^3 \operatorname{asinh} \frac{l_n + \rho_n \cos(2\beta)}{\rho_n \sin(2\beta)} d\beta \}$$

Finally, the mutual inductance between two coplanar ordinary disks is,

$$M = \frac{\mu_0 N_1 N_2}{3(R_2 - R_1)(R_4 - R_3)} \sum_{n=1}^{n=4} (-1)^n \{ l_n \rho_n (l_n + \rho_n) E(k_n) -$$

$I_n \}$ (6) where,

$$I_n = \int_0^{\pi/2} \left\{ l_n^3 \operatorname{asinh} \frac{\rho_n + l_n \cos(2\beta)}{l_n \sin(2\beta)} + \rho_n^3 \operatorname{asinh} \frac{l_n + \rho_n \cos(2\beta)}{\rho_n \sin(2\beta)} \right\} d\beta$$

$$k_n^2 = \frac{4\rho_n l_n}{(\rho_n + l_n)^2}$$

$$\rho_1 = \rho_2 = R_1, \quad \rho_3 = \rho_4 = R_2, \quad l_1 = l_4 = R_3, \quad l_2 = l_3 = R_4$$

$E(k)$ – elliptic integral of the second kind [19-20].

The mutual inductance is obtained in remarkably simple form over the elliptic integral of the second kind and one simple integral whose kernel function is the continue and integrable over the domain of the integration so that (6) is applicable in the regular or the singular cases (disks are in the contact or overlap).

For calculating the self-inductance of the ordinary disk coil one can use (2) or directly putting in (6) $R_1 = R_3$, $R_2 = R_4$ and $N_1 = N_2 = N$ that gives,

$$L = \frac{2\mu_0 N^2}{3(R_2 - R_1)^2} \{ (R_1^3 + R_2^3)(2G - 1) + R_2 R_1 (R_2 + R_1) E(k) - I_0 \} \quad (7)$$

where,

$$I_0 = \int_0^{\pi/2} \left\{ R_1^3 \operatorname{asinh} \frac{R_2 + R_1 \cos(2\beta)}{R_1 \sin(2\beta)} + R_2^3 \operatorname{asinh} \frac{R_1 + R_2 \cos(2\beta)}{R_2 \sin(2\beta)} \right\} d\beta$$

$$k^2 = \frac{4R_1 R_2}{(R_1 + R_2)^2}$$

$G = 0.91596559417721901505460351493238411 \dots$, Catalan's constant

We put $R_2 = \alpha R_1$ ($\alpha > 1$) and (7) becomes,

$$L = \frac{2\mu_0 N^2 R_1}{3(\alpha - 1)^2} \{ (\alpha^3 + 1)(2G - 1) + \alpha(\alpha + 1)E(k) - I_0(\alpha) \} \quad (8)$$

where,

$$I_0 = \int_0^{\pi/2} \left\{ \operatorname{asinh} \frac{\alpha + \cos(2\beta)}{\sin(2\beta)} + \alpha^3 \operatorname{asinh} \frac{1 + \alpha \cos(2\beta)}{\alpha \sin(2\beta)} \right\} d\beta$$

$$k^2 = \frac{4\alpha}{(\alpha + 1)^2}$$

From (2) or (7) it is possible to obtain the self-inductance of the full disk ($R_1 = 0$ and $R_2 = R$) as follows,

$$L = \frac{2\mu_0 N^2 R_1}{3} (2G - 1) \quad (9)$$

3.2. Concentric coplanar Bitter disks

Before the first integration let us take the following substitution $\theta = \pi - 2\beta$ so that (3) becomes,

$$M_B = \frac{-2\mu_0 N_1 N_2}{\ln \frac{R_2}{R_1} \ln \frac{R_4}{R_3}} \int_0^\pi \int_{R_1}^{R_2} \int_{R_3}^{R_4} \frac{\cos(2\beta) dr_1 dr_2 d\beta}{\sqrt{r_1^2 + 2r_1 r_2 \cos(2\beta) + r_2^2}} \quad (10)$$

