

# Dark matter on the closed region connected with black hole by wormhole

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In this letter, instead of choosing the Einstein Rosen bridge between two black holes as in ER=EPR, we consider a wormhole between a black hole and a closed edge of the wormhole. We assume that information in a black hole travels through a wormhole, turns to mass (dark matter) in the closed region. This study is in contradiction with the existence of the white hole in our Universe. We replace the notion of the white hole with the massive closed region. We prove the metric of the closed region by the Hopf fibration, this new metric generalizes the  $AdS_5$  metric.

## I. INTRODUCTION

The observations of the rotation of galaxies and gravitational lenses indicate the presence of dark matter (DM) hiding in galaxies, which does not interact with radiation and matter, which can be detected by its gravitational effect. The  $\Lambda$ CDM model designates a cosmological model parametrized by a cosmological constant  $\Lambda$  associated with cold dark matter. However, the  $\Lambda$ CDM model presents several problems, as the cosmological constant, fine-tuning problem and the problem of cosmic coincidence [1]. The deflection angle in gravitational lensing is proportional to the Schwarzschild radius, which will allow us to study massive regions invisible in the Universe, and connect these regions with the black hole by a wormhole. Recently investigators have examined the Hopf fibration to study the wormholes [2]. In  $ER = EPR$  [3, 4] the entangled particles are connected through a wormhole or Einstein–Rosen bridge. The information paradox opposing the laws of quantum mechanics to those of general relativity. Indeed, the general relativity implies that the information could fundamentally disappear in a black hole, following the evaporation of this one. This loss of information implies a non-reversibility (the same state can come from several different states), and a non-unitary evolution of quantum states, in fundamental contradiction with the postulates of quantum mechanics [5]. In 2019, Penington and al. [6] discovered a class of semi-classical space-time geometries that had been overlooked by Hawking and later researchers. Penington et al. calculate entropy using the cue trick and show that for sufficiently old black holes. We must consider solutions in which the aftershocks are connected by wormholes. The inclusion of these wormhole geometries prevents entropy from increasing indefinitely [6, 7].

## II. GRAVITATIONAL LENSING OF CLOSED REGIONS

Light rays emitted from the source  $S$  are deflected by the lens  $L$ , then observed by the observer  $O$ , in the form of two images  $S_1$  and  $S_2$  of the source. If the observed

source is perfectly aligned with the celestial body acting as a gravitational lens concerning the observer, the mirage can take the form of an Einstein ring by the Einstein radius is

$$\theta_E = \sqrt{\frac{4G_N M}{c^2} \frac{D_{LS}}{D_L D_S}} \quad (1)$$

where  $D_L$ ,  $D_S$ ,  $D_{LS}$  are the distance between  $O$  and  $L$ , and the distance between  $O$  and  $S$  and the distance between  $L$  and  $S$ , respectively. The angle of curvature we would like to point out here is that the deflection angle is given as

$$\tilde{\alpha}(\xi) = \frac{4G_N M_\xi}{c^2} \frac{1}{\xi} \quad (2)$$

where  $M_\xi$  is the mass inside a radius  $\xi$ . We take two entangled regions; regions  $A$  represent the black hole surface and an invisible closed region  $B$  of the radius  $\xi$  and a surface  $A_\xi$ , which also supports a constant positive Ricci curvature. Firstly, we associate with each region a quantum state of two particles.

$$|\Psi_A\rangle \sim |+\rangle |-\rangle + |-\rangle |+\rangle \quad (3)$$

$$|\Psi_B\rangle \sim |+\rangle |-\rangle - |-\rangle |+\rangle \quad (4)$$

the maximal entanglement between the states  $|\Psi_A\rangle$  and  $|\Psi_B\rangle$  describes the maximal entanglement between two particles. The first particle is on the black hole surface and the second particle is on the closed region surface. The entanglement between the two particles describes quantum information passes from the black hole surface  $A$  into the closed region  $B$ . General Relativity also has its non-local features. In particular, there are solutions to Einstein's equations in which a pair of arbitrarily distant black holes are connected by a wormhole or Einstein-Rosen bridge (ERB) [3]. We assume that the information from the black hole surface transforms by the wormhole [4], towards a mass which will gather in an extreme region of the wormhole, this extremal part is the closed region, which exists in a compact dimension. We propose that the closed regions exist in 5-dimensions, with 4-dimensions of space-time and a 5th compact dimension. These regions are a sub-manifold of space-time

