Competing Conventions with Costly Information Acquisition

Roberto Rozzi*

Abstract

We consider an evolutionary model of social coordination in a 2x2 game where two groups of agents prefer to coordinate on different actions. Agents can pay a cost to learn their opponent's type: conditional on this decision, they can play different actions with different types. We assess the stability of outcomes in the long-run using stochastic stability analysis. We find that three elements matter for the equilibrium selection: the group size, the strength of preferences, and the information's cost. If the cost is too high, agents never learn the type of their opponents in the long-run. If one group is stronger in preferences for its favorite action than the other, or its size is sufficiently large compared to the other group, every agent plays that action. If both groups are strong enough in preferences, or if none of the group's size is large enough, agents play their favorite actions and miscoordinate in inter-group interactions. When the cost is sufficiently low, agents always coordinate. In inside-group interactions, agents always coordinate on their favorite action. In inter-group interactions, they coordinate on the favorite action of the group that is stronger in preferences or large enough.

1 Introduction

Since the seminal contribution of Kandori *et al.* [1], evolutionary game theorists have used stochastic stability analysis and 2x2 coordination games to study the formation of social conventions.[1] Some of these works focus on coordination games such as the battle of sexes: a class that describes situations in which two groups of people attach value zero to miscoordination but prefer to coordinate on different actions. In this framework, the long-run convention may depend on how easily people can apprehend other people's preferences.

Think about Bob, who wants to hang out with Andy: Bob must choose between proposing to Andy to go to a football match or the cinema. Bob prefers football over cinema, but he does not know what Andy prefers. In certain contexts, learning Andy's preferences may require too much effort for Bob. In these cases, if Bob knows that everybody usually goes to the cinema, he will ask Andy to go to the cinema. In

*Ca' Foscari University of Venice, Dorsoduro 3246, 30123 Venezia Italia, email: roberto.rozzi@unive.it.

[1] Lewis [2] and Bicchieri [3] are classical references on social conventions from philosophy, while for economics see Schelling [4] Young [5], and Young [6].

other situations, Bob may learn Andy's preferences with a small effort, for instance, watching Andy's Facebook feed. In this case, if Bob finds out that Andy prefers football, he will propose to him to see a football match together.

In this work, we contribute to the literature on coordination games. We show which conventions establish between two groups of people different in preferences if people can learn other people's preferences by exerting an effort. We do so, formalizing the example previously made and studying the evolution of conventions in a dynamic setting. We model the coordination problem as a repeated language game (Neary [7]): we use evolutionary game theory solution concepts and characterize the long-run equilibrium as the stochastically stable state (see Foster & Young [8], Kandori *et al.* [1] and Young [9]).
We consider a population divided into two groups types, which repeatedly play a 2x2 coordination game. The two types differ in preferences towards the coordination outcomes. Agents can learn the type of their opponent if they pay a cost. Such a cost represents the effort to exert if they want to learn their opponent's type. If they pay this cost, they can play a different action with respect to the type they meet. If they do not pay it, they can only play the same action with every agent. Given this change in the strategic set, we introduce a new possible perturbation. Agents can make a mistake in the information choice and a mistake in the coordination choice. We assume that one group is larger than the other because we are interested in capturing the effect of such an asymmetry in groups' size. We model two situations: a complete information scenario, where agents always learn their opponent's type, and an incomplete information one, where agents can learn their opponent's type conditional on paying a cost. Agents decide myopically their best reply based on the current state, which is always observable. However, in the incomplete information case, a player does not learn her/his opponent's type unless s/he pays for information. We say that a type is stronger in preferences for its favorite action than the other if it assigns higher payoffs to its favorite outcome or lower payoffs to the other outcome compared to the other group.

We find that cost level, strength in preferences, and group size are crucial drivers of the long-run stability of outcomes. Two different scenarios can happen, depending on the cost. Firstly, when the cost is zero or sufficiently low, agents always coordinate on their favorite action with agents of their same type. If one group is stronger in preferences for its favorite action or its size is sufficiently large compared to the other, every agent plays the action preferred by that group in inter-group interactions. Interestingly, agents from the group that is stronger in preferences never buy the information, while agents from the other group always buy it.
A second outcome occurs when the cost is high. In this case, agents never learn the type of their opponents, and they play the same action with every agent. Some agents coordinate on one action that they do not like, even with agents of their type. Indeed, we find that when one group is stronger in preferences than the other for its favorite action, or if its size is sufficiently large compared to the other, every agent coordinates on that group's favorite action. Even worse, the two types may play their favorite action and miscoordinate in inter-group interactions. We find that this occurs when both types are strong enough in preferences for their favorite action or if the two groups are sufficiently close in size.

Neary [7] considers a similar model, where each agent decides one single action valid for both types. Hence, it is as if learning the type of an opponent requires too much effort, and no agent ever learns it. In this scenario, if one type is enough stronger in preferences than the other for its favorite action, then every agent coordinates on the action preferred by that type in the long-run. Depending on payoffs and group size, miscoordination between the types can prevail in the long-run.
It is helpful to highlight our analysis with respect to the one proposed by Neary, from which we started. When the cost is high enough, our results are the same as the previous model. We further show what happens when agents can learn the opponent's type at a low cost. In this case, only states where all the agents in one group condition their action on the type can be stochastically stable. This result was not possible in the analysis of Neary.
Moreover, comparing the high cost case with the low cost case enrich the previous analysis. In this sense, we prove that miscoordination does not occur without incomplete information and a high cost. Unlike in Neary, strength in preferences or group size alone does not cause miscoordination. Indeed, when the cost is low, the two types always coordinate.

The paper is organized as follows: In Section 2, we explain the model's basic features. In Section 3, we determine the results for the complete information case where the cost is 0. In Section 4, we derive the results for the case with incomplete information and costly acquisition. We distinguish between 2 cases: c low enough and c high enough. In Section 5, we discuss results, and in Section 6, we conclude. We give all proofs in Appendix A.

2 The Model

	a	b
a	Π_A, Π_A	$0, 0$
b	$0, 0$	π_A, π_A

Table 1: Interactions inside group A.

	a	b
a	π_B, π_B	$0, 0$
b	$0, 0$	Π_B, Π_B

Table 2: Interactions inside group B.

	a	b
a	Π_A, π_B	$0, 0$
b	$0, 0$	π_A, Π_B

Table 3: Inter-group Interactions.

We consider N agents divided into two groups A and B, $N = N_A + N_B$. We assume $N_A > N_B + 1$ and $N_B > 1$. Each agent in group A is of type A, and each agent in group B is of type B. Throughout the paper, we will use types and groups as synonyms. Agents are randomly matched in pairs to play the 2x2 coordination game represented in Matrix 1 to 3. Matching occurs with uniform probability, regardless of type. Matrix 1 and 2 represent inside-group interactions, while Matrix 3 represents

inter-group interactions (A type row player and B type column player). We assume that $\Pi_A > \pi_A$ and thus, we name a the favorite action of type A. Equally, we assume $\Pi_B > \pi_B$ and hence, b is the favorite action of type B. We do not assume any particular order between Π_B, and Π_A. However, without loss of generality, we assume that $\Pi_A + \pi_A = \Pi_B + \pi_B$. We say that group A is stronger in preferences for its favorite action than group B if $\Pi_A > \Pi_B$ or $\pi_A < \pi_B$.
Before choosing between action a and b, agents choose whether to pay a cost to learn their opponent's type or not. If they do not pay it, they do not learn the type of their opponent, and they play one single action valid for both types. If they pay it, they can condition the action on the two types. We call information choice the first and coordination choice the second.

Consider player $i \in K$, with $K \in \{A, B\}$, and $K' \neq K \in \{A, B\}$. τ_i is the information choice of player i: if $\tau_i = 0$ player i does not learn the type of her/his opponent. If $\tau_i = 1$, player i pays a cost c, and learns the type. We assume $c \geq 0$. $x_{0i} \in \{a, b\}$ is the coordination choice when player i chooses $\tau_i = 0$. If $\tau_i = 1$, $x_{1i}^K \in \{a, b\}$ is the coordination choice when player i meets type K, while $x_{1i}^{K'} \in \{a, b\}$ is the coordination choice when player i meets type K'.

A pure strategy of an agent consists of her/his information choice, τ_i, and of her/his coordination choices conditioned on the information choice, *i.e.*

$$s_i = \left(\tau_i, x_{0i}, x_{1i}^K, x_{1i}^{K'}\right) \in \mathcal{S} = \{0, 1\} \times \{a, b\}^3.$$

Each player has sixteen strategies. However, we can safely neglect some strategies because they are both payoff equivalent (a player earns the same payoff disregarding which strategy s/he chooses) and behaviorally equivalent (a player earns the same payoff independently from which strategy the other players play against her/him).

We consider a model of noisy best response learning in discrete time (see Kandori *et al.* [1], Young [9]).
Each period $t = 0, 1, 2, \ldots$, independently from previous events, there is a positive probability $p \in (0, 1)$ that an agent is given the opportunity to revise her/his strategy. When such an event occurs, each agent chooses with positive probability a strategy that maximizes her/his payoff at time t. $s_i(t)$ is the strategy played by player i at time t. $U_s^i(s', s_{-i})$ is the payoff of player i that chooses strategy s' against the strategy profile s_{-i} played by all the other agents except i. Such a payoff depends on the random matching assumption and on the payoffs of the underlying 2x2 game. At time $t + 1$, player i chooses

$$s_i(t + 1) \in \arg\max_{s' \in \mathcal{S}} U_s^i(s', s_{-i}(t)).$$

If there is more than one strategy that maximizes the payoff, player i assigns the same probability to each of these strategies.
We group the sixteen strategies into six analogy classes that we call behaviors. We name behavior a (b) as the set of strategies when player $i \in K$ chooses $\tau_i = 0$, and $x_{0i} = a$ (b). We name behavior ab as the set of strategies when player i chooses $\tau_i = 1$, $x_{1i}^K = a$, and $x_{1i}^{K'} = b$, and so on and so forth. Z is the set of possible behaviors: $Z = (a, b, ab, ba, aa, bb)$. $z_i(t)$ is the behavior played by player i at time t as implied from $s_i(t)$. $z_{-i}(t)$ is the behavior profile played by all the other agents except i at

period t as implied from $s_{-i}(t)$. Note that behaviors catch all the relevant information as defined when agents are myopic best repliers. $U^i_z(z', z_{-i}(t))$ is the payoff for player i that chooses behavior z' against the behavior profile $z_{-i}(t)$. Such a payoff depends on the random matching assumption and the payoffs of the underlying 2x2 game. The dynamics of behaviors as implied by strategies coincide with the dynamics of behaviors, assuming that agents myopically best reply to a behavior profile. We formalize the result in the following lemma.

Lemma 1. *Given the dynamics of $z_i(t+1)$ as implied by $s_i(t+1)$, it holds that $z_i(t+1) \in \underset{z' \in Z}{argmax}\, U^i_z(z', z_{-i}(t))$.*

We provide the proof in the appendix, and we give an example here. Consider a player $i \in A$ such that the best thing to do for her/him is to play a with every player s/he meets regardless of the type. In this case, both $(0, a, a, b)$ and $(0, a, b, b)$ maximize her/his payoff. Differently, $(0, b, a, b)$, does not maximize her/his payoff since in this case, s/he plays b with every agent s/he meets. Moreover, the payoff of player i is equal whether $s_{-i} = (0, a, a, b)^{N-1}$ or $s_{-i} = (0, a, b, b)^{N-1}$, but different if $s_{-i} = (0, b, a, b)^{N-1}$. Therefore, all the strategies that belong to the same behavior are payoff equivalent and behaviorally equivalent.

A further reduction is possible because aa (bb) is behaviorally equivalent to a (b) for each player. The last observation and the fact that we are interested in the number of agents playing a with each group lead us to introduce the following state variable. We denote with n^{AA} (n^{BB}) the number of players of type A (B) playing action a with type A (B), and n^{AB} (n^{BA}) the number of players of type A (B) playing action a with type B (A). We define states as vectors of four components: $\omega = \left\{n^{AA}, n^{AB}, n^{BA}, n^{BB}\right\}$, with Ω being the state space, and $\omega_t = \left\{n^{AA}_t, n^{AB}_t, n^{BA}_t, n^{BB}_t\right\}$ the state at time t. At each t, all the agents know all the components of ω_t. Consider player i playing behavior $z_i(t)$ at time t. $U^i_{z_i(t)}(z', \omega_t)$ is the payoff of i if s/he chooses behavior z' at time $t+1$ against the state ω_t. All that matters for a decision-maker is ω_t and $z_i(t)$. We formalize the result in the following lemma.

Lemma 2. *Given the dynamics of ω_{t+1} generated by $z_i(t+1)$, it holds that $U^i_z(z', z_{-i}(t)) = U^i_{z_i(t)}(z', \omega_t)$. Moreover, $U^i_a(z', \omega_t) = U^i_{aa}(z', \omega_t) = U^i_{ab}(z', \omega_t)$, and $U^i_b(z', \omega_t) = U^i_{bb}(z', \omega_t) = U^i_{ba}(z', \omega_t)$.*

We prove the result in the appendix, and we give a short explanation here. If players are randomly matched, it is as if each agent plays against the entire population. Therefore, agents myopically best respond to the current period by looking at how many players of each type play action a with their group. Moreover, the player that is given the revision opportunity subtracts her/himself from the component of ω_t where s/he belongs. If $i \in K$ is playing behavior a, aa or ab at time t, s/he knows that $n^{KK}_t - 1$ players of group K are playing action a with type K at time t.
Define with θ_{t+1} the vector of players that are given the revision opportunity at time t. Given Lemma 2, it holds that ω_{t+1} depends on ω_t and on θ_{t+1}. That is, we can define a map $F(\cdot)$ such that $\omega_{t+1} = F(\omega_t, \theta_{t+1})$. The vector θ_{t+1} reveals whether the player who is given the revision opportunity is playing a behavior between a, aa, and ab, or a behavior between b, bb, and ba. In the first case we should look at U^i_a, while in the second at U^i_b.