The first integration in (10), [19-20] gives:

$$I_{r_1} = \int_{R_1}^{R_2} \frac{dr_1}{\sqrt{r_1^2 + 2r_1 r_2 \cos(2\beta) + r_2^2}} = \left\{ \operatorname{asinh} \frac{r_1 + r_2 \cos(2\beta)}{r_2 \sin(2\beta)} \right\}_{R_1}^{R_2}$$

The second integration in (10), [19-20] gives:

$$I_{r_2} = \int_{R_3}^{R_4} I_{r_1} dr_2 = \sum_{n=1}^{n=4} (-1)^{n-1} \left\{ l_n \operatorname{asinh} \frac{\rho_n + l_n \cos(2\beta)}{l_n \sin(2\beta)} + \operatorname{asinh} \frac{l_n + \rho_n \cos(2\beta)}{\rho_n \sin(2\beta)} \right\}$$

where,

$$\rho_1 = \rho_2 = R_1, \rho_3 = \rho_4 = R_2, l_1 = l_4 = R_3, l_2 = l_3 = R_4$$

The third integration in (10), [19-20] gives:

$$I_3 = \int_{R_3}^{R_4} \cos(2\beta) I_{r_2} d\theta = \sum_{n=1}^{n=4} (-1)^{n-1} (l_n + \rho_n) E(k_n)$$

Finally, the mutual inductance between two coplanar Bitter disks is,

$$M_B =$$

$$\frac{2\mu_0 N_1 N_2}{\ln \frac{R_2}{R_1} \ln \frac{R_4}{R_3}} \sum_{n=1}^{n=4} (-1)^n (l_n + \rho_n) E(k_n) \quad (11)$$

where,

$$k_n^2 = \frac{4\rho_n l_n}{(\rho_n + l_n)^2}$$

$$\rho_1 = \rho_2 = R_1, \rho_3 = \rho_4 = R_2, l_1 = l_4 = R_3, l_2 = l_3 = R_4$$

The mutual inductance is obtained in amazingly simple form over the elliptic integral of the second kind. It is applicable in the regular or the singular cases (disks are in the contact or overlap).

For calculating the self-inductance of the Bitter disk coil one can use (4) or directly putting in (6) $R_1 = R_3$, $R_2 = R_4$ and $N_1 = N_2 = N$ that gives,

$$L_B = \frac{4\mu_0 N^2 (R_1 + R_2)}{\left[\ln \frac{R_2}{R_1} \right]^2} [E(k) - 1] \quad (12)$$

where,

$$k^2 = \frac{4R_1 R_2}{(R_1 + R_2)^2}$$

Thus, either the mutual inductance or the self-inductance for the Bitter Disks are obtained in the close form over the elliptic integral of the second kind.

4. Numerical Validation

Example 1.

In [8] Conway give the calculation of normalized self-inductance in terms of the shape factor $R_2 = \alpha R_1$ as $L_N = L/N^2/R_1$, (Table 1).

Table 1. Self-inductance given as $L_N = L/N^2/R_1$ in $\mu\text{H/m}$

α	[4]	[8] Equ-(58)	This work (8)
1.5	3.9375	3.937556957309482	3.937556957309482
3	4.1202	4.120247770949786	4.120247770949786
4	4.6535	4.653592975459867	4.653592975459867
7	6.5440	6.544182694169381	6.544182694169381
9	7.8795	7.879438694940537	7.879438694940537
19	14.740	14.73964824153050	14.73964824153050
39	28.628	28.62677689946369	28.62677689946369

Table 1 shows the comparison between inductance given by Kalantarov [4] and calculated in [8] with (58) as well as with (8) from this work. The agreement is excellent in all cases.

Example 2.

Table 2 shows calculations of the self-inductance of the ordinary disk for various shape factors and for values remarkably close to the logarithmic singularity as $\alpha \rightarrow 1$. These are compared to two sets of results given by Spielrein method [5] and [8] with (58).

Table 2. Comparison of calculations for the self-inductance for values of the shape factor both close and far from the singularity $\alpha \rightarrow 1$ Results are in $\mu\text{H/m}$.