that is closed on the 5th dimension. We can represent the wormhole by the Einstein Rosen bridge between the black hole and the closed region because according to [4]; the Einstein Rosen bridge is associated with each entangled state  $|\Psi_A\rangle$  and  $|\Psi_B\rangle$ . On the black hole region  $A$ , we define the Bekenstein-Hawking entropy is equal to

$$S_{BH} = \frac{k_B c^3}{4\hbar G_N} A \quad (5)$$

Next, we take  $k_B = c = G_N = \hbar = 1$ . We can represent entropy geometrically by the surface  $A = 4\pi r^2$  of radius  $r$ . In every moment, a small portion of area  $A$  is shifting towards the closed region. The area of the closed region increases at the expense of the black hole area, for this we connect the variation of  $A_\xi$  with the deflection angle, like [8]. At each instant  $t$  we define a black hole surface  $A(t) = A_\xi(t) + \delta A_\xi$ , with  $\delta A_\xi$  is a part of surface which transforms from  $A$  to  $A_\xi \sim 4\pi\xi^2$  at  $t$ . The infinitesimal surface  $\delta A_\xi$  depend on the variation of the mass  $M_\xi$  in the region of the radius  $\xi$ . We take  $\delta A_\xi$  as a cross product of two vector in the closed region with the deflection angle  $\tilde{\alpha}$  ( $\sin \tilde{\alpha} \approx \tilde{\alpha}$ ). To express this variation, we will use the deflection angle

$$\delta A_\xi \equiv \tilde{\alpha} A_\xi = 16\pi M_\xi \xi \quad (6)$$

Eq.(5) can be rewritten as

$$S_{BH} = \pi\xi^2 + 4\pi M_\xi \xi \quad (7)$$

the entropy is viewed as a measure of quantum entanglement of the maximally entangled particles in each region  $A$  and  $B$ . We introduced the condition:  $M_\xi \succeq -\xi/4$ . According to this condition, the mass  $M_\xi$  is always greater than a minimum value. By comparison with the Schwarzschild radius, the condition is equivalent to

$$M_\xi \equiv M - \xi/2 \quad (8)$$

where  $M$  is the black hole mass. We can use Eq.(8) to search by the method of gravitational lensing for region  $(\xi, M_\xi)$  which is connected by the black hole mass. Now let us express  $S_{BH}$  as a function of  $M_\xi$  and  $M$

$$S_{BH} = 4\pi [M^2 - M_\xi^2] \quad (9)$$

the black hole entropy is proportional to the closed region mass. If we assume that the mass of the closed region is zero, then we get an entropy  $S_{BH}$  only for a black hole. The entropy  $S_{BH}$  connects each black hole of the mass  $M$  with a closed region of mass  $M_\xi$ . Hence, there is no loss of the information in a black hole, but there is a transformation of the information during a wormhole to the closed region, the information is registered on the closed region as a mass  $M_\xi$ . We remark in the entropy (9) that when the black hole mass transforms entirely to the closed region, the black hole entropy will be zero. The mass  $M_\xi$  describes dark matter

in the closed region. The mass in closed regions does not interact with the electromagnetic waves; because these regions are closed and compact, nothing can enter or leave these regions, except by the wormholes which are open from the side of the black hole. The place to test for the presence of dark matter in the closed regions is black holes. However, the dark matter does not exist inside black holes, but the black holes are gates that can help us to describe the closed regions.