From now on, we will refer to behaviors and states following the simplifications described above.

We illustrate here the general scheme of our presentation. We divide the analysis into two cases: complete information and incomplete information. For each case, we consider unperturbed dynamics (agents choose the best reply behavior with probability 1) and perturbed dynamics (agents choose a random behavior with a small probability). First, we help the reader understand how each player evaluates her/his best reply behavior and which states are absorbing. Second, we highlight the general structure of the dynamics with perturbation and then determine the stochastically stable states. We provide the proofs of all the results in the appendix and their intuition in the main text. In the next section, we analyze the case with complete information, hence, when the cost is zero.

3 Complete Information with Free Acquisition

In this section, we assume that each player can freely learn the type of her/his opponent when randomly matched with her/him. Without loss of generality, we assume that agents always learn the type of their opponent in this case. We refer to this condition as free information acquisition. Each player has four possible behaviors as defined in the previous section. $Z = \{aa, ab, ba, bb\}$, with $a = aa$, and $b = bb$ in this case.

Define $\pi_a^K = \begin{cases} \Pi_A & \text{if } K = A \\ \pi_B & \text{if } K = B \end{cases}$ and $\pi_b^K = \begin{cases} \pi_A & \text{if } K = A \\ \Pi_B & \text{if } K = B \end{cases}$.

Equation (1) to (4) are the payoffs for a player $i \in K$ playing aa or ab at time t.

$$U_a^i(aa, \omega_t) = \frac{N_K - 1}{N - 1}\frac{n_t^{KK} - 1}{N_K - 1}\pi_a^K + \frac{N_{K'}}{N-1}\frac{n_t^{K'K}}{N_{K'}}\pi_a^K, \tag{1}$$

$$U_a^i(ab, \omega_t) = \frac{N_K - 1}{N - 1}\frac{n_t^{KK} - 1}{N_K - 1}\pi_a^K + \frac{N_{K'}}{N-1}\frac{N_{K'} - n_t^{K'K}}{N_{K'}}\pi_b^K, \tag{2}$$

$$U_a^i(ba, \omega_t) = \frac{N_K - 1}{N - 1}\frac{N_K - n_t^{KK}}{N_K - 1}\pi_b^K + \frac{N_{K'}}{N-1}\frac{n_t^{K'K}}{N_{K'}}\pi_a^K, \tag{3}$$

$$U_a^i(bb, \omega_t) = \frac{N_K - 1}{N - 1}\frac{N_K - n_t^{KK}}{N_K - 1}\pi_b^K + \frac{N_{K'}}{N-1}\frac{N_{K'} - n_t^{K'K}}{N_{K'}}\pi_b^K. \tag{4}$$

The two ratios $\frac{N_K - 1}{N-1}$ and $\frac{N_{K'}}{N-1}$ express how much frequently a player meets type K or K'.

3.1 Unperturbed Dynamics

We begin the analysis for complete information by studying the dynamics of the system when agents play their best reply behavior with probability one.

We can separate the dynamics of the system into 3 different dynamics. The two regarding inside-group interactions *i.e.* n_t^{AA} and n_t^{BB} and the one regarding inter-group interaction, *i.e.* n_t^{AB} and n_t^{BA}. We call this subset of states $n_t^I = \left(n_t^{AB}, n_t^{BA}\right)$. Both n_t^{AA} and n_t^{BB} are one-dimensional; n_t^I instead is two-dimensional.

Lemma 3. *Under free information acquisition,* $n_{t+1}^{AA} = F_1(n_t^{AA}, \theta_{t+1})$*,* $n_{t+1}^{BB} = F_4(n_t^{BB}, \theta_{t+1})$ *and* $(n_{t+1}^{AB}, n_{t+1}^{BA}) = F_{2,3}(n_t^{AB}, n_t^{BA}, \theta_{t+1})$*.*

The intuition behind the result is as follows. If agents always learn their opponents' type, the inter-group dynamics does not interfere with the inside-group and vice-versa. If player $i \in K$ is given the revision opportunity, s/he chooses x_{1i}^K only based on n_t^{KK}.

Consider a subset of 8 states: $\omega^R = \{(N_A, N_A, N_B, N_B)$, $(0, N_A, N_B, N_B)$, $(N_A, N_A, N_B, 0)$, $(N_A, 0, 0, N_B)$, $(0, N_A, N_B, 0)$, $(N_A, 0, 0, 0)$, $(0, 0, 0, N_B)$ and $(0, 0, 0, 0)\}$.

Lemma 4. *Under free information acquisition, the states in* ω^R *are the unique absorbing states of the system.*

We call (N_A, N_A, N_B, N_B) and $(0, 0, 0, 0)$ Monomorphic States (MS from now on). Specifically, we refer to the first one as MS_a and to the second as MS_b. We label the remaining six as Polymorphic States (PS from now on). We call $(N_A, N_A, N_B, 0)$ PS_a and $(N_A, 0, 0, 0)$ PS_b. In MS, every agent plays the same action with any other agent; in PS, at least one type is differentiating the action. In MS_a, every agent plays aa, in MS_b, every agent plays bb. In PS_a, A types play aa and B types play ba. In PS_b, A types play ab while B types play bb. Both in PS_a and PS_b, all players coordinate on their favorite action with their similar.

In the model of Neary, only three absorbing states were possible: the two MS and a Type Monomorphic State where A types play a, and B types play b. The PS were not present in the previous analysis. We observe these absorbing states in our analysis, thanks to the possibility of conditioning the action on the type.
We can break the absorbing states in ω^R into the three dynamics in which we are interested. This simplification helps in understanding why only these states are absorbing. For instance in inter-group interactions there are just two possible absorbing states, namely (N_A, N_B) and $(0, 0)$. For what concerns inside-group interactions, N_A and 0 matters for n_t^{AA}, N_B and 0 for n_t^{BB}. For each dynamics, the states where every agent plays a or where every agent plays b with one type are absorbing. In this simplification, we can see the importance of Lemma 3. As a matter of fact, in all the dynamics we are studying, there are just two candidates to be stochastically stable. This result simplifies the stochastic stability analysis.

3.2 Perturbed Dynamics

We now introduce perturbations in the model presented in the previous section. We use tools and concepts developed by Freidlin & Wentzell [10], and refined by Ellison [11]. Agents can experiment while choosing their behaviors: there is a small probability that an agent does not choose her/his best response behavior when s/he is given the revision opportunity.
Given perturbations, ω_{t+1} depends on ω_t, θ_{t+1}, and on which players experiment among those who are given the revision opportunity. We define with ψ_{t+1} the set of players that do not choose their best reply behavior. Formally, $\omega_{t+1} = F(\omega_t, \theta_{t+1}, \psi_{t+1})$.
We use the uniform error model for mistakes: the probability of experimenting is equal for every agent and every state. At each step, if an agent is given the revision opportunity, s/he experiments with probability ε. In this section, we assume

that agents make mistakes only in the coordination choice: assuming $c = 0$, adding mistakes also in the information choice would not influence the analysis. Note that Lemma 3 is still valid under this specification.

If we consider a sequence of transition matrices $\{P^{\varepsilon}\}_{\varepsilon>0}$, with associated stationary distributions $\{\mu^{\varepsilon}\}_{\varepsilon>0}$, by continuity the accumulation point of $\{\mu^{\varepsilon}\}_{\varepsilon>0}$ that we call $\mu^{\star}$, is a stationary distribution of $P := \lim_{\varepsilon\to 0} P^{\varepsilon}$. Mutations guarantee the ergodicity of the Markov process and the uniqueness of the invariant distribution. We are interested in states which have positive probability in $\mu^{\star}$.

Definition 1. *A state $\bar{\omega}$ is stochastically stable if $\mu^{\star}(\bar{\omega}) > 0$ and it is uniquely stochastically stable if $\mu^{\star}(\bar{\omega}) = 1$.*

We define some useful concepts from Ellison [11]. Let $\bar{\omega}$ be an absorbing state of the unperturbed process. $D(\bar{\omega})$ is the basin of attraction of $\bar{\omega}$: the set of initial states from which the unperturbed Markov process converges to $\bar{\omega}$ with probability one. The Radius of the basin of attraction of $\bar{\omega}$ is the number of errors needed to leave $D(\bar{\omega})$, when the system starts in $\bar{\omega}$. Define a path from state $\bar{\omega}$ to state ω' as a sequence of distinct states $(\omega_1, \omega_2, \ldots, \omega_T)$, with $\omega_1 = \bar{\omega}$ and $\omega_T = \omega'$. $\Upsilon(\bar{\omega}, \omega')$ is the set of all paths from $\bar{\omega}$ to ω'. Define $r(\omega_1, \omega_2, \ldots, \omega_T)$ as the resistance of the path $(\omega_1, \omega_2, \ldots, \omega_T)$, namely the number of mistakes that occurs to pass from state $\bar{\omega}$ to state ω'. The Radius of $\bar{\omega}$ is then

$$R(\bar{\omega}) = \min_{(\omega_1,\omega_2,\ldots,\omega_T)\in\Upsilon(\bar{\omega},\Omega-D(\bar{\omega}))} r(\omega_1, \omega_2, \ldots, \omega_T).$$

Now define the Coradius of $\bar{\omega}$ as

$$CR(\bar{\omega}) = \max_{\omega\notin D(\bar{\omega})} \min_{(\omega_1,\omega_2,\ldots,\omega_T)\in\Upsilon(\omega,D(\bar{\omega}))} r(\omega_1, \omega_2, \ldots, \omega_T)$$

Thanks to Theorem 1 in Ellison [11], we know that if $R(\bar{\omega}) > CR(\bar{\omega})$, then $\bar{\omega}$ is uniquely stochastically stable.
We are ready to calculate the stochastically stable states under complete information.

Theorem 1. *Under free information acquisition, for N large enough, if $\frac{\pi_B}{\pi_A} < \frac{N_B}{N_A}$, then PS_b is uniquely stochastically stable. If $\frac{\pi_B}{\pi_A} > \frac{N_B}{N_A}$, then PS_a is uniquely stochastically stable.*

When the cost is null, agents can freely learn the type of their opponent. Therefore, in the long-run, they succeed in coordinating on their favorite action with their similar. Hence, n_t^{AA} always converge to N_A and n_t^{BB} always converge to 0. This result rules out Monomorphic States and other 4 Polymorphic States: only PS_a and PS_b are left. Which of the two is selected depends on strength in preferences and group size. Two effects determine the results in the long-run. Firstly, if $\pi_A = \pi_B$, PS_a is uniquely stochastically stable. The majority prevails in inter-group interactions if the two groups are equally strong in preferences.
Secondly, if $\pi_A \neq \pi_B$, there is a trade-off between strength in preferences and group size. If $\frac{\pi_B}{\pi_A} > \frac{N_B}{N_A}$, either type A is stronger in preferences than type B, or group A is sufficiently larger than group B. In both of the two situations, the number of mistakes necessary to leave PS_a is bigger than the one to leave PS_b: in a sense, more errors are needed to make b best reply for A players than to make a best reply for

B players. Therefore, every agent will play action a in inter-group interactions. A similar reasoning applies if $\frac{\pi_B}{\pi_A} < \frac{N_B}{N_A}$.
Interestingly, in both cases, only agents of one group need to learn their opponents' type: the agents from the group that is weaker in preferences or sufficiently smaller than the other.

Unlike in the analysis of Neary, if learning the opponent's type is costless, the Monomorphic States are never stochastically stable. This result is a consequence of the possibility to condition the action on the type. Indeed, if agents can freely learn the opponent's type, they will always play their favorite action inside-group.

We provide two numerical examples to explain how the model works in Figure 1 and 2. We represent just n_t^I, hence, a two-dimensional dynamics. Red states represent the basin of attraction of $(0,0)$, while green states the one of (N_A, N_B). From grey states there are paths of zero resistance both to $(0,0)$ and to (N_A, N_B). Any path that involves more players playing a within red states has a positive resistance. Every path that involves fewer people playing a within green states has a positive resistance. The Radius of $(0,0)$ is equal to the Coradius of (N_A, N_B), and it is the minimum error path from $(0,0)$ to grey states. The Coradius of $(0,0)$ is equal to the Radius of (N_A, N_B), and it is the minimum error path from (N_A, N_B) to grey states.

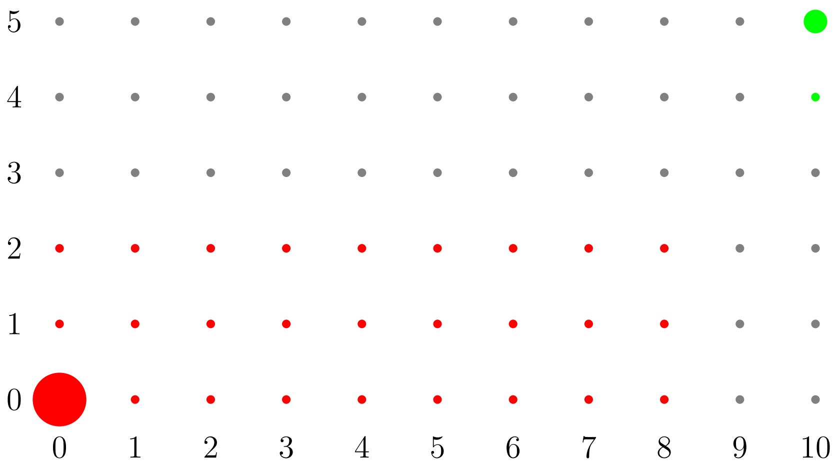

Figure 1: $PS_b = (0,0)$ is uniquely stochastically stable: $\frac{\pi_B}{\pi_A} < \frac{N_B}{N_A}$.

