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Article

Cosmology of Barrow Holographic Dark Energy as Nojiri-Odintsov Holographic Dark Energy with Specific Cut-off and the Thermodynamics

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Abstract: Motivated by the work of Saridakis (*Phys. Rev. D* **102**, 123525 (2020)), the present study reports the cosmological consequences of Barrow holographic dark energy (HDE) and its thermodynamics. Literatures demonstrate that Dark Energy (DE) may result from electroweak symmetry breaking that triggers a phase transition from early inflation to late time acceleration. In the present study, we incorporated viscosity in the Barrow HDE. A reconstruction scheme is presented for the parameters associated with Barrow holographic dark energy under the purview of viscous cosmology. Equation of state (EoS) parameter is reconstructed in this scenario and quintessence behaviour is observed. Considering Barrow HDE as a specific case of Nojiri-Odintsov (NO) HDE, we have observed quintom behaviour of the EoS parameter and for some values of n the EoS has been observed to be very close to -1 for the current universe. The generalised second law of thermodynamics has come out to be valid in all the scenarios under consideration. Physical viability of considering Barrow HDE as a specific case of NO HDE is demonstrated in this study.

Keywords: Barrow holographic dark energy; bulk viscosity; thermodynamics.

0. Introduction

Gerard 't Hooft proposed the famous Holographic Principle (HP) inspired by black-hole thermodynamics [1,2]. HP states that all the information contained in a volume of space can be represented as a hologram, which corresponds to a theory located on the boundary of that space [3]. It is widely believed that HP is a fundamental principle of quantam gravity.

In the late 90's, Reiss et al. [4] and Perlmutter et al. [5] independently reported that the current universe is passing through a phase of accelerated expansion. This started a new era in Modern Cosmology. The authors of [4,5] proved this by observational data. This was further supported by other observational studies [6–10]. Characterised by negative pressure to some exotic matter is thought to be responsible for this acceleration. The exotic matter is dubbed as "Dark Energy" (DE) [11,12]. It is described by an equation of state (EoS) parameter defined as $w = \frac{p}{\rho}$, where p is the pressure and ρ is the density due to DE. One can easily verify from Friedmann's equations that $w < \frac{-1}{3}$ is a necessary condition for the accelerated expansion of the universe. The simplest candidate of DE is cosmological constant (Λ), characterised by EoS parameter $w = -1$ [14]. Various DE models have been reviewed in the literatures [11–13,33–38,46–51]. Note that around

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68.3% of the total energy density of the present observable universe is contributed by DE. Remaining density are due to dark matter (DM), baryonic matter and radiation. The contributions of baryonic matter and radiation are negligible with respect to the total density of the universe. Dimopoulos and Markkanen [59] have demonstrated that it is possible to obtain DE from the interplay of Higgs boson and inflation and it has further been demonstrated that a key element for the same is the electroweak symmetry breaking that can lead to a transition to inflation to late time acceleration.
One of the broad type of DE candidate is Holographic DE (HDE), which is discussed in the references [15–18,28–30]. The principle of HDE is HP. Its density is given by $\rho_\Lambda = 3c^2 {M_p}^2 L^{-2}$ [16,31,32], where c^2 represents a dimensionless constant, M_p is the reduced Planck mass and L stands for infrared (IR) cut-off. Till date there are different modifications in IR cut-off being made. In this paper, we will study the Barrow Holographic DE.
In the pandemic Covid19, Barrow was very much inspired by its illustrations and deduced that intricate, fractal features on the black-hole structured may be introduced by the quantam - gravitational effects [19]. This complex structure leads to infinite / finite area but with finite volume. Therefore, the entropy expression to a deformed black-hole is [19]

$$S_B = \left(\frac{A}{A_0}\right)^{\Delta+1}, \tag{1}$$

where A is the standard horizon area and A_0 is the Planck area. The quantam gravitational deformation is quantified by Δ. $\Delta = 0$, corresponds to the standard Bekenstein - Hawking entropy. $\Delta = 1$, corresponds to the most intricate and fractal structure. Note that the usual " quantam - corrected" entropy with logarithmic corrections is very much different than the "above quantam - gravitationally corrected entropy ". No doubt, the involved foundation and physical principles are completely different but resemble to Tsallis non- extensive entropy.
As it is known, Viscosity refers to the resistance to flow. By considering many components in the cosmology, there is a contribution of bulk viscosity in the thermodynamic pressure [25], which also plays a very important and crucial role in accelerating the universe. The term bulk viscosity arises because of different cooling rates of the components. We can affirm that the bulk viscous pressure in cosmic media emerges as a result of coupling among the different component of the cosmic substratum [26,27,39–43].
Gordon M. Barrow quoted "Thermodynamics should be built on energy not on heat and work" [20–24]. The standard HDE is given by $\rho_{DE} L^4 \leq S$, where $L =$ horizon length and $S \propto A \propto L^2$. Therefore, by using the Barrow entropy Eq.(1) lead to

$$\rho_{DE} = CL^{-2(1-\Delta)}, \tag{2}$$

where C is the parameter with dimension $[L]^{-2(\Delta+1)}$. When $\Delta = 0$, the expression (2) will be standard HDE i.e., $\rho_{DE} = 3c^2 {M_p}^2 L^{-2}$ (M_p is the Planck mass and L is IR cut-off) where $C = 3c^2 {M_p}^2$ and c^2 is the model parameter. When the deformation effects quantified by Δ, Barrow HDE will deviate from standard HDE and hence leading to different cosmological consequences. It is very interesting to note that in the limiting case of $\Delta \to 1$, the above expression becomes the constant i.e., $\rho_{DE} = constant$.
Nojiri and Odintsov [52,53] developed cosmological models, where the DE and DM were treated as imperfect fluids. Viscous fluids represent one particular case of what was presented in [52,53]. In the paper, we will incorporate the viscosity term in the various parameters of Barrow HDE. The paper is organised as follows: In Section 1, we will reconstruct the density, thermodynamic pressure of Barrow HDE. We will also reconstruct effective pressure, effective EoS of Viscous Barrow HDE. We will also calculate viscous pressure of Barrow HDE. We will plot density versus redshift z versus Δ; effective EoS versus redshift z and bulk viscous pressure of viscous Barrow HDE versus redshift z versus C_1. We will study accordingly. In Section 2, we will study the

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generalised second law of thermodynamics of viscous Barrow HDE using Barrow entropy. In Section 3, we will reconstruct the density, EoS parameter of Barrow HDE as a Specific NO HDE. We will plot EoS versus redshift z in this case and will study the outcomes. In Subsection 3.1, we will study the validity of generalised second law of thermodynamics for Barrow HDE with NO cut-off. Here, we will plot the total entropy of the Barrow HDE with NO cut-off against the cosmic time t. We give our conclusions in Concluding Remarks.

