Competing Conventions with Costly Acquisition of Information

Abstract

We consider an evolutionary model of social coordination in a 2x2 game where two groups of agents prefer to coordinate on different actions. Agents can pay a cost to learn their opponent's type: conditional on this decision, they can play different actions with different types. We assess the stability of outcomes in the long-run using stochastic stability analysis. We find that three elements matter for the equilibrium selection: group size, the strength of preferences, and the information's cost. If the cost is too high, agents never learn the type of their opponents in the long-run. If one group is stronger in preferences for its favorite action than the other, or its size is large enough compared to the other group, every agent plays that action. If both groups are strong enough in preferences, or if none of the group's size is large enough, agents play their favorite actions, and they miscoordinate in inter-group interactions. When the cost is sufficiently low, agents always learn the type of their opponent in the long-run. Therefore, they always coordinate. In inside-group interactions, agents always coordinate on their favorite action. In inter-group interactions, agents coordinate on the favorite action of the group that is stronger in preferences or large enough.

1 Introduction

Social scientists usually describe conventions as situations where every person acts in the same way with everybody. How an agent behaves mostly depends on what s/he expects other people to do and marginally on her/his preferences.[1] This is why game theorists usually represent social conventions as the outcome of coordination games. Specifically, since the seminal contribution of Kandori, Mailath, and Rob 1993, evolutionary game theorists have used stochastic stability analysis and 2x2 coordination games to study the formation of social conventions. Some of these works focus on coordination games such as the battle of sexes: a class that describes situations in which two groups of people attach value zero to miscoordination but prefer to coordinate on different actions. In these situations, it is not clear which convention establishes in the long-run.

Let us think about a person who wants to hang out with a friend: s/he has to choose between proposing her/him going to see a football match or going to the cinema, and s/he does not know what her/his friend prefers. Imagine being in a world without

[1] Lewis 1969 and Bicchieri 2005 are classical references from philosophy, while for economics see Schelling 1980 Young 1996, and Young 2020.

social networks: that person has to pay a high cognitive cost to learn what her/his friend fancies. If s/he knows that everybody goes to the cinema, s/he asks her/his friend to go to the cinema, even if s/he favors football matches. However, why does everybody go to the cinema?

Neary 2012 considers a similar situation to the one described above. He considers a population divided into two groups/types. The two types differ in preferences, and each agent decides one single action valid for both types. Hence, it is as if learning the type of an opponent is too costly, and no agent ever learns it. His study finds that if one type is enough stronger in preferences than the other for its favorite action, then every agent coordinates on the action preferred by that type in the long-run. It also gives conditions on payoffs and group size for the prevalence in the long-run of a state, which causes miscoordination between the types. Such a state is a situation where every agent plays her/his favorite action.

However, in certain circumstances, it is reasonable to assume that agents can learn the type of their opponents at a small cost. In the context of the previous example, a lower cost means, for instance, having social networks, where everyone can learn what their friends prefer from their feeds. In this case, all the agents bear this cost, learn the type of their opponents, and coordinate on their favorite actions with their similar. For instance, those who like cinema learn what the other players prefer through their feeds, and they go to the cinema with those who prefer cinema. Similar for those who fancy football. However, expanding the behavior to an entire population: what do people who prefer cinema and people who prefer football do together? That is, which convention does prevail in inter-group interactions when the cost is that low? More generally, how does the cost influence the formation of conventions?

We answer the above questions, formalizing the example previously made and studying the evolution of conventions in a dynamic setting. We model the coordination problem as a repeated language game (Neary 2012): we use evolutionary game theory solution concepts and characterize the long-run equilibrium as the stochastically stable state (see Foster and Young 1990, Kandori, Mailath, and Rob 1993 and Young 1993). What we do differently from Neary 2012 is the following. Agents can learn the type of their opponent if they pat a cost. If they pay it, they can play a different action with respect to the type they meet. If they do not pay it, they can only play the same action with every agent. Given this change in the strategic set, we introduce a new possible perturbation. Agents can make a mistake in the information choice and a mistake in the coordination choice. We model two situations: a complete information scenario, where agents always learn their opponent's type, and an incomplete information one, where agents can learn their opponent's type conditional on paying a cost. In the latter case, we follow a field of the literature regarding costly acquisition of information (see Simon 1955, or Grossman and Stiglitz 1980). Agents decide myopically their best reply based on the current state, which is always observable. However, in the incomplete information case, a player does not learn her/his opponent's type unless s/he pays for information.

We say that a type is stronger in preferences for its favorite action than the other if it assigns higher payoffs to its favorite outcome or lower payoffs to the other outcome compared to the other group. Cost level, strength in preferences, and group size are crucial drivers of the long-run stability of outcomes. We find that two different scenarios can happen, depending on the cost. Firstly, when the cost is zero or suffi-

ciently low, agents always learn their opponents' type, and they always coordinate. In this case, they also coordinate on their favorite action with agents of their same type. If one group is stronger in preferences for its favorite action or its size is large enough compared to the other, every agent plays the action preferred by that group in inter-group interactions. A second outcome occurs when the cost is high. In this case, agents never learn the type of their opponents, and they play the same action with every agent. They risk coordinating on one action that they do not like, even with agents of their type. Indeed, we find that when one group is stronger in preferences than the other for its favorite action, or if its size is large enough compared to the other, every agent coordinates on that group's favorite action. Even worse, when the cost is high, the two types may play their favorite action and miscoordinate in inter-group interactions. We find that this occurs when both types are strong enough in preferences for their favorite action or if the two groups are sufficiently close in size.
It is helpful to highlight our analysis with respect to the one proposed by Neary 2012, from which we started. When the cost is high enough, our results are the same as the previous model. We further show what happens when agents can learn the opponent's type at a low cost. Comparing these two cases enrich the previous analysis: in this sense, we prove that miscoordination does not occur without incomplete information and a high cost. Strength in preferences and group size alone does not cause miscoordination. Indeed, when the cost is low, the two types always coordinate.

The paper is organized as follows: In Section 2, we explain the model's basic features. In Section 3, we determine the results for the complete information case where the cost is 0. In Section 4, we derive the results for the case with incomplete information and costly acquisition. We distinguish between 2 cases: c high enough and c low enough. In Section 5, we discuss results, and in Section 6, we conclude. We give all proofs in Appendix A.

2 The Model

	a	b
a	Π_A, Π_A	$0, 0$
b	$0, 0$	π_A, π_A

Table 1: Interactions inside group A.

	a	b
a	π_B, π_B	$0, 0$
b	$0, 0$	Π_B, Π_B

Table 2: Interactions inside group B.

	a	b
a	Π_A, π_B	$0, 0$
b	$0, 0$	π_A, Π_B

Table 3: Inter-group Interactions.

We consider N agents divided into two groups A and B, $N = N_A + N_B$. We assume $N_A > N_B + 1$ and $N_B > 1$. Each agent in group A owns type A, and each agent

in group B owns type B. Throughout the paper, we will use types and groups as synonyms. Agents are randomly matched in pairs to play the 2x2 coordination game represented in Matrix 1 to 3. Matching occurs with uniform probability, regardless of type. Matrix 1 and 2 represent inside-group interactions, while Matrix 3 represents inter-group interactions (A type row player and B type column player). We assume that $\Pi_A > \pi_A$, and thus, we name a the favorite action of type A. Equally, we assume $\Pi_B > \pi_B$, and hence, b is the favorite action of type B. We do not assume any particular order between Π_B, and Π_A. However, without loss of generality, we assume that $\Pi_A + \pi_A = \Pi_B + \pi_B$. We say that group A is stronger in preferences for its favorite action than group B if $\Pi_A > \Pi_B$ (or if $\pi_B > \pi_A$).
Before choosing between action a and b, agents choose whether to pay a cost to learn their opponent's type. If they do not pay it, they do not learn the type of their opponent, and they play one single action valid for both types. If they pay it, they can differentiate the action for the two types. We call information choice the first kind of choice and coordination choice the second one. Consider player $i \in K = \{A, B\}$, with $K' \neq K, = \{A, B\}$. τ_i is the information choice of player i: if $\tau_i = 0$ player i does not learn the type of her/his opponent. If $\tau_i = 1$, player i pays a cost c, and learns the type. We assume $c \geq 0$. $x_{0i} \in \{a, b\}$ is the coordination choice when player i chooses $\tau_i = 0$. If $\tau_i = 1$, $x_{1i}^K \in \{a, b\}$ is the coordination choice when player i meets type K, while $x_{1i}^{K'} \in \{a, b\}$ is the coordination choice when player i meets type K'. A pure strategy of an agent consists of her/his information choice, τ_i, and of her/his coordination choices conditioned on the information choice, *i.e.*

$$s_i = \left(\tau_i, x_{0i}, x_{1i}^K, x_{1i}^{K'}\right) \in \mathcal{S}_i = \{0, 1\} \times \{a, b\}^3.$$

For example, $s_i' = \{0, a, a, b\}$ is a strategy when player i decides not to buy the information and plays a with every type of player. The third and fourth entries do not matter since the agent is not learning the type of her/his opponent. Each player has sixteen strategies.

We consider a model of noisy best response learning in discrete time (see Kandori, Mailath, and Rob 1993, Young 1993).
Each period $t = 0, 1, 2, \ldots$, independently from previous events, there is a positive probability $p \in (0, 1)$ that an agent is given the opportunity to revise her/his strategy. When such an event occurs, each agent chooses with positive probability a strategy that maximizes her/his payoff at time t. $s_i(t)$ is the strategy played by player i at time t. $U^i(s_i, s_{-i})$ is the payoff of player i from playing strategy s_i against the strategy profile s_{-i} played by all the other agents except i. At time $t + 1$, player i chooses

$$s_i(t + 1) \in \arg\max_{s_i \in \mathcal{S}_i} U^i(s_i, s_{-i}(t)).$$

If there is more than one strategy that maximizes the payoff, player i assigns the same probability to each of these strategies. We define $n^{AA}(n^{BB})$ as the number of players of type $A(B)$ currently playing action a with type $A(B)$, and $n^{AB}(n^{BA})$ as the number of players of type $A(B)$ currently playing action a with type $B(A)$. Under the assumption of myopic best reply, the payoff of playing strategy s_i at time t depends on n_t^{KK} and on $n_t^{K'K}$, namely, it depends on how many players are playing a with the type of player i. In other words, whether the other agents are buying the information or not does not matter for a decision maker. We define states as vectors

of four components: $\omega = \{n^{AA}, n^{AB}, n^{BA}, n^{BB}\}$, with Ω being the state space, and $\omega_t = \{n_t^{AA}, n_t^{AB}, n_t^{BA}, n_t^{BB}\}$ being the state at time t. We assume that each agent knows all the components of the state vectors at each t. Given myopic best reply, ω_t is all that a decision-maker has to know to make a decision. Therefore, we can characterize the dynamics with this simplification of the states.
Moreover, given the myopic best reply and our definition of states, we can group the 16 strategies in 6 behaviors affecting the dynamics and the individual payoffs in the same way. The logic passages are the following: if players are myopic best repliers, we can consider simplified states. Given the states and myopia of agents, some strategies affect the dynamics and the individual payoffs in the same way. Therefore, we can group strategies concerning the effect they have on the dynamics and individual payoffs.
We name behavior $a(b)$ as the set of strategies when player i chooses $\tau_i = 0$, and $x_{0i} = a(b)$. We name ab the strategies when player i chooses $\tau_i = 1$, $x_{1i}^K = a$, and $x_{1i}^{K'} = b$, and so on and so forth. We call Z_i the set of possible behaviors of player i: $Z_i = (a, b, ab, ba, aa, bb)$. It is easy to show that each strategy in a behavior in Z affects the dynamics and the individual payoffs in the same way.
Consider a player $i \in A$ playing $s_i' = (0, a, a, b)$, who is given the opportunity to revise her/his strategy at time t. If s/he chooses $s_i'' = (0, a, b, b)$ her/his payoff does not change: s/he is still playing a with every agent s/he meets. Moreover, also the state does not change. Differently, if player i chooses $s_i''' = (0, b, a, b)$, her/his payoff changes, and also the state. Indeed, both n_{t+1}^{AA} and n_{t+1}^{AB} decreases of one unit.
From now on, we will refer to behaviors and states following the simplifications described above.

We illustrate here the general scheme of our presentation. We divide the analysis into two cases: complete information and incomplete information. For each case, we consider unperturbed dynamics (agents choose the best reply behavior with probability 1) and perturbed dynamics (agents choose a random behavior with a small probability). First, we help the reader understand how each player evaluates her/his best reply behavior and which states are absorbing. Second, we highlight the general structure of the dynamics with perturbation and then determine the stochastically stable state. We provide the proofs of all results in the appendix and their intuition in the main text. In the next section, we analyze the case with complete information, hence, when the cost is zero.