Firstly, consider the example in Figure 1. $N_A = 10$, $N_B = 5$, $\pi_A = 8$, $\Pi_A = 10$, $\pi_B = 3$, $\Pi_B = 15$. Clearly, $\frac{\pi_B}{\pi_A} = \frac{3}{8} < \frac{5}{10} = \frac{N_B}{N_A}$. In this case $R(10,5) = CR(0,0) = 1$, while $R(0,0) = CR(10,5) = 3$. Hence, $(0,0)$ is the uniquely stochastically stable state. We give here a short intuition. Starting from $(0,0)$, the minimum error path to grey states is the one that reaches $(0,3)$. The minimum error path from $(10,5)$ to grey states is the one that reaches $(9,5)$. Hence, fewer mistakes are needed to exit from the green states than to exit from the red states. This is why $PS_b = (10,0,0,0)$ is uniquely stochastically stable.

Secondly, consider the example in Figure 2. $N_A = 10$, $N_B = 5$, $\pi_A = 3$, $\Pi_A = 15$, $\pi_B = 8$, $\Pi_B = 10$. Note that $\frac{\pi_B}{\pi_A} = \frac{8}{3} > \frac{5}{10} = \frac{N_B}{N_A}$. In this case, $R(10,5) = CR(0,0) = 4$, $CR(10,5) = R(0,0) = 1$. Hence, $PS_a = (10,5)$ is uniquely stochastically stable. In this case, the minimum error path to exit green states is the one that reaches $(6,5)$ or $(10,1)$. The one to exit the red states is the one that reaches $(0,1)$.

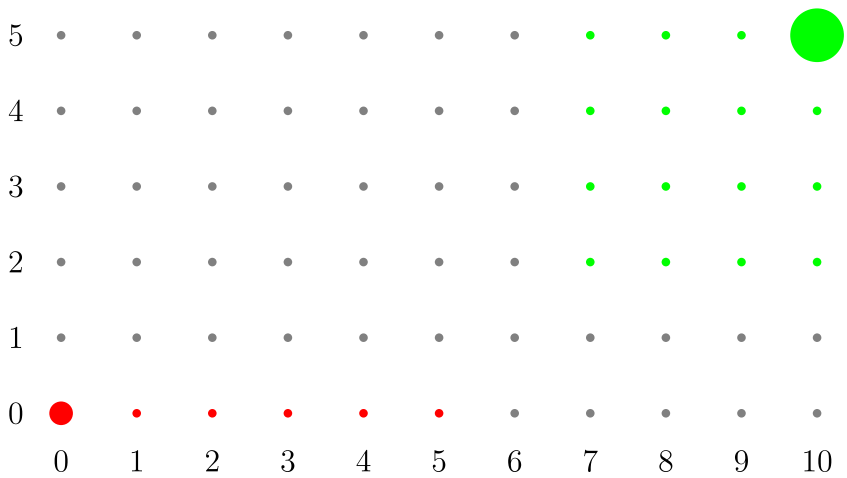

Figure 2: $PS_a = (10, 5)$ is uniquely stochastically stable: $\frac{\pi_B}{\pi_A} > \frac{N_B}{N_A}$.

4 Incomplete Information with Costly Acquisition

In this section, we assume that each player can not freely learn the type of her/his opponent. Each agent can buy this information at cost $c > 0$. We refer to this condition as costly information acquisition.[2]
This time $Z = \{a, b, ab, ba, aa, bb\}$, $\forall i \in N$. It is trivial to show that there are 4 strictly dominant behavior out of the 6 behaviors, indeed, $U^i(aa) = U^i(a) - c$ and $U^i(bb) = U^i(b) - c$. Hence, for all $i \in N$, $U^i(aa) < U^i(a)$ and $U^i(bb) < U^i(b)$, $\forall c > 0$. We define strictly dominant behaviors as $Z^o = \{a, b, ab, ba\}$, $\forall i \in N$, with z_i^o being a strictly dominant behavior of player i.
Equation (5) to (8) are the payoffs at time t, for a player $i \in K$ currently playing a or ab.

$$U_a^i(a, \omega_t) = \frac{n_t^{KK} + n_t^{K'K} - 1}{N - 1} \pi_a^K, \tag{5}$$

$$U_a^i(b, \omega_t) = \frac{N - n_t^{KK} - n_t^{K'K}}{N - 1} \pi_b^K, \tag{6}$$

$$U_a^i(ab, \omega_t) = \frac{N_K - 1}{N - 1} \frac{n_t^{KK} - 1}{N_K - 1} \pi_a^K + \frac{N_{K'}}{N - 1} \frac{N_{K'} - n_t^{K'K}}{N_{K'}} \pi_b^K - c, \tag{7}$$

$$U_a^i(ba, \omega_t) = \frac{N_K - 1}{N - 1} \frac{N_K - n_t^{KK}}{N_K - 1} \pi_b^K + \frac{N_{K'}}{N - 1} \frac{n_t^{K'K}}{N_{K'}} \pi_a^K - c. \tag{8}$$

To help the reader visualize the differences between this section and Section 3, we did not explicit in Equation (5) and (6) the frequencies of meetings. Note that if $c = 0$, then $aa = a$ and $bb = b$.
We begin the analysis again with the unperturbed dynamics.

[2]It is trivial to notice that Lemma 3 is not valid anymore. Indeed, since agents learn the type of their opponent conditional on paying a cost, not every player pays it, and the dynamics are no longer separable.

4.1 Unperturbed Dynamics

State	Condition on group size and payoffs	Conditions on c
MS_a	none	none
MS_b	none	none
TS	$\frac{\pi_B}{\Pi_B} < \frac{N_B-1}{N_A}$	$c > \max\left\{\frac{N_B}{N-1}\pi_A, \frac{N_A}{N-1}\pi_B\right\}$
PS_b	none	$c < \frac{N_B}{N-1}\pi_A$
PS_a	1) $\frac{\pi_B}{\Pi_B} > \frac{N_B-1}{N_A}$ 2) $\frac{\pi_B}{\Pi_B} < \frac{N_B-1}{N_A}$	1) $c < \frac{N_B-1}{N-1}\Pi_B$ 2) $c < \frac{N_A}{N-1}\pi_B$
$(0, N_A, N_B, N_B)$	1) $\frac{\pi_A}{\Pi_A} < \frac{N_B}{N_A-1}$ 2) $\frac{\pi_A}{\Pi_A} > \frac{N_B}{N_A-1}$	1) $c < \frac{N_A-1}{N-1}\pi_A$ 2) $c < \frac{N_B}{N-1}\Pi_A$
$(N_A, 0, 0, N_B)$	none	$c < \min\left\{\frac{N_B}{N-1}\pi_A, \frac{N_B-1}{N-1}\pi_B\right\}$
$(0, N_A, N_B, 0)$	1) $\frac{\pi_A}{\Pi_A} < \frac{N_A-1}{N_B}$ and $\frac{\pi_B}{\Pi_B} > \frac{N_B-1}{N_A}$ 2) $\frac{\pi_A}{\Pi_A} > \frac{N_A-1}{N_B}$ and $\frac{\pi_B}{\Pi_B} > \frac{N_B-1}{N_A}$ 3) $\frac{\pi_A}{\Pi_A} < \frac{N_A-1}{N_B}$ and $\frac{\pi_B}{\Pi_B} < \frac{N_B-1}{N_A}$ 4) $\frac{\pi_A}{\Pi_A} > \frac{N_A-1}{N_B}$ and $\frac{\pi_B}{\Pi_B} < \frac{N_B-1}{N_A}$	1) $c < \min\left\{\frac{N_A-1}{N-1}\pi_A, \frac{N_B-1}{N-1}\Pi_B\right\}$ 2) $c < \min\left\{\frac{N_B}{N-1}\Pi_A, \frac{N_B-1}{N-1}\Pi_B\right\}$ 3) $c < \min\left\{\frac{N_A-1}{N-1}\pi_A, \frac{N_A}{N-1}\pi_B\right\}$ 4) $c < \min\left\{\frac{N_B}{N-1}\Pi_A, \frac{N_A}{N-1}\pi_B\right\}$
$(0, 0, 0, N_B)$	none	$c < \frac{N_B-1}{N-1}\pi_B$

Table 4: Necessary and sufficient conditions for absorbing states.

So far, there are no more random elements with respect to Section 3. Therefore, $\omega_{t+1} = F(\omega_t, \theta_{t+1})$. Nine states can be absorbing under this specification.

Lemma 5. *Under costly information acquisition, there are nine possible absorbing states:* $\omega^R \cup (N_A, N_A, 0, 0)$.

We summarize all the relevant information in Table 4. The reader can note two differences with respect to Section 3: firstly, some states are absorbing if and only if some conditions hold, and secondly, there is one more possible absorbing state, that is $(N_A, N_B, 0, 0)$. Such an absorbing state was also possible in Neary under the same conditions on payoffs and group size.

Where we write "none", we mean that a state is always absorbing for every value of group size, payoffs, and/or the cost. We name $(N_A, N_B, 0, 0)$ the Type Monomorphic State (TS from now on): in this state, each type is playing its favorite action, causing miscoordination in inter-group interactions. Monomorphic States are absorbing states for every value of group size, payoffs, and cost. When every player is playing one action with any other player, agents do not need to learn their opponent's type (the information cost does not matter). They best reply to these states by playing the same action.

Polymorphic States are absorbing if and only if the cost is low enough: if the cost is too high, buying the information is too expensive, and agents best reply to Polymorphic States by playing a or b. The Type Monomorphic State is absorbing if type B is either sufficiently large compared to group A or strong enough in preferences for its favorite action and if the cost is high enough. The intuition is the following. On the one hand, if type B is weak in preferences or small enough, every player of type B best replies to TS by playing a if the cost is high. On the other hand, if the cost is low enough, every player best replies to this state by buying the information and differentiating the action.

4.2 Perturbed Dynamics

We now introduce perturbed dynamics. In this case, we assume that agents can make two types of errors: they can make a mistake in the information choice and in the coordination choice. Choosing the wrong behavior, in this case, can mean both. We say that with probability η, an agent that is given the revision opportunity at time t chooses to buy the information when it is not optimal. With probability ε, s/he makes a mistake in the coordination choice. We could have chosen to set only one probability of experimenting with a different behavior or strategy.
The logic behind our assumption is to capture behaviorally relevant errors. We assume a double punishment mechanism for players choosing by mistake the information level and the coordination action. Specifically, our error counting is not influenced by our definition of behaviors. We could have made the same assumption starting from the standard definition of strategies assuming that agents can make separate mistakes in choosing the two actions that constitute the strategy. Our assumption is in line with works such as Jackson & Watts [12] and Bhaskar & Vega-Redondo [13], which assume errors in the coordination choice and the link choice.
Formally, $\omega_{t+1} = F(\omega_t, \theta_{t+1}, \psi^c_{t+1})$. Where $\psi^c_{t+1} = \{\psi^\varepsilon_{t+1}, \psi^\eta_{t+1}\}$ is the set of players that make a mistake at time t. ψ^ε_{t+1} is the set of players that make a mistake in the coordination choice, and ψ^η_{t+1} the set of players that make a mistake in the information choice.
Since we are assuming two types of errors, the concept of resistance changes, we then need to consider three types of resistances. We call $r_\varepsilon(\omega_t, \ldots, \omega_s)$ the path from state ω_t to state ω_s with ε errors (players make a mistake in the coordination choice). We call $r_\eta(\omega_t, \ldots, \omega_s)$ the path with η errors (players make a mistake in the information choice). Finally, we call $r_{\varepsilon\eta}(\omega_t, \ldots, \omega_s)$ the path with errors both in the coordination choice and the information choice. Since we do not make further assumptions on ε and η (probability of making errors uniformly distributed), we can assume $\eta \propto \varepsilon$.
We count each error in the path of both ε and η errors as 1, however, $r_{\varepsilon\eta}(\omega_t, \ldots, \omega_s)$ is always double since it implies a double error. Indeed, we can see this kind of error as the sum of two components, one in η and the other in ε, namely $r_{\varepsilon\eta}(\omega_t, \ldots, \omega_s) = r_{\varepsilon\eta|_\varepsilon}(\omega_t, \ldots, \omega_s) + r_{\varepsilon\eta|_\eta}(\omega_t, \ldots, \omega_s)$.
For example, think about $\omega_t = MS_a$, and that one player from B is given the revision opportunity. Consider the case where s/he makes a mistake both in the information choice and in the coordination choice. For example, s/he learns the type and s/he plays a with A and b with B. This error delineates a path from MS_a to the state $(N_A, N_A, N_B, N_B - 1)$ of resistance $r_{\varepsilon\eta}(MS_a, \ldots, (N_A, N_A, N_B, N_B - 1)) = 2$. Next, think about $\omega_t = TS$: the transition from TS to $(N_A, N_A - 1, 0, 0)$ happens with one η error. One player from A should make a mistake in the information choice and optimally choosing ab. In this case, $r_\eta(TS, \ldots, (N_A, N_A - 1, 0, 0)) = 1$. With a similar reasoning, $r_\varepsilon(MS_a, \ldots, (N_A - 1, N_A - 1, N_B, N_B)) = 1$: a player of type A makes a mistake in the coordination choice and chooses b.

Before giving the results, we explain why using behaviors instead of strategies does not influence the stochastic stability analysis. Let us consider all the sixteen strategies as presented in Section 2, and just one kind of mistake in the choice of the strategy. Let us take two strategies $s', s'' \in z'$ and a third strategy $s''' \in z''$. Now consider the state $\bar{\omega}$, where $s_i = s'$, $\forall i \in N$ and the state ω', where $s_i = s'$, $\forall i \in \{0, \ldots, N-m-1\}$

and $s_j = s''$, $\forall j \in \{N-m, \dots, N\}$. Since s' and s'' are both payoff equivalent and behaviorally equivalent, s' and s'' are the best reply strategies $\forall i \in N$, in both states $\bar{\omega}$ and ω'. Therefore at each step, every player who is given the revision opportunity in state $\bar{\omega}$ or ω' chooses s' and s'' with equal probability. Now let us consider the state $\bar{\omega}'$ where $s_i = s'''$, $\forall i \in N$. When considering the transition between $\bar{\omega}$ and $\bar{\omega}'$, the number of mistakes necessary for this transition is the same whether the path passes through ω' or not because the best reply strategy is the same in both ω' and $\bar{\omega}$. Therefore, when computing the stochastically stable state we can neglect s'' and ω'.