1. Viscous Barrow Holographic Dark Energy

In this section we study the effect of viscosity in Barrow HDE. We will reconstruct the thermodynamic pressure of Barrow HDE with viscosity. Let us assume R_h be the radius of event horizon, then it is given by

$$\dot{R}_h = HR_h - 1, \tag{3}$$

or,

$$R_h \equiv a\int_t^\infty \frac{dt}{a} = a\int_a^\infty \frac{da}{Ha^2}. \tag{4}$$

Let us assume infrared (IR) cut-off as the event horizon. Therefore replacing L in Eq.(2)with R_h, we get the density of Barrow HDE ρ_{DE} as

$$\rho_{DE} = C{R_h}^{2(\Delta-1)}. \tag{5}$$

where R_h = radius of event horizon and C is constant. The deformation effect is quantified by Δ. As the DM is in the form of dust particle, so we can consider it as pressureless DM i.e., $p_m = 0$. The two Friedmann equations are $3H^2 = \rho_{DE} + \rho_m$ and $6\frac{\ddot{a}}{a} = -(\rho_{DE} + \rho_m + 3(p_{DE} + \Pi))$, where Π = Viscous Pressure $= -3H\xi$ and $\xi = \xi_o + \xi_1 H + \xi_2(\dot{H} + H^2)$.
Conservation equation for pressureless DM is $\dot{\rho}_m + 3H\rho_m = 0$. By solving the expression, we get

$$\rho_m = \rho_{m0}a^{-3}. \tag{6}$$

Now we are introducing density parameters Ω_m and Ω_{DE} and are given by

$$\Omega_m \equiv \frac{1}{3H^2}\rho_m, \tag{7}$$

and

$$\Omega_{DE} \equiv \frac{1}{3H^2}\rho_{DE}. \tag{8}$$

For $\Delta = 1$, the scenario coincides with ΛCDM cosmology with $\rho_{DE} = constant = \Lambda$. Using density parameters (7) and (8) in the expressions (4) and (5), we obtain

$$\int_x^\infty \frac{dx}{Ha} = \frac{1}{a}\left(\frac{C}{3H^2\Omega_{DE}}\right)^{\frac{1}{2(1-\Delta)}}. \tag{9}$$

Using ρ_m from Eq. (6) in Eq.(7), we obtain

$$\Omega_m = \Omega_{m0}\frac{{H_0}^2}{a^3H^2}. \tag{10}$$

where $\Omega_{m0}{H_0}^2 = \frac{\rho_{m0}}{3}$. Now using the Friedmann Equation $\Omega_m + \Omega_{DE} = 1$ and also using Eq.(8) and Eq.(10), we get

$$\frac{1}{aH} = \frac{\sqrt{a(1-\Omega_{DE})}}{H_0\sqrt{\Omega_{m0}}}. \tag{11}$$

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Inserting Eq.(11) into Eq.(9) it results

$$\int_x^\infty \frac{\sqrt{a(1-\Omega_{DE})}}{H_0\sqrt{\Omega_{m0}}}dx = \frac{1}{a}\left(\frac{C}{3H^2\Omega_{DE}}\right)^{\frac{1}{2(1-\Delta)}}. \tag{12}$$

Differentiating Eq.(12) with respect to $x = \ln a$ one gets

$$\frac{\Omega'_{DE}}{\Omega_{DE}(1-\Omega_{DE})} = 2\Delta + 1 + Q(1-\Omega_{DE})^{\frac{\Delta}{2(\Delta-1)}}(\Omega_{DE})^{\frac{1}{2(1-\Delta)}}e^{\frac{3\Delta}{2(\Delta-1)}x}. \tag{13}$$

where $Q \equiv 2(1-\Delta)(\frac{C}{3})^{\frac{1}{2(\Delta-1)}}(H_0\sqrt{\Omega_{m0}})^{\frac{\Delta}{1-\Delta}}$. Eq.(13) is the evolution of Barrow HDE in a flat universe for dust matter. For $\Delta = 0$, it coincides with the usual HDE i.e., $\Omega'_{DE}]_{\Delta=0} = \Omega_{DE}(1-\Omega_{DE})(1+2\sqrt{\frac{3\Omega_{DE}}{C}})$. Now from Eq.(11), we have

$$H = \frac{H_0\sqrt{\Omega_{m0}}}{a\sqrt{a(1-\Omega_{DE})}}. \tag{14}$$

From Eq.(3) taking H from Eq.(14), we get R_h as

$$R_h = \frac{a\sqrt{-a(\Omega_{DE}-1)}}{H_0\sqrt{\Omega_{m0}}} + e^{\frac{H_0\sqrt{\Omega_{m0}}t}{a\sqrt{-a(\Omega_{DE}-1)}}}C_1. \tag{15}$$

Now using this R_h in Eq.(5), we obtain reconstructed density of Barrow HDE $\rho_{DE,rec}$ as

$$\rho_{DE,rec} = C\left(C_1 e^{\frac{H_0\sqrt{\Omega_{m0}}t}{a\sqrt{a-a\Omega_{DE}}}} + \frac{a\sqrt{a-a\Omega_{DE}}}{H_0\sqrt{\Omega_{m0}}}\right)^{2(-1+\Delta)}. \tag{16}$$