3 Complete Information with Free Acquisition

In this section, we assume that each player can freely learn the type of her/his opponent when randomly matched with her/him. Without loss of generality, we assume that agents always learn the type of their opponent in this case. We refer to this condition as free acquisition of information. Each player has four possible behaviors as defined in the previous section. $Z_i = \{aa, ab, ba, bb\}$, with $a = aa$, and $b = bb$ in this case.[2]

[2]Under this specification, it is like there is no information choice. Hence, behaviors and strategies coincide. Therefore, our grouping of the strategies does not influence the dynamics.

Consider $i \in K$. Define $\pi_a^K = \begin{cases} \Pi_A & \text{if } K = A \\ \pi_B & \text{if } K = B \end{cases}$ and $\pi_b^K = \begin{cases} \pi_A & \text{if } K = A \\ \Pi_B & \text{if } K = B \end{cases}$.
Equation (1) to (4) are the payoffs for a player $i \in K$ currently playing aa or ab.

$$U^i(aa, z_{-i}(t)) = \frac{N_K - 1}{N - 1}\frac{n_t^{KK} - 1}{N_K - 1}\pi_a^K + \frac{N_{K'}}{N - 1}\frac{n_t^{K'K}}{N_{K'}}\pi_a^K, \tag{1}$$

$$U^i(ab, z_{-i}(t)) = \frac{N_K - 1}{N - 1}\frac{n_t^{KK} - 1}{N_K - 1}\pi_a^K + \frac{N_{K'}}{N - 1}\frac{N_{K'} - n_t^{K'K}}{N_{K'}}\pi_b^K, \tag{2}$$

$$U^i(ba, z_{-i}(t)) = \frac{N_K - 1}{N - 1}\frac{N_K - n_t^{KK}}{N_K - 1}\pi_b^K + \frac{N_{K'}}{N - 1}\frac{n_t^{K'K}}{N_{K'}}\pi_a^K, \tag{3}$$

$$U^i(bb, z_{-i}(t)) = \frac{N_K - 1}{N - 1}\frac{N_K - n_t^{KK}}{N_K - 1}\pi_b^K + \frac{N_{K'}}{N - 1}\frac{N_{K'} - n_t^{K'K}}{N_{K'}}\pi_b^K. \tag{4}$$

The two ratios $\frac{N_K-1}{N-1}$ and $\frac{N_{K'}}{N-1}$ express how much frequently a player meets type K or K'. Note that when an agent calculates her/his payoffs, it does not matter what s/he is doing with the other group, as s/he does not count her/himself in that group.

3.1 Unperturbed Dynamics

We begin the analysis for complete information by studying the dynamics of the system when agents play their best reply behavior with probability one. In this case, what happens at time $t + 1$ depends on the state at time t (agents myopically best reply to the current state) and on who is given the revision opportunity. We define formally the dynamical system as $\omega_{t+1} = F(\omega_t, \theta_{t+1})$. θ_{t+1} is the set of players who received the opportunity to revise their behavior at time t.
We can separate the dynamics of the system into 3 different dynamics. The two regarding inside-group interactions *i.e.* n_t^{AA} and n_t^{BB} and the one regarding inter-group interaction, *i.e.* n_t^{AB} and n_t^{BA}. We call this subset of states $n_t^I = \left(n_t^{AB}, n_t^{BA}\right)$. Both n_t^{AA} and n_t^{BB} are one-dimensional dynamics; n_t^I instead is a two-dimensional dynamics.

Lemma 1. *Under free acquisition of information,* $n_{t+1}^{AA} = F_1(n_t^{AA}, \theta_{t+1})$, $n_{t+1}^{BB} = F_4(n_t^{BB}, \theta_{t+1})$ *and* $(n_{t+1}^{AB}, n_{t+1}^{BA}) = F_{2,3}(n_t^{AB}, n_t^{BA}, \theta_{t+1})$.

The intuition behind the result is as follows. If agents always learn their opponents' type, the inter-group dynamics does not interfere with the inside-group. If player $i \in K$ is given the revision opportunity, s/he chooses x_{1i}^K based only on n_t^{KK}.

Consider a subset of 8 states: $\omega^R = \{(N_A, N_A, N_B, N_B)$, $(0, N_A, N_B, N_B)$, $(N_A, N_A, N_B, 0)$, $(N_A, 0, 0, N_B)$, $(0, N_A, N_B, 0)$, $(N_A, 0, 0, 0)$, $(0, 0, 0, N_B)$ and $(0, 0, 0, 0)\}$.

Lemma 2. *Under free acquisition of information, the states in* ω^R *are the unique absorbing states of the system.*

We call (N_A, N_A, N_B, N_B) and $(0, 0, 0, 0)$ Monomorphic States (MS from now on). Specifically, we refer to the first one as MS_a and to the second as MS_b. We label the remaining six as Polymorphic States (PS from now on). We call $(N_A, N_A, N_B, 0)$ PS_a and $(N_A, 0, 0, 0)$ PS_b. In MS, every agent plays the same action with any other agent; in PS, at least one type is differentiating the action. In MS_a, every agent plays aa, in MS_b, every agent plays bb. In PS_a, A type plays aa and B type plays ba. In PS_b, A type plays ab while B type plays bb. Both in PS_a and PS_b, all players coordinate on their favorite action with their similar. We can break these absorbing states into the three dynamics in which we are interested. This simplification helps in understanding why only these states are absorbing. For instance in inter-group interactions there are just two possible absorbing states, namely (N_A, N_B) and $(0, 0)$. For what concerns inside-group interactions, N_A and 0 matters for n_t^{AA}, N_B and 0 for n_t^{BB}. For each dynamics, the states where every agent plays a or where every agent plays b with one type are absorbing. In this simplification, we can see the importance of Lemma 1. As a matter of fact, in all the dynamics we are studying, there are just two candidates to be stochastically stable. This result simplifies the stochastic stability analysis.

3.2 Perturbed Dynamics

We now introduce perturbations in the model presented in the previous section. We use tools and concepts developed by Freidlin and Wentzell 1984, and refined by Ellison 2000. Agents can experiment while choosing their behaviors: there is a small probability that an agent does not choose her/his best response behavior when s/he is given the revision opportunity. We use the uniform error model for mistakes: the probability of experimenting is equal for every agent and every state. At each step, if an agent is given the revision opportunity, s/he experiments with probability ε. In this section, we assume that agents make mistakes only in the coordination choice: assuming $c = 0$, adding mistakes also in the information choice would not influence the analysis. Note that Lemma 1 is still valid under this specification.

If we consider a sequence of transition matrices $\{P^\varepsilon\}_{\varepsilon>0}$, with associated stationary distributions $\{\mu^\varepsilon\}_{\varepsilon>0}$, by continuity the accumulation point of $\{\mu^\varepsilon\}_{\varepsilon>0}$ that we call $\mu^\star$, is a stationary distribution of $P := \lim_{\varepsilon\to 0} P^\varepsilon$. Mutations guarantee the ergodicity of the Markov process and the uniqueness of the invariant distribution. We are interested in states which have positive probability in $\mu^\star$.

Definition 1. *A state $\bar{\omega}$ is stochastically stable if $\mu^\star(\bar{\omega}) > 0$ and it is uniquely stochastically stable if $\mu^\star(\bar{\omega}) = 1$.*

We define some useful concepts from Ellison 2000. Let $\bar{\omega}$ be an absorbing state of the unperturbed process. $D(\bar{\omega})$ is the basin of attraction of $\bar{\omega}$: the set of initial states from which the unperturbed Markov process converges to $\bar{\omega}$ with probability one. The Radius of the basin of attraction of $\bar{\omega}$ is the number of errors needed to leave $D(\bar{\omega})$, when the system starts in $\bar{\omega}$. Define a path from state $\bar{\omega}$ to state ω' as a sequence of distinct states $(\omega_1, \omega_2, \dots, \omega_T)$, with $\omega_1 = \bar{\omega}$ and $\omega_T = \omega'$. $\Upsilon(\bar{\omega}, \omega')$ is the set of all paths from $\bar{\omega}$ to ω'. Define $r(\omega_1, \omega_2, \dots, \omega_T)$ as the resistance of the path $(\omega_1, \omega_2, \dots, \omega_T)$, namely the number of mistakes that occurs to pass from state $\bar{\omega}$ to state ω'. The Radius of $\bar{\omega}$ is then

$$R(\bar{\omega}) = \min_{(\omega_1,\omega_2,\dots,\omega_T)\in\Upsilon(\bar{\omega},\Omega-D(\bar{\omega}))} r(\omega_1, \omega_2, \dots, \omega_T).$$

Now define the Coradius of $\bar{\omega}$ as

$$CR(\bar{\omega}) = \max_{\omega \notin D(\bar{\omega})} \min_{(\omega_1,\omega_2,\ldots,\omega_T)\in\Upsilon(\omega,D(\bar{\omega}))} r(\omega_1, \omega_2, \ldots, \omega_T)$$

Thanks to Theorem 1 in Ellison 2000, we know that if $R(\bar{\omega}) > CR(\bar{\omega})$, then $\bar{\omega}$ is uniquely stochastically stable.
We are ready to calculate the stochastically stable states under complete information.

Theorem 1. *Under free acquisition of information, for N large enough, if $\frac{\pi_B}{\pi_A} < \frac{N_B}{N_A}$, then PS_b is uniquely stochastically stable. If $\frac{\pi_B}{\pi_A} > \frac{N_B}{N_A}$, then PS_a is uniquely stochastically stable.*

When the cost is null, agents always learn the type of their opponent. Therefore, they always coordinate on their favorite action in inside-group interactions. Hence, n_t^{AA} always converge to N_A and n_t^{BB} always converge to 0. This result rules out Monomorphic States and other 4 Polymorphic States: only PS_a and PS_b are left. Which of the two is selected depends on strength in preferences and group size. Two effects determine the results in the long-run. Firstly, if $\pi_A = \pi_B$, PS_a is uniquely stochastically stable. The majority prevails in inter-group interactions if the two groups are equally strong in preferences. Secondly, if $\pi_A \neq \pi_B$, there is a trade-off between strength in preferences and group size. If $\frac{\pi_B}{\pi_A} > \frac{N_B}{N_A}$, either type A is stronger in preferences than type B, or group A is enough larger than group B. In both of the two situations, the number of mistakes necessary to leave PS_a is bigger than the one to leave PS_b: in a sense, more errors are needed to make b best reply for A players than to make a best reply for B players. Therefore, every agent will play action a in inter-group interactions. A similar reasoning applies if $\frac{\pi_B}{\pi_A} < \frac{N_B}{N_A}$.
We provide two numerical examples to explain how the model works in Figure 1 and 2. We represent just n_t^I, hence, a two-dimensional dynamics.[3] Red states represent the basin of attraction of $(0, 0)$, while green states the one of (N_A, N_B). From grey states there are paths of zero resistance both to $(0, 0)$ and to (N_A, N_B). Any path that involves more players playing a within red states has a positive resistance. Every path that involves fewer people playing a within green states has a positive resistance. The Radius of $(0, 0)$ is equal to the Coradius of (N_A, N_B), and it is the minimum error path from $(0, 0)$ to grey states. The Coradius of $(0, 0)$ is equal to the Radius of (N_A, N_B), and it is the minimum error path from (N_A, N_B) to grey states.
Firstly, consider the example in Figure 1. $N_A = 10$, $N_B = 5$, $\pi_A = 8$, $\Pi_A = 10$, $\pi_B = 3$, $\Pi_B = 15$. Clearly, $\frac{\pi_B}{\pi_A} = \frac{3}{8} < \frac{5}{10} = \frac{N_B}{N_A}$. In this case $R(10, 5) = CR(0, 0) = 1$, while $R(0, 0) = CR(10, 5) = 3$. Hence, $(0, 0)$ is the uniquely stochastically stable state. We give here a short intuition. Starting from $(0, 0)$, the minimum error path to grey states is the one that reaches $(0, 3)$. The minimum error path from $(10, 5)$ to grey states is the one that reaches $(9, 5)$. Hence, fewer mistakes are needed to exit from the green states than to exit from the red states. This is why $PS_b = (10, 0, 0, 0)$ is uniquely stochastically stable.
Secondly, consider the example in Figure 2. $N_A = 10$, $N_B = 5$, $\pi_A = 3$, $\Pi_A = 15$, $\pi_B = 8$, $\Pi_B = 10$. Note that $\frac{\pi_B}{\pi_A} = \frac{8}{3} > \frac{5}{10} = \frac{N_B}{N_A}$. In this case, $R(10, 5) = CR(0, 0) = 4$, $CR(10, 5) = R(0, 0) = 1$. Hence, $PS_a = (10, 5)$ is uniquely stochastically stable. In this case, the minimum error path to exit green states is the one that reaches $(6, 5)$ or $(10, 1)$. The one to exit the red states is the one that reaches $(0, 1)$.

[3] For a more exhaustive treatment of lattices, see Appendix A of Neary 2012.