α	[5]	[8] Equ-(58)	This work (8)
50	36.2822050627	36.28220506268449	36.28220506268449
10	8.5558078657	8.555807865723495	8.555807865723494
3	4.1202478984	4.120247770949786	4.120247770949786
1.5	3.9375565536	3.937556957309482	3.937556957309482
1.1	5.1875898298	5.187589829874826	5.187589829874826
1.01	7.8169836166	7.816983616632973	7.816983616632973
1.001	10.6712873756	10.67128737563754	10.67128737563754
1.00001	16.4524421475	16.45244214746880	16.45244214746880
1.000001	19.3458776688	19.34587766869611	19.34587766869611
1.0000001	22.2393823064	22.23938230721058	22.23938230721058

Again, the excellent agreement of this work (8) with (58) in [1] and very well close with [5]. Equation (8) gives much better accurate results (for $\alpha - 1 \rightarrow 10^{-5}, 10^{-6}, \dots$) then (3) in [12].

Example 3.

In this example we show the performance of the formula (8) when the singularity is approached ($\alpha - 1 \rightarrow 10^{-16}$). We compare the results of the formula (8) with those obtained by the asymptotic formula (20) and (58) in [8], Table 3.

Table 3. Comparison of (8) and (58), [8] with asymptotic formula (20), [8] as the singularity at unit shape factor is approached. Results are in $\mu\text{H/m}$.

$\alpha - 1$	[8] Equ-(20)	[8] Equ-(58)	This work (8)
10^{-1}	4.998068846886786	5.187589829874826	5.187589829874826

10^{-2}	7.784316262102376	7.816983616632973	7.816983616632973
10^{-3}	10.66658208170639	10.67128737563754	10.67128737563754
10^{-6}	19.34586862407991	19.34587766869611	19.34587766869611
10^{-8}	25.13289490994222	25.13289502932351	25.13289502932351
10^{-10}	30.91992242743389	30.91992242891705	30.91992242891705
10^{-12}	36.70694995724185	36.70694995725958	36.70694995725958
10^{-15}	45.38749125213915	45.38749125213918	45.38749125213918
10^{-16}	48.28100501710534	48.28100501710534	48.28100501710534

All results are in remarkably good agreement. Thus, the formula (8) has the large range of application.

Example 4.

Calculate the self-inductance of the full disk coil for which $R = 1 (m)$ and $N = 100$.

From (9) the self-inductance of the full disk is,

$$L_{Full-Disk} = 6.969570425670744(\text{mH})$$

To verify this result we use (7) and $R_1 = 10^{-17}(m)$, $R_2 = 1(m)$, $N = 100$ that gives,

$$L_{Full-Disk} = 6.969570425670744(\text{mH})$$

We obtain the identical result.

From previous examples we show that the self-inductance (8) is applicable in large range of α .

Thus, remarkably simple formula (8) can be used as the general formula for calculating the self-inductance of the ordinary disk coil (pancake).

Example 5.

Table 4 gives the calculations of the mutual inductance in millihenrys for two disk coils where the radii and are given the fixed values: $R_2 = 1(m)$, $R_3 = 0.4(m)$, and $R_4 = 0.6(m)$, whereas R_1 varies between 0.1 and 0.9 m. The turn numbers and are both specified to be 100, [8]. This covers all the generic overlap cases.

Table 4. Mutual inductance in mH and CPU in seconds for all overlap cases.

$R_1(m)$	$M_{12}(mH)$ [8]	CPU (sec) [8]	$M_{12}(mH)$ (6) This work	CPU (sec) This work
0.1	8.3452226401271722326	2.262	8.3452226401271722326	0.0
0.2	9.2662149362835422651	2.215	9.2662149362835422651	0.0
0.3	10.178800706150685814	1.778	10.178800706150685814	0.0
0.4	10.751261394420867987	1.092	10.751261394420867987	0.0