As we are having now $\rho_{DE,rec}$ (Eq.(16)), H (Eq.(14)) and let us take $p_{eff} = p_{DE} + \Pi$ and using these in the conservation equation $\dot{\rho}_{DE,rec} + 3H(\rho_{DE,rec} + p_{eff}) = 0$, we obtain

$$p_{eff} = \frac{1}{3H_0\sqrt{\Omega_{m0}}}a\sqrt{a(1-\Omega_{DE})}\left(-\frac{3CH_0\left(C_1 e^{\frac{H_0\sqrt{\Omega_{m0}}t}{a\sqrt{a-a\Omega_{DE}}}} + \frac{a\sqrt{a-a\Omega_{DE}}}{H_0\sqrt{\Omega_{m0}}}\right)^{2(-1+\Delta)}\sqrt{\Omega_{m0}}}{a\sqrt{a(1-\Omega_{DE})}} - \frac{2CC_1 e^{\frac{H_0\sqrt{\Omega_{m0}}t}{a\sqrt{a-a\Omega_{DE}}}}H_0\left(C_1 e^{\frac{H_0\sqrt{\Omega_{m0}}t}{a\sqrt{a-a\Omega_{DE}}}} + \frac{a\sqrt{a-a\Omega_{DE}}}{H_0\sqrt{\Omega_{m0}}}\right)^{-3+2\Delta}\sqrt{\Omega_{m0}}(-1+\Delta)}{a\sqrt{a-a\Omega_{DE}}}\right). \tag{17}$$

Therefore, thermodynamic pressure $p_{DE} = p_{eff} - \Pi$. Hence,

$$p_{DE} = \left(\frac{1}{3a\sqrt{a-a\Omega_{DE}}}\right)\left(-aCH_0{}^2\sqrt{a-a\Omega_{DE}}\left(C_1 e^{\frac{H_0\sqrt{\Omega_{m0}}t}{a\sqrt{a-a\Omega_{DE}}}} + \frac{a\sqrt{a-a\Omega_{DE}}}{H_0\sqrt{\Omega_{m0}}}\right)^{2\Delta}\Omega_{m0}\left(3a\sqrt{a-a\Omega_{DE}} + C_1 e^{\frac{H_0\sqrt{\Omega_{m0}}t}{a\sqrt{a-a\Omega_{DE}}}}H_0\sqrt{\Omega_{m0}}(1+2\Delta)\right)\left(a\sqrt{a-a\Omega_{DE}} + C_1 e^{\frac{H_0\sqrt{\Omega_{m0}}t}{a\sqrt{a-a\Omega_{DE}}}}H_0\sqrt{\Omega_{m0}}\right)^{-3} + 9H_0\sqrt{\Omega_{m0}}\xi\right). \tag{18}$$

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which is the thermodynamic pressure of DE involving the viscous term ξ. As, viscous term is involved here, so we can take p_{DE} = effective pressure. Hence, effective pressure p_{eff} is Eq.(18). As we know that effective Eos, $w_{eff} = \frac{p_{eff}}{\rho_{DE,rec}}$. There, using p_{eff} from Eq.(18) and $\rho_{DE,rec}$ from Eq.(16), we get effective EoS as

$$w_{eff} = \frac{\left(C_1 e^{\frac{H_0\sqrt{\Omega_{m0}}t}{a\sqrt{a-a\Omega_{DE}}}} + \frac{a\sqrt{a-a\Omega_{DE}}}{H_0\sqrt{\Omega_{m0}}}\right)^{2-2\Delta}}{3aC\sqrt{a-a\Omega_{DE}}} \left(-aCH_0{}^2\sqrt{a-a\Omega_{DE}}\left(C_1 e^{\frac{H_0\sqrt{\Omega_{m0}}t}{a\sqrt{a-a\Omega_{DE}}}} + \frac{a\sqrt{a-a\Omega_{DE}}}{H_0\sqrt{\Omega_{m0}}}\right)^{2\Delta} \Omega_{m0}\left(3a\sqrt{a-a\Omega_{DE}} + C_1 e^{\frac{H_0\sqrt{\Omega_{m0}}t}{a\sqrt{a-a\Omega_{DE}}}} H_0\sqrt{\Omega_{m0}}(1+2\Delta)\right) \left(a\sqrt{a-a\Omega_{DE}} + C_1 e^{\frac{H_0\sqrt{\Omega_{m0}}t}{a\sqrt{a-a\Omega_{DE}}}} H_0\sqrt{\Omega_{m0}}\right)^{-3} + 9H_0\sqrt{\Omega_{m0}}\xi\right). \tag{19}$$

Now we will insert Δ in Π, to make it a viscous pressure in Barrow HDE. Now using $\rho_{DE,rec}$ from Eq.(16), w_{eff} from Eq.(19) in the conservation equation $\dot{\rho}_{DE,rec} + 3H\rho_{DE,rec}(1 + w_{eff}) = 0$, we get H. Let us name it as H_{rec} and is given by

