Article

Category-Theoretic Formulation of Model-Based Systems Architecting as a Concept-Model-Graph-View-Concept Transformation Cycle

Yaniv Mordecai [1,*], James Fairbanks [2], Edward F. Crawley [3]

[1] Massachusetts Institute of Technology https://orcid.org/0000-0001-5768-1044
[2] University of Florida https://orcid.org/0000-0002-1778-3350
[3] Massachusetts Institute of Technology https://orcid.org/0000-0002-3006-1512
* Correspondence: yanivm@mit.edu

Abstract: We introduce the Concept→Model→Graph→View Cycle (CMGVC). The CMGVC facilitates coherent architecture analysis, reasoning, insight, and decision making based on conceptual models that are transformed into a generic, robust graph data structure (GDS). The GDS is then transformed into multiple views of the model, which inform stakeholders in various ways. This GDS-based approach decouples the view from the model and constitutes a powerful enhancement of model-based systems engineering (MBSE). The CMGVC applies the rigorous foundations of Category Theory, a mathematical framework of representations and transformations. We show that modeling languages are categories, drawing an analogy to programming languages. The CMGVC architecture is superior to direct transformations and language-coupled common representations. We demonstrate the CMGVC to transform a conceptual system architecture model built with the Object Process Modeling Language (OPM) into dual graphs and a stakeholder-informing matrix that stimulates system architecture insight.

Keywords: Model-Based Systems Engineering; Category Theory; Object-Process Methodology; Model Analytics; Concept-Model-Graph-View Cycle; Graph Data Structures

Acronyms and Glossary Terms

Acronym	Full Term
CMGVC	Concept→Model→Graph→View Cycle
DSM	Design Structure Matrix
GDS	Graph Data Structure
GR	Generic Representation
MBSE	Model-Based Systems Engineering
MGV	Model→Graph→View

Acronym	Full Term
ML	Modeling Language
OPM	Object-Process Methodology
OPML	Object-Process Modeling Language
SAM	System Architecture Matrix
SIM	Stakeholder-Informing Matrix
UUID	Universal Unique ID

1. Introduction

Models represent concepts by capturing the ideas, observations, notions, theories, insights, and intents that people conceive of and wish to express and communicate to others [1]. Conceptual models use syntax and semantics of a modeling language to describe the concepts. A popular example for concept modeling depicts Isaac Newton's observation and conception of gravity in the falling of an apple from a tree, and the ensuing formulation of Newton's Theory of Gravity. Models represent scientific concepts, as projections of theories on phenomena; they describe and abstract phenomena, while relying on theories that may be hypothetical or partially validated [2]. Likewise, models represent engineered, man-made systems in their current or future state [3].

Models also inform concepts: we can enrich our concept, perception, and subject matter understanding, by capturing, manipulating, contextualizing, composing, and analyzing concepts and aspects. Conceptual models play a central and growingly more critical role in complex systems development and operation. Models are supposed to capture and represent various relations and

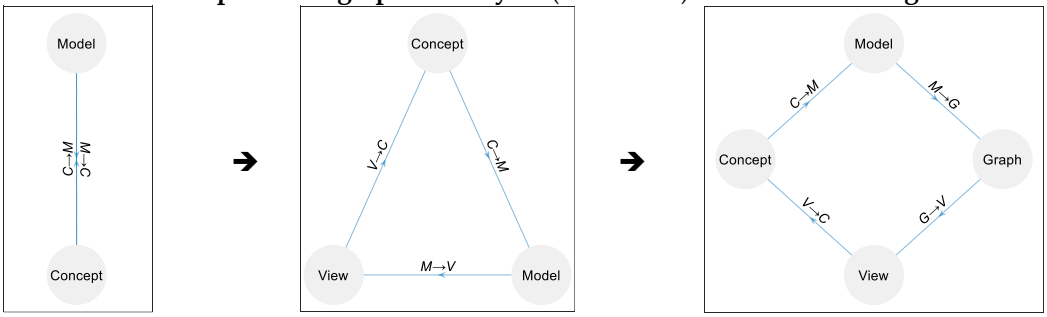
interactions, whose understanding is fundamental to comprehending solutions, designs, or operational processes. Models are also important for facilitating stakeholder communication and discussion. The concept-model duo is therefore a binary self-enhancing system.