We divide this part of the analysis into two cases, the first one where the cost is low and the second one when the cost is high.

4.2.1 Low Cost

In this section, we discuss the case when c is as low as possible but greater than 0.

Corollary 1. *Under costly information acquisition, if* $0 < c < \min\{\frac{\pi_A}{N-1}, \frac{\pi_B}{N-1}\}$, MS *and* PS *are absorbing states, while* TS *is not an absorbing state.*

The proof is straightforward from Table 4. In this case, there are 8 candidates to be stochastically stable equilibria.

Theorem 2. *Under costly information acquisition, for* N *large enough, take* $0 < c < \min\{\frac{\pi_A}{N-1}, \frac{\pi_B}{N-1}\}$. *If* $\frac{\pi_B}{\pi_A} < \frac{N_B}{N_A}$, *then* PS_b *is uniquely stochastically stable. If* $\frac{\pi_B}{\pi_A} > \frac{N_B}{N_A}$, *then* PS_a *is uniquely stochastically stable.*

The conditions are the same as in Theorem 1. When the cost is low enough, whenever a player can buy the information, s/he does it. Consequently, the basins of attraction of Polymorphic States have the dimension they had under free information acquisition. So does for both Monomorphic States. Due to these two effects, the results are the same as under free information acquisition. This result is not surprising per se but serves as a robustness check of the results of Section 3.2.

4.2.2 High Cost

In this part of the analysis, we focus on a case when only MS and TS are absorbing states.

Define the following set of values:

$$\Xi_{PS} = \left\{ \frac{N_B}{N-1}\pi_A, \frac{N_A}{N-1}\pi_B, \frac{N_B-1}{N-1}\Pi_B, \frac{N_A-1}{N-1}\pi_A, \frac{N_B}{N-1}\Pi_A, \frac{N_B-1}{N-1}\pi_B \right\}.$$

Corollary 2. *Under costly information acquisition, if* $c > max\{\Xi_{PS}\}$ *and* $\frac{\pi_B}{\Pi_B} < \frac{N_B-1}{N_A}$, *then only* MS *and* TS *are absorbing states. If* $\frac{\pi_B}{\Pi_B} \geq \frac{N_B-1}{N_A}$, *then only* MS *are absorbing states.*

The proof is straightforward from Table 4 and therefore, we omit it. We previously give the intuition behind this result. Let us firstly consider the case in which TS is not an absorbing state, hence, the case when $\frac{\pi_B}{\Pi_B} \geq \frac{N_B-1}{N_A}$.

Theorem 3. *Under costly information acquisition, for N large enough, take $\frac{\pi_B}{\Pi_B} \geq \frac{N_B-1}{N_A}$ and $c > max\{\Xi_{PS}\}$. If $N_A > \frac{2N\pi_A+\Pi_A-\pi_A}{\Pi_A+\pi_A}$, then MS_a is uniquely stochastically stable. If $N_A < \frac{2N\pi_A+\Pi_A-\pi_A}{\Pi_A+\pi_A}$, then MS_b is uniquely stochastically stable.*

If group A is sufficiently large or strong enough in preferences, the minimum number of errors to exit from the basin of attraction of MS_a is higher than the minimum number of errors to exit from the one of MS_b. Therefore, MS_a is uniquely stochastically stable: every agent plays behavior a in the long-run.
Now we analyze the case when also TS is a strict equilibrium.

Theorem 4. *Under costly information acquisition, for N large enough, take $\frac{\pi_B}{\Pi_B} < \frac{N_B-1}{N_A}$ and $c > max\{\Xi_{PS}\}$.*

- *If $N(\pi_B - \pi_A) > N_B\Pi_B - N_A\pi_B - \Pi_B + \pi_B + \Pi_A$, then MS_a is uniquely stochastically stable.*

- *If $N(\pi_A - \pi_B) > N_A\Pi_A - N_B\pi_A - \Pi_A + \Pi_B + \pi_A$, then MS_b is uniquely stochastically stable.*

- *If $\min\{N_A\Pi_A - N_B\pi_A + \pi_A, N_B\Pi_B - N_A\pi_B + \pi_B\} - \Pi_A - \Pi_B > N(\pi_A + \pi_B)$, then TS is uniquely stochastically stable.*

Moreover, when all the above conditions do not hold simultaneously:

- *If $N(\pi_B - \pi_A) > N_B(\Pi_B + \pi_A) - N_A(\Pi_A + \pi_B) + \Pi_A - \pi_A + \pi_B - \Pi_B$, then MS_a is uniquely stochastically stable.*

- *If $N(\pi_A - \pi_B) > N_A(\Pi_A + \pi_B) - N_B(\Pi_B + \pi_A) - \Pi_A + \pi_A - \pi_B + \Pi_B$, then MS_b is uniquely stochastically stable.*

- *If $N(\pi_A - \pi_B) = N_A(\Pi_A + \pi_B) - N_B(\Pi_B + \pi_A) - \Pi_A + \pi_A - \pi_B + \Pi_B$, then both MS_a and MS_b are stochastically stable.*

We divide the statement of the theorem into two parts for technical reasons. However, the reader can understand the results from the first three conditions. The first condition expresses a situation where type A is stronger in preferences than type B or group A is sufficiently larger than group B. In this case, there is an asymmetry in the two costs of exiting the two basins of attraction of MS_a and MS_b. Exit from the first requires more errors than exit from the second. Moreover, reaching MS_a from TS requires less errors than reaching MS_b from TS. For this reason $R(MS_a) > CR(MS_a)$ and MS_a is uniquely stochastically stable in this case. Similar reasoning applies to the second condition.
The third condition expresses a case where both types are strong enough in preferences, or the two groups have sufficiently similar sizes. Numerous errors are required to exit from TS, compared to how many errors are required to reach TS from the two MS. Indeed, TS is the state where both types are playing their favorite action. Therefore, in this case, all the agents play their favorite action in the long-run, but they miscoordinate in inter-group interactions.

The results of Theorem 3 and 4 reach the same conclusions as Neary. However, our analysis allows us to affirm that only with a high cost, the MS or the TS are stochastically stable. This result enriches the previous analysis.
As a further contribution, comparing these results with those in Section 4.2.1, we can give the two conditions for miscoordination to happen in the long-run. First, the cost to pay to learn the opponent's type should be so high that agents never learn their opponents' type. Second, both types should be strong enough in preferences or sufficiently close in size. The following lemma states what happens when the cost takes medium values.

Lemma 6. *If* $\min\left\{\frac{\pi_A}{N-1}, \frac{\pi_B}{N-1}\right\} < c < \max\{\Xi_{PS}\}$, *then the stochastically stable states must be in the set* $M = \{PS_a, PS_b, MS_a, MS_b\}$.

When the cost lowers a tiny quantity from the level of Section 4.2.2, TS is not absorbing anymore. Therefore, only PS and MS can be stochastically stable when the cost is in the interval above. However, not all the PS can be stochastically stable, only the two where all the agents play their favorite action in inside-group interactions. The intuition of this result is simple: if agents condition their action on the types in the long-run, they play their favorite action with their similar.
We do not study when do MS are stochastically stable and when do PS are: we leave this question for future analysis. Nevertheless, given the results of Section 4.2.1 and 4.2.2, we expect that for higher levels of the cost MS are stochastically stable, and for lower levels, PS are stochastically stable.

5 Discussion

The results of our model involve three fields of the literature. Firstly, we contribute to the literature on social conventions. Secondly, we contribute to the literature on stochastic stability analysis, and lastly, we contribute to the literature on costly information acquisition.
For what concerns social conventions, many works in this field study the existence in the long-run of heterogeneous strategy profiles. We started from the original model of Neary [7], which considers agents heterogeneous in preferences, but with a smaller strategic set than ours.[3] Neary's model gives conditions for the stochastic stability of a heterogeneous strategy profile that causes miscoordination in inter-group interactions in a random matching case. Neary & Newton [19] expands the previous idea to investigate the role of different classes of graphs on the long-run result. It finds conditions on graphs such that a heterogeneous strategy profile is stochastically stable. It also considers the choice of a social planner that wants to induce heterogeneous or homogeneous behavior in a population.
Carvalho [20] considers a similar model, where agents choose their actions from a set of culturally constrained possibilities, and the heterogeneous strategy profile is labeled as miscoordination. It finds that cultural constraints drive miscoordination in the long-run. Michaeli & Spiro [21] studies a game between agents with heterogeneous preferences and who feel pressure from behaving differently. Such a study

[3]Heterogeneity has been discussed in previous works such as Smith & Price [14], Friedman [15], Cressman *et al.* [16], Cressman *et al.* [17] or Quilter *et al.* [18].

characterizes the circumstances under which a biased norm can prevail on a non-biased norm. Tanaka *et al.* [22] studies how local dialects survive in a society with an official language. Naidu *et al.* [23] studies the evolution of egalitarian and inegalitarian conventions in a framework with asymmetry similar to the language game. Likewise, Belloc & Bowles [24] examines the evolution and the persistence of inferior cultural conventions.
We introduce the assumption that agents can condition the action on the type if they pay a cost. This assumption helps to understand the conditions for the stability of the Type Monomorphic State, where agents miscoordinate in inter-group interactions. We show that incomplete information, high cost, strength in preferences, and group size are key drivers for miscoordination. Like many works in this literature, we show the importance of strength in preferences and group size in the equilibrium selection. Concerning network formation literature, Goyal *et al.* [25] experiments the language game, testing whether agents segregate or conform to the majority. van Gerwen & Buskens [26] suggests a variant of the language game similar to our version but in a model with networks to study the influence of partner-specific behavior on coordination. Concerning auctions theory, He [27] studies a framework where each individual of a population divided into two types has to choose between two skills: a "majority" and a "minority" one. It finds that minorities are advantaged in competition context rather than in coordination one. He & Wu [28] tests the role of compromise in the battle of sexes with an experiment.
Like these works, we show that group size and strength in preferences matter for the long-run equilibrium selection. The states where the action preferred by the minority is played in most of the interactions (MS_b or PS_b) are stochastically stable provided that the minority is strong enough in preferences or sufficiently large.
A parallel field is the one of bilingual games such as the one proposed by Goyal & Janssen [29] or Galesloot & Goyal [30]: these models consider situations in which agents are homogeneous in preferences but can become bilingual at a given cost.

Nyborg *et al.* [31] has recently suggested the applicability of tipping points theories to policy and interventions. This field could produce explanations and further research questions for language games (see Neary & Newton [19] again). Indeed, in our model, there are situations in which the majority conforms to the action preferred by the minority. This fact happens even in inside-group interactions.

Concerning the technical literature on stochastic stability, we contribute by applying standard stochastic stability techniques to an atypical context, such as the costly information acquisition. Specifically, we show that with low cost levels, Polymorphic States where all agents in one group condition their action on the type are stochastically stable. Interestingly only one group of agents learns their opponents' type. With high cost levels, Monomorphic States where no agent conditions her/his action on the type are stochastically stable. Since the seminal works by Bergin & Lipman [32], and Blume [33], many studies have focused on testing the role of different error models in equilibrium selection. We use a uniform error model, and introducing different models could be an interesting exercise for future studies.
Among the many models that can be used, there are three relevant variants: payoff/cost-dependent mistakes (Sandholm [34], Dokumacı & Sandholm [35] and Klaus & Newton [36], Blume [37] and Myatt & Wallace [38]), intentional mistakes (Naidu *et al.* [39] and Hwang *et al.* [40]) and condition dependent mistakes (Bilancini & Boncinelli [41]).

Important experimental works in this literature have been done by Lim & Neary [42], Hwang *et al.* [43], Mäs & Nax [44], and Bilancini *et al.* [45].

Other works contribute to the literature on stochastic stability from the theoretical perspective (see Newton [46] for an exhaustive review of the field). Recently, Newton [47] has expanded the domain of behavioral rules regarding the results of stochastic stability. Sawa & Wu [48] shows that with loss aversion individuals, the stochastic stability of risk dominant equilibria is no longer guaranteed. Sawa & Wu [49] introduces reference-dependent preferences and analyzes the stochastic stability of best response dynamics. Staudigl [50] examines stochastic stability in an asymmetric binary choice coordination game.

For what concerns the literature on costly information acquisition, many works interpret the information's cost as a cognition cost (see the seminal contributions by Simon [51], or Grossman & Stiglitz [52]). Our paper is one of those. Many studies place this framework in a sender-receiver game. This is the case of Dewatripont & Tirole [53], which builds a model of costly communication in a sender-receiver setup. More recent contributions in this literature are Dewatripont [54], Caillaud & Tirole [55], Tirole [56] and Butler *et al.* [57]. Bilancini & Boncinelli [58] applies this model to persuasion games with labeling. Both Bilancini & Boncinelli [59] and Bilancini & Boncinelli [60] consider coarse thinkers receivers, combining costly information acquisition with the theory of Jehiel [61].[4] To the best of our knowledge, we are the first to use costly information acquisition in an evolutionary model.
Many works use similar concepts of cost in the evolutionary game theory literature. Staudigl & Weidenholzer [63] considers a model, where agents can pay a cost to form links with each other. The main finding is that if a small number of players play the Payoff-Dominant action, other players connect with them and play the Payoff-Dominant action.
The work by Bilancini & Boncinelli [64] extends Staudigl & Weidenholzer [63]. The model introduces the fact that interacting with a different type might be costly for an agent. It finds that when the cost is low, the Payoff-Dominant strategy is the stochastically stable one. When the cost is high, the two types in the population coordinate on two different strategies: one on the Risk-Dominant and the other on the Payoff-Dominant. Similarly, Bilancini *et al.* [65] studies the role of cultural intolerance and assortativity in a coordination context. In the model, there is a population divided into two cultural groups, and each group sustains a cost from interacting with the other group. It finds interesting conditions under which cooperation can emerge even with cultural intolerance.