### III. QUATERNIONIC HOPF FIBRATION OF WORMHOLE

There are many methods to geometrize the set {black hole and closed region}, for example, a non-orientable surface like the Klein bottle, stereographic projection, or the Bloch sphere. Here we propose that the black hole surfaces and the closed region are topologically equivalent to a spatio-temporal sphere. The black hole is topologically equivalent to a spatio-temporal sphere  $S^3$ . On the other hand, the surface of the closed region is equivalent to the spatio-temporal sphere  $S^4$ , since we can't see directly the closed regions. To go from the sphere  $S^3$ (black hole) to  $S^4$ (closed region), there is a mathematical technique called Hopf fibration. We define the Hopf fibration  $\pi$  by a map that transports each element of  $S^4$  to an element of  $S^3 \cong SU(2)$ . By Adams's theorem [9] there are only 4 possible paths to make a Hopf fibration. Consequently, there is only one possibility to go from  $S^3$  to  $S^4$ : the fiber space  $S^3$  is embedded in the total space  $S^7$ , and the Hopf fibration  $\pi$  starts from  $S^7$  to  $S^4$

$$S^3 \hookrightarrow S^7 \xrightarrow{\pi} S^4 \quad (10)$$

We suppose that the Hopf fibration is an information transformation between the black hole and the closed region. We notice that the geometry  $S^7$  is equivalent to a wormhole that passes through  $S^3$  to  $S^4$ , The information is saved in  $S^4$  as a mass. We know that the topology of a pair of entangled two-level systems [9] is given by the Hopf fibration of Eq.(10).

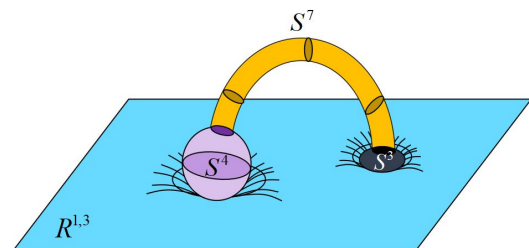


FIG. 1: The geometry of space-time with a wormhole that transforms information from  $S^3$  to  $S^4$ .

We take a space of the quaternions  $H$ , we know that  $S^4 \cong PH$  (quaternions projective space). The vector of

Minkowski space-time  $(t, x, y, z) \in R^{1,3}$ , is interpreted as a quaternion  $q = t + ix + jy + kz \in H$ .

The Hopf fibration  $\pi : S^7 \rightarrow S^4$  [10, 11], its explicit form can be written as:

$$\pi(p, q) = \frac{1}{|q|^2 + 1} \left( 2\frac{q}{p}, \left| \frac{q}{p} \right|^2 - 1 \right) = (2pq, |q|^2 - |p|^2) \quad (11)$$

where  $p, q \in H$ ,  $|q| = \sqrt{t^2 + x^2 + y^2 + z^2}$ ,  $\bar{q} = t + ix + jy + kz$ . Eq.(11) can be rewritten as

$$\pi(p, q) = (X_0, X_1, X_2, X_3, X_5) \quad (12)$$

where

$$X_0 = p\bar{q} + q\bar{p} \quad (13)$$

$$iX_1 + jX_2 + kX_3 = p\bar{q} - q\bar{p} \quad (14)$$

$$X_5 = \delta_{\mu\nu}x^\mu x^\nu - \delta_{\alpha\beta}y^\alpha y^\beta = |q|^2 - |p|^2 \quad (15)$$

the field  $X_5$  is the only one among the others that is real. The Hopf fibration creates a Minkowski space-time geometry and four quantum states. The fields Eq.(13,14) are equivalent to entangled states Eq.(3,4). The pure state in two-qubit system is described by rays which represent state vectors in the complex Hilbert space modulo a  $U(1)$  phase is explicitly given by

$$|\Psi_A\rangle \sim |p\rangle_+ |q\rangle_- + |q\rangle_+ |p\rangle_- \quad (16)$$

$$|\Psi_B\rangle \sim |p\rangle_+ |q\rangle_- - |q\rangle_+ |p\rangle_- \quad (17)$$