Model-Based Systems Engineering (MBSE) is the formalized application of models in systems engineering [4]. MBSE facilitates the digital transformation of systems engineering: generating and communicating digitally-encoded and interchangeable systems engineering deliverables up and down the value chain [5]. Digital systems engineering (DSE) increases the dependency on MBSE, which gradually evolves from a model-focused practice to a value-focused approach and to a critical asset for systems engineering in digital enterprises [6–9].

Generating stakeholder value out of model-based platforms is a primary expected outcome of DSE. Transforming system models into analyzable and reusable artifacts, decision-supporting information, and machine-generated insight is therefore a critical contribution to enterprise-wide DSE. A DSE ecosystem architecture defined in [10] specifies modeling, infrastructure, data services, simulation, testing, analysis, and repositories (MIDSTAR1) as primary DSE services, and management tools, interoperability services, digital representations, systems, things, auditing, and reporting services (MIDSTAR2) as primary interfaces. We shall refer collectively to such results or artifacts following model processing or analysis as *views* of those models.

Model-based views abstract, highlight, or pivot specific aspects, and become increasingly more important as models grow bigger and more complicated. Observing, reasoning about, processing, analyzing, and deciding according to such views results in ongoing concept revisions [11]. The concept-model-view triangle is therefore a self-enhancing system that extends the concept-model duo. However, the potential combination of modeling languages (MLs), models, and views explodes: if every model were to be represented by every necessary view, the number of mappings would be a product of the number of models M by the number of views V, i.e., $M \cdot V$.

In the absence of dynamic and robust model-to-view transforms, concept understanding is shaped by ML-bound views. A generic representation (GR) across a wide variety of models that MLs could map into, and views could map out of, can make a difference. Graph data structures (GDS) can serve as GRs [12–15]. A GDS is simple, robust, scalable, and amenable to both relational and graph-theoretic algorithms. Graph and GDS are not completely interchangeable concepts: A graph is a visual representation of a subset of the GDS, and a GDS is a logical data structure that represents relations, which can be illustrated in graphs. Nevertheless, the term *graph* is used for brevity and intuition for the GDS. A graph rendition of any portion of a GDS, whether raw or processed, is indeed a view of the model. A robust transformation of models into GDS, and generation of views from GDS, shall make models and views interoperable and interchangeable, and extend the collection of views that support model analysis. A concept-model-graph-view quadratic cycle is potentially a more effective, extensive, and efficient cognitive-computational concept modeling and analysis process than the concept-model-view trio. The evolution from the concept-model duo through concept-model-view trio to **concept-model-graph-view cycle** (CMVGVC) is illustrated in **Figure 1**.



(a) Concept-Model Duo (b) Concept-Model-View Trio (c) Concept-Model-Graph-View Cycle

Figure 1. (a) The Model⇔Concept duo extended to (b) Model→View→Concept trio extended as a (c) Concept→Model→Graph→View Cycle (CMGVC).

Matrices are useful views of models. Matrices facilitate analyzing quantitative and qualitative relations. A matrix is a bidimensional discrete data structure that lends itself to various mathematical

or logical analyses. Matrices are common, intuitive, and easy to work with for scientists, engineers, and analysts. As graphic MLs evolved, matrices had not found a place as formal representation modalities, although matrix-like structures or layouts remained present to some extent in modeling frameworks. The IDEF0 notation advocates a diagonal layout of blocks which resembles the N^2 matrix of inter-component dependencies [16]. UML Activity Diagram assumes a "swimlane" layout that helps divide complex multi-participant activities with columns. Sequence Diagrams apply a similar idea, using vertically-aligned lifelines, as anchors for input and output exchange steps [17].

Simple matrices are two-dimensional and non-hierarchical. This construct is insufficient for describing complex architectures. Requirements or structural elements are leaves in an asymmetric hierarchy, which may differ in depth and detail level. Therefore, the matrix kernel must be integrated with hierarchical representations of row and column entities. Advanced matrix representations for systems engineering include the Verification Cross-Reference Matrix (VCRM), Design Structure Matrix (DSM), and System Architecture Matrix (SAM). The Department of Defense Architecture Framework (DoDAF) advocates the use of matrices in some of its views [18].

We collectively refer to advanced purpose-driven matrices as described above as Stakeholder-Informing Matrices (SIMs). A SIM is any matrix, matrix-like, or matrix-based data representation, whose purpose is to inform stakeholders, decision-makers, analysts, and other readers, to help them reason, make better decisions, and take informed action. SIMs organize, summarize, and contextualize information that may be scattered across multiple models or sections of models.