6 Conclusions

We can summarize our results as follows. When agents learn the type of their opponents at a low cost, they always coordinate. They play their favorite action with their similar, while in inter-group interactions, they play the favorite action of the group that is stronger in preferences or with size large enough. If the cost is high,

[4] A recent field of the literature concerns rational inattention, which is a way of endogenizing the cost of information (see Mackowiak *et al.* [62] for an exhaustive review). We assume that the cost is exogenous and homogeneous for each player.

agents never learn the type of their opponents. Either all the agents play the same action with every agent, or all the agents play their favorite action.

Comparing Section 4.2.2 and 4.2.1, we can see the impact of varying the cost levels on the long-run results. A change in the cost level produces two effects that need perhaps a further investigation. The first effect concerns the change in the payoff from the interactions between agents. The second concerns the change in the purchase of the information.

Consider a starting situation where the cost is low. Agents always coordinate on their favorite action in inside-group interactions. If the cost increases, agents will stop learning their opponent's type (hence, they stop paying the cost), and they will begin to play the same action with any other player. If this happens, either Monomorphic States establish in the long-run, or the Type Monomorphic State emerges. In the first case, a group of agents coordinates on its second best option, even in inside-group interactions. For this group, there could be a certain loss in terms of welfare. In the second case, agents miscoordinate in inter-group interactions, and hence, all of them could have a certain loss in welfare.

Nevertheless, when the cost is low, there is a "free-riding" behavior that vanishes if the cost increases. In fact, with low cost levels, only one type pays the cost, and the other never pays it. In one case, A types play their favorite action both in inside-group and inter-group interactions, and they never pay the cost, while B types afford it. In the other case, the opposite happens. Hence, when the cost increases, one of the two groups will benefit from not paying for the information anymore. Future studies could address the implications of this trade-off between successful coordination and the possibility of not paying the cost.

We conclude with a short comparison of our result with the one of Neary [7]. It is worthwhile to mention a contrast that is a consequence of the possibility of conditioning the action on the type of the player. In the model of Neary, a change in the strength of preferences or the group size of one type does not affect the behavior of the other type. We can find this effect even in our model when the cost is high. For example, when MS_a is stochastically stable and type B becomes strong enough in preferences or sufficiently large, the new stochastically stable state becomes TS. This means that A type does not change its behavior. However, when the cost is sufficiently low, the change in payoffs or group size of one type influences the other type's behavior in inter-group interactions. For instance, when PS_a is stochastically stable, if type B becomes strong enough in preferences or sufficiently large, PS_b becomes stochastically stable. Both types change the way they behave in inter-group interactions.

Nevertheless, we can interpret similarly the passing from MS_a to TS and the one from PS_a to PS_b. Both groups keep playing their favorite action in inside-group interactions, and what happens in inter-group interactions depends on strength in preferences and group size. Therefore, under this aspect, the behavioral interpretation of our results is similar to Neary's.

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A Proofs

Proof of Lemma 1

We have to formally show that each strategy inside the same behavior is behaviorally and payoff equivalent for each player. Consider a player $i \in K$. Define $g_i^K(s_{-i})$ and $g_i^{K'}(s_{-i})$ as the frequencies of successful coordination for i on action a with group K and K' given strategy profile s_{-i}.

$$U_s^i((0,a,a,a),s_{-i}) = U_s^i((0,a,b,b),s_{-i}) =$$
$$U_s^i((0,a,b,a),s_{-i}) = U_s^i((0,a,a,b),s_{-i}) = \frac{N_K-1}{N-1} g_i^K(s_{-i})\pi_a^K + \frac{N_{K'}}{N-1} g_i^{K'}(s_{-i})\pi_a^K.$$

Therefore, if $(0,a,a,a)$ is the maximizer, then also $(0,a,a,b)$, $(0,a,b,a)$ and $(0,a,b,b)$ are so. Hence, in this case, i maximizes her/his payoff choosing behavior a. Moreover, consider $s'_{-i} = (0,a,a,b)^{N-1}$, and $s''_{-i} = (0,a,a,a)^{N-1}$. In this case $g_i^K(s'_{-i}) = g_i^K(s''_{-i})$, so for $g_i^{K'}$. Contrarily, if $s'''_{-i} = (0,b,a,a)^{N-1}$, $g_i^K(s'_{-i}) \neq g_i^K(s'''_{-i})$, so for $g_i^{K'}$. Therefore, $U_s^i(a,s'_{-i}) = U_s^i(a,s''_{-i}) = U_z^i(a,a^{N-1})$. Thanks to symmetry in payoff matrix the argument stands for all strategies and behaviors. This passage completes the proof.

$\square$

Proof of Lemma 2

Consider a player $i \in K$ currently playing behavior a that is given the revision opportunity at time t. $g_i^K(z_{-i}(t))$ is the frequency of successful coordinations of player i on action a with group K at time t, given $z_{-i}(t)$. In this case, $U_z^i(a,z_{-i}(t)) = \frac{N_K-1}{N-1} g_i^K(z_{-i}(t))\pi_a^K + \frac{N_{K'}}{N-1} g_i^{K'}(z_{-i}(t))\pi_a^K$. Note that $g_i^K(z_{-i}(t)) = g_i^K(z_i(t),\omega_t)$ and $g_i^{K'}(z_{-i}(t)) = g_i^K(z_i(t),\omega_t)$. Where $g_i^K(z_i(t),\omega_t)$ is the frequency of successful coordinations of player i on action a with group K at time t, given ω_t and that player

i is currently playing $z_i(t)$. Therefore, $U^i_z(a, z_{-i}(t)) = U^i_{z_i(t)}(a, \omega_t)$. With $z_i(t) = a$ in our case.
Note that $g^K_i(a, \omega_t) = \frac{n^{KK}_t - 1}{N_K - 1}$, and $g^K_i(b, \omega_t) = \frac{n^{KK}_t}{N_K - 1}$. Moreover, $g^K_i(a, \omega_t) = g^K_i(aa, \omega_t) = g^K_i(ab, \omega_t)$, and $g^K_i(b, \omega_t) = g^K_i(bb, \omega_t) = g^K_i(ba, \omega_t)$. Contrarily, $g^{K'}_i(z_i(t), \omega_t) = g^{K'}_i(\omega_t) = \frac{n^{K'K}_t}{N_{K'}}$, $\forall z_i(t) \in Z$.
Therefore, $U^i_a(a, \omega_t) = U^i_{aa}(a, \omega_t) = U^i_{ab}(a, \omega_t)$. Equally, $U^i_b(a, \omega_t) = U^i_{bb}(a, \omega_t) = U^i_{ba}(a, \omega_t)$. Thanks to symmetry in payoff matrix the argument stands for all strategies and behaviors.

□

A.1 Proofs of Section 3

Proof of Lemma 3:

Consider a player $i \in K$ currently playing aa who is given the revision opportunity at time t. On the one hand, $\forall n^{KK}_t$, $U^i_a(ab, \omega_t) = U^i_a(aa, \omega_t)$. On the other hand, $\forall n^{K'K}_t$, $U^i_a(ba, \omega_t) = U^i_a(aa, \omega_t)$. Therefore, player i chooses aa or ab depending on $n^{K'K}_t$, and ba or aa depending on n^{KK}_t.
Moreover, if player i chooses ab instead of aa, $n^{KK}_{t+1} = n^{KK}_t$, but $n^{K'K}_{t+1} < n^{K'K}_t$. If player i chooses ba instead of aa, $n^{KK}_{t+1} < n^{KK}_t$, but $n^{K'K}_{t+1} = n^{K'K}_t$. This completes the proof.

□

With abuse of notation, we call best reply (BR), the action optimally taken by a player in one of the three dynamics. For example, if a type A earns the highest payoff by playing a against a player of type B, we say that a is her/his BR. We do this in the context of complete information because of the separability of the dynamics.

Proof of Lemma 4:

Thanks to Lemma 3, we can consider the 3 separated dynamics: n^{AA}_t, n^{BB}_t, and n^I_t.

Inside-group interactions.

Firstly, we prove the result for n^{AA}_t and then the argument stands for n^{BB}_t thanks to symmetry of payoff matrix. We have to show that all the states in ω^R have an absorbing component for n^{AA}_t, that is 0 or N_A. When $n^{AA} = N_A$, $\forall i \in A$, a is BR against type A at time t. Hence, $F_1(N_A, \theta_{t+1}) = N_A$. Symmetrically if $n^{AA} = 0$, b is always BR and so, $F_1(0, \theta_{t+1}) = 0$. Therefore, N_A and 0 are fixed points for n^{AA}_t. We need to show that these states are absorbing, that all the other states are transient, and that there are no cycles. Consider player $i \in A$ who is given the revision opportunity at time t. We define $\bar{n}^A$ as the minimum number of A players such that a is BR, and $\underline{n}^A$ as the maximum number of A players such that b is BR. From Equation (1) to (4), we know that $\bar{n}^A = \frac{N_A \pi_A + \Pi_A}{\Pi_A + \pi_A}$, and that $\underline{n}^A = \frac{N_A \pi_A - \pi_A}{\Pi_A + \pi_A}$. Assume $n^{AA}_t \geq \bar{n}^A$. There is always a positive probability that a player not playing a is given the revision opportunity. Hence, $F_1(n^{AA}_t, \theta_{t+1}) \geq n^{AA}_t$. Symmetrically, we can say that if $n^{AA}_t < \underline{n}^A$, $F_1(n^{AA}_t, \theta_{t+1}) \leq n^{AA}_t$.

We now prove that if $n_t^{AA} \leq \underline{n}^A \neq 0$,

$$Pr\left(\lim_{t\to\infty} F_1(n_t^{AA}, \theta_{t+1}) = n_t^{AA}\right) = 0.$$

Equally, if $n_t^{AA} \geq \bar{n}^A \neq N_A$,

$$Pr\left(\lim_{t\to\infty} F_1(n_t^{AA}, \theta_{t+1}) = n_t^{AA}\right) = 0.$$

We prove the first case, and the result stands for the second, thanks to symmetry in payoff matrices. Consider to be at period s in a state $n_s^{AA} < \underline{n}^A \neq 0$. For every player, b is BR. Define $Pr\left(n_{s+1}^{AA} = n_s^{AA}\right) = p$. Such a probability represents the event that only players playing b are given the revision opportunity. $Pr\left(n_{s+2}^{AA} = n_s^{AA}\right) = p^2$, $Pr\left(n_{s+k}^{AA} = n_s^{AA}\right) = p^k$. If $k \to \infty$, $Pr\left(n_{s+k}^{AA} = n_s^{AA}\right) = 0$. Therefore,

$$\text{If } n_0^{AA} \leq \underline{n}^A, \quad Pr\left(\lim_{t\to\infty} F_1(n_t^{AA}, \theta_{t+1}) = 0\right) = 1,$$

$$\text{If } n_0^{AA} \geq \bar{n}^A, \quad Pr\left(\lim_{t\to\infty} F_1(n_t^{AA}, \theta_{t+1}) = N_A\right) = 1.$$

Next, consider $\underline{n}^A < n_0^{AA} < \bar{n}^A$. For every i playing a, b is BR while, for every i' playing b, a is BR. There are no absorbing states between these states. If only agents playing a are given the revision opportunity, they all choose b, and if enough of them are given the revision opportunity, $n_1^{AA} < \underline{n}^A$. The opposite happens if only players playing b are given the revision opportunity.

Inter-group interactions.

We now pass to the analysis of n_t^I. We define 4 important values for n^{AB} and n^{BA}: $T_A = min\left\{n^{BA} | n^{BA} > \frac{\pi_A N_B}{\Pi_A + \pi_A}\right\}$, $T_B = min\left\{n^{AB} | n^{AB} > \frac{\Pi_B N_A}{\Pi_B + \pi_B}\right\}$,
$D_A = max\left\{n^{BA} | n^{BA} < \frac{\pi_A N_B}{\Pi_A + \pi_A}\right\}$, and $D_B = max\left\{n^{AB} | n^{AB} < \frac{\Pi_B N_A}{\Pi_B + \pi_B}\right\}$.
Given these values we also define two sets of states, Ω_I^b and Ω_I^a:
$\Omega_I^a = \left\{n^I | n^{BA} \geq T_A \text{ and } n^{AB} \geq T_B\right\}$ and $\Omega_I^b = \left\{n^I | n^{BA} \leq D_A \text{ and } n^{AB} \leq D_B\right\}$.
With similar computation as for n_t^{AA}, we can say that $(0, 0)$ and (N_A, N_B) are two fixed points for n_t^I. Are they absorbing states?
Consider the choice of a player $i \in A$ against player $j \in B$ and vice-versa. There can be four possible combinations of states. States in which a is BR for every agent, states in which b is BR for every agent. States, in which $\forall i \in A$, a is the best reply, and b is the best reply $\forall j \in B$, and states for which the opposite is true. Let us call the third situation as Ω_I^{ab} and the fourth as Ω_I^{ba}.
Firstly, we prove that Ω_I^a and Ω_I^b are the regions where a and b are BR for every agent. Secondly, we prove that there is no other absorbing state in Ω_I^a than (N_A, N_B), and no other absorbing state in Ω_I^b than $(0, 0)$.
Assume that player $i \in A$ is given the revision opportunity at period t. From Equation (1) to (4), a is the BR against type B if $n_t^{BA} > \frac{\pi_A N_B}{\Pi_A + \pi_A}$. Since T_A is defined as the minimum value s.t. the latter holds, $\forall n_t^{BA} \geq T_A$, $\forall i \in A$, a is BR against B types. Now, assume that $j \in B$ is given the revision opportunity, a is the BR against type A if $n_t^{AB} > \frac{\Pi_B N_A}{\Pi_B + \pi_B}$. Since T_B is defined as the minimum values s.t. this relation is true, $\forall n_t^{AB} \geq T_B$, a is the best reply $\forall j \in B$. Therefore, if $n_0^I \in \Omega_I^a$, $n_s^I \in \Omega_I^a, \forall s \geq 0$. Similarly, if $n_0^I \in \Omega_I^b$, $n_s^I \in \Omega_I^b, \forall s \geq 0$.