the entanglement between  $|\Psi_A\rangle$  and  $|\Psi_B\rangle$ , shows that the field  $X_0$  exists at the same time on the black hole and the closed region and the fields  $(X_1, X_2, X_3)$  exist on the closed region. We can describe the fields  $(X_1, X_2, X_3)$  by the state  $|\Psi_B\rangle$ , this state is entangled with the state  $|\Psi_A\rangle$  which describes the field  $X_0$ . The Hopf fibration (12) summarizes the information stored in the closed region. The closed region appears as a single surface field  $X_5$  in space-time. On the other hand, the four other dimensions of the closed region are hidden at the quantum scale in the form of the fields  $(X_0, X_1, X_2, X_3)$ . These four fields are quaternions. We can represent these quaternions by the  $SU(1, 1)$  matrix

$$\begin{pmatrix} X_0 + iX_1 & X_2 + iX_3 \\ -X_2 + iX_3 & X_0 - iX_1 \end{pmatrix} \quad (18)$$

Subsequently wants to study the transformation of the mass on the wormhole by the Hopf fibration. we remark in Eq.(7) that the entropy  $S_{BH}$  is proportional to the spatial parameter  $\xi^2$  (surface), and since the field  $X_5$  Eq.(15) also represents a surface over space-time. This shows that there is an equivalence between  $S_{BH}$  and  $X_5$ .

By comparing Eq.(9) and Eq.(11), we can write a second Hopf fibration concerning the masses

$$X_0 = m_\xi \bar{m} + m \bar{m}_\xi \quad (19)$$

$$iX_1 + jX_2 + kX_3 = m_\xi \bar{m} - m \bar{m}_\xi \quad (20)$$

$$S_{BH}/4\pi \equiv X_5 = M^2 - M_\xi^2 \quad (21)$$

where  $|m| = M$  and  $|m_\xi| = M_\xi$ . Let's make  $H$  act on  $C^2 \approx H$  by left multiplication, this action is  $C$ -linear. It is also faithful, so defines a morphism of injective algebras  $H \hookrightarrow \text{End}_C(H) \approx M_2(C)$ . The matrix associated with the quaternion will be equivalent to the matrix (18), which contains unit matrices, which are the basis of the Lie algebra of the group  $SU(2)$ . Indeed, we can represent the 5 fields by one writing

$$(m + m_\xi)(\bar{m} + \bar{m}_\xi) = M^2 + M_\xi^2 + X_0 \quad (22)$$

$$(m + m_\xi)(\bar{m} - \bar{m}_\xi) = X_5 + iX_1 + jX_3 + kX_3 \quad (23)$$

the problem here is that we can't describe the 5-fields by single writing with a quaternion product. We define another surface in space-time  $X_6 = M^2 + M_\xi^2$ . This field will replace the  $X_5$  field in Eq.(23) to have an equivalence between the entangled states on equivalent surfaces. By connecting  $X_5$  and the black hole entropy, we can connect the last two equations to have a single description:

$$|m + m_\xi|^2 |\bar{m} + \bar{m}_\xi|^2 = |X_6 + X_0|^2 \quad (24)$$

$$|m + m_\xi|^2 |\bar{m} - \bar{m}_\xi|^2 = X_1^2 + X_2^2 + X_3^2 + X_5^2 \quad (25)$$

We use the subadditivity of the quantity above  $|X_6 + X_0|^2 \leq |X_6|^2 + |X_0|^2$ , and we choose the maximum value of the quantity Eq.(24), one can obtain

$$F |X_6|^2 = -F |X_0|^2 + \frac{1}{F} (X_1^2 + X_2^2 + X_3^2 + X_5^2) \quad (26)$$

where  $F = \left| \frac{\bar{m} - \bar{m}_\xi}{\bar{m} + \bar{m}_\xi} \right| \in R$  is a function. If we accept that the first member of Eq.(26) is a metric, we must first define the term  $F |X_6|^2$ . We know that the Hopf fibration (12) generates 5 dimensional by 8 dimensional, all the fields  $(X_0, X_1, X_2, X_3, X_5)$  therefore represents the dimensions of the space-time of the closed region. We also remark that the second member of the equation (26) looks like the Schwarzschild metric and  $AdS$  metric. Then, the term  $F |X_6|^2$  represents the metric of the closed region. Since  $X_5^2 \equiv S_{BH} \sim \xi^2$ , which shows that  $\frac{1}{F} X_5^2$  is the 5th dimension of the closed region since  $\xi$  is a parameter that describes the closed region. Therefore, the first member of Eq.(26) describes the passage