Various attempts to represent models with SIMs were mostly model-, aspect-, language-, or visualization-specific (see Section 2). A mathematically sound and robust framework for transforming models to views is needed. In fact, since models are instantiations of MLs, and modeling consists of instantiations of ML patterns, we ought to transform the ML's set of patterns into a superset of representable data structures, rather than from a specific model to a specific view.

This paper defines the CMGVC and introduces both the cognitive and computational transforms it includes. Our framework relies on Category Theory [19]. A mathematical category consists of objects and morphisms. An Object represents a type, and a morphism is a mapping between types. The Curry-Howard-Lambek Correspondence states that categories, theories, and programming languages are equivalent, and that writing a software program is like defining a mathematical category and like proving a mathematical theory [20, 21]. The observation that software programming is like theory building has also been discussed by Naur, explaining the importance of explicit encoding of the underlying knowledge and concept as part of the software program, rather than as implicit in the designer's mind [22].

The categorical equivalence of programs, concepts, and theories has inspired us to assert that modeling languages are categories, and that mappings within and between modeling categories are powerful means of robust model transformations for various applications. This assertion substantiates a formal, holistic approach for perceiving and implementing concept transformations, particularly in complex systems. Such an approach is instrumental for reasoning about complex concepts using the reasoning mechanisms of people's choice, rather than the representation mechanisms originally used for capturing those concepts. In complex systems, this idea of concept representation interchangeability is critical for a paradigm shift, cohesion of modeling and reasoning practices, and enabling digital systems engineering [10].

This paper advances the state of the art in several ways: a) asserting and proving that **modeling languages are categories**; b) augmenting the emerging model-based systems engineering paradigm with robust foundations using Category Theory, rather than supplanting such system representation and analysis approaches with categorical or otherwise-algebraic representations; c) offering a holistic systems perspective over the entire life cycle, metamorphosis, and evolution of the concept, from its inception, through modeling, generic representation, and informative rendering; d) providing mathematical evidence that this approach is superior to both direct and language-bound mappings, e) synergistically integrating and fusing category theory, graph theory, modeling theory, systems theory, and informatics – to facilitate a multi-faceted pipeline of representations, transformations, and visualizations – which is a critical building-block for the digital engineering paradigm; and f)

providing a concrete example for a concept transformation cycle that renders conceptual models built with Object-Process Methodology (OPM) as graph data structures, and graph-data structures as stakeholder-informing matrices, in a significantly more structured and robust manner, which allows for both additional visualization of OPM models and matrix visualization of graphs of other MLs.

The rest of this paper is organized as follows: we review related work, relevant MLs, Graph Theory, and Category Theory in Section 2. We describe the methods to conduct the CMGVC in section 3. We assess the framework in section 4. We demonstrate our method in section 5, and discuss the results in section 6. Conclusions, and potential, ongoing, and future research appear in section 7.

2. Related Work

In this section we review the methods, theories, and concepts that are necessary for constructing a robust CMGVC. We introduce Object-Process Methodology (OPM), the ML used in this study, in section 2.1. We briefly review basic concepts in Graph Theory and graph data representation in section 2.2. We discuss stakeholder-informing matrices for systems engineering applications in section 2.3. We review Category Theory and specifically its applications and derivatives for systems engineering and MBSE in section 2.4. We discuss model transformation to graph in section 2.5.

2.1 *Object-Process Methodology (OPM)*

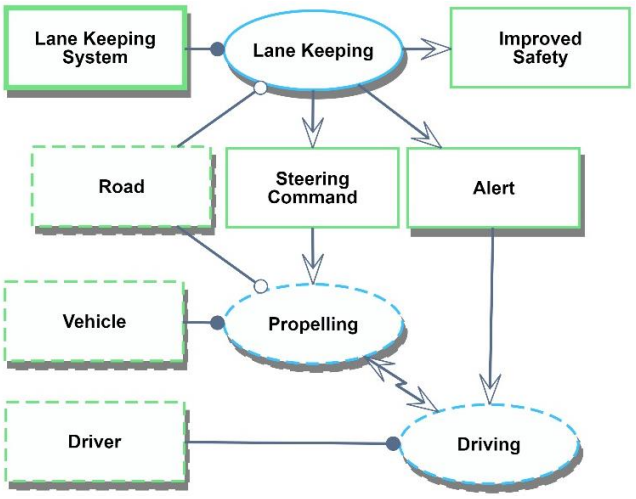
OPM is a leading ML and MBSE framework [4], standardized as ISO 19450 [23], applied in thousands of studies in research and industry. OPM relies on a minimal universal ontology, comprising *objects* (things that exist), *states* (which characterize *objects*), *processes* (things that occur), and *relations* among them. OPM has a visual modality and a textual modality. The visual representation is a set of hierarchically organized Object-Process Diagrams (OPDs). All OPDs use the same notation, which means that OPM has only one kind of diagram, which accommodates structural, procedural, and functional aspects. OPM's visual modality uses rectangles to represent objects, ovals to represent processes, and few more shapes and symbols for relations.