$$H_{rec} = -\left(2CC_1 e^{\frac{H_0\sqrt{\Omega_{m0}}t}{a\sqrt{a-a\Omega_{DE}}}} H_0{}^3 \left(C_1 e^{\frac{H_0\sqrt{\Omega_{m0}}t}{a\sqrt{a-a\Omega_{DE}}}} + \frac{a\sqrt{a-a\Omega DE}}{H_0\sqrt{\Omega_{m0}}}\right)^{2\Delta} \Omega_{m0}{}^{3/2}(-1+\Delta)\right) \left(-9a^4(-1+\Omega_{DE})\sqrt{a-a\Omega_{DE}}\xi - 27a^3 C_1 e^{\frac{H_0\sqrt{\Omega_{m0}}t}{a\sqrt{a-a\Omega_{DE}}}} H_0(-1+\Omega_{DE})\sqrt{\Omega_{m0}}\xi + 9C_1{}^3 e^{\frac{3H_0\sqrt{\Omega_{m0}}t}{a\sqrt{a-a\Omega_{DE}}}} H_0{}^3\Omega_{m0}{}^{3/2}\xi + aC_1 e^{\frac{H_0\sqrt{\Omega_{m0}}t}{a\sqrt{a-a\Omega_{DE}}}} H_0{}^2\sqrt{a-a\Omega_{DE}}\Omega_{m0} \left(-2C\left(C_1 e^{\frac{H_0\sqrt{\Omega_{m0}}t}{a\sqrt{a-a\Omega_{DE}}}} + \frac{a\sqrt{a-a\Omega_{DE}}}{H_0\sqrt{\Omega_{m0}}}\right)^{2\Delta}(-1+\Delta) + 27C_1 e^{\frac{H_0\sqrt{\Omega_{m0}}t}{a\sqrt{a-a\Omega_{DE}}}}\xi\right)\right)^{-1}. \tag{20}$$

Let us assume power-law form of scale factor as $a(t) = a(t-t_0)^n$. As, we know that $H = \frac{\dot{a}}{a}$. Hence , by using the power-law form of scale factor, we get H. Let us denote this H by H_{recc} and is given by

$$H_{recc} = \frac{n}{t-t_0}. \tag{21}$$

Now, by using H_{recc} in place of H in ξ i.e., $\xi = \xi_0 + \xi_1 H_{recc} + \xi_2(\dot{H}_{recc} + H^2_{recc})$. The, using this ξ and H as H_{rec} from Eq.(20) in $\Pi = -3H\xi$, we obtain the viscous pressure in Barrow HDE as

$$\Pi = \left(6CC_1 e^l H_0{}^3\Omega^{3/2}_{m0}\left(C_1 e^l + j\right)^{2\Delta}(-1+\Delta)(k+(-1+n)n\xi_2)\right) \left(9C_1{}^3 e^{3l} H_0{}^3\Omega^{3/2}_{m0}(k+(-1+n)n\xi_2) - 27a_0{}^3 C_1 e^l H_0(-1+\Omega_{DE})\sqrt{\Omega_{m0}}(t-t_0)^{3n} (k+(-1+n)n\xi_2) + 9a_0{}^3(t-t_0)^{3n}(-a_0(-1+\Omega_{DE})(t-t_0)^n)^{3/2}(k+(-1+n)n\xi_2) + a_0 C_1 e^{2l} H_0{}^2\Omega_{m0}(t-t_0)^n\sqrt{-a_0(-1+\Omega_{DE})(t-t_0)^n} \left(-2Ce^{-l}\left(C_1 e^l + j\right)^{2\Delta}(t-t_0)^2(-1+\Delta) + 27C_1(k+(-1+n)n\xi_2)\right)\right)^{-1}, \tag{22}$$

where, $l = -\frac{H_0(-1+\Omega_{DE})\sqrt{\Omega_{m0}}t}{(-a_0(-1+\Omega_{DE})(t-t_0)^n)^{3/2}}$,
$k = (t-t_0)(t\xi_0 - t_0\xi_0 + n\xi_1)$,

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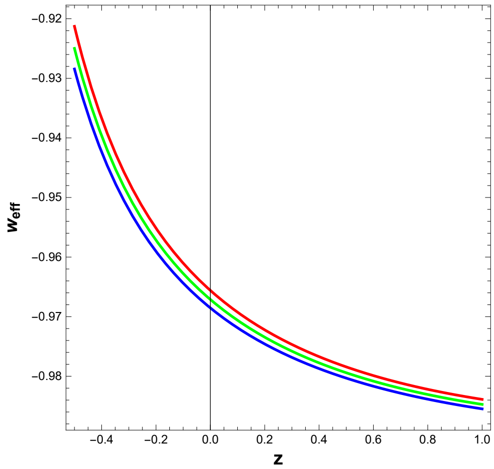

Figure 1. Evolution of effective EoS (Eq.(19)) of viscous Barrow Holographic Dark Energy against redshift z. We consider $a_0 = 0.001$, $C = 0.09$, $C_1 = 0.00015$, $H_0 = 0.999$, $\Omega_{m0} = 0.002$, $t_0 = 0.20$, $\xi_0 = 0.000005$, $\xi_1 = 0.00001$, $\xi_2 = 0.92$, $\Delta = 0.04$ and the red, green and blue lines correspond to $n = 0.9, 0.91, 0.92$ respectively.

$j = \frac{a_0(t-t_0)^n\sqrt{-a_0(-1+\Omega_{DE})(t-t_0)^n}}{H_0\sqrt{\Omega_{m0}}}$. Now we will reconstruct a thermodynamic DE pressure i.e., $p_{DE,rec}$ to make it thermodynamic pressure of viscous Barrow HDE. So, in the conservation equation $\dot{\rho}_{DE,rec} + 3H(\rho_{DE,rec} + p_{DE,rec} + \Pi) = 0$, using $\rho_{DE,rec}$ from Eq.(16), Π from Eq.(22), H as H_{rec} from Eq.(20), we obtain $p_{DE,rec}$, which is a thermodynamic pressure of viscous Barrow HDE. Now using Taylor series expansion in the term $\sqrt{1-\Omega_{DE}}$ of Eq.(11) and ignoring higher order derivatives, we obtain $\frac{1}{aH} = \frac{\sqrt{a}}{H_0\sqrt{\Omega_{m0}}}(1-\frac{1}{2}\Omega_{DE})$. From the above equation, we get Ω_{DE} as

$$\Omega_{DE} = 2 - \frac{2H_0\sqrt{\Omega_{m0}}}{\dot{a}\sqrt{a}}. \tag{23}$$

Now, using Ω_{DE} from Eq.(23) in Eq.(19), we get w_{eff} and plotted the evolution of effective EoS (19) of viscous Barrow HDE against the redshift z in Fig.1.