of information through the black hole to the closed region. We can represent the closed region metric by the coordinates  $(t, r, \theta, \varphi_1, \varphi_2)$  as

$$ds^2 = -F dt^2 + \frac{1}{F} dr^2 + r^2 d\Omega_4^2 \quad (27)$$

where  $d\Omega_4^2 = d\Omega_4^2(\theta, \varphi_1, \varphi_2)$ .

according to this metric, the geometry of the closed-region is a copy of the black hole geometry. The copy-paste of information influence the geometry of the closed region. If we take only 4-dimensions of the closed-region we notice that the closed-region geometry is a reflection symmetry of the black hole geometry. We can see this transformation differently; if the information on the black hole geometry represents a face, the closed region geometry will be a mirror that copies the information of the black hole geometry. From the transformation (10), we can't talk about the geometry of the closed region, unless there is a Hopf fibration (information transformation). With this guesswork, we can compare this transition to the Ryu-Takayanagi prescription [13], which is given by the area of a minimal surface with less area than the horizons. The holography claims that the degrees of freedom in  $(d + 2)$ -dimensional quantum gravity is comparable to those of a quantum system in  $(d + 1)$ -dimensions. Since information travels from 4-dimensions to 5-dimensions. The function  $F$  is expressed by

$$F(r) = \sqrt{\frac{X_6 - 2L(r)}{X_6 + 2L(r)}} \quad (28)$$

where  $L$  is a local function, in the case of  $AdS$  metric  $L \equiv r^2$ . The expression of  $F$ , shows that there is no singularity on the closed region. We notice from Eq.(28) that  $X_6 \geq 2L$ . For  $X_6 \gg 2L$ , we obtain

$$F(r) = \frac{X_6 - L(r)}{X_6 + L(r)} \quad (29)$$

the function  $F$  shows that the metric (27) is a unification of the  $AdS_5$  metric and the Schwarzschild metric. To see this generalization, we can use approximation:

$$F^{-1}(r) \sim 1 + \frac{L(r)}{X_6} \quad (30)$$

Eq.(30) is equivalent with the inverse of the factor that present in the  $AdS$  metric. The approximation of  $F$  Eq.(29) is equivalent to the spatial term of the  $AdS$  metric, which shows that there is a change between space and time in the closed region. On the other hand, if  $L(r)/X_6 = 2M/r$ , the approximation (30) represents the temporal factor of the Schwarzschild metric.

#### IV. CONCLUSION

Here, instead of choosing a wormhole between a region with two edges, the first edge represents a black hole and the second edge represents a closed-region. We have shown that the black hole mass or energy transformed into information that passes through the wormhole, that it stabilizes as a mass (dark matter) in the closed-region. The closed region is compact in space-time, except by a wormhole made by a black hole. Nothing can enter or exit the closed region except through a black hole, which explains why the dark matter does not interact with radiations. To determine the geometry of a wormhole with a closed edge, we used the Hopf fibration, which shows that if the black hole geometry is topologically equivalent to the space-time sphere  $S^3$ , then there is only one passage to create a closed edge of the wormhole. This study has identified that the wormhole is equivalent to the sphere  $S^7$  (8 dimensions) and the closed region is equivalent to  $S^4$  (5 dimensions). The quaternionic Hopf fibration shows that the closed region behaves like a chameleon geometry, which changes concerning the black hole geometry. Since the closed region lives in  $(4 + 1)$ -dimensions and the black hole exists with 4-dimensions, which shows that the information on the black hole projected on the region closed by the holography. This holography is done through a wormhole. We can calculate the closed-region metric from the Hopf fibration, which generalizes the  $AdS_5$  metric.

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