OPM's textual representation consists of sentences in Object-Process Language, OPL – a quasi-natural language. Each sentence corresponds to an OPD construct – a set of linked entities – and vice versa. Each OPD is accompanied by an OPL paragraph – a set of OPL sentences. OPL is formal, machine-readable, and color-coded. A comprehensive model-level OPL Specification (OPS) includes a set of distinct OPL sentences, such that each sentence appears in the OPS only once regardless of the number of appearances in the model (e.g. in different diagrams). Accordingly, every statement can be made only once anywhere in the model in order to hold for the entire model. Thus, modelers cannot specify two syntactically contradicting statements in two different places in the same model.

The logical data representation of the model captures all entities and relations in a graphics-agnostic manner, while the graphic engine renders the visual representation according to the graphical conventions.

OPM has been broadly applied as a system architecting and design, stakeholder engagement and communication, and concept validation paradigm [3, 24–27]. OPM models can be easily built using OPCloud, a cloud-based modeling studio [28]. OPCloud has been applied in medicine [29], industry [30], aerospace [31] and digital transformation [10]. We used OPCloud to build the model, generate a specification, and submit it for analysis by the proposed approach.

An OPM model of a Lane Keeping System for passenger vehicles, based on a Ford concept [32] is illustrated in **Figure 2**, including both the graphical (left) and textual (right) modalities.

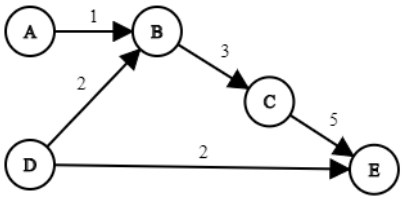


1	Driver is environmental.
2	Vehicle is environmental.
3	Road is environmental.
4	Driving is environmental.
5	Driver handles Driving.
6	Driving consumes Alert.
7	Driving invokes Propelling.
8	Lane Keeping System handles Lane Keeping.
9	Lane Keeping requires Road.
10	Lane Keeping yields Alert, Improved Safety, and Steering Command.
11	Propelling is environmental.
12	Vehicle handles Propelling.
13	Propelling requires Road.
14	Propelling consumes Steering Command.
15	Propelling invokes Driving.

Figure 2. OPM model (OPD on the left, OPL on the right) of a Lane Keeping system, loosely based on [32]. The system handles a Lane Keeping function that provided Improved Safety. Lane Keeping generates Alerts, which inform the Driving process, and Steering Commands, which affect the Vehicle's Propelling function. The Road constitutes an instrument for both Propelling and Lane Keeping. Driving and Propelling affect each other regardless of the Lane Keeping system.

2.2 *Graph Data Structures (GDS)*

A graph is defined as a set of nodes or vertices, with arcs or edges connecting pairs of nodes. In a directed graph (digraph) edges have directions (hence the common synonym "arrows"), such that a tuple <N_1, R, N_2> reads: "N_1 R N_2", "R connects N_1 to N_2", or "the direction of R is from N_1 to N_2". Arrows can have types and additional attributes like values. **Figure 3** (i) shows a simple directed graph with one type of arrow and a value index. Graphs lend themselves to various representations and visualizations, including matrices. **Figure 3** (ii) presents the graph in (i) as an adjacency matrix. The rows and columns of the adjacency matrix are the nodes of the graph, and the cells represent the values or labels of the edges.



(i) directed graph

	A	B	C	D	E
A		1			
B			3		
C					5
D		2			2
E					

(ii) adjacency matrix

S	T	V
A	B	1
B	C	3
C	E	5
D	B	2
D	E	2

(iii) set of tuples

Figure 3. Equivalent representations of relations between pairs of objects as: (i) a digraph, (ii) an adjacency matrix (cell (r,c) holds the arrow value of row object (r) to column object (c), and (iii) a set of tuples (triplets of source object (S), target object (T), and value (V), relating S to T.