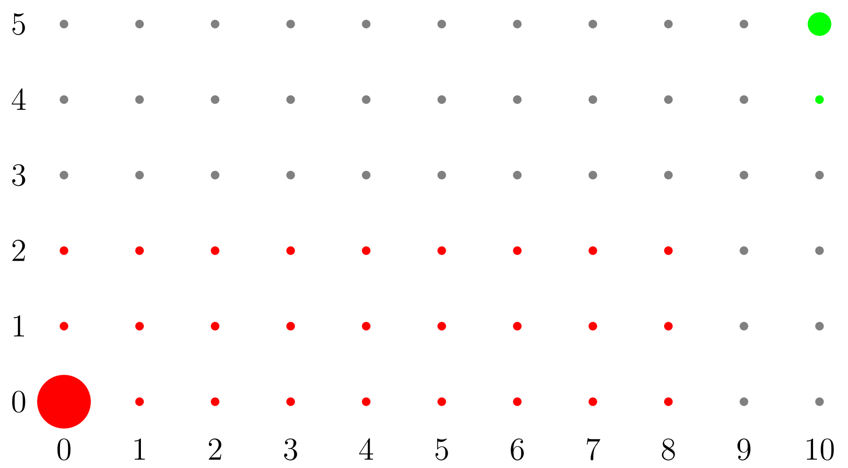

Figure 1: $PS_b = (0,0)$ is uniquely stochastically stable: $\frac{\pi_B}{\pi_A} < \frac{N_B}{N_A}$.

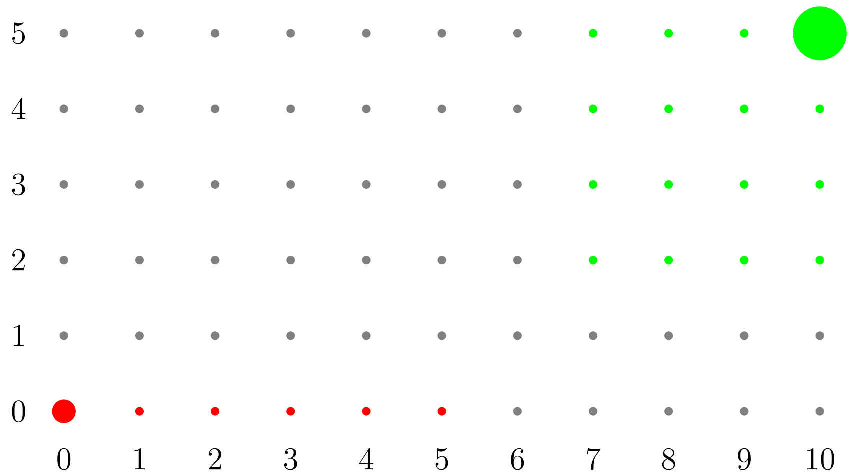

Figure 2: $PS_a = (10,5)$ is uniquely stochastically stable: $\frac{\pi_B}{\pi_A} > \frac{N_B}{N_A}$.

4 Incomplete Information with Costly Acquisition

In this section, we assume that each player can not freely learn the type of her/his opponent. Each agent can buy this information at a strictly positive cost. We refer to this condition as costly acquisition of information.[4] This time $Z_i = \{a, b, ab, ba, aa, bb\}$, $\forall i \in N$. It is trivial to show that there are 4 optimizing behavior out of the 6 behaviors, indeed, $U^i(aa) = U^i(a) - c$ and $U^i(bb) = U^i(b) - c$. Hence, for all $i \in N$, $U^i(aa) < U^i(a)$ and $U^i(bb) < U^i(b)$, $\forall c > 0$. We define optimizing behaviors as $Z_i^o = \{a, b, ab, ba\}$, $\forall i \in N$, with z_i^o being an optimizing behavior of player i.
Equation (5) to (8) are the payoffs at time t, for a player $i \in K$ currently playing a or ab.

$$U^i(a, z_{-i}(t)) = \frac{n_t^{KK} + n_t^{K'K} - 1}{N-1}\pi_a^K, \tag{5}$$

[4]It is trivial to notice that Lemma 1 is not valid anymore. Indeed, since agents learn the type of their opponent conditional on paying a cost, not every player pays it, and the dynamics are no longer separable.

$$U^i(b, z_{-i}(t)) = \frac{N - n_t^{KK} - n_t^{K'K}}{N-1}\pi_b^K, \quad (6)$$

$$U^i(ab, z_{-i}(t)) = \frac{N_K - 1}{N-1}\frac{n_t^{KK} - 1}{N_K - 1}\pi_a^K + \frac{N_{K'}}{N-1}\frac{N_{K'} - n_t^{K'K}}{N_{K'}}\pi_b^K - c, \quad (7)$$

$$U^i(ba, z_{-i}(t)) = \frac{N_K - 1}{N-1}\frac{N_K - n_t^{KK}}{N_K - 1}\pi_b^K + \frac{N_{K'}}{N-1}\frac{n_t^{K'K}}{N_{K'}}\pi_a^K - c. \quad (8)$$

To help the reader visualize the differences between this section and Section 3, we did not explicit in Equation (5) and (6) the frequencies of meetings. Note that if $c = 0$, then $aa = a$ and $bb = b$.
We begin the analysis again with the unperturbed dynamics, where agents choose their best reply behavior with probability one.

4.1 Unperturbed Dynamics

State	Condition on group size and payoffs	Conditions on c
MS_a	none	none
MS_b	none	none
TS	$\frac{\pi_B}{\Pi_B} < \frac{N_B-1}{N_A}$	$c > \max\left\{\frac{N_B}{N-1}\pi_A, \frac{N_A}{N-1}\pi_B\right\}$
PS_b	none	$c < \frac{N_B}{N-1}\pi_A$
PS_a	1) $\frac{\pi_B}{\Pi_B} > \frac{N_B-1}{N_A}$ 2) $\frac{\pi_B}{\Pi_B} < \frac{N_B-1}{N_A}$	1) $c < \frac{N_B-1}{N-1}\Pi_B$ 2) $c < \frac{N_A}{N-1}\pi_B$
$(0, N_A, N_B, N_B)$	1) $\frac{\pi_A}{\Pi_A} < \frac{N_B}{N_A-1}$ 2) $\frac{\pi_A}{\Pi_A} > \frac{N_B}{N_A-1}$	1) $c < \frac{N_A-1}{N-1}\pi_A$ 2) $c < \frac{N_B}{N-1}\Pi_A$
$(N_A, 0, 0, N_B)$	none	$c < \min\left\{\frac{N_B}{N-1}\pi_A, \frac{N_B-1}{N-1}\pi_B\right\}$
$(0, N_A, N_B, 0)$	1) $\frac{\pi_A}{\Pi_A} < \frac{N_A-1}{N_B}$ and $\frac{\pi_B}{\Pi_B} > \frac{N_B-1}{N_A}$ 2) $\frac{\pi_A}{\Pi_A} > \frac{N_A-1}{N_B}$ and $\frac{\pi_B}{\Pi_B} > \frac{N_B-1}{N_A}$ 3) $\frac{\pi_A}{\Pi_A} < \frac{N_A-1}{N_B}$ and $\frac{\pi_B}{\Pi_B} < \frac{N_B-1}{N_A}$ 4) $\frac{\pi_A}{\Pi_A} > \frac{N_A-1}{N_B}$ and $\frac{\pi_B}{\Pi_B} < \frac{N_B-1}{N_A}$	1) $c < \min\left\{\frac{N_A-1}{N-1}\pi_A, \frac{N_B-1}{N-1}\Pi_B\right\}$ 2) $c < \min\left\{\frac{N_B}{N-1}\Pi_A, \frac{N_B-1}{N-1}\Pi_B\right\}$ 3) $c < \min\left\{\frac{N_A-1}{N-1}\pi_A, \frac{N_A}{N-1}\pi_B\right\}$ 4) $c < \min\left\{\frac{N_B}{N-1}\Pi_A, \frac{N_A}{N-1}\pi_B\right\}$
$(0, 0, 0, N_B)$	none	$c < \frac{N_B-1}{N-1}\pi_B$

Table 4: Necessary and sufficient conditions for absorbing states.

So far, there are no more random elements with respect to Section 3. What happens at time $t + 1$ depends on the state at time t and on the players that are given the revision opportunity. Nine states can be absorbing under this specification.

Lemma 3. *Under costly acquisition of information, there are nine possible absorbing states:* $\omega^R \cup (N_A, N_A, 0, 0)$.

We summarize all the relevant information in Table 4. The reader can note two differences with respect to Section 3: firstly, there is one more possible absorbing state, that is $(N_A, N_B, 0, 0)$, and secondly, some states are absorbing if and only if some conditions hold. Where we write "none", we mean that a state is always absorbing for every value of group size, payoffs, and/or the cost. We name $(N_A, N_B, 0, 0)$ the

Type Monomorphic State (TS from now on): in this state, each type is playing its favorite action, causing miscoordination in inter-group interactions. Monomorphic States are absorbing states for every value of group size, payoffs, and cost. When every player is playing one action with any other player, agents do not need to learn their opponent's type (the information cost does not matter). They best reply to these states by playing the same action.
Polymorphic States are absorbing if and only if the cost is low enough: if the cost is too high, buying the information is too expensive, and agents best reply to Polymorphic States by playing a or b. The Type Monomorphic State is absorbing if type B is either large enough compared to group A or strong enough in preferences for its favorite action and if the cost is high enough. The intuition is the following. On the one hand, if type B is weak in preferences or small enough, every player of type B best replies to TS by playing a if the cost is high. On the other hand, if the cost is low enough, every player best replies to this state by buying the information and differentiating the action.

4.2 Perturbed Dynamics

We now introduce perturbed dynamics. In this case, we assume that agents can make two types of errors: they can make a mistake in the information choice and in the coordination choice. Choosing the wrong behavior, in this case, can mean both. We say that with probability η, an agent that is given the revision opportunity at time t chooses to buy the information when it is not optimal. With probability ε, s/he makes a mistake in the coordination choice. Alternatively, we could have chosen to set only the probability of experimenting with a different behavior or strategy.
The logic behind our assumption is to capture behaviorally relevant errors. We assume a double punishment mechanism for players choosing by mistake the information level and the coordination action. Specifically, our error counting is not influenced by our definition of behaviors. We could have made the same assumption starting from the standard definition of strategies assuming that agents can make separate mistakes in choosing the two actions that constitute a strategy. Our assumption is in line with works such as Jackson and Watts 2002 and Bhaskar and Vega-Redondo 2004, which assume errors in the coordination choice and the link choice.
Since we are assuming two types of errors, the concept of resistance changes, we then need to consider three types of resistances. We call $r_{\varepsilon}(\omega_t, \dots, \omega_s)$ the path from state ω_t to state ω_s with ε errors (players make a mistake in the coordination choice). We call $r_{\eta}(\omega_t, \dots, \omega_s)$ the path with η errors (players make a mistake in the information choice). Finally, we call $r_{\varepsilon\eta}(\omega_t, \dots, \omega_s)$ the path with errors both in the coordination choice and the information choice. Since we do not make further assumptions on ε and η (probability of making errors uniformly distributed), we can assume $\eta \propto \varepsilon$.
We count each error in the path of both ε and η errors as 1, however, $r_{\varepsilon\eta}(\omega_t, \dots, \omega_s)$ is always double since it implies a double error. Indeed, we can see this kind of error as the sum of two components, one in η and the other in ε, namely $r_{\varepsilon\eta}(\omega_t, \dots, \omega_s) = r_{\varepsilon\eta|_{\varepsilon}}(\omega_t, \dots, \omega_s) + r_{\varepsilon\eta|_{\eta}}(\omega_t, \dots, \omega_s)$.
For example, think about $\omega_t = MS_a$, and that one player from B is given the revision opportunity. Consider the case where s/he makes a mistake both in the information choice and in the coordination choice. For example, s/he learns the type and s/he

plays a with A and b with B. This error delineates a path from MS_a to the state $(N_A, N_A, N_B - 1, N_B)$ of resistance $r_{\varepsilon\eta}(MS_a, \ldots, (N_A, N_A, N_B - 1, N_B)) = 2$. Next, think about $\omega_t = TS$: the transition from TS to $(N_A, N_A - 1, 0, 0)$ happens with one η error. One player from A should make a mistake in the information choice and optimally choosing ab. In this case, $r_\eta(TS, \ldots, (N_A, N_A - 1, 0, 0)) = 1$. With a similar reasoning, $r_\varepsilon(MS_a, \ldots, (N_A - 1, N_A - 1, N_B, N_B)) = 1$: a player of type A makes a mistake in the coordination choice and chooses b.

We explain why our grouping of strategies does not influence the stochastic stability analysis before we give the results. Let us consider all the sixteen strategies as presented in Section 2, and just one kind of mistake in the choice of the strategy. Let us think about two different states: $\bar{\omega}$ where every agent is playing strategy s', and ω', where m agents are playing s''', and all the others are playing s'. Consider $s', s'' \in z'$, and $s''' \in z''$. Strategy s' is the best reply for every player in state $\bar{\omega}$, and trivially, $\bar{\omega} \neq \omega'$. Consider $\omega_0 = \bar{\omega}$. If m players choose s_i'' at time 0, $\omega_1 = \bar{\omega}$. If m players choose s_i''' at time 1, $\omega_2 = \omega'$. The cost of this transition is $2m$. However, there is a lower error path from $\bar{\omega}$ to ω'. This path is the one with m errors directly towards strategy s_i'''. Such a path happens with cost m. Therefore, we would never consider the first path for stochastic stability analysis. This result is true for every absorbing state: there is never a minimum resistance path involving errors towards strategies grouped in the same behavior from one absorbing state to another. Consequently, our grouping of strategies does not influence the results.