0.5	10.014392107083652030	1.373	10.014392107083652030	0.0
0.6	8.2517781770123374536	0.094	8.2517781770123374536	0.0
0.7	7.1724760612076564190	0.140	7.1724760612076564190	0.0
0.8	6.5028431844372044556	1.109	6.5028431844372044556	0.0
0.9	6.0061106298597996959	0.094	6.0061106298597996959	0.0

Table 4 shows that all results are identical, obtained by [9] and (6) from this work but the CPU times by this work is practically zero. In Mathematica programing [21] using the instruction ‘Timing’ the computational time is 0 seconds. Using the instruction ‘Absolute Timing’ the computational time is $3 \cdot 10^{-7}$ seconds. We confirmed that relatively simple equation (6) covers all overlap cases for the negligible computation time and with the excellent accuracy

Example 6.

Calculate the mutual inductance between two disk coils for which is, $R_1 = 1 (m)$, $R_2 = 2 (m)$, $R_3 = 1 (m)$, $R_4 = 2 (m)$ and $N_1 = N_2 = N = 1000$. Obviously, the disks completely overlap it is the case of the self-inductance. Let us apply equation (6) for the mutual inductance of the disk coils. It gives,

$$M = L = 3.794801607308613(H)$$

Applying the formula (7) for the self-inductance of the disk coil we obtain,

$$L = 3.794801607308613 \quad (H)$$

We obtain the identical result. Thus, we show that the simple formula (6) is general for any case either for the regular or for the singular. It is confirmed in the previous examples.

Example 7.

Table 5 gives the calculations of mutual inductance in millihenrys for two disk coils where the radii and are given the fixed values: $R_2 = 10(m)$, $R_3 = 4(m)$, and $R_4 = 6(m)$, whereas R_1 varies between 1 and 9 m. The turn numbers and are both specified to be 1000. This covers all the overlap cases.

Table 5. Mutual inductance in H for all overlap cases.

$R_1(m)$	This work (11) $M_B(H)$	$M_B(H)$ [14 – 15]
1	6.176953480899784	6.176890611593179
2	8.442109820575922	8.441929573292805
3	10.32822091283136	10.32810057022684
4	11.43930901799091	11.44063208654759
5	10.40642626084792	10.40633203780527
6	8.263044997910772	8.263044761562908
7	7.051548638612127	7.051548623257631
8	6.338568884058207	6.338568878810852

9	5.83274907953413	5.832749077642109
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From Table 5 we can see the very good agreement of all results. There is the negligible discrepancy between presented results because the filament method [14-15] is approximative and depends on the number of the divisions of the disk coils. Many disk divisions are related with the computational time.

Example 8.

Calculate the self-inductance of the Bitter disk coil using equations (1) and (12).
The disk dimensions and the number of turns is as follows, $R_1 = 1(m)$, $R_2 = 2(m)$, $N = 1000$.

Equation (11) calculate the mutual inductance between two Bitter disk coils. Let us take $R_1 = 1(m)$, $R_2 = 2(m)$, $R_3 = 1(m)$, $R_4 = 2(m)$, $N_1 = N_2 = 1000$. . Applying (11) we obtain,

$$M_B = L_B = 3.569912886724817 \text{ (H)}$$

Applying the formula for the self-inductance of the Bitter coil we have,

$$L_B = 3.569912886724817(H)$$

Thus, we obtain the same expected results.

6. Conclusion

In this paper we presented remarkably simple solutions for calculating the self – and mutual inductance of the ordinary concentric coplanar and the Bitter disk coils. All cases either the regular or the singular (disks overlap) are cover with obtained formulas in the semi-analytical and analytical form. We show that with formulas for the mutual inductance is possible to obtained directly results for the corresponding self-inductances. The presented formulas are much familiar and easier to use for the potential readers that those given in [8]. In [8] are given more complicated functions with which more people are not familiar. In this work all calculations are made by Mathematica [21]. From presented examples one can see that all compared results are practically identical or in the excellent agreement.

Author Contributions: Conceptualization, methodology, software, validation, formal analysis, investigation, re-sources, writing—original draft preparation, writing—review and editing, visualization.

Funding: This research received no external funding.

Conflicts of Interest: The author declares no conflict of interest.

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