Consider being in a generic state $(T_B+d, T_A+d') \in \Omega_I^a$ at time t, with $d \in [0, N_A-T_B)$ and $d' \in [0, N_B - T_A)$. In such a state, there is always a probability p that a player not playing a is given the revision opportunity.
Therefore, if $n_t^I \in \Omega_I^a \setminus (N_A, N_B), Pr\left(F_{2,3}\left(n_t^I, \theta_{t+1}\right) \geq n_t^I\right) > p$.[5] Similar to what we proved before,

$$\text{if } n_t^I \in \Omega_I^a \setminus (N_A, N_B), \quad Pr\left(\lim_{t\to\infty} F_{2,3}(n_t^I, \theta_{t+1}) = n_t^I\right) = 0,$$

$$\text{if } n_t^I \in \Omega_I^b \setminus (0, 0), \quad Pr\left(\lim_{t\to\infty} F_{2,3}(n_t^I, \theta_{t+1}) = n_t^I\right) = 0.$$

Consequently,

$$\text{If } n_0^I \in \Omega_I^a \quad Pr\left(\lim_{t\to\infty} F_{2,3}(n_t^I, \theta_{t+1}) = (N_A, N_B)\right) = 1,$$

$$\text{if } n_0^I \in \Omega_I^b, \quad Pr\left(\lim_{t\to\infty} F_{2,3}(n_t^I, \theta_{t+1}) = (0, 0)\right) = 1.$$

We now consider Ω_I^{ab} and Ω_I^{ba}. Take an $n_0^I \in \Omega_I^{ab}$: at each step, there is a positive probability that only agents of type A are given the revision opportunity, since for them a is the best reply, in the next period, there will be more or equal agents in A playing a. Hence, if enough players of A that are currently playing b are given the revision opportunity, $n_1^I \in \Omega_I^a$. By the same reasoning, there is also a positive probability that only agents from B are given the revision opportunity, hence, that $n_1^I \in \Omega_I^b$. The same can be said for every state in Ω_I^{ba}. Hence, starting from every state in $\Omega_I^{ab} \bigcup \Omega_I^{ba}$, there is always a positive probability to end up in Ω_I^a or Ω_I^b.

□

Lemma 7. *Under complete information,*
$Pr\left(\lim_{t\to\infty} n_t^I = (N_A, N_B)\right) = 1 - Pr\left(\lim_{t\to\infty} n_t^I = (0, 0)\right)$.
$Pr\left(\lim_{t\to\infty} n_t^{AA} = N_A\right) = 1 - Pr\left(\lim_{t\to\infty} n_t^{AA} = 0\right)$.
$Pr\left(\lim_{t\to\infty} n_t^{BB} = N_B\right) = 1 - Pr\left(\lim_{t\to\infty} n_t^{BB} = 0\right)$.

Proof:

We prove the result for n_t^I, and the argument stands for the two other dynamics thanks to symmetry in the payoff matrix. Firstly, note that whenever the process starts in $\Omega_I^a \cup \Omega_I^b$, the lemma is always true thanks to the proof of Lemma 4. We need to show that this is the case, also when the process starts inside $\Omega_I^{ab} \bigcup \Omega_I^{ba}$. We prove the result for Ω_I^{ab} using the same logic, and the result stands for Ω_I^{ba} for symmetry of payoff matrix.
Take $n_0^I \in \Omega_I^{ab}$, define as p_a the probability of extracting m agents from A that are currently playing b, and that would change action a if given the revision opportunity. Define as p_b the probability of picking m agents from B currently choosing a that would change action to b if given the revision opportunity. The probability $1-p_a-p_b$ defines all the other possibilities.
Let us take k steps forward in time:

[5] Meaning that $n_t^{'I} > n_t^{''I}$ if either $n_t^{'AB} > n_t^{''AB}$ and $n_t^{'BA} = n_t^{''BA}$ or $n_t^{'BA} > n_t^{''BA}$ and $n_t^{'AB} = n_t^{''AB}$ or both $n_t^{'BA} > n_t^{''BA}$ and $n_t^{'AB} > n_t^{''AB}$.

$$Pr\left(n_k^I \in \Omega_I^a\right) \geq (p_a)^k$$

$$Pr\left(n_k^I \in \Omega_I^b\right) \geq (p_b)^k$$

$$Pr\left(n_k^I \in \Omega_I^{ab} \bigcup \Omega_I^{ba}\right) \leq (1 - p_a - p_b)^k.$$

Consider period $k + d$:

$$Pr\left(n_{k+d}^I \in \Omega_I^a\right) \geq (p_a)^k$$

$$Pr\left(n_{k+d}^I \in \Omega_I^b\right) \geq (p_b)^k$$

$$Pr\left(n_{k+d}^I \in \Omega_I^{ab} \bigcup \Omega_I^{ba}\right) \leq (1 - p_a - p_b)^{k+d}.$$

Clearly, the probability of being in $\Omega_I^a(\Omega_I^b)$ is now greater or equal than $(p_a)^k((p_b)^k)$: we know that once in $\Omega_I^a(\Omega_I^b)$ the system stays there. The probability of being in $\Omega_I^{ab} \bigcup \Omega_I^{ba}$ consequently, is lower than $(1 - p_a - p_b)^{k+d}$.
Taking the limit for d that goes to infinity

$$\lim_{d\to\infty}\left(Pr\left(n_{k+d}^I \in \Omega_I^{ab} \bigcup \Omega_I^{ba}\right)\right) = 0.$$

This means that if we start in a state in Ω_I^{ab} there is no way of ending up in $\Omega_I^{ab} \bigcup \Omega_I^{ba}$ in the long-run; hence, the system ends up either in Ω_I^a or in Ω_I^b, but given this, we know that it ends up either in $(0, 0)$ or in (N_A, N_B).

□

Corollary 3. *Under complete information,*
$Pr\left(\lim_{t\to\infty} n_t^I = (N_A, N_B)\right) = 1$ *IFF* $n_0^I \in \Omega_I^a$.
$Pr\left(\lim_{t\to\infty} n_t^I = (0, 0)\right) = 1$ *IFF* $n_0^I \in \Omega_I^b$.
$Pr\left(\lim_{t\to\infty} n_t^{AA} = N_A\right) = 1$ *IFF* $n_0^{AA} \in \left[\bar{n}^A, N_A\right]$*, and*
$Pr\left(\lim_{t\to\infty} n_t^{AA} = 0\right) = 1$ *IFF* $n_0^{AA} \in \left[0, \underline{n}^A\right]$.
$Pr\left(\lim_{t\to\infty} n_t^{BB} = N_B\right) = 1$ *IFF* $n_0^{BB} \in \left[\bar{n}^B, N_B\right]$*, and*
$Pr\left(\lim_{t\to\infty} n_t^{BB} = 0\right) = 1$ *IFF* $n_0^{BB} \in \left[0, \underline{n}^B\right]$.

This result is a consequence of the previous lemmas, and therefore, the proof is omitted. Since the only two absorbing states in the dynamics of n_t^I are $(0, 0)$ and (N_A, N_B), they are the only two candidates to be stochastically stable states. From now on we call $(0, 0)$ as I_n^b and (N_A, N_B) as I_n^a. We define as 0_A the state where all agents of type A play b with type A and 0_B the state where all agents of type B play b with B type.

Let us call E_A and E_B the two values for which agents in A and in B are indifferent in playing a or b in inter-group interactions. $E_A = \left\lceil \frac{N_B \pi_A}{\Pi_A + \pi_A} \right\rceil$ and $E_B = \left\lceil \frac{N_A \Pi_B}{\Pi_B + \pi_B} \right\rceil$. From now on we often use values of N large enough to compare the arguments inside ceiling functions.

Lemma 8. *Under free information acquisition, for* N *large enough,* $R(I_n^b) = CR(I_n^a) = E_A$ *for all values of payoffs and sizes of groups, while*

$$R(I_n^a) = CR(I_n^b) = \begin{cases} N_A - E_B & \textit{if } \frac{\pi_B}{\Pi_A} < \frac{N_B}{N_A} \\ N_B - E_A & \textit{if } \frac{\pi_B}{\Pi_A} > \frac{N_B}{N_A} \end{cases}$$

Proof:

Firstly we know from Ellison [11] that if there are just two absorbing states, the Radius of one is the Coradius of the other and vice-versa. Hence, $R(I_n^b) = CR(I_n^a)$, and $R(I_n^a) = CR(I_n^b)$. Moreover, from the proof of Lemma 4, we know that $D(I_n^a) = \Omega_I^a$ and $D(I_n^b) = \Omega_I^b$.
We prove that the minimum error path to exit the basin of attraction of I_n^b is the one that reaches $(E_B, 0)$ or $(0, E_A)$, and that the one to exit the basin of attraction of I_n^a is the one that reaches either (E_B, N_B) or (N_A, E_A). To prove this statement for I_n^b, firstly, note that once inside Ω_I^b every step which involves a passage to a state with more people playing a requires an error. Secondly, note that in a state that is out of Ω_I^b at least one of the two types is indifferent in playing b or a. In other words, in a state where either $n^{AB} = E_B$ or $n^{BA} = E_A$ or both. Hence, the minimum resistance path to exit from I_n^b is the one either to $(E_B, 0)$ or to $(0, E_A)$. It is straightforward to show that all the other paths have greater resistance than the two above. Since we use uniform mistakes, every mutation counts the same value, and without loss of generality, we can count each of them as 1. Since every resistance counts as 1, then $R(I_n^b) = \min\{E_B; E_A\} = E_A$. Similarly, $R(I_n^a) = \min\{N_A - E_B; N_B - E_A\}$, and

$$N_A - E_B < N_B - E_A \iff \frac{\pi_B}{\Pi_A} < \frac{N_B}{N_A}.$$

□

Lemma 9. *Under free information acquisition, for* N *large enough,* $R(0_A) = \left\lceil \frac{N_A \pi_A - \pi_A}{\Pi_A + \pi_A} \right\rceil$, $R(N_A) = \left\lceil \frac{N_A \Pi_A + \Pi_A}{\Pi_A + \pi_A} \right\rceil$, $R(0_B) = \left\lceil \frac{N_B \Pi_B + \Pi_B}{\Pi_B + \pi_B} \right\rceil$ *and* $R(N_B) = \left\lceil \frac{N_B \pi_B - \pi_B}{\Pi_B + \pi_B} \right\rceil$.

Proof:

The proof is straight forward, indeed, the minimum path in terms of error required to reach one absorbing state starting from the other one is the cost of exit from the basin of attraction of the first. As a matter of fact, let us consider $R(0_A)$, we know from the proof of Lemma 4 that we are out of the basin of attraction of 0_A when we reach the state $\underline{n}^A$. Hence, $R(0_A) = \left\lceil \frac{N_A \pi_A - \pi_A}{\Pi_A + \pi_A} \right\rceil$. The same applies to the other states.

□

Proof of Theorem 1:

We divide the proof for the three dynamics described so far: for what concerns n_t^{AA}, N_A is uniquely stochastically stable and for what concerns n_t^{BB}, 0_B is uniquely stochastically stable, this proof follows directly from Lemma 9, and therefore is omitted. Let us pass to n_t^I. We know from Lemma 8 that $R(I_n^b) = E_A$ and that the value of $R(I_n^a)$ depends on payoffs and group size. Let us firstly consider the case when $\frac{\pi_B}{\Pi_A} < \frac{N_B}{N_A}$ and $R(I_n^a) = N_A - E_B$. It is sufficient that $E_A > N_A - E_B$ for I_n^b to be uniquely stochastically stable. Indeed, if this happens, $R(I_n^b) > CR(I_n^b)$. This is the case IFF

$$\frac{\pi_A N_B}{\Pi_A + \pi_A} > \frac{\pi_B N_A}{\Pi_B + \pi_B} \iff \frac{\pi_B}{\pi_A} < \frac{N_B}{N_A}. \tag{9}$$

To complete the proof, we show that whenever $\frac{\pi_B}{\pi_A} > \frac{N_B}{N_A}$, then I_n^a is the uniquely stochastically stable state. Firstly, note that $\frac{\pi_B}{\Pi_A} < \frac{\pi_B}{\pi_A}$, hence, for $\frac{\pi_B}{\pi_A} > \frac{N_B}{N_A} > \frac{\pi_B}{\Pi_A}$, $R(I_n^a) = N_A - E_B$ and $E_A = R(I_n^b)$. However, Equation (9) is reversed, so, I_n^a is uniquely stochastically stable. For $\frac{\pi_B}{\pi_A} > \frac{\pi_B}{\Pi_A} > \frac{N_B}{N_A}$, $R(I_n^a) = N_B - E_A$ and still $R(I_n^b) = E_A$. In this case, I_n^a is the uniquely stochastically stable if $E_A < N_B - E_A$, hence, IFF

$$\frac{\pi_A N_B}{\Pi_A + \pi_A} < \frac{\Pi_A N_B}{\Pi_A + \pi_A}.$$

This happens for every value of the payoffs (given that $\Pi_A > \pi_A$) and of the group size. We conclude that whenever $\frac{\pi_B}{\pi_A} < \frac{N_B}{N_A}$, PS_b is uniquely stochastically stable and when $\frac{\pi_B}{\pi_A} > \frac{N_B}{N_A}$, PS_a is uniquely stochastically stable.

□

A.2 Proofs of Section 4

For convenience, we call behavior 1 the optimal behavior when a player decides to acquire the information: $1 = \max(ab, ba, aa, bb)$.