We divide this part of the analysis into two cases, the first one where the cost is high and the second one when the cost is low.

4.2.1 High Cost

In this part of the analysis, we focus on a case when only MS and TS are absorbing states.

Define the following set of values:

$$\Xi_{PS} = \left\{ \frac{N_B}{N-1}\pi_A, \frac{N_A}{N-1}\pi_B, \frac{N_B - 1}{N-1}\Pi_B, \frac{N_A - 1}{N-1}\pi_A, \frac{N_B}{N-1}\Pi_A, \frac{N_B - 1}{N-1}\pi_B \right\}.$$

Corollary 1. *Under costly acquisition of information, if $c > max\{\Xi_{PS}\}$ and $\frac{\pi_B}{\Pi_B} < \frac{N_B - 1}{N_A}$, then only MS and TS are absorbing states. If $\frac{\pi_B}{\Pi_B} \geq \frac{N_B - 1}{N_A}$, then only MS are absorbing states.*

The proof is straightforward from Table 4 and therefore, we omit it. We previously give the intuition behind this result. Let us firstly consider the case in which TS is not an absorbing state, hence, the case when $\frac{\pi_B}{\Pi_B} \geq \frac{N_B - 1}{N_A}$.

Theorem 2. *Under costly acquisition of information, for N large enough, take $\frac{\pi_B}{\Pi_B} \geq \frac{N_B - 1}{N_A}$ and $c > max\{\Xi_{PS}\}$. If $N_A > \frac{2N\pi_A + \Pi_A - \pi_A}{\Pi_A + \pi_A}$, then MS_a is uniquely stochastically stable. If $N_A < \frac{2N\pi_A + \Pi_A - \pi_A}{\Pi_A + \pi_A}$, then MS_b is uniquely stochastically stable.*

If group A is large enough or strong enough in preferences, the minimum number of errors to exit from the basin of attraction of MS_a is higher than the minimum number

of errors to exit from the one of MS_b. Therefore, MS_a is uniquely stochastically stable: every agent plays behavior a in the long-run.
Now we analyze the case when also TS is a strict equilibrium.

Theorem 3. *Under costly acquisition of information, for N large enough, take $\frac{\pi_B}{\Pi_B} < \frac{N_B-1}{N_A}$ and $c > max\{\Xi_{PS}\}$.*

- *If $N(\pi_B - \pi_A) > N_B\Pi_B - N_A\pi_B - \Pi_B + \pi_B + \Pi_A$, then MS_a is uniquely stochastically stable.*
- *If $N(\pi_A - \pi_B) > N_A\Pi_A - N_B\pi_A - \Pi_A + \Pi_B + \pi_A$, then MS_b is uniquely stochastically stable.*
- *If $\min\{N_A\Pi_A - N_B\pi_A + \pi_A, N_B\Pi_B - N_A\pi_B + \pi_B\} - \Pi_A - \Pi_B > N(\pi_A + \pi_B)$, then TS is uniquely stochastically stable.*

Moreover when the conditions of the following system hold

$$\begin{cases} N(\pi_B - \pi_A) \leq N_B\Pi_B - N_A\pi_B - \Pi_B + \pi_B + \Pi_A \\ N(\pi_A - \pi_B) \leq N_A\Pi_A - N_B\pi_A - \Pi_A + \Pi_B + \pi_A \\ \min\{N_A\Pi_A - N_B\pi_A + \pi_A, N_B\Pi_B - N_A\pi_B + \pi_B\} - \Pi_A - \Pi_B \leq N(\pi_A + \pi_B) \end{cases}$$

- *If $N(\pi_B - \pi_A) > N_B(\Pi_B + \pi_A) - N_A(\Pi_A + \pi_B) + \Pi_A - \pi_A + \pi_B - \Pi_B$, then MS_a is uniquely stochastically stable.*
- *If $N(\pi_A - \pi_B) > N_A(\Pi_A + \pi_B) - N_B(\Pi_B + \pi_A) - \Pi_A + \pi_A - \pi_B + \Pi_B$, then MS_b is uniquely stochastically stable.*
- *If $N(\pi_A - \pi_B) = N_A(\Pi_A + \pi_B) - N_B(\Pi_B + \pi_A) - \Pi_A + \pi_A - \pi_B + \Pi_B$, then both MS_a and MS_b are stochastically stable.*

We divide the statement of the theorem into two parts for technical reasons. However, the reader can understand the results from the first three conditions. The first condition expresses a situation where type A is stronger in preferences than type B or group A is enough larger than group B. In this case, there is an asymmetry in the two costs of exiting the two basins of attraction of MS_a and MS_b. Exit from the first requires more errors than exit from the second. Moreover, reaching MS_a from TS requires less errors than reaching MS_b from TS. This is why $R(MS_a) > CR(MS_a)$ and MS_a is uniquely stochastically stable in this case. Similar reasoning applies to the second condition.
The third condition expresses a case where both types are strong enough in preferences, or the two groups have sufficiently similar sizes. Many errors are required to exit from TS, compared to how many errors are required to reach TS from the two MS. TS is the state where both types are playing their favorite action. Therefore, if the two groups are symmetric in the strength in preferences or their sizes, agents play their favorite action in the long-run. However, they miscoordinate in inter-group interactions. We conclude from this section that two conditions must hold for miscoordination to happen in the long-run. First, the cost to pay to learn the opponent's type should be so high that agents never learn their opponents' type. Second, both types should be strong enough in preferences or enough close in size.

4.2.2 Low Cost

In this section, we discuss the case when c is as low as possible but greater than 0.

Corollary 2. *Under costly acquisition of information, if* $0 < c < \min\{\frac{\pi_A}{N-1}, \frac{\pi_B}{N-1}\}$*, MS and PS are absorbing states, while TS is not an absorbing state.*

The proof is straightforward from Table 4. In this case, there are 8 candidates to be stochastically stable equilibria.

Theorem 4. *Under costly acquisition of information, for N large enough, take* $0 < c < \min\{\frac{\pi_A}{N-1}, \frac{\pi_B}{N-1}\}$*. If $\frac{\pi_B}{\pi_A} < \frac{N_B}{N_A}$, then PS_b is uniquely stochastically stable. If $\frac{\pi_B}{\pi_A} > \frac{N_B}{N_A}$, then PS_a is uniquely stochastically stable.*

The conditions are the same as in Theorem 1. When the cost is low enough, whenever a player can buy the information, s/he does it. Consequently, the basins of attraction of Polymorphic States enlarge: they reach the dimension they had under free acquisition of information. Furthermore, the basins of attraction of both Monomorphic States contract. Due to these two effects, the results are the same as under free acquisition of information.

5 Discussion

The results of our model involve three fields of the literature. Firstly, we contribute to the literature on social conventions. Secondly, we contribute to the literature on stochastic stability analysis, and lastly, we contribute to the literature on costly acquisition of information.
For what concerns social conventions, we contribute to the literature on language games or, more generally, to the one on coordination games. Many works in this field are concerned about the existence in the long-run of heterogeneous strategy profiles. We started from the original model of Neary 2012, which considers agents heterogeneous in preferences, but with a smaller strategic set. His work gives conditions for the stochastic stability of heterogeneous strategy profiles that causes miscoordination in inter-group interactions in a random matching case.[5] Neary and Newton 2017 expands the previous idea to investigate the role of different classes of graphs on the long-run result. They find conditions on graphs such that a heterogeneous strategy profile is stochastically stable. They also consider the choice of a social planner that wants to induce heterogeneous or homogeneous behavior in a population. Carvalho 2017 considers a similar model, where agents choose their actions from a set of culturally constrained possibilities. The author associates the heterogeneous strategy profile with miscoordination. He finds that cultural constraints are a crucial driver for miscoordination. Michaeli and Spiro 2017 studies a game between agents with heterogeneous preferences and who feel pressure from behaving differently. They characterize the circumstances under which a biased norm can prevail on a non-biased norm. Tanaka, Lee, and Iwasa 2018 studies how local dialects survive in a society

[5]Heterogeneity has been discussed in previous works such as Smith and Price 1973, Friedman 1998, Cressman, Garay, and Hofbauer 2001, Cressman, Ansell, and Binmore 2003 or Quilter et al. 2007.

with an official language. Naidu, Hwang, and Bowles 2017 studies the evolution of egalitarian and inegalitarian conventions. To do so, they consider a framework with asymmetry similar to the language game. Likewise, Belloc and Bowles 2013 examines the evolution and the persistence of inferior cultural conventions.
We give the conditions for the stability of the Type Monomorphic State, where agents miscoordinate in inter-group interactions. We show that in our framework, incomplete information, high cost, strength in preferences, and group size are key drivers for miscoordination.
Many works propose a version of the language game in a network context. Goyal, Hernández, et al. 2017 experiments the language game, testing whether agents segregate or conform to the majority. van Gerwen and Buskens 2018 suggests a variant of the language game similar to our version but in a model with networks to study the influence of partner-specific behavior on coordination outcomes. Concerning auctions theory, He 2019 studies a framework where each individual of a population divided into two types has to choose between two skills: a "majority" and a "minority" one. She finds that minorities are advantaged in competition context rather than in coordination one. He and Wu 2020 tests the role of compromise in the battle of sexes with an experiment. A parallel field is the one of bilingual games such as the one proposed by Goyal and Janssen 1997 or Galesloot and Goyal 1997: these models consider situations in which agents are homogeneous in preferences but can become bilingual at a given cost.

Nyborg et al. 2016 has recently suggested the applicability of tipping points theories to policy and interventions. This field could produce explanations and further research questions for language games (see Neary and Newton 2017 again). Indeed, in our model, there are situations in which the majority conforms to the action preferred by the minority. This fact happens even in inside-group interactions.

Concerning the technical literature on stochastic stability, we contribute by applying standard stochastic stability techniques to an atypical context, such as the costly acquisition of information. Since the seminal works by Bergin and Lipman 1996, and Blume 2003, many studies have focused on testing the role of different error models in equilibrium selection. We used a uniform error model, and we believe that introducing different models could be an interesting exercise for future studies. Among the many models that can be used, there are four relevant variants: payoff-dependent mistakes (Sandholm 2010, Dokumacı and Sandholm 2011 and Klaus and Newton 2016), cost-dependent mistakes (Blume 1993 and Myatt and Wallace 2003), intentional mistakes (Naidu, Hwang, and Bowles 2010 and Hwang, Naidu, and Bowles 2016) and condition dependent mistakes (Bilancini and Boncinelli 2020). Important experimental works in this literature have been done by Lim and Neary 2016, Hwang, Lim, et al. 2018, Mäs and Nax 2016, and Bilancini, Boncinelli, and Nax 2020.

Other works contribute to the literature on stochastic stability from the theoretical perspective (see Newton 2018 for an exhaustive review of the field). Recently, Newton 2020 has expanded the domain of behavioral rules regarding the result of stochastic stability. Sawa and Wu 2018a shows that with loss aversion individuals, the stochastic stability of risk dominant equilibria is no longer guaranteed. Sawa and Wu 2018b introduces reference-dependent preferences and analyzes the stochastic stability of best response dynamics. Staudigl 2012 examines stochastic stability in an asymmetric

binary choice coordination game.
Future extensions could break down a common assumption in many evolutionary game theory models, such as the random matching a là Kandori, Mailath, and Rob 1993. In our model, every agent knows the current state. Robson and Vega-Redondo 1996 is one of the most famous works breaking down this property. It would be interesting to go deep into this assumption and modify the way agents estimate the state. Such an analysis would exploit the role of bounded rationality in the formation of social conventions. Specifically, we are interested in applying models such as the one of Jehiel 2005.

For what concerns the literature on costly acquisition of information, many works interpret the information's cost as a cognition cost (see the seminal contributions by Simon 1955, or Grossman and Stiglitz 1980). Our paper is one of those. Many studies place this framework in a sender-receiver game. This is the case of Dewatripont and Tirole 2005, who builds a model of costly communication in a sender-receiver setup. More recent contributions in this literature are Dewatripont 2006, Caillaud and Tirole 2007, Tirole 2009 and Butler, Guiso, and Jappelli 2013. Bilancini and Boncinelli 2016 applies this model to persuasion games with labeling, Bilancini and Boncinelli 2018a studies these kinds of models to dual-process theories in psychology. Finally, Bilancini and Boncinelli 2018b, is the first to apply costly acquisition of information to the analogy-based reasoning theory developed by Jehiel 2005.[6] To the best of our knowledge, we are the first to use costly acquisition of information in an evolutionary model.
Many works use a similar concept of cost in the evolutionary game theory literature: the link formation one. Staudigl and Weidenholzer 2014 introduces the possibility that agents can establish costly links with other players. The main finding is that if a small number of players play the Payoff-Dominant action, other players connect with them and play the Payoff-Dominant action. The work by Bilancini and Boncinelli 2018c extends Staudigl and Weidenholzer 2014. They firstly twine evolutionary game theory and costly interactions. This model introduces the fact that interacting with a different type might be costly for an agent. They find that when the cost is low, the Payoff-Dominant strategy is also the stochastically stable one. When the cost is high, the two types in the population coordinate on two different strategies. One on the Risk-Dominant and the other on the Payoff-Dominant. Similarly, Bilancini, Boncinelli, and Wu 2018 studies the role of cultural intolerance and assortativity in a coordination context. They divide the population into two cultural groups who sustain a cost from interacting with the other group. They find interesting conditions under which cooperation can emerge even with cultural intolerance.