Graphs can also be represented as flat sets of tuples <S,T,V> with S as the source node, T as the target node, and V as the value, as shown in **Figure 3** (iii), or <R,S,T> tuples where R is a relation. A graph data structure (GDS) is a set of tuples <R,S,T>, potentially with additional tuple attributes like identity and value. The GDS is a common exchange protocol.

2.3 *Stakeholder-Informing Matrices (SIM)*

In this section we review several examples for the theory and application of SIMs, including design structure matrices, multi-domain matrices, system architecting matrices, and pivot tables.

The Department of Defense Architecture Framework (DoDAF) encourages the use of matrices in some viewpoints [18]. However, there are no formal requirements for the structure or layout of the

matrices. In some cases, the term *matrix* is used to describe other tabular structures that are not matrixial in essence. For instance, a table of inputs with some attributes is not a matrix. The Unified Architecture Framework (UAF), a profile of UML, is a model-based extension of DoDAF [33]. UAF advocates that aspects of the architecture specification's traceability to operational specifications, services, resources, standards, activities, etc. are represented as matrices, however there are no data structures or illustrations to demonstrate how matrices should be extracted from the profiles.

Design Structure Matrices / Dependency Structure Matrices (DSMs) are powerful tools for analyzing interrelated aspects, comprehension, and decision making [34]. Transforming models into DSMs is studied in [35–37]. DSMs require explicit definition of row and column ontologies. This is not trivial for sematic ontology, which reflects the modeled system or domain.

Multi-Domain Matrices (MDMs) extend DSMs by including multiple sets on each dimension and composing multiple matrices [38, 39]. MDMs are difficult to construct and visualize due to their size. MDMs require an underlying definition of a) a single super-set of concept terms, b) a set-to-set mapping for all set pairs, and c) a member-member mapping for each pair of set members.

System Architecture Matrices (SAMs) are special MDMs for system architecture analysis, e.g., a 2-domain SAM that maps {Systems, Processes} to {Systems, Processes} [37]. The mapping relations can be determined per sub-SAM according to the desired analysis. An example is illustrated in **Table 1** (a). The mappings *Instrument*, *Effect*, *Generation*, and *Consumption* are drawn from the set of OPM relations. Additionally, the mappings *Attribute*, *Input*, *Output*, and *ID* are defined as indirect yet more intuitive relations. For example, *Attribute*(S) is the set of model entities E_i that participate in an *Exhibition-Characterization* relation to S: *Exhibition*(S,E_i) or *Characterization*(E_i,S). *Input*(P_{column}) is the set of entities E_i with an *Instrument*(E_i,P_{column}) or *Consumption*(E_i,P_{column}) relation to P_{column}.

A 3x3 SAM, consisting of Processes, Operands (inputs, resources, and outputs), and Components (structural objects), is illustrated in **Table 1** (b), based on [24]. Each sub-SAM has a relation with which the row item relates to the column item, which can be static or dynamic. A row-cell-column tuple in the matrix would read: <row item (iRow)> <cell item (iRow, iCol)> <column item(iCol)>. For example: [Process P2] *requires* [Operand O2]; [Component C3] *exhibits* [Process P2].

Table 1. System Architecture Matrices

(a) 2-2 SAM mapping of systems and processes [37]

	System (S)	**Process (P)**
System	*ID*(S_{row})	*Instrument*(S,P)
Process	*Effect*(P,*Attribute*(S))	*Result*(P_{row}, Input(P_{column})) *Consumption*(Output(P_{row}),P_{column})

(b) 3-3 SAM of processes, operands, and system components [24]

		[P]	**[O]**	**[C]**
Process	[P]	[PP]	[PO]	[PC]
Operand	[O]	[OP]	[OO]	[OC]
Component	[C]	[CP]	[CO]	[CC]

Classifying model entities as appropriate set members (Process, Operand, Component) is a challenge. A ML that defines syntactic types simplifies identification of type-corresponding set members. However, the problem domain or solution domain may include types that exceed the ML's syntax. Even common terms such as *Stakeholder* and *Operand* are not included in syntactically-rich MLs like SysML [40]. In addition, defining direct and indirect relations in semantic terms (e.g. referring to whole-part relations as either organization structures, activity breakdowns, or product/structure breakdown) is also a challenge, but it is critical for informative SIM renditions.

Generating SIMs automatically is another challenge. Transforming model data into analyzable matrixial representations is not trivial. It mandates transforming any model into a uniform, common, language-agnostic representation that is robust enough to accommodate any domain-specific syntax and semantics, and powerful enough to facilitate any sort of analysis [35, 38].