From the figure we observe that behaviour of the effective EoS parameter w_{eff} (19) is quintessence. Now we will study the behaviour of $\rho_{DE,rec}$ (Eq.(16)) when $\Delta \to -1$. We have plotted the reconstructed density of Barrow HDE against redshift z in Fig.2 for a range of values of Δ. From Fig.2, we obtain that as Δ approaches to 1, the density of Barrow HDE is also increasing. This indicates that at that point we can study the evolution of the universe at its large due to Dark Energy. Using the expression of Ω_{DE} from Eq.(23) in the expression of bulk viscous pressure of Barrow HDE Π i.e., Eq.(22), we have plotted Π versus redshift z versus C_1 in Fig.3. We have found that with the passage of time, the effect of bulk viscous pressure is decreasing.

2. Generalised Second Law of Thermodynamics of Viscous Barrow HDE

In this section, we will study the generalised second law of thermodynamics using barrow entropy [24]. We consider the universe horizon to be the boundary of thermodynamical system . We can take it as an apparent horizon as it the most appropriate one. There are many choices in the literature and we chose here the apparent horizon [20–22]. Apparent horizon is given by

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}}. \tag{24}$$

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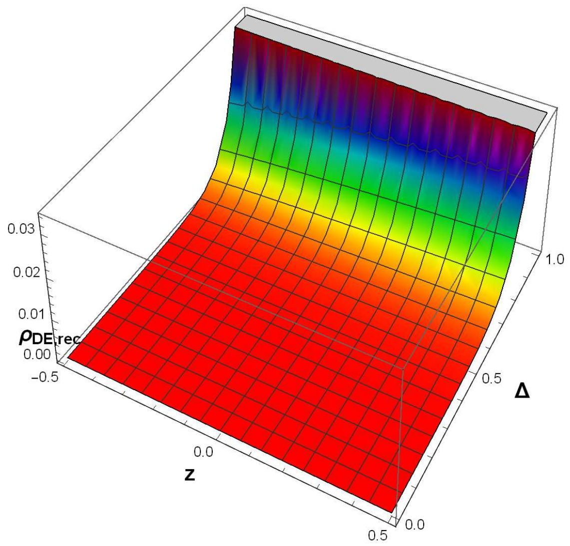

Figure 2. Evolution of density of Barrow HDE (Eq.(16)) against redshift z and against Δ. We consider $a_0 = 0.001$, $C = 0.09$, $C_1 = 0.00015$, $H_0 = 0.999$, $\Omega_{m0} = 0.002$, $t_0 = 0.20$, $n = 0.9$
.

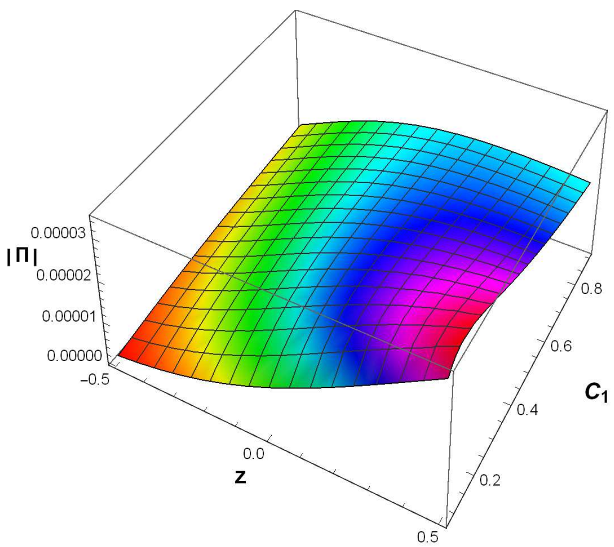

Figure 3. Evolution of bulk viscous pressure of Barrow HDE (Eq.(22)) against redshift z and against C_1. We consider $a_0 = 0.001$, $C_2 = 0.09$, $\Delta = 0.5$, $H_0 = 0.999$, $\Omega_{m0} = 0.002$, $t_0 = 0.20$, $n = 0.9$, $\xi_0 = 0.02$, $\xi_1 = 0.005$, $\xi_2 = 0.03$
.

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where k quantifies the spatial curvature and hence $k = 0$ as we considered the universe to be flat. Therefore Eq.(24) becomes

$$\tilde{r}_A = \frac{1}{H}. \tag{25}$$

From the first Friedmann equation $3H^2 = \rho_m + \rho_{DE}$ and Eq.(25), we get

$$\frac{1}{\tilde{r}_A^2} = \frac{1}{3}(\rho_m + \rho_{DE}). \tag{26}$$

Using $\rho_m = \rho_{m0}a^{-3}$, $a = a_0(t - t_0)^n$ and $\rho_{DE,rec}$ from Eq.(16) in place of ρ_{DE} in Eq.(26), we get apparent horizon $\tilde{r}_A$.
Now we will check whether the total entropy of the system i.e., sum of the entropy enclosed by apparent horizon plus entropy of the apparent horizon of the system is non-decreasing function of time or not. Apparent horizon $\tilde{r}_A$ is dependent on time . So, changes in apparent horizon $d\tilde{r}_A$ in time interval dt will contribute a change in volume dV. Hence, the energy and entropy of the system will change by dE and dS respectively. The first law of Thermodynamics is $TdS = dE + PdV$. Therefore the dark enery entropy and dark matter entropy will be [23]:

$$dS_{DE} = \frac{1}{T}(P_{DE}dV + dE_{DE}), \tag{27}$$

$$dS_m = \frac{1}{T}(P_m dV + dE_m). \tag{28}$$

where dS_{DE} = DE entropy, dS_m = DM entropy, P_{DE} = DE pressure, P_m = DM pressure. V is the universe volume bounded by apparent horizon and is given by $V = \frac{4\pi\tilde{r}_A^3}{3}$. Therefore, $dV = 4\pi\tilde{r_A}^2 d\tilde{r}_A$. We assume the system to be in equilibrium, so we can consider the temperature of the universe fluids to be same. Dividing Eqns. (27), (28) by t, we get

$$\dot{S}_{DE} = \frac{1}{T}(P_{DE}4\pi\tilde{r}_A^2\dot{\tilde{r}}_A + \dot{E_{DE}}), \tag{29}$$

$$\dot{S}_m = \frac{1}{T}(P_m 4\pi\tilde{r_A}^2\dot{\tilde{r}}_A + \dot{E_m}). \tag{30}$$