6 Conclusions

We can summarize our results as follows. When agents learn the type of their opponents at a low cost, they always coordinate. They play their favorite action with their similar, while in inter-group interactions, they play the favorite action of the

[6] A recent field of the literature concerns rational inattention, which is a way of endogenizing the cost of information (see Mackowiak, Matejka, Wiederholt, et al. 2020 for an exhaustive review). We assume that the cost is exogenous and homogeneous for each player.

group that is stronger in preferences or with size large enough. If the cost is high, agents never learn the type of their opponents. Either all the agents play the same action with every agent, or all the agents play their favorite action.

Comparing Section 4.2.1 and 4.2.2, we can see the impact of varying the cost levels on the long-run results. A change in the cost level produces two effects that need perhaps a further investigation. The first effect concerns the change in the payoff from the interactions between agents. The second concerns the change in the purchase of the information.
Consider a starting situation where the cost is low. Agents always coordinate on their favorite action in inside-group interactions. If the cost increases, agents will stop learning their opponent's type (hence, they stop paying the cost), and they will begin to play the same action with any other player. If this happens, either Monomorphic States establish in the long-run, or the Type Monomorphic State emerges. In the first case, a group of agents coordinates on its second best option, even in inside-group interactions. For this group, there will be a certain loss in terms of welfare. In the second case, agents miscoordinate in inter-group interactions, and hence, all of them will have a certain loss in welfare.
Nevertheless, when the cost is low, there is a "free-riding" behavior that vanishes if the cost increases. In fact, with low cost levels, only one type pays the cost, and the other never pays it. In one case, A always plays its favorite action and never pays the cost, while B affords it. In the other case, the opposite happens. Hence, when the cost increases, one of the two groups will benefit from not paying the information anymore. However, given that we characterize the equilibria just for cases when the cost is low, this seems not relevant.

We conclude with a short comparison of our result with the one of Neary 2012. Our results are not qualitatively different from that work, but it is worthwhile to mention a contrast that is a consequence of introducing the possibility to differentiate the action. In the model of Neary 2012, a change in the strength of preferences of one type does not affect the behavior of the other type. We can find this effect even in our model when the cost is high. For example, when MS_a is stochastically stable, and type B becomes enough stronger in preferences, the new stochastically stable state becomes TS. This means that A type does not change its behavior. However, when the cost is sufficiently low, the change in payoffs of one type influences the other type's behavior in inter-group interactions. For instance, when PS_b is stochastically stable, if type A becomes strong enough in preferences, PS_a becomes stochastically stable. Both types change the way they behave in inter-group interactions.
Nevertheless, this is a quantitative difference. From the qualitative point of view, our results are similar to the findings of Neary 2012. If the payoffs of one type change, the other keeps playing its favorite action in inside-group interactions, but it changes the action in inter-group ones. The same reasoning can be done with group size.

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A Proofs

A.1 Proofs of Section 3

Proof of Lemma 1:

Consider a player $i \in K$ currently playing aa, who is given the revision opportunity at time t. On the one hand, $\forall n_t^{KK}$, $U^i(ab, z_{-i}(t)) = U^i(aa, z_{-i}(t))$. On the other hand, $\forall n_t^{K'K}$, $U^i(ba, z_{-i}(t)) = U^i(aa, z_{-i}(t))$. Therefore, player i chooses aa or ab depending on $n_t^{K'K}$, and ba or aa depending on n_t^{KK}.
Moreover, if player i chooses ab instead of aa, $n_{t+1}^{KK} = n_t^{KK}$, but $n_{t+1}^{K'K} < n_t^{K'K}$. If player i chooses ba instead of aa, $n_{t+1}^{KK} < n_t^{KK}$, but $n_{t+1}^{K'K} = n_t^{K'K}$. This completes the proofs.

□

With abuse of notation, we call best reply (BR), the action optimally taken by a player in one of the three dynamics. For example, if a type A earns the highest payoff by playing a against a player of type B, we say that a is her/his BR. We do this in the context of complete information because of the separability of the dynamics.

Proof of Lemma 2:

Thanks to Lemma 1, we can consider the 3 separated dynamics: n_t^{AA}, n_t^{BB}, and n_t^I.

Inside-group interactions.

Firstly, we prove the result for n_t^{AA} and then the argument stands for n_t^{BB} thanks to symmetry of payoff matrix. We have to show that all the states in ω^R have an absorbing component for n_t^{AA}, that is 0 or N_A. When $n^{AA} = N_A$, $\forall i \in A$, a is BR against type A at time t. Hence, $F_1(N_A, \theta_{t+1}) = N_A$. Symmetrically if $n^{AA} = 0$, b is always BR and so, $F_1(0, \theta_{t+1}) = 0$. Therefore, N_A and 0 are fixed points for n_t^{AA}.
We need to show that these states are absorbing, that all the other states are transient, and that there are no cycles. Consider player $i \in A$ who is given the revision opportunity at time t. We define $\bar{n}^A$ as the minimum number of A players such that a is BR, and $\underline{n}^A$ as the maximum number of A players such that b is BR. From Equation (1) to (4), we know that $\bar{n}^A = \frac{N_A\pi_A + \Pi_A}{\Pi_A + \pi_A}$, and that $\underline{n}^A = \frac{N_A\pi_A - \pi_A}{\Pi_A + \pi_A}$. Assume $n_t^{AA} \geq \bar{n}^A$. There is always a positive probability that a player not playing a is given the revision opportunity. Hence, $F_1(n_t^{AA}, \theta_{t+1}) \geq n_t^{AA}$. Symmetrically, we can say that if $n_t^{AA} < \underline{n}^A$, $F_1(n_t^{AA}, \theta_{t+1}) \leq n_t^{AA}$.
We now prove that if $n_t^{AA} \leq \underline{n}^A \neq 0$,

$$Pr\left(\lim_{t\to\infty} F_1(n_t^{AA}, \theta_{t+1}) = n_t^{AA}\right) = 0.$$

Equally, if $n_t^{AA} \geq \bar{n}^A \neq N_A$,

$$Pr\left(\lim_{t\to\infty} F_1(n_t^{AA}, \theta_{t+1}) = n_t^{AA}\right) = 0.$$

We prove the first case, and the result stands for the second, thanks to symmetry in payoff matrices. Consider to be at period s in a state $n_s^{AA} < \underline{n}^A \neq 0$. For every player, b is BR. Define $Pr\left(F_1(n_s^{AA}, \theta_{s+1}) = n_s^{AA}\right) = p$. Such a probability represents the event that only players playing b are given the revision opportunity. $Pr\left(F_1(n_{s+1}^{AA}, \theta_{s+2}) = n_{s+1}^{AA}\right) = p^2$, $Pr\left(F_1(n_{s+k-1}^{AA}, \theta_{s+k}) = n_{s+k-1}^{AA}\right) = p^k$. If $k \to \infty$, $Pr\left(F_1(n_{s+k-1}^{AA}, \theta_{s+k}) = n_{s+k-1}^{AA}\right) = 0$. Therefore,

$$\text{If } n_0^{AA} \leq \underline{n}^A, \quad Pr\left(\lim_{t\to\infty} F_1(n_t^{AA}, \theta_{t+1}) = 0\right) = 1,$$

$$\text{If } n_0^{AA} \geq \bar{n}^A, \quad Pr\left(\lim_{t\to\infty} F_1(n_t^{AA}, \theta_{t+1}) = N_A\right) = 1.$$

Next, consider $\underline{n}^A < n_0^{AA} < \bar{n}^A$. For every i playing a, b is BR while, for every i' playing b, a is BR. There are no absorbing states between these states. If only agents playing a are given the revision opportunity, they all choose b, and if enough of them are given the revision opportunity, $n_1^{AA} < \underline{n}^A$. The opposite happens if only players playing b are given the revision opportunity.

Inter-group interactions.

We now pass to the analysis of n_t^I. We define 4 important values for n^{AB} and n^{BA}:
$T_A = min\left\{n^{BA} | n^{BA} > \frac{\pi_A N_B}{\Pi_A + \pi_A}\right\}$, $T_B = min\left\{n^{AB} | n^{AB} > \frac{\Pi_B N_A}{\Pi_B + \pi_B}\right\}$,
$D_A = max\left\{n^{BA} | n^{BA} < \frac{\pi_A N_B}{\Pi_A + \pi_A}\right\}$, and $D_B = max\left\{n^{AB} | n^{AB} < \frac{\Pi_B N_A}{\Pi_B + \pi_B}\right\}$.
Given these values we also define two sets of states, Ω_I^b and Ω_I^a:
$\Omega_I^a = \left\{n^I | n^{BA} \geq T_A \text{ and } n^{AB} \geq T_B\right\}$ and $\Omega_I^b = \left\{n^I | n^{BA} \leq D_A \text{ and } n^{AB} \leq D_B\right\}$.
With similar computation as for n_t^{AA}, we can say that $(0,0)$ and (N_A, N_B) are two fixed points for n_t^I. Are they absorbing states?
Consider the choice of a player $i \in A$ against player $j \in B$ and vice-versa. There can be four possible combinations of states. States in which a is BR for every agent, states in which b is BR for every agent. States, in which $\forall i \in A$, a is the best reply, and b is the best reply $\forall j \in B$, and states for which the opposite is true. Let us call the third situation as Ω_I^{ab} and the fourth as Ω_I^{ba}.
Firstly, we prove that Ω_I^a and Ω_I^b are the regions where a and b are BR for every agent. Secondly, we prove that there is no other absorbing state in Ω_I^a than (N_A, N_B), and no other absorbing state in Ω_I^b than $(0,0)$.
Assume that player $i \in A$ is given the revision opportunity at period t. From Equation (1) to (4), a is the BR against type B if $n_t^{BA} > \frac{\pi_A N_B}{\Pi_A + \pi_A}$. Since T_A is defined as the minimum value s.t. the latter holds, $\forall n_t^{BA} \geq T_A$, $\forall i \in A$, a is BR with B types. Now, assume that $j \in B$ is given the revision opportunity, a is the BR with type A if $n_t^{AB} > \frac{\Pi_B N_A}{\Pi_B + \pi_B}$. Since T_B is defined as the minimum values s.t. this relation is true, $\forall n_t^{AB} \geq T_B$, a is the best reply $\forall j \in B$. Therefore, if $n_0^I \in \Omega_I^a$, $n_s^I \in \Omega_I^a, \forall s \geq 0$. Similarly, if $n_0^I \in \Omega_I^b$, $n_s^I \in \Omega_I^b, \forall s \geq 0$.
Consider being in a generic state $(T_B + d, T_A + d') \in \Omega_I^a$ at time t, with $d \in [0, N_A - T_B)$ and $d' \in [0, N_B - T_A)$. In such a state, there is always a probability p that a player not playing a is given the revision opportunity.

Therefore, if $n_t^I \in \Omega_I^a \setminus (N_A, N_B)$, $Pr\left(F_{2,3}\left(n_t^I, \theta_{t+1}\right) \geq n_t^I\right) > p$.[7] Similar to what we proved before,

$$\text{if } n_t^I \in \Omega_I^a \setminus (N_A, N_B), \quad Pr\left(\lim_{t\to\infty} F_{2,3}(n_t^I, \theta_{t+1}) = n_t^I\right) = 0,$$

$$\text{if } n_t^I \in \Omega_I^b \setminus (0,0), \quad Pr\left(\lim_{t\to\infty} F_{2,3}(n_t^I, \theta_{t+1}) = n_t^I\right) = 0.$$

Consequently,

$$\text{If } n_0^I \in \Omega_I^a \quad Pr\left(\lim_{t\to\infty} F_{2,3}(n_t^I, \theta_{t+1}) = (N_A, N_B)\right) = 1,$$

$$\text{if } n_0^I \in \Omega_I^b, \quad Pr\left(\lim_{t\to\infty} F_{2,3}(n_t^I, \theta_{t+1}) = (0,0)\right) = 1.$$

We now consider Ω_I^{ab} and Ω_I^{ba}. Take an $n_0^I \in \Omega_I^{ab}$: at each step, there is a positive probability that only agents of type A are given the revision opportunity, since for them a is the best reply, in the next period, there will be more or equal agents in A playing a. Hence, if enough players of A that are currently playing b are given the revision opportunity, $n_1^I \in \Omega_I^a$. By the same reasoning, there is also a positive probability that only agents from B are given the revision opportunity, hence, that $n_1^I \in \Omega_I^b$. The same can be said for every state in Ω_I^{ba}. Hence, starting from every state in $\Omega_I^{ab} \bigcup \Omega_I^{ba}$, there is always a positive probability to end up in Ω_I^a or Ω_I^b.