Pivot tables are powerful data analysis tools, typically used to aggregate, summarize, cluster, slice, and dice relational data. They are massively used in Business Intelligence applications, typically over operational datasets classified by time, geography, identity, organizational unit, etc. Pivot tables are embedded in common spreadsheet tools like Microsoft Excel and Google Sheets. Pivot tables are powerful because they allow the analyst to dynamically manipulate perspectives on the original

dataset, experiment with various perspectives, and reach informative configurations. Pivot tables are scarce in systems engineering [41–43], perhaps due to insufficient data analysis background.

2.4 *Category Theory in Systems Engineering*

2.4.1 What is a category?

Category Theory is an abstract mathematical formalism of representations and transformations that describes and deals with the structure of mathematical or mathematically-specifiable concepts. A mathematical category is an abstract structure that consists of *objects* and *morphisms*. Objects are types of entities and morphisms map objects to each other. We shall use the common term *type* instead of *object*, because object semantics in programming and engineering relates to instances rather than classes. A morphism $m: X \rightarrow Y$ has a domain X and codomain Y which are types, and maps each $x \in X$ to a $y \in Y$. Morphisms can be composed if the former's codomain is the latter's domain:

$$\text{if } f: X \rightarrow Y, \text{ and } g: Y \rightarrow Z, \text{ then } f \cdot g = g \circ f: X \rightarrow Z.$$

This sequential composition executes g after f. A category with three types, two explicit morphisms and one composed morphism is illustrated in **Figure 4**. A morphism denotes a mapping of a cartesian product of two types to a third type. The morphism $f: X \otimes Y \rightarrow Z$ maps each combination of $x \in X$ and $y \in Y$ to $z \in Z$. Z can be any structure including a multi-dimensional tuple, or tensor.

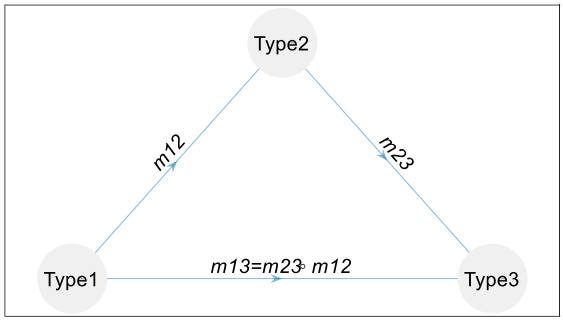


Figure 4. A category with three types: Type1, Type2, and Type3. The morphisms m12: Type1→Type2 and m23: Type2→Type3 are specified. The morphism m13 is a composition of m12 and m23. We can transform Type1 to Type3 by transforming Type1 to Type2 and then Type2 to Type3.

A *functor* is a structure-preserving mapping between two categories, which maps both the types and the morphisms from one category to another. Functors are the categorical equivalents of functions in algebra. $F: C_1 \rightarrow C_2$ denotes a functor that transfers the types and morphisms in C_1 to types and morphisms in C_2. Defining functors from one category to another is important for establishing categorical foundations for scientific and engineering theories as it maps the concepts in a theory domain to concepts in a codomain that is possible to work with.

2.4.2 How Are Systems, Models, and Categories Related?

The Curry-Howard-Lambek Correspondence [20, 21] asserts that programming languages and type systems are categories, in which the morphisms are rules or patterns that map types to each other. A software program is an instance of a programming language that reflects or represents some real-world system. Several software programs may represent the same real-world system and even solve the same real-world problem in different ways or using different programming languages. Programs can also work together to solve higher-level problems and represent higher-level systems.

Wymore defines a system as a morphism of input to output (IO), composed of a morphism of input to state (IS) and state to output (SO) [44]. A *system homomorphism* is a functional equivalence of two system designs (or models) in terms of their state-sets, their input-sets, and output-sets. A system $Z1$, with a state-set $SZ1$, an input set IZ1 and an output set OZ1, is a homomorphic image of a system $Z2$ (with respective sets $SZ2$, $IZ2$, and $OZ2$) if and only if it maps the states of $Z1$ to those of $Z2$, the inputs of $Z1$ to those of $Z2$, the outputs of $Z1$ to those of $Z2$, the input-state transforms in $Z1$ to input-state transformations of pairs to states in $Z2$, and the transformations of state to outputs in $Z2$ to transformations of states to outputs in $Z1$. Accordingly, in a category of system models, the objects are system models, and the morphisms are