We will use in some proofs the concept of Modified Coradius from Ellison [11]. We write here the formal definition. Suppose $\bar{\omega}$ is an absorbing state and $(\omega_1, \omega_2, \ldots \omega_T)$ is a path from state ω' to $\bar{\omega}$. Let $L_1, L_2, \ldots, L_r = \bar{\omega}$ be the sequence of limit sets through which the path passes consecutively. The modified resistance is the original resistance minus the Radius of the intermediate limit sets through which the path passes,

$$r^*(\omega_1, \omega_2, \ldots \omega_T) = r(\omega_1, \omega_2, \ldots \omega_T) - \sum_{i=2}^{r-1} R(L_i).$$

Define

$$r^*(\omega', \bar{\omega}) = \min_{(\omega_1, \omega_2, \ldots \omega_T) \in \Upsilon(\omega', \bar{\omega})} r^*(\omega_1, \omega_2, \ldots \omega_T),$$

the Modified Coradius is defined as follows

$$CR^*(\bar{\omega}) = \max_{\omega' \neq \bar{\omega}} r^*(\omega', \bar{\omega}).$$

Note that $CR^*(\bar{\omega}) \leq CR(\bar{\omega})$. Thanks to theorem 2 in Ellison [11], we know that when $R(\bar{\omega}) > CR^*(\bar{\omega})$, $\bar{\omega}$ is uniquely stochastically stable.

Proof of Lemma 5:

We first show that the nine states are effectively strict equilibria, that there is no other possible equilibrium, and that a state is absorbing if and only if it is a strict equilibrium.

Monomorphic States.

It is easy to show that (N_A, N_A, N_B, N_B) and $(0, 0, 0, 0)$ are two strict equilibria. We take the first case, and the argument stands also for the second, thanks to the symmetry of the payoff matrix. Consider player $i \in K$ who is given the revision opportunity at time t:

$$U_a^i(a, \omega_t) = \frac{N_K + N_{K'} - 1}{N - 1}\pi_a^K = \pi_a^K,$$

$$U_a^i(b, \omega_t) = \frac{N - N_K - N_{K'}}{N - 1}\pi_b^K = 0,$$

$$U_a^i(1, \omega_t) = \frac{N_K + N_{K'} - 1}{N - 1}\pi_a^K - c = \pi_a^K - c.$$

(N_A, N_A, N_B, N_B) is a strict equilibrium since $\pi_a^K > 0$ and $c > 0$.

Polymorphic States.

Firstly let us consider the case of $(N_A, 0, 0, N_B)$. Since in this case, every player is playing ab, the state is a strict equilibrium IFF $\max z_i^o = ab$, $\forall i \in N$. If player $i \in K$ is given the revision opportunity at time t:

$$U_a^i(a, \omega_t) = \frac{N_K - 1}{N - 1}\pi_a^K,$$

$$U_a^i(b, \omega_t) = \frac{N_{K'}}{N - 1}\pi_b^K,$$

$$U_a^i(1, \omega_t) = \frac{N_K - 1}{N - 1}\pi_a^K + \frac{N_{K'}}{N - 1}\pi_b^K - c.$$

For type A players, $U_a^i(a, \omega_t) > U_a^i(b, \omega_t)$ since $\frac{N_B}{N-1}\pi_A < \frac{N_A - 1}{N-1}\Pi_A$. For type B players, $U_a^i(b, \omega_t) > U_a^i(a, \omega_t)$ as $\frac{N_B - 1}{N-1}\pi_B < \frac{N_A}{N-1}\Pi_B$. $U_a^i(1, \omega_t)$ is the highest of the three $\forall i \in N$ IFF $c < \min\left\{\frac{N_B}{N-1}\pi_A, \frac{N_B - 1}{N-1}\pi_B\right\}$.
Consider the case of $(0, N_A, N_B, 0)$, since every agent is playing ba, it must be that $\max z_i^o = ba$. $i \in K$ faces the following payoffs

$$U_b^i(a, \omega_t) = \frac{N_{K'}}{N - 1}\pi_a^K,$$

$$U_b^i(b, \omega_t) = \frac{N_K - 1}{N - 1}\pi_b^K,$$

$$U_b^i(1, \omega_t) = \frac{N_K - 1}{N - 1}\pi_b^K + \frac{N_{K'}}{N - 1}\pi_a^K - c.$$

Note that $U_b^i(a,\omega_t) > U_b^i(b,\omega_t)$ IFF $\frac{\pi_b^K}{\pi_a^K} < \frac{N_{K'}}{N_K - 1}$, and therefore ba is the best reply behavior in this case if $c < \frac{N_K - 1}{N-1}\pi_b^K$. When the opposite happens and so, $\frac{\pi_b^K}{\pi_a^K} > \frac{N_{K'}}{N_K - 1}$, ba is the best reply behavior if $c < \frac{N_{K'}}{N-1}\pi_a^K$. These conditions take the form of the ones in Table 4.

Consider the remaining 4 PS, they are characterised by the following fact $BR(n^{KK}) = BR(n^{K'K})$ but $BR(n^{K'K'}) \neq BR(n^{KK'})$. In this case it must be that $\tau_i = 0$ is optimal for $i \in K$ while $\tau_j = 1$ is optimal for $j \in K'$. Thanks to the symmetry in payoff matrices we can say that the argument to prove the results for these 4 states is similar to the one for $(N_A, 0, 0, N_B)$ and $(0, N_A, N_B, 0)$. All the conditions are listed in Table 4.

Type Monomorphic State.

$(N_A, N_A, 0, 0)$ is a strict equilibrium if a is the BR $\forall i \in A$ and b, $\forall j \in B$. Consider a player $i \in A$, who is given the revision opportunity at time t:

$$U_a^i(a,\omega_t) = \frac{N_A - 1}{N-1}\Pi_A,$$

$$U_a^i(b,\omega_t) = \frac{N_B}{N-1}\pi_A,$$

$$U_a^i(1,\omega_t) = \frac{N_A - 1}{N-1}\Pi_A + \frac{N_B}{N-1}\pi_A - c.$$

Given that $U_a^i(a,\omega_t) > U_a^i(b,\omega_t)$, a is the best reply behavior IFF $c > \frac{N_B}{N-1}\pi_A$. Consider player $j \in B$:

$$U_b^j(a,\omega_t) = \frac{N_A}{N-1}\pi_B,$$

$$U_b^j(b,\omega_t) = \frac{N_B - 1}{N-1}\Pi_B,$$

$$U_b^j(1,\omega_t) = \frac{N_A}{N-1}\pi_B + \frac{N_B - 1}{N-1}\Pi_B - c.$$

In this case when $\frac{\pi_B}{\Pi_B} > \frac{N_B - 1}{N_A}$, b is never best reply and a is best reply hence, the state can not be a strict equilibrium. When $\frac{\pi_B}{\Pi_B} < \frac{N_B - 1}{N_A}$, $U_b^j(b,\omega_t) > U_b^j(a,\omega_t)$, and $U_b^j(b,\omega_t) > U_b^j(1,\omega_t)$ IFF $c > \frac{N_A}{N-1}\pi_B$.

No other state is a strict equilibrium.

For what concerns states where not all players of a type are playing the same action with the same type, this is easy to prove. Indeed, by definition, in these states, either not all players are playing their best reply action, or players are indifferent between two or more behaviors. In the first case, the state is not a strict equilibrium by definition; in the second case, there is no strictness of the equilibrium since there is not one best reply, but more behaviors can be best reply simultaneously. Hence, such states can not be strict equilibria. We are left with the 7 states where every player of one type is doing the same thing against the same type. Such states are: $(0, 0, N_B, N_B)$, $(0, N_A, 0, N_B)$, $(N_A, 0, N_B, 0)$, $(0, 0, N_B, 0)$, $(N_A, N_A, 0, N_B)$, $(0, N_A, 0, 0)$, and $(N_A, 0, N_B, N_B)$. It is easy to prove that these states enter in the category of states where not every player is playing her/his best reply. Therefore, they can not be strict equilibria.

Strict equilibria are always absorbing states.

We first prove the sufficient and necessary conditions to be a fixed point, and second that every fixed point is an absorbing state. To prove the sufficient part we rely on the definition of strict equilibrium. In a strict equilibrium, every player is playing her/his BR, and no one has the incentive to deviate. Whoever is given the revision opportunity does not change her/his behavior. Therefore, $F(\omega_t, \theta_{t+1}) = \omega_t$. To prove the necessary condition think about being in a state that is not a strict equilibrium; in this case, by definition, we know that not all the players are playing their BR. Among them, there are states in which there are no indifferent players, in this case, with positive probability one or more agents who are not playing their BR are given the revision opportunity and they change action, therefore, $F(\omega_t, \theta_{t+1}) \neq \omega_t$ for some realization of θ_{t+1}. In states where some players are indifferent between two or more behaviors, thanks to the tie rule, there is always a positive probability that the indifferent agent changes her/his action since s/he is randomizing her/his choice. Moreover, there is also a positive probability to select an agent indifferent between two or more behaviors. In this case, s/he changes the one that is currently playing with a positive probability too. Knowing this, we are sure that no state outside strict equilibria can be a fixed point. In our case, a fixed point is also an absorbing state by definition. Indeed, every fixed point absorbs at least one state: the one where all players except one are playing the same behavior. In this case, if that player is given the revision opportunity s/he changes for sure her/his behavior into the one played by every agent.

□

Here we prove the results of the stochastic stability analysis of Section 4.

Proof of Theorem 2:

We split the absorbing states into 2 sets and then apply Theorem 1 by Ellison [11]. Define the following two sets of states: $M_1 = \{PS_a, PS_b\}$ and $M_2 = (PS \setminus M_1) \cup MS$. Similarly, define $M_1' = PS_b$ and $M_2' = MS \cup (PS \setminus M_1')$.

Analysis with M_1 and M_2.

$R(M_1)$ is the minimum number of errors to escape the basins of attraction of both PS_a and PS_b. The dimension of these basins of attraction is determined by the value of c. In a state inside $D(PS_a)$, ba is BR for B, and a is BR for A. Similarly, ab is optimal for A inside $D(PS_b)$ and b is optimal for B. The minimum error paths that starts in PS_a, and PS_b and exit from their basins of attraction involve ε errors.
We calculate the dimension of these basins of attraction for $0 < c < \min\{\frac{\pi_A}{N-1}, \frac{\pi_B}{N-1}\}$. We start from PS_a and the argument stands for the other states in PS for symmetry of payoffs matrix.
Firstly, we consider the minimum number of errors that makes a BR for B players. Consider the choice of a B player inside a category of states where $n^{BB} \in$

$\left[0, \frac{N_B\Pi_B - \Pi_B}{\Pi_B+\pi_B}\right)$ and $n^{AB} \in \left(\frac{N_A\Pi_B}{\Pi_B+\pi_B}, N_A\right]$. Referring to Equation (5) to (8), the optimal level of c s.t. 1 is the best reply for B players is

$$c < \min\left\{\frac{N_B\Pi_B - n^{BB}(\Pi_B+\pi_B) - \Pi_B}{N-1}, \frac{n^{AB}(\Pi_B+\pi_B) - N_A\Pi_B}{N-1}\right\}.$$

If $0 < c < \min\{\frac{\pi_A}{N-1}, \frac{\pi_B}{N-1}\}$, whenever $n^{BB} \in \left[0, \frac{N_B\Pi_B-\Pi_B}{\Pi_B+\pi_B}\right)$ and $n^{AB} \in \left(\frac{N_A\Pi_B}{\Pi_B+\pi_B}, N_A\right]$, 1 is the BR for B. Therefore, a path towards a state where $n^{BB} \geq \frac{N_B\Pi_B+\pi_B}{\Pi_B+\pi_B}$, is a transition out of the basin of attraction of PS_a. Starting from $n^{BB} = 0$, the cost of this transition is $\frac{N_B\Pi_B+\pi_B}{\Pi_B+\pi_B}$. This cost is determined by ε errors, since once in PS_a it is sufficient that a number of B plays by mistake b. Another possible path is to make ba BR for A. The cost of this transition is $\frac{N_A\Pi_A+\pi_A}{\Pi_A+\pi_A}$. With similar arguments, it is possible to show that the cost of exit from M_1 starting from PS_b is the same. For this reason, $R(M_1) = \min\left\{\frac{N_B\Pi_B+\pi_B}{\Pi_B+\pi_B}, \frac{N_A\Pi_A+\pi_A}{\Pi_A+\pi_A}\right\}$.
We can show that the minimum error path to exit from the basin of attraction of M_2 reaches either PS_a from MS_a, or PS_b from MS_b. Therefore, $R(M_2) = \min\left\{\frac{N_A\pi_A+\Pi_A}{\Pi_A+\pi_A}, \frac{N_B\pi_B+\Pi_B}{\Pi_B+\pi_B}\right\}$. $R(M_1) > R(M_2)$ for every value of payoffs and group size: the stochastically stable state must be in M_1.

Analysis with M_1' and M_2'.

Let us consider the path that goes from M_1' to PS_a. Starting in PS_b, it is sufficient that $\frac{N_B\pi_A}{\Pi_A+\pi_A}$ players from A play a for a transition from PS_b to $D(PS_a)$ to happen. Since $\frac{N_B\pi_A}{\Pi_A+\pi_A} < \min\left\{\frac{N_B\Pi_B+\pi_B}{\Pi_B+\pi_B}, \frac{N_A\Pi_A+\pi_A}{\Pi_A+\pi_A}\right\}$, we can say that $R(M_1') = \frac{N_B\pi_A}{\Pi_A+\pi_A}$. With a similar argument it can be shown that $R(M_2') = \frac{N_A\pi_B}{\Pi_B+\pi_B}$. When $R(M_2') > R(M_1')$, PS_a is uniquely stochastically stable. When $R(M_1') > R(M_2')$, PS_b is uniquely stochastically stable.
$R(M_2') \lesseqgtr R(M_1')$ when $\frac{N_B}{N_A} \lesseqgtr \frac{\pi_B}{\pi_A}$.

□

Proof of Theorem 3:

In this case $R(MS_a) = CR(MS_b)$ and $R(MS_b) = R(MS_a)$. Therefore, we just need to calculate the two Radius.

Radius of each state.