□

Lemma 4. *Under complete information,*
$Pr\left(\lim_{t\to\infty} n_t^I = (N_A, N_B)\right) = 1 - Pr\left(\lim_{t\to\infty} n_t^I = (0,0)\right)$.
$Pr\left(\lim_{t\to\infty} n_t^{AA} = N_A\right) = 1 - Pr\left(\lim_{t\to\infty} n_t^{AA} = 0\right)$.
$Pr\left(\lim_{t\to\infty} n_t^{BB} = N_B\right) = 1 - Pr\left(\lim_{t\to\infty} n_t^{BB} = 0\right)$.

Proof:

We prove the result for n_t^I, and the argument stands for the two other dynamics thanks to symmetry in the payoff matrix. Firstly, note that whenever the process starts in $\Omega_I^a \cup \Omega_I^b$, the lemma is always true thanks to the proof of Lemma 2. We need to show that this is the case, also when the process starts inside $\Omega_I^{ab} \bigcup \Omega_I^{ba}$. We prove the result for Ω_I^{ab} using the same logic, and the result stands for Ω_I^{ba} for symmetry of payoff matrix.
Take $n_0^I \in \Omega_I^{ab}$, define as p_a the probability of extracting m agents from A that are currently playing b, and that would change action a if given the revision opportunity. Define as p_b the probability of picking m agents from B currently choosing a that would change action to b if given the revision opportunity. The probability $1-p_a-p_b$ defines all the other possibilities.
Let us take k steps forward in time:

$$Pr\left(n_k^I \in \Omega_I^a\right) \geq (p_a)^k$$

[7] Meaning that $n_t^{'I} > n_t^{''I}$ if either $n_t^{'AB} > n_t^{''AB}$ and $n_t^{'BA} = n_t^{''BA}$ or $n_t^{'BA} > n_t^{''BA}$ and $n_t^{'AB} = n_t^{''AB}$ or both $n_t^{'BA} > n_t^{''BA}$ and $n_t^{'AB} > n_t^{''AB}$.

$$Pr\left(n_k^I \in \Omega_I^b\right) \geq (p_b)^k$$

$$Pr\left(n_k^I \in \Omega_I^{ab} \bigcup \Omega_I^{ba}\right) \leq (1 - p_a - p_b)^k.$$

Consider period $k + d$:

$$Pr\left(n_{k+d}^I \in \Omega_I^a\right) \geq (p_a)^k$$

$$Pr\left(n_{k+d}^I \in \Omega_I^b\right) \geq (p_b)^k$$

$$Pr\left(n_{k+d}^I \in \Omega_I^{ab} \bigcup \Omega_I^{ba}\right) \leq (1 - p_a - p_b)^{k+d}.$$

Clearly, the probability of being in $\Omega_I^a(\Omega_I^b)$ is now greater or equal than $(p_a)^k((p_b)^k)$: we know that once in $\Omega_I^a(\Omega_I^b)$ the system stays there. The probability of being in $\Omega_I^{ab} \bigcup \Omega_I^{ba}$ consequently, is lower than $(1 - p_a - p_b)^{k+d}$.
Taking the limit for d that goes to infinity

$$\lim_{d\to\infty} \left(Pr\left(n_{k+d}^I \in \Omega_I^{ab} \bigcup \Omega_I^{ba}\right)\right) = 0.$$

This means that if we start in a state in Ω_I^{ab} there is no way of ending up in $\Omega_I^{ab} \bigcup \Omega_I^{ba}$ in the long-run; hence, the system ends up either in Ω_I^a or in Ω_I^b, but given this, we know that it ends up either in $(0, 0)$ or in (N_A, N_B).

□

Corollary 3. *Under complete information,*
$Pr\left(\lim_{t\to\infty} n_t^I = (N_A, N_B)\right) = 1$ *IFF* $n_0^I \in \Omega_I^a$.
$Pr\left(\lim_{t\to\infty} n_t^I = (0, 0)\right) = 1$ *IFF* $n_0^I \in \Omega_I^b$.
$Pr\left(\lim_{t\to\infty} n_t^{AA} = N_A\right) = 1$ *IFF* $n_0^{AA} \in \left[\bar{n}^A, N_A\right]$, *and*
$Pr\left(\lim_{t\to\infty} n_t^{AA} = 0\right) = 1$ *IFF* $n_0^{AA} \in \left[0, \underline{n}^A\right]$.
$Pr\left(\lim_{t\to\infty} n_t^{BB} = N_B\right) = 1$ *IFF* $n_0^{BB} \in \left[\bar{n}^B, N_B\right]$, *and*
$Pr\left(\lim_{t\to\infty} n_t^{BB} = 0\right) = 1$ *IFF* $n_0^{BB} \in \left[0, \underline{n}^B\right]$.

This result is a consequence of the previous lemmas, and therefore, the proof is omitted. Since the only two absorbing states in the dynamics of n_t^I are $(0, 0)$ and (N_A, N_B), they are the only two candidates to be stochastically stable states. From now on we call $(0, 0)$ as I_n^b and (N_A, N_B) as I_n^a. We define as 0_A the state where all agents of type A play b with type A and 0_B the state where all agents of type B play b with B type.

Formally, adding the set of players that choose the behavior by mistake, we obtain a new dynamical system: $\omega_{t+1} = F(\omega_t, \theta_{t+1}, \psi_{t+1})$. Where ψ_{t+1} is the set of players that do not choose their best reply behavior.
Let us call E_A and E_B the two values for which agents in A and in B are indifferent in playing a or b; $E_A = \left\lceil \frac{N_B \pi_A}{\Pi_A + \pi_A} \right\rceil$ and $E_B = \left\lceil \frac{N_A \Pi_B}{\Pi_B + \pi_B} \right\rceil$. From now on we often use values of N large enough to compare the arguments inside ceiling functions.

Lemma 5. *Under free acquisition of information, for N large enough, $R(I_n^b) = CR(I_n^a) = E_A$ for all values of payoffs and sizes of groups, while*

$$R(I_n^a) = CR(I_n^b) = \begin{cases} N_A - E_B & \text{if } \frac{\pi_B}{\Pi_A} < \frac{N_B}{N_A} \\ N_B - E_A & \text{if } \frac{\pi_B}{\Pi_A} > \frac{N_B}{N_A} \end{cases}$$

Proof:

Firstly we know from Ellison 2000 that if there are just two absorbing states, the Radius of one is the Coradius of the other and vice-versa. Hence, $R(I_n^b) = CR(I_n^a)$, and $R(I_n^a) = CR(I_n^b)$. Moreover, from the proof of Lemma 2, we know that $D(I_n^a) = \Omega_I^a$ and $D(I_n^b) = \Omega_I^b$.
We prove that the minimum error path to exit the basin of attraction of I_n^b is the one that reaches $(E_B, 0)$ or $(0, E_A)$, and that the one to exit the basin of attraction of I_n^a is the one that reaches either (E_B, N_B) or (N_A, E_A). To prove this statement for I_n^b, firstly, note that once inside Ω_I^b every step which involves a passage to a state with more people playing a requires an error. Secondly, note that in a state that is out of Ω_I^b at least one of the two types is indifferent in playing b or a. In other words, in a state where either $n^{AB} = E_B$ or $n^{BA} = E_A$ or both. Hence, the minimum resistance path to exit from I_n^b is the one either to $(E_B, 0)$ or to $(0, E_A)$. It is straightforward to show that all the other paths have greater resistance than the two above. Since we use uniform mistakes, every mutation counts the same value, and without loss of generality, we can count each of them as 1. Since every resistance counts as 1, then $R(I_n^b) = \min\{E_B; E_A\} = E_A$. Similarly, $R(I_n^a) = \min\{N_A - E_B; N_B - E_A\}$, and

$$N_A - E_B < N_B - E_A \iff \frac{\pi_B}{\Pi_A} < \frac{N_B}{N_A}.$$

□

Lemma 6. *Under free acquisition of information, for N large enough, $R(0_A) = \left\lceil \frac{N_A\pi_A - \pi_A}{\Pi_A + \pi_A} \right\rceil$, $R(N_A) = \left\lceil \frac{N_A\Pi_A + \Pi_A}{\Pi_A + \pi_A} \right\rceil$, $R(0_B) = \left\lceil \frac{N_B\Pi_B + \Pi_B}{\Pi_B + \pi_B} \right\rceil$ and $R(N_B) = \left\lceil \frac{N_B\pi_B - \pi_B}{\Pi_B + \pi_B} \right\rceil$.*

Proof:

The proof is straight forward, indeed, the minimum path in terms of error required to reach one absorbing state starting from the other one is the cost of exit from the basin of attraction of the first. As a matter of fact, let us consider $R(0_A)$, we know from the proof of Lemma 2 that we are out of the basin of attraction of 0_A when we reach the state $\underline{n}^A$. Hence, $R(0_A) = \left\lceil \frac{N_A\pi_A - \pi_A}{\Pi_A + \pi_A} \right\rceil$. The same applies to the other states.

□

Proof of Theorem 1:

We divide the proof for the three dynamics described so far: for what concerns n_t^{AA}, N_A is uniquely stochastically stable and for what concerns n_t^{BB}, 0_B is uniquely stochastically stable, this proof follows directly from Lemma 6, and therefore is omitted. Let us pass to n_t^I. We know from Lemma 5 that $R(I_n^b) = E_A$ and that the value of $R(I_n^a)$ depends on payoffs and group size. Let us firstly consider the case when $\frac{\pi_B}{\Pi_A} < \frac{N_B}{N_A}$ and $R(I_n^a) = N_A - E_B$. It is sufficient that $E_A > N_A - E_B$ for I_n^b to be uniquely stochastically stable. Indeed, if this happens, $R(I_n^b) > CR(I_n^b)$. This is the case IFF

$$\frac{\pi_A N_B}{\Pi_A + \pi_A} > \frac{\pi_B N_A}{\Pi_B + \pi_B} \iff \frac{\pi_B}{\pi_A} < \frac{N_B}{N_A}. \tag{9}$$

To complete the proof, we show that whenever $\frac{\pi_B}{\pi_A} > \frac{N_B}{N_A}$, then I_n^a is the uniquely stochastically stable state. Firstly, note that $\frac{\pi_B}{\Pi_A} < \frac{\pi_B}{\pi_A}$, hence, for $\frac{\pi_B}{\pi_A} > \frac{N_B}{N_A} > \frac{\pi_B}{\Pi_A}$, $R(I_n^a) = N_A - E_B$ and $E_A = R(I_n^b)$. However, Equation (9) is reversed, so, I_n^a is uniquely stochastically stable. For $\frac{\pi_B}{\pi_A} > \frac{\pi_B}{\Pi_A} > \frac{N_B}{N_A}$, $R(I_n^a) = N_B - E_A$ and still $R(I_n^b) = E_A$. In this case, I_n^a is the uniquely stochastically stable if $E_A < N_B - E_A$, hence, IFF

$$\frac{\pi_A N_B}{\Pi_A + \pi_A} < \frac{\Pi_A N_B}{\Pi_A + \pi_A}.$$

This happens for every value of the payoffs (given that $\Pi_A > \pi_A$) and of the group size. We conclude that whenever $\frac{\pi_B}{\pi_A} < \frac{N_B}{N_A}$, PS_b is uniquely stochastically stable and when $\frac{\pi_B}{\pi_A} > \frac{N_B}{N_A}$, PS_a is uniquely stochastically stable.

□

A.2 Proofs of Section 4

We define formally the dynamical system as $\omega_{t+1} = F_c(\omega_t, \theta_{t+1}, \psi_{t+1}^c)$. Where $\psi_{t+1}^c = \{\psi_{t+1}^\varepsilon, \psi_{t+1}^\eta\}$ is the set of players that make a mistake at time t. ψ_{t+1}^ε is the set of players that make a mistake in the coordination choice, and ψ_{t+1}^η the set of players that make a mistake in the information choice.
For convenience, we call behavior 1 the optimal behavior when a player decides to acquire the information: $1 = \max(ab, ba, aa, bb)$.
We will use in some proofs the concept of Modified Coradius from Ellison 2000. We write here the formal definition. Suppose $\bar{\omega}$ is an absorbing state and $(\omega_1, \omega_2, \ldots \omega_T)$ is a path from state ω' to $\bar{\omega}$. Let $L_1, L_2, \ldots, L_r = \bar{\omega}$ be the sequence of limit sets through which the path passes consecutively. The modified resistance is the original resistance minus the Radius of the intermediate limit sets through which the path passes,

$$r^*(\omega_1, \omega_2, \ldots \omega_T) = r(\omega_1, \omega_2, \ldots \omega_T) - \sum_{i=2}^{r-1} R(L_i).$$

Define

$$r^*(\omega', \bar{\omega}) = \min_{(\omega_1, \omega_2, \ldots \omega_T) \in \Upsilon(\omega', \bar{\omega})} r^*(\omega_1, \omega_2, \ldots \omega_T),$$

the Modified Coradius is defined as follows

$$CR^*(\bar{\omega}) = \max_{\omega' \neq \bar{\omega}} r^*(\omega', \bar{\omega}).$$

Note that $CR^*(\bar{\omega}) \leq CR(\bar{\omega})$. Thanks to theorem 2 in Ellison 2000, we know that when $R(\bar{\omega}) > CR^*(\bar{\omega})$, $\bar{\omega}$ is uniquely stochastically stable.