To consider the relationship between thermodynamical quantities $\dot{E}_{DE}$ and $\dot{E}_m$ with cosmological quantities ρ_{DE} and ρ_m, we use

$$E_{DE} = \frac{4\pi}{3}\tilde{r}_A^3\rho_{DE}, \tag{31}$$

$$E_m = \frac{4\pi}{3}\tilde{r}_A^3\rho_m. \tag{32}$$

Now we have $\tilde{r}_A$, so we can find $\dot{\tilde{r}}_A$, E_{DE} from Eq.(31), E_m from Eq.(32) and hence $\dot{E}_{DE}$ and $\dot{E}_m$. We consider $T \approx$ horizon temperature $(T_h) = \frac{1}{2\pi\tilde{r}_A}$. Therefore, we can calculate $\dot{S}_{DE}$ and $\dot{S}_m$ from Eqns. (29) and (30), respectively. Now we will calculate horizon entropy $\dot{S}_h$. Applying entropy expression to a deformed black hole Eq.(1) with standard horizon area $A = 4\pi\tilde{r}_A^2$, we get $S_h = \gamma\tilde{r}_A^{2(\Delta+1)}$, where $\gamma \equiv \left(\frac{4\pi}{A_0}\right)^{1+\Delta}$. Therefore horizon entropy is given by

$$\dot{S}_h = \gamma 2(1+\Delta)\tilde{r}_A^{2\Delta+1}\dot{\tilde{r}}_A. \tag{33}$$

Therefore, $\dot{S}_{total} = \dot{S}_{DE} + \dot{S}_m + \dot{S}_h$. After calculating $\dot{S}_{total}$, we plotted it in Fig.4. From the figure we have seen that $\dot{S}_{total}$ is positive and is non-decreasing. Hence, it satisfies the second law of thermodynamics. This implies the validity of the generalised second

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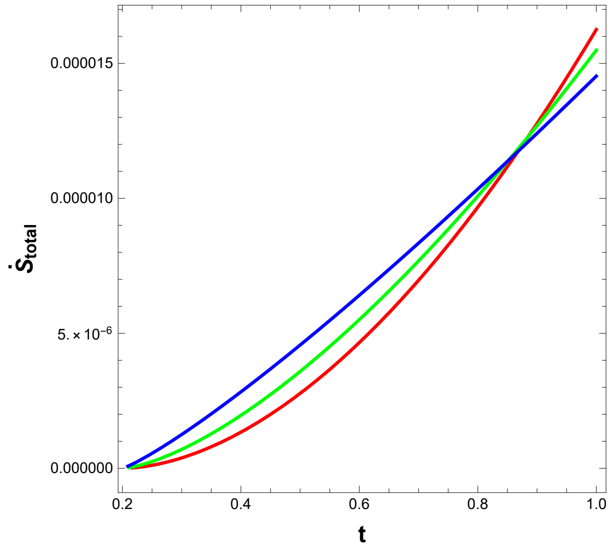

Figure 4. Plot of $\dot{S}_{total}$ of viscous Barrow HDE against the cosmic time t. We consider $a_0 = 0.001$, $C = 0.09$, $C_1 = 0.00015$, $H_0 = 0.999$, $\Omega_{m0} = 0.002$, $t_0 = 0.20$, $\xi_0 = 0.000005$, $\xi_1 = 0.00001$, $\xi_2 = 0.92$, $\Delta = 0.04$, $\rho_{m0} = 0.32$, $A_0 = 0.00905$. The red, green and blue lines correspond to $n = 0.9, 0.8$ and 0.7 respectively.

law of thermodynamics in the case of viscous Barrow HDE. It is further observed that with evolution of the universe $\dot{S}_{total}$ is increasing. This indicates that the validity of the generalised second law of thermodynamics is expected to occur with the evolution of the universe in case of viscous Barrow HDE.

3. Barrow HDE as a Specific NO HDE

Nojiri and Odintsov [54] demonstrated a unifying approach to the early and late time universe through a phantom cosmology. They considered a gravity-scalar system containing usual potential and scalar coupling function within the kinetic term. Their study [54] resulted in the possibility of phantom - non-phantom transition in such a manner that the universe could have the phantom EoS in the early as well as in the late time. Contrary to the study of [54], our work, a specific case of NO HDE, has led to a quintessence behaviour with no crossing of phantom boundary, see Fig.1. In this connection, we further note that the generalised HDE with NO cut-off as proposed in [54] suggested unified cosmological scenario for tachyon phantoms and for time-dependent phantomic EoS. We further take into account the study of Nojiri and Odintsov [55], where a generalised HDE was proposed with infrared cut-off identified with the combination of the FRW universe parameters. Their study took into account the Hubble rate $H(t) = f_0|t_s - t|^{\alpha}$. However in our study, we have taken into consideration a Hubble rate $H = \frac{n}{t-t_0}$, for which we could get a universe where generalised second law of thermodynamics has come out to be valid. Hence, we can state that the Barrow HDE, a specific case of more general NO HDE can lead to a universe where generalised second law of thermodynamics is valid. Nojiri et. al. [56] established that at late times, the effective fluid can act as the driving force behind the accelerated expansion in absence of cosmological constant. Consistent with the findings of [56] in our work on a specific form of NO HDE the generalised second law came out to be valid without any cosmological constant. In this context let us mention the work of Nojiri et. al. [57] that applied the HP at early times to realise the bounce scenario. The current study with a specific NO HDE cut-off can further be extanded to check the realisation of holographic bounce and to study the mechanism of holographic preheating [57] under this framework. Lastly, let us mention the study of Nojiri et. al. [58] that confronted the cosmological scenario arising from the application of non extensive thermodynamics

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with varying exponent. Their study could provide a description of both inflation and late time acceleration with the same choices of parameters. We further re-iterate that the current Barrow HDE can be examined for its realisation for early inflation and late time acceleration as a specific case of NO HDE.
In this section, we consider Barrow HDE as a particular case of NO HDE. The NO HDE was proposed in the work of Nojiri and Odintsov [54]. This was further studied in [11]. The DE density for NO HDE is defined as

$$\rho_{NO} = \frac{3c^2}{L^2}, \tag{34}$$

with

$$\frac{c}{L} = \frac{1}{R_h}(\alpha_0 + \alpha_1 R_h + \alpha_2 {R_h}^2), \tag{35}$$

where R_h is the future event horizon discussed in the equations (3) and (4). For the choice of power law form of scale factor $a(t) = a_0(t-t_0)^n$, we have IR cut off L as

$$L = \frac{c}{\frac{(n-1)\alpha_0}{t+C_2(n-1)(t-t_0)^n - t_0} + \alpha_1 + \frac{(t-t_0)\alpha_2}{n-1} + C_2(t-t_0)^n\alpha_2}. \tag{36}$$

In equations (34), (35) and (36), c, α_0, α_1 and α_2 are numerical constants and C_2 is the constant of integration. Equation (36) represents the NO cut-off as a function of cosmic time t. Now, we consider this NO cut-off as the cut-off for Barrow HDE and from this consideration, we get Barrow HDE generalised by NO HDE and hence, we get the density for Barrow HDE generalised through NO cut-off $\rho_{BarrowHDE}$ is

$$\rho_{BarrowHDE} = C\left(\frac{c}{\frac{(n-1)\alpha_0}{t+C_2(n-1)(t-t_0)^n - t_0} + \alpha_1 + \frac{(t-t_0)\alpha_2}{n-1} + C_2(t-t_0)^n\alpha_2}\right)^{2(-1+\Delta)}. \tag{37}$$

We will find the thermodynamic pressure for Barrow HDE generalised through NO cut-off i.e., $p_{BarrowHDE}$ from the conservation equation $\dot{\rho}_{BarrowHDE} + 3H(\rho_{BarrowHDE} + p_{BarrowHDE}) = 0$, we get $p_{BarrowHDE}$. Hence, EoS parameter for Barrow HDE generalised through NO cut-off i.e., $w_{BarrowHDE}$ can be calculated by using $\rho_{BarrowHDE}$ from Eq.(37) and $p_{BarrowHDE}$ on equation $w_{BarrowHDE} = \frac{p_{BarrowHDE}}{\rho_{BarrowHDE}}$. In Fig.5, we have plotted the EoS parameter for Barrow HDE as a specific case of NO HDE. In this figure, the evolution of the reconstructed EoS parameter is demonstrated for $\Delta = 0.4$ and the range of values of $1.5 \leq n \leq 2$. It is apparent from this figure that for smaller values of n, the transition from quintessence to phantom is happening at an earlier stage of the universe. However, for $n \approx 2$, the transition is happening at a later stage. Therefore, in general we can say that the EoS parameter for Barrow HDE reconstructed through NO HDE is characterise by quintom behaviour. Moreover, for $n \approx 1.56$, we have $w_{BarrowHDE} \approx -1$ for $z = 0$ and hence, it is consistent with observation. Hence, we can conclude that as the IR cut-off for Barrow HDE is reconstructed through NO HDE the transition from quintessence to phantom is available. It further indicates that under this reconstruction scheme the universe may end with a Big-Rip in the future.

3.1. Generalised Second Law of Thermodynamics for Barrow HDE with NO Cut-off

In this subsection, we have studied the generalised second law of thermodynamics for Barrow HDE with NO cut-off using Barrow entropy likewise in Section 2. We have proceeded similarly as Section 2 just by taking the scale factor as $a(t) = a_0(t-t_0)^n$, with $n > 0$. We have calculated the total entropy of the Barrow HDE with NO cut-off i.e., $\dot{S}_{total,BarrowHDE}$ and plotted this in Fig.6 against the cosmic time t. The Fig.6 indicates that $\dot{S}_{total,BarrowHDE}$ is positive and non-decreasing. Therefore, we have observed

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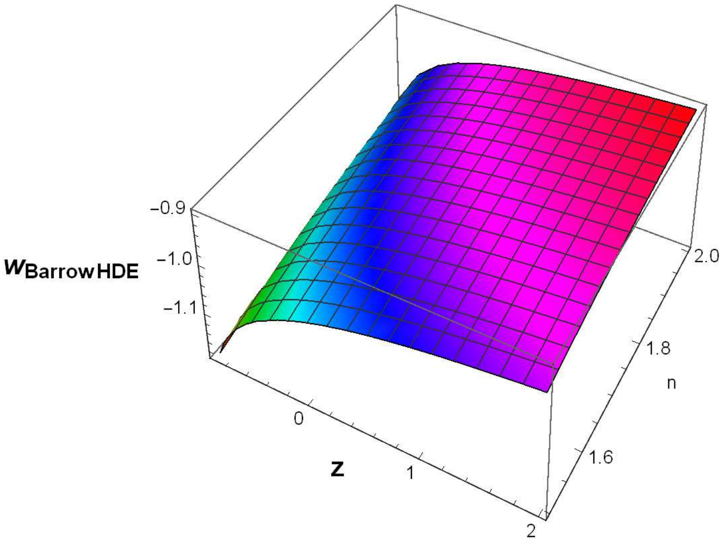

Figure 5. Evolution of EoS parameter for Barrow HDE generalised through NO cut-off i.e., $w_{BarrowHDE}$ against the redshift z and against n. We consider $a_0 = 0.05$, $c = 0.06$, $C_2 = 0.09$, $C = 0.9$, $t_0 = 0.02$, $\alpha_0 = 0.41$, $\alpha_1 = 0.135$, $\alpha_2 = 0.003$, $\Delta = 0.4$.

the validity of the generalised second law of thermodynamics when Barrow HDE is considered as a specific case of NO HDE.