Let us consider $R(MS_a)$. Since the basin of attraction of MS_a is a region where a is the best reply behavior for both types, many players should make a mistake such that b becomes BR for one of the two types. For b to be BR for B players, it must be that $n^{AB} + n^{BB} \leq \frac{N\Pi_B - \Pi_B}{\Pi_B+\pi_B}$. This state can be reached with ε mutations, at cost $\frac{N\pi_B+\Pi_B}{\Pi_B+\pi_B}$. In a state where $n^{AA} + n^{BA} \leq \frac{N\pi_A-\pi_A}{\Pi_A+\pi_A}$, b is BR for A, this path happens at cost $\frac{N\Pi_A+\pi_A}{\Pi_A+\pi_A}$. In principle $\frac{N\Pi_A+\pi_A}{\Pi_A+\pi_A} > \frac{N\pi_B+\Pi_B}{\Pi_B+\pi_B}$, hence, $R(MS_a)$ should be $\frac{N\pi_B+\Pi_B}{\Pi_B+\pi_B}$. However, it may not be sufficient to reach such a state.
Consider to reach a state s.t. $n^{AB} + n^{BB} = \frac{N\Pi_B-\Pi_B}{\Pi_B+\pi_B}$, since $\frac{N\Pi_B-\Pi_B}{\Pi_B+\pi_B} > \frac{N\pi_A-\pi_A}{\Pi_A+\pi_A}$, it must be that a is still the best reply $\forall i \in A$, and therefore there is a path of

zero resistance to MS_a. Nevertheless, once in that state, it can happen that only B players are given the revision opportunity, and that they all choose behavior b. This creates a path of zero resistance to a state $(\bar{n}^{AA}, \bar{n}^{AB}, 0, 0)$. Once in this state, if $\bar{n}^{AA} < \frac{N\pi_A - \pi_A}{\Pi_A + \pi_A}$, the state is in the basin of attraction of MS_b. This happens only if $\frac{N\pi_A - \pi_A}{\Pi_A + \pi_A} + N_B = \frac{N\Pi_B - \Pi_B}{\Pi_B + \pi_B}$. More generally, considering $k \geq 0$, this happens if $\frac{N\pi_A - \pi_A}{\Pi_A + \pi_A} + N_B = \frac{N\Pi_B - \Pi_B}{\Pi_B + \pi_B} - k$. Fixing payoffs and groups size, $k = \frac{N\Pi_B - \Pi_B}{\Pi_B + \pi_B} - \frac{N\pi_A - \pi_A}{\Pi_A + \pi_A} - N_B$, hence, the cost of this path would be

$$\frac{N\pi_B + \Pi_B}{\Pi_B + \pi_B} + \frac{N\Pi_B - \Pi_B}{\Pi_B + \pi_B} - \frac{N\pi_A - \pi_A}{\Pi_A + \pi_A} - N_B = N_A - \frac{N\pi_A - \pi_A}{\Pi_A + \pi_A}.$$

With a similar reasoning $R(MS_b) = \frac{N\pi_A + \Pi_A}{\Pi_A + \pi_A}$.

We prove that all the other paths with η errors are costlier than ones with ε. We know that a is the BR for every state inside the basin of attraction of MS_a, nobody in the basin of attraction of MS_a optimally buys the information, and every player once bought the information (by mistake) plays behavior aa. Every path with an η error also involves an ε error, and hence, is double that of the one described above.

Conditions for stochastically stable states.

MS_a is stochastically stable IFF $N_A - \frac{N\pi_A - \pi_A}{\Pi_A + \pi_A} > \frac{N\pi_A + \Pi_A}{\Pi_A + \pi_A}$, this is verified when $N_A > \frac{2N\pi_A + \Pi_A - \pi_A}{\Pi_A + \pi_A}$. Therefore, we conclude that MS_a is stochastically stable in the above scenario, while if the opposite happens, MS_b is stochastically stable.

□

Proof of Theorem 4

We first calculate Radius, Coradius, and Modified Coradius for the three states we are interested in, and then we compare them to draw inference about stochastic stability.

Radius of each state.

The Radius of MS_a is the minimum number of errors that makes b BR for B players. This number is $\frac{N\pi_B + \Pi_B}{\Pi_B + \pi_B}$. The alternative is to make b BR for A: hence, a path to state $(0, 0, N_B, N_B)$, and then to $(0, 0, 0, 0)$. The number of ε errors for this path is $\frac{N\Pi_A + \pi_A}{\Pi_A + \pi_A}$. Therefore, $R(MS_a) = \frac{N\pi_B + \Pi_B}{\Pi_B + \pi_B}$. With a similar reasoning we can conclude that $R(MS_b) = \frac{N\pi_A + \Pi_A}{\Pi_A + \pi_A}$.
Consider TS: the minimum error path to exit from its basin of attraction reaches either MS_a or MS_b, depending on payoffs. In other words, the minimum number of errors to exit from $D(TS)$ is the one that makes either a or b as BR. Consider the path from TS to MS_a: in this case, some errors are needed to make a BR for B. The state in which a is BR for B depends on payoffs and group size. In a state (N_A, N_A, k', k'), a is BR for every player in B if $(N_A + k' - 1)\pi_B > (N - N_A - k')\Pi_B$. This inequality is obtained declining Equation (5) to (8), comparing B playing a/ab or b/ba. Fixing payoffs, we can calculate the exact value of k' that is $\frac{N_B\Pi_B - N_A\pi_B + \pi_B}{\Pi_B + \pi_B}$, this would be the cost of the minimum error transition from TS to MS_a. With a similar argument, the cost of the minimum error transition from TS to MS_b is $\frac{N_A\Pi_A - N_B\pi_A + \pi_A}{\Pi_A + \pi_A}$.
There are no paths involving η errors that are lower than the two proposed above. The intuition is the following. Consider a situation in which m players of A are given

the revision opportunity at one step and they all choose to buy the information. In this case, they all optimally choose behavior ab. This means that at the cost of n there is a path to a state in which $N_A - m$ players are playing b against B, in this state b is still the BR for B type, while a is still the BR for A. Hence, from that state, there is a path of zero resistance to TS. The same happens when B players choose by mistake to buy the information. Therefore, $R(TS) = \min\left\{\frac{N_B\Pi_B - N_A\pi_B + \pi_B}{\Pi_B+\pi_B}, \frac{N_A\Pi_A - N_B\pi_A + \pi_A}{\Pi_A+\pi_A}\right\}$.

Coradius of each state

We start from TS: in this case, we have to consider the two minimum paths to reach it from MS_a and MS_b. Therefore, $\frac{N\pi_A+\Pi_A}{\Pi_A+\pi_A}$ and $\frac{N\pi_B+\Pi_B}{\Pi_B+\pi_B}$. Firstly, the argument to prove that these two are the minimum error paths to reach TS from MS_a and MS_b are given by the previous part of the proof. Secondly, we have to prove that this path is the maximum among the minimum paths starting from any other state and ending in TS. There are three regions from which we can start and end-up in TS: the basin of attraction of MS_b, the one of MS_a, and all the other states which are not in the basins of attraction of the three states considered. We can say that from this region, there is always a positive probability to end up in MS_a, MS_b, or TS. Hence, we can consider as 0 the cost to reach TS from this region. The other two regions are the one considered above, and since we are taking the maximum path to reach TS from any other state we have to take the sum of this two. Hence, $CR(TS) = \frac{N\pi_A+\Pi_A}{\Pi_A+\pi_A} + \frac{N\pi_B+\Pi_B}{\Pi_B+\pi_B}$.
Let us think about MS. Similarly to the two previous proofs we can focus only on ε paths. Note that in this case, TS is always placed between the two MS. Let us start from MS_b: in this case we can consider 3 different path starting from any state and arriving to MS_b. The first one starts in TS, the second starts in every state outside the basin of attractions of the three absorbing states, and the last starts in MS_a. In the second case there is at least one transition of zero resistance to MS_b. Next, assume to start in TS: the minimum number of errors to reach MS_b from TS is the one that makes b BR for A players. Therefore, $\frac{N_A\Pi_A - N_B\pi_A + \pi_A}{\Pi_A+\pi_A}$.
Now, we need to consider the case of starting in MS_a. Firstly, consider the minimum number of errors to make b BR for A players. This number is $\frac{N\Pi_A+\pi_A}{\Pi_A+\pi_A}$. Secondly, consider the minimum number of errors to make b BR for B players, and then once reached TS the minimum that makes b BR for A players.

$$\min r(MS_a, MS_b) = \min\left\{\frac{N\Pi_A + \pi_A}{\Pi_A + \pi_A}, \frac{N\pi_B + \Pi_B}{\Pi_B + \pi_B} + \frac{N_A\Pi_A - N_B\pi_A + \pi_A}{\Pi_A + \pi_A}\right\}.$$

Since the two numbers in the expression are all greater than $\frac{N_A\Pi_A - N_B\pi_A + \pi_A}{\Pi_A+\pi_A}$ we can say that $CR(MS_b) = \min r(MS_a, MS_b)$.
Reaching a state where b is BR for type A from TS is for sure less costly than reaching it from MS_a, since in TS there are more people playing b. Therefore, $\frac{N\Pi_A+\pi_A}{\Pi_A+\pi_A} \geq \frac{N\pi_B+\Pi_B}{\Pi_B+\pi_B} + \frac{N_A\Pi_A - N_B\pi_A + \pi_A}{\Pi_A+\pi_A}$, hence, $CR(MS_b) = \frac{N\pi_B+\Pi_B}{\Pi_B+\pi_B} + \frac{N_A\Pi_A - N_B\pi_A + \pi_A}{\Pi_A+\pi_A}$.
With a similar reasoning, $CR(MS_a) = \frac{N\pi_A+\Pi_A}{\Pi_A+\pi_A} + \frac{N_B\Pi_B - N_A\pi_B + \pi_B}{\Pi_B+\pi_B}$.

Modified Coradius of each state.

Firstly, note that $CR(TS) = CR^*(TS)$, since between MS and TS there are no intermediate states. Formally,

$$CR^*(TS) = \min r^*(MS_a, \dots, TS) + \min r^*(MS_b, \dots, TS) = \frac{N\pi_A + \Pi_A}{\Pi_A + \pi_A} + \frac{N\pi_B + \Pi_B}{\Pi_B + \pi_B}.$$

The maximum path of minimum resistance from each MS to the other MS passes through TS. Hence, for each MS, we need to subtract from the Coradius the cost of passing from TS to the other MS. Let us consider $CR^*(MS_a)$, we need to subtract to the Coradius the cost of passing from TS to MS_b: this follows from the definition of Modified Coradius. Hence,

$$CR^*(MS_a) = \frac{N\pi_A + \Pi_A}{\Pi_A + \pi_A} + \frac{N_B\Pi_B - N_A\pi_B + \pi_B}{\Pi_B + \pi_B} - \frac{N_A\Pi_A - N_B\pi_A + \pi_A}{\Pi_A + \pi_A}.$$

Similarly,

$$CR^*(MS_b) = \frac{N\pi_B + \Pi_B}{\Pi_B + \pi_B} + \frac{N_A\Pi_A - N_B\pi_A + \pi_A}{\Pi_A + \pi_A} - \frac{N_B\Pi_B - N_A\pi_B + \pi_B}{\Pi_B + \pi_B}.$$

Note that $CR^*(MS_a) < CR(MS_a)$ and $CR^*(MS_b) < CR(MS_b)$.

Conditions for stochastically stable states.

By comparing all the possibilities it is possible to verify that if $R(MS_a) > CR(MS_a)$, both $R(MS_b) < CR(MS_b)$ and $R(TS) < CR(TS)$. Similar for $R(MS_b) > CR(MS_b)$ or $R(TS) > CR(TS)$. When $R(MS_a) \leq CR(MS_a)$, $R(MS_b) \leq CR(MS_b)$, and $R(TS) \leq CR(TS)$, we need to use Modified Coradius. Given that $CR(TS) = CR^*(TS)$ it will never be that $R(TS) > CR^*(TS)$. We can show that when $R(MS_a) > CR^*(MS_a)$, then $R(MS_b) < CR^*(MS_b)$ and vice-versa.
When $R(MS_a) = CR^*(MS_a)$, it is also possible that $R(MS_b) = CR^*(MS_b)$. Thanks to Theorem 3 in Ellison [11], we know that either both states are stochastically stable, or none of the two is. Note that for the ergodicity of our process the second case is impossible, hence, it must be that when both $R(MS_a) = CR^*(MS_a)$ and $R(MS_b) = CR^*(MS_b)$, both $\mu^\star(MS_b) > 0$ and $\mu^\star(MS_a) > 0$.

□

Proof of Lemma 6

Recall from Section 3 that $\omega^R = \{PS, MS\}$. Firstly, if $\min\left\{\frac{\pi_A}{N-1}, \frac{\pi_B}{N-1}\right\} < c < \max\{\Xi_{PS}\}$, TS is not an absorbing state (see Corollary 2), all PS are absorbing states (see Corollary 1), and MS are absorbing states (see Table 4). Secondly, consider the set $M = \{PS_a, PS_b, MS_a, MS_b\}$ and the set $\omega^R \setminus M$ containing all the PS not in M. If $R(M) > R(\omega^R \setminus M)$ then the stochastically stable state must be in M. Since the level of the cost is not fixed the Radius of these two sets depend on the cost level. Following the same logic as in Theorem 2 but computing the result as a function of c we can calculate the two Radius.

$$R(M) = \min\left\{\frac{N_A\pi_B + c(N-1)}{\Pi_A + \pi_A}, \frac{N_B\pi_A + c(N-1)}{\Pi_A + \pi_A}, \frac{N\Pi_A + \pi_A}{\Pi_A + \pi_A}, \frac{N\Pi_B + \pi_B}{\Pi_B + \pi_B}\right\}$$

$$R(\omega^R \backslash M) = \min\left\{\frac{N_B\pi_B - c(N-1) + \Pi_B}{\Pi_B + \pi_B}, \frac{N_B\pi_A - c(N-1)}{\Pi_A + \pi_A}, \frac{N_A\pi_B - c(N-1)}{\Pi_B + \pi_B}\right\}.$$

By comparing all the twelve possibilities case by case, it is possible to show that for every value of payoffs, group size, and of the cost $R(M) > R(\omega^R \setminus M)$. Therefore the stochastically stable state must be in the set M.

□