Proof of Lemma 3:

We first show that the nine states are effectively strict equilibria, that there is no other possible equilibrium, and that a state is absorbing if and only if it is a strict equilibrium.

Monomorphic States.

It is easy to show that (N_A, N_A, N_B, N_B) and $(0,0,0,0)$ are two strict equilibria. We take the first case, and the argument stands also for the second, thanks to the symmetry of the payoff matrix. Consider player $i \in K$ who is given the revision opportunity at time t:

$$U^i(a, z_{-i}(t)) = \frac{N_K + N_{K'} - 1}{N-1}\pi_a^K = \pi_a^K,$$

$$U^i(b, z_{-i}(t)) = \frac{N - N_K - N_{K'}}{N-1}\pi_b^K = 0,$$

$$U^i(1, z_{-i}(t)) = \frac{N_K + N_{K'} - 1}{N-1}\pi_a^K - c = \pi_a^K - c.$$

(N_A, N_A, N_B, N_B) is a strict equilibrium since $\pi_a^K > 0$ and $c > 0$.

Polymorphic States.

Firstly let us consider the case of $(N_A, 0, 0, N_B)$. Since in this case, every player is playing ab, the state is a strict equilibrium IFF $\max z_i^o = ab$, $\forall i \in N$. If player $i \in K$ is given the revision opportunity at time t:

$$U^i(a, z_{-i}(t)) = \frac{N_K - 1}{N-1}\pi_a^K,$$

$$U^i(b, z_{-i}(t)) = \frac{N_{K'}}{N-1}\pi_b^K,$$

$$U^i(1, z_{-i}(t)) = \frac{N_K - 1}{N-1}\pi_a^K + \frac{N_{K'}}{N-1}\pi_b^K - c.$$

For type A players, $U^i(a, z_{-i}(t)) > U^i(b, z_{-i}(t))$ since $\frac{N_B}{N-1}\pi_A < \frac{N_A-1}{N-1}\Pi_A$. For type B players, $U^i(b, z_{-i}(t)) > U^i(a, z_{-i}(t))$ as $\frac{N_B-1}{N-1}\pi_B < \frac{N_A}{N-1}\Pi_B$. $U^i(1, z_{-i}(t))$ is the highest of the three $\forall i \in N$ IFF $c < \min\left\{\frac{N_B}{N-1}\pi_A, \frac{N_B-1}{N-1}\pi_B\right\}$.
Consider the case of $(0, N_A, N_B, 0)$, since every agent is playing ba, it must be that $\max z_i^o = ba$. $i \in K$ faces the following payoffs

$$U^i(a, z_{-i}(t)) = \frac{N_K'}{N-1}\pi_a^K,$$

$$U^i(b, z_{-i}(t)) = \frac{N_K - 1}{N-1}\pi_b^K,$$

$$U^i(1, z_{-i}(t)) = \frac{N_K - 1}{N-1}\pi_b^K + \frac{N_{K'}}{N-1}\pi_a^K - c.$$

Note that $U^i(a, z_{-i}(t)) > U^i(b, z_{-i}(t))$ IFF $\frac{\pi_b^K}{\pi_a^K} < \frac{N'_K}{N_K - 1}$, and therefore ba is the best reply behavior in this case if $c < \frac{N_K-1}{N-1}\pi_b^K$. When the opposite happens and so, $\frac{\pi_b^K}{\pi_a^K} > \frac{N'_K}{N_K-1}$, ba is the best reply behavior if $c < \frac{N'_K}{N-1}\pi_a^K$. These conditions take the form of the ones in Table 4.

Consider the remaining 4 PS, they are characterised by the following fact $BR(n^{KK}) = BR(n^{K'K})$ but $BR(n^{K'K'}) \neq BR(n^{KK'})$. In this case it must be that $\tau_i = 0$ is optimal for $i \in K$ while $\tau_j = 1$ is optimal for $j \in K'$. Thanks to the symmetry in payoff matrices we can say that the argument to prove the results for these 4 states is similar to the one for $(N_A, 0, 0, N_B)$ and $(0, N_A, N_B, 0)$. All the conditions are listed in Table 4.

Type Monomorphic State.

$(N_A, N_A, 0, 0)$ is a strict equilibrium if a is the BR $\forall i \in A$ and b, $\forall j \in B$. Consider a player $i \in A$, who is given the revision opportunity at time t:

$$U^i(a, z_{-i}(t)) = \frac{N_A - 1}{N-1}\Pi_A,$$

$$U^i(b, z_{-i}(t)) = \frac{N_B}{N-1}\pi_A,$$

$$U^i(1, z_{-i}(t)) = \frac{N_A - 1}{N-1}\Pi_A + \frac{N_B}{N-1}\pi_A - c.$$

Given that $U^i(a, z_{-i}(t)) > U^i(b, z_{-i}(t))$, a is the best reply behavior IFF $c > \frac{N_B}{N-1}\pi_A$. Consider player $j \in B$:

$$U^j(a, z_{-i}(t)) = \frac{N_A}{N-1}\pi_B,$$

$$U^j(b, z_{-i}(t)) = \frac{N_B - 1}{N-1}\Pi_B,$$

$$U^j(1, z_{-i}(t)) = \frac{N_A}{N-1}\pi_B + \frac{N_B - 1}{N-1}\Pi_B - c.$$

In this case when $\frac{\pi_B}{\Pi_B} > \frac{N_B-1}{N_A}$, b is never best reply and a is best reply hence, the state can not be a strict equilibrium. When $\frac{\pi_B}{\Pi_B} < \frac{N_B-1}{N_A}$, $U^j(b, z_{-i}(t)) > U^j(a, z_{-i}(t))$, and $U^j(b, z_{-i}(t)) > U^j(1, z_{-i}(t))$ IFF $c > \frac{N_A}{N-1}\pi_B$.

No other state is a strict equilibrium.

For what concerns states where not all players of a type are playing the same action with the same type, this is easy to prove. Indeed, by definition, in these states, either not all players are playing their best reply action, or players are indifferent between two or more behaviors. In the first case, the state is not a strict equilibrium by definition; in the second case, there is no strictness of the equilibrium since there is not one best reply, but more behaviors can be best reply simultaneously. Hence, such states can not be strict equilibria. We are left with the 7 states where every player of one type is doing the same thing against the same type. Such states are: $(0, 0, N_B, N_B)$, $(0, N_A, 0, N_B)$, $(N_A, 0, N_B, 0)$, $(0, 0, N_B, 0)$, $(N_A, N_A, 0, N_B)$, $(0, N_A, 0, 0)$, and $(N_A, 0, N_B, N_B)$. It is easy to prove that these states enter in the category of states where not every player is playing her/his best reply. Therefore, they can not be strict equilibria.

Strict equilibria are always absorbing states.

We first prove the sufficient and necessary conditions to be a fixed point, and second that every fixed point is an absorbing state. To prove the sufficient part we rely on the definition of strict equilibrium. In a strict equilibrium, every player is playing her/his BR, and no one has the incentive to deviate. Whoever is given the revision opportunity does not change her/his behavior. Therefore, $F_c(\omega_t, \theta_{t+1}) = \omega_t$. To prove the necessary condition think about being in a state that is not a strict equilibrium; in this case, by definition, we know that not all the players are playing their BR. Among them, there are states in which there are no indifferent players, in this case, with positive probability one or more agents who are not playing their BR are given the revision opportunity and they change action, therefore, $F_c(\omega_t, \theta_{t+1}) \neq \omega_t$ for some realization of θ_{t+1}. In states where some players are indifferent between two or more behaviors, thanks to the tie rule, there is always a positive probability that the indifferent agent changes her/his action since s/he is randomizing her/his choice. Moreover, there is also a positive probability to select an agent indifferent between two or more behaviors. In this case, s/he changes the one that is currently playing with a positive probability too. Knowing this, we are sure that no state outside strict equilibria can be a fixed point. In our case, a fixed point is also an absorbing state by definition. Indeed, every fixed point absorbs at least one state: the one where all players except one are playing the same behavior. In this case, if that player is given the revision opportunity s/he changes for sure her/his behavior into the one played by every agent.

□

Here we prove the results of the stochastic stability analysis of Section 4.

Proof of Theorem 2:

In this case $R(MS_a) = CR(MS_b)$ and $R(MS_b) = R(MS_a)$. Therefore, we just need to calculate the two Radius.

Radius of each state.

Let us consider $R(MS_a)$. Since the basin of attraction of MS_a is a region where a is the best reply behavior for both types, many players should make a mistake such that b becomes BR for one of the two types. For b to be BR for B players, it must be that $n^{AB} + n^{BB} \leq \frac{N\Pi_B - \Pi_B}{\Pi_B + \pi_B}$. This state can be reached with ε mutations, at cost $\frac{N\pi_B + \Pi_B}{\Pi_B + \pi_B}$. In a state where $n^{AA} + n^{BA} \leq \frac{N\pi_A - \pi_A}{\Pi_A + \pi_A}$, b is BR for A, this path happens

at cost $\frac{N\Pi_A+\pi_A}{\Pi_A+\pi_A}$. In principle $\frac{N\Pi_A+\pi_A}{\Pi_A+\pi_A} > \frac{N\pi_B+\Pi_B}{\Pi_B+\pi_B}$, hence, $R(MS_a)$ should be $\frac{N\pi_B+\Pi_B}{\Pi_B+\pi_B}$. However, it may not be sufficient to reach such a state.
Consider to reach a state s.t. $n^{AB} + n^{BB} = \frac{N\Pi_B-\Pi_B}{\Pi_B+\pi_B}$, since $\frac{N\Pi_B-\Pi_B}{\Pi_B+\pi_B} > \frac{N\pi_A-\pi_A}{\Pi_A+\pi_A}$, it must be that a is still the best reply $\forall i \in A$, and therefore there is a path of zero resistance to MS_a. Nevertheless, once in that state, it can happen that only B players are given the revision opportunity, and that they all choose behavior b. This creates a path of zero resistance to a state $(\bar{n}^{AA}, \bar{n}^{AB}, 0, 0)$. Once in this state, if $\bar{n}^{AA} < \frac{N\pi_A-\pi_A}{\Pi_A+\pi_A}$, the state is in the basin of attraction of MS_b. This happens only if $\frac{N\pi_A-\pi_A}{\Pi_A+\pi_A}+N_B = \frac{N\Pi_B-\Pi_B}{\Pi_B+\pi_B}$. More generally, considering $k \geq 0$, this happens if $\frac{N\pi_A-\pi_A}{\Pi_A+\pi_A} + N_B = \frac{N\Pi_B-\Pi_B}{\Pi_B+\pi_B} - k$. Fixing payoffs and groups size, $k = \frac{N\Pi_B-\Pi_B}{\Pi_B+\pi_B} - \frac{N\pi_A-\pi_A}{\Pi_A+\pi_A} - N_B$, hence, the cost of this path would be

$$\frac{N\pi_B+\Pi_B}{\Pi_B+\pi_B} + \frac{N\Pi_B-\Pi_B}{\Pi_B+\pi_B} - \frac{N\pi_A-\pi_A}{\Pi_A+\pi_A} - N_B = N_A - \frac{N\pi_A-\pi_A}{\Pi_A+\pi_A}.$$

With a similar reasoning $R(MS_b) = \frac{N\pi_A+\Pi_A}{\Pi_A+\pi_A}$.
We prove that all the other paths with η errors are costlier than ones with ε. We know that a is the BR for every state inside the basin of attraction of MS_a, nobody in the basin of attraction of MS_a optimally buys the information, and every player once bought the information (by mistake) plays behavior aa. Every path with an η error also involves an ε error, and hence, is double that of the one described above.

Conditions for stochastically stable states.

MS_a is stochastically stable IFF $N_A - \frac{N\pi_A-\pi_A}{\Pi_A+\pi_A} > \frac{N\pi_A+\Pi_A}{\Pi_A+\pi_A}$, this is verified when $N_A > \frac{2N\pi_A+\Pi_A-\pi_A}{\Pi_A+\pi_A}$. Therefore, we conclude that MS_a is stochastically stable in the above scenario, while if the opposite happens, MS_b is stochastically stable.

□

Proof of Theorem 3

We first calculate Radius, Coradius, and Modified Coradius for the three states we are interested in, and then we compare them to draw inference about stochastic stability.

Radius of each state.