4. Concluding remarks

Motivated by the work of Saridakis [19], the present study is attempted to probe the cosmological consequences of Barrow HDE and its thermodynamics. In the first phase of the study, we have studied the effect of bulk viscosity in presence of Barrow HDE. We have reconstructed the density of Barrow HDE as $\rho_{DE,rec}$ in Eq.(16). We also found effective pressure p_{eff} of viscous Barrow HDE as in Eq.(18). After finding p_{eff} (Eq.(18)), we have derived effective EoS of viscous Barrow HDE w_{eff} as in Eq.(19). Thereafter, we calculated viscous pressure Π in Eq.(22) and we also reconstructed thermodynamic pressure $p_{DE,rec}$ of viscous Barrow HDE. In Fig.1, we have plotted w_{eff} (Eq.(19)) versus redshift z. From the Fig.1, we observed that the behaviour of w_{eff} (Eq.(19)) is quintessence. Next, we studied the behaviour of $\rho_{DE,rec}$ (Eq.(16)) when $\Delta \to 1$. We have plotted $\rho_{DE,rec}$ (Eq.(16)) against redshift z for a range of values of $\Delta \in [0, 1]$ in Fig.2. It is apparent from this figure that there is an increasing tendency of $\rho_{DE,rec}$ (Eq.(16)) as $\Delta \to 1$, which indicates that we can study the evolution of the universe at its large due to DE. Also, we have studied the behaviour of the bulk viscous pressure Π (Eq.(22)) under the purview of Barrow HDE with the evolution of the universe for a range of values of C_1 in Fig.3. The study demonstrated above shows the decaying effect of bulk viscous pressure with the evolution of the universe. This is in contrast with the finding of [44], where the effect of bulk viscosity was found to have an increasing pattern under the purview of holographic Ricci DE.

In Section 2, we have demonstrated the generalised second law of thermodynamics under the purview of the bulk-viscosity of the Barrow HDE. Here, for the study we have taken apparent horizon as the enveloping horizon of the universe. We have calculated the total entropy $\dot{S}_{total}$ of the system. The $\dot{S}_{total}$ has been plotted in Fig.4. From Fig. 4 it results that with the evolution of the universe $\dot{S}_{total}$ of the viscous Barrow HDE is increasing and is staying at positive level. Therefore, we conclude that the generalised second law of thermodynamics is obeyed by this model [45]. This finding is consistent with the study of [45], where the validity of generalised second law of thermodynamics was examined in presence of viscous DE and it was observed that the generalised second law of thermodynamics is fulfilled in presence of bulk viscosity. However, the

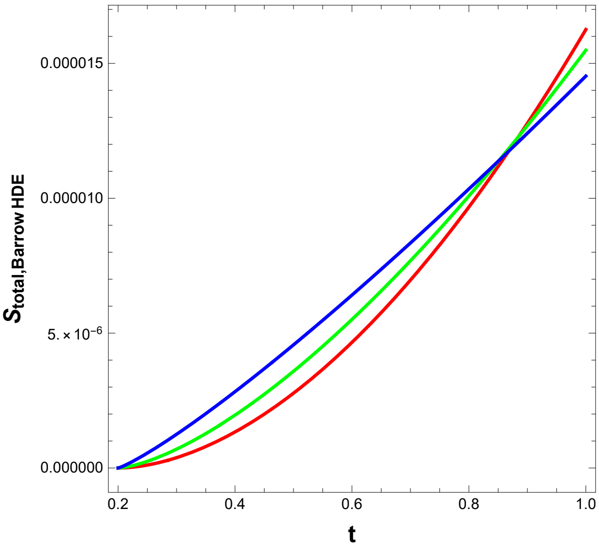

Figure 6. Plot of $\dot{S}_{total,BarrowHDE}$ of Barrow HDE with NO cut-off against the cosmic time t. We consider $a_0 = 0.001$, $c = 0.06$, $\alpha_0 = 0.004$, $\alpha_1 = 0.005$, $\alpha_2 = 0.0003$, $C = 0.00015$, $C_2 = 0.09$, $t_0 = 0.20$, $\Delta = 0.04$, $\rho_{m0} = 0.32$, $A_0 = 0.00905$. There, green, blue lines correspond to $n = 0.9, 0.8, 0.7$ respectively.

approach of the current study differs from Setare and Sheykhi [45] in the sense that the standard Eckart approach is adopted here.
In Section 3, we have demonstrated reconstructed schemes of Barrow HDE as a specific NO HDE. We have reconstructed the density i.e., $\rho_{BarrowHDE}$ in Eq.(37) for Barrow HDE generalised through NO cut-off. We have also reconstructed the EoS parameter $w_{BarrowHDE}$ for Barrow HDE generalised through NO cut-off and plotted it in Fig.5. This figure shows the quintom behaviour of $w_{BarrowHDE}$. Moreover, $w_{BarrowHDE} \approx -1$ at $z = 0$ for some values of n. It also suggests that the universe may end with a Big-Rip in the future. Finally, for this reconstructed Barrow HDE we have demonstrated the generalised second law of thermodynamics. For the Barrow HDE with NO cut-off it is observed that (see Fig.6) the time derivative of the total entropy is staying at positive level and hence, it is concluded that the generalised second law holds if we consider Barrow HDE as a specific case of NO HDE. In the future study, we proposed to carry out the similar viscous cosmology under the purview of modified theories of gravity with the background evolution as Barrow HDE.

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Abbreviations

The following abbreviations are used in this manuscript:

DE	Dark Energy
DM	Dark Matter
HP	Holographic Principle
HDE	Holographic Dark Energy
IR	Infrared
EoS	Equation of State
NO	Nojiri-Odintsov

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