The Radius of MS_a is the minimum number of errors that makes b BR for B players. This number is $\frac{N\pi_B+\Pi_B}{\Pi_B+\pi_B}$. The alternative is to make b BR for A: hence, a path to state $(0, 0, N_B, N_B)$, and then to $(0, 0, 0, 0)$. The number of ε errors for this path is $\frac{N\Pi_A+\pi_A}{\Pi_A+\pi_A}$. Therefore, $R(MS_a) = \frac{N\pi_B+\Pi_B}{\Pi_B+\pi_B}$. With a similar reasoning we can conclude that $R(MS_b) = \frac{N\pi_A+\Pi_A}{\Pi_A+\pi_A}$.
Consider TS: the minimum error path to exit from its basin of attraction reaches either MS_a or MS_b, depending on payoffs. In other words, the minimum number of errors to exit from $D(TS)$ is the one that makes either a or b as BR. Consider the path from TS to MS_a: in this case, some errors are needed to make a BR for B. The state in which a is BR for B depends on payoffs and group size. In a state (N_A, N_A, k', k'), a is BR for every player in B if $(N_A + k' - 1)\pi_B > (N - N_A - k')\Pi_B$. This inequality is obtained declining Equation (5) to (8), comparing B playing a/ab or b/ba. Fixing payoffs, we can calculate the exact value of k' that is $\frac{N_B\Pi_B-N_A\pi_B+\pi_B}{\Pi_B+\pi_B}$, this would be

the cost of the minimum error transition from TS to MS_a. With a similar argument, the cost of the minimum error transition from TS to MS_b is $\frac{N_A\Pi_A - N_B\pi_A + \pi_A}{\Pi_A+\pi_A}$.
There are no paths involving η errors that are lower than the two proposed above. The intuition is the following. Consider a situation in which m players of A are given the revision opportunity at one step and they all choose to buy the information. In this case, they all optimally choose behavior ab. This means that at the cost of n there is a path to a state in which $N_A - m$ players are playing b against B, in this state b is still the BR for B type, while a is still the BR for A. Hence, from that state, there is a path of zero resistance to TS. The same happens when B players choose by mistake to buy the information. Therefore, $R(TS) = \min\left\{\frac{N_B\Pi_B - N_A\pi_B + \pi_B}{\Pi_B+\pi_B}, \frac{N_A\Pi_A - N_B\pi_A + \pi_A}{\Pi_A+\pi_A}\right\}$.

Coradius of each state

We start from TS: in this case, we have to consider the two minimum paths to reach it from MS_a and MS_b. Therefore, $\frac{N\pi_A+\Pi_A}{\Pi_A+\pi_A}$ and $\frac{N\pi_B+\Pi_B}{\Pi_B+\pi_B}$. Firstly, the argument to prove that these two are the minimum error paths to reach TS from MS_a and MS_b are given by the previous part of the proof. Secondly, we have to prove that this path is the maximum among the minimum paths starting from any other state and ending in TS. There are three regions from which we can start and end-up in TS: the basin of attraction of MS_b, the one of MS_a, and all the other states which are not in the basins of attraction of the three states considered. We can say that from this region, there is always a positive probability to end up in MS_a, MS_b, or TS. Hence, we can consider as 0 the cost to reach TS from this region. The other two regions are the one considered above, and since we are taking the maximum path to reach TS from any other state we have to take the sum of this two. Hence, $CR(TS) = \frac{N\pi_A+\Pi_A}{\Pi_A+\pi_A} + \frac{N\pi_B+\Pi_B}{\Pi_B+\pi_B}$.
Let us think about MS. Similarly to the two previous proofs we can focus only on ε paths. Note that in this case, TS is always placed between the two MS. Let us start from MS_b: in this case we can consider 3 different path starting from any state and arriving to MS_b. The first one starts in TS, the second starts in every state outside the basin of attractions of the three absorbing states, and the last starts in MS_a. In the second case there is at least one transition of zero resistance to MS_b. Next, assume to start in TS: the minimum number of errors to reach MS_b from TS is the one that makes b BR for A players. Therefore, $\frac{N_A\Pi_A - N_B\pi_A + \pi_A}{\Pi_A+\pi_A}$.
Now, we need to consider the case of starting in MS_a. Firstly, consider the minimum number of errors to make b BR for A players. This number is $\frac{N\Pi_A+\pi_A}{\Pi_A+\pi_A}$. Secondly, consider the minimum number of errors to make b BR for B players, and then once reached TS the minimum that makes b BR for A players.

$$\min r(MS_a, MS_b) = \min\left\{\frac{N\Pi_A + \pi_A}{\Pi_A + \pi_A}, \frac{N\pi_B + \Pi_B}{\Pi_B + \pi_B} + \frac{N_A\Pi_A - N_B\pi_A + \pi_A}{\Pi_A + \pi_A}\right\}.$$

Since the two numbers in the expression are all greater than $\frac{N_A\Pi_A - N_B\pi_A + \pi_A}{\Pi_A+\pi_A}$ we can say that $CR(MS_b) = \min r(MS_a, MS_b)$.
Reaching a state where b is BR for type A from TS is for sure less costly than reaching it from MS_a, since in TS there are more people playing b. Therefore, $\frac{N\Pi_A+\pi_A}{\Pi_A+\pi_A} \geq \frac{N\pi_B+\Pi_B}{\Pi_B+\pi_B} + \frac{N_A\Pi_A - N_B\pi_A + \pi_A}{\Pi_A+\pi_A}$, hence, $CR(MS_b) = \frac{N\pi_B+\Pi_B}{\Pi_B+\pi_B} + \frac{N_A\Pi_A - N_B\pi_A + \pi_A}{\Pi_A+\pi_A}$.
With a similar reasoning, $CR(MS_a) = \frac{N\pi_A+\Pi_A}{\Pi_A+\pi_A} + \frac{N_B\Pi_B - N_A\pi_B + \pi_B}{\Pi_B+\pi_B}$.

Modified Coradius of each state.

Firstly, note that $CR(TS) = CR^*(TS)$, since between MS and TS there are no intermediate states. Formally,

$$CR^*(TS) = \min r^*(MS_a, \dots, TS) + \min r^*(MS_b, \dots, TS) = \frac{N\pi_A + \Pi_A}{\Pi_A + \pi_A} + \frac{N\pi_B + \Pi_B}{\Pi_B + \pi_B}.$$

The maximum path of minimum resistance from each MS to the other MS passes through TS. Hence, for each MS, we need to subtract from the Coradius the cost of passing from TS to the other MS. Let us consider $CR^*(MS_a)$, we need to subtract to the Coradius the cost of passing from TS to MS_b: this follows from the definition of Modified Coradius. Hence,

$$CR^*(MS_a) = \frac{N\pi_A + \Pi_A}{\Pi_A + \pi_A} + \frac{N_B\Pi_B - N_A\pi_B + \pi_B}{\Pi_B + \pi_B} - \frac{N_A\Pi_A - N_B\pi_A + \pi_A}{\Pi_A + \pi_A}.$$

Similarly,

$$CR^*(MS_b) = \frac{N\pi_B + \Pi_B}{\Pi_B + \pi_B} + \frac{N_A\Pi_A - N_B\pi_A + \pi_A}{\Pi_A + \pi_A} - \frac{N_B\Pi_B - N_A\pi_B + \pi_B}{\Pi_B + \pi_B}.$$

Note that $CR^*(MS_a) < CR(MS_a)$ and $CR^*(MS_b) < CR(MS_b)$.

Conditions for stochastically stable states.

It is easy to verify that if $R(MS_a) > CR(MS_a)$, both $R(MS_b) < CR(MS_b)$ and $R(TS) < CR(TS)$. Similar for $R(MS_b) > CR(MS_b)$ or $R(TS) > CR(TS)$. When $R(MS_a) \leq CR(MS_a)$, $R(MS_b) \leq CR(MS_b)$, and $R(TS) \leq CR(TS)$, we need to use Modified Coradius. Given that $CR(TS) = CR^*(TS)$ it will never be that $R(TS) > CR^*(TS)$. We can show that when $R(MS_a) > CR^*(MS_a)$, then $R(MS_b) < CR^*(MS_b)$ and vice-versa.
When $R(MS_a) = CR^*(MS_a)$, it is also possible that $R(MS_b) = CR^*(MS_b)$. Thanks to Theorem 3 in Ellison 2000, we know that either both states are stochastically stable, or none of the two is. Note that for the ergodicity of our process the second case is impossible, hence, it must be that when both $R(MS_a) = CR^*(MS_a)$ and $R(MS_b) = CR^*(MS_b)$, both $\mu^\star(MS_b) > 0$ and $\mu^\star(MS_a) > 0$.

□

Proof of Theorem 4:

We split the absorbing states into 2 sets and then apply Theorem 1 by Ellison 2000. Define the following two sets of states: $M_1 = \{PS_a, PS_b\}$ and $M_2 = (PS \setminus M_1) \cup MS$. Similarly, define $M_1' = PS_b$ and $M_2' = MS \cup (PS \setminus M_1')$.

Analysis with M_1 and M_2.

$R(M_1)$ is the minimum number of errors to escape the basins of attraction of both PS_a and PS_b. The dimension of these basins of attraction is determined by the value of c. In a state inside $D(PS_a)$, ba is BR for B, and a is BR for A. Similarly, ab is

optimal for A inside $D(PS_b)$ and b is optimal for B. The minimum error paths that starts in PS_a, and PS_b and exit from their basins of attraction involve ε errors.
We calculate the dimension of these basins of attraction for $0 < c < \min\{\frac{\pi_A}{N-1}, \frac{\pi_B}{N-1}\}$. We start from PS_a and the argument stands for the other states in PS for symmetry of payoffs matrix.
Firstly, we consider the minimum number of errors that makes a BR for B players. Consider the choice of a B player inside a category of states where $n^{BB} \in \left[0, \frac{N_B\Pi_B - \Pi_B}{\Pi_B+\pi_B}\right)$ and $n^{AB} \in \left(\frac{N_A\Pi_B}{\Pi_B+\pi_B}, N_A\right]$. Referring to Equation (5) to (8), the optimal level of c s.t. 1 is the best reply for B players is

$$c < \min\left\{\frac{N_B\Pi_B - n^{BB}(\Pi_B+\pi_B) - \Pi_B}{N-1}, \frac{n^{AB}(\Pi_B+\pi_B) - N_A\Pi_B}{N-1}\right\}.$$

If $0 < c < \min\{\frac{\pi_A}{N-1}, \frac{\pi_B}{N-1}\}$, whenever $n^{BB} \in \left[0, \frac{N_B\Pi_B-\Pi_B}{\Pi_B+\pi_B}\right)$ and $n^{AB} \in \left(\frac{N_A\Pi_B}{\Pi_B+\pi_B}, N_A\right]$, 1 is the BR for B. Therefore, a path towards a state where $n^{BB} \geq \frac{N_B\Pi_B+\pi_B}{\Pi_B+\pi_B}$, is a transition out of the basin of attraction of PS_a. Starting from $n^{BB} = 0$, the cost of this transition is $\frac{N_B\Pi_B+\pi_B}{\Pi_B+\pi_B}$. This cost is determined by ε errors, since once in PS_a it is sufficient that a number of B plays by mistake b. Another possible path is to make ba BR for A. The cost of this transition is $\frac{N_A\Pi_A+\pi_A}{\Pi_A+\pi_A}$. With similar arguments, it is possible to show that the cost of exit from M_1 starting from PS_b is the same. For this reason, $R(M_1) = \min\left\{\frac{N_B\Pi_B+\pi_B}{\Pi_B+\pi_B}, \frac{N_A\Pi_A+\pi_A}{\Pi_A+\pi_A}\right\}$.
We can show that the minimum error path to exit from the basin of attraction of M_2 reaches either PS_a from MS_a, or PS_b from MS_b. Therefore, $R(M_2) = \min\left\{\frac{N_A\pi_A+\Pi_A}{\Pi_A+\pi_A}, \frac{N_B\pi_B+\Pi_B}{\Pi_B+\pi_B}\right\}$. $R(M_1) > R(M_2)$ for every value of payoffs and group size: the stochastically stable state must be in M_1.

Analysis with M_1' and M_2'.

Let us consider the path that goes from M_1' to PS_a. Starting in PS_b, it is sufficient that $\frac{N_B\pi_A}{\Pi_A+\pi_A}$ players from A play a for a transition from PS_b to $D(PS_a)$ to happen. Since $\frac{N_B\pi_A}{\Pi_A+\pi_A} < \min\left\{\frac{N_B\Pi_B+\pi_B}{\Pi_B+\pi_B}, \frac{N_A\Pi_A+\pi_A}{\Pi_A+\pi_A}\right\}$, we can say that $R(M_1') = \frac{N_B\pi_A}{\Pi_A+\pi_A}$. With a similar argument it can be shown that $R(M_2') = \frac{N_A\pi_B}{\Pi_B+\pi_B}$. When $R(M_2') > R(M_1')$, PS_a is uniquely stochastically stable. When $R(M_1') > R(M_2')$, PS_b is uniquely stochastically stable.
$R(M_2') \lesseqgtr R(M_1')$ when $\frac{N_B}{N_A} \lesseqgtr \frac{\pi_B}{\pi_A}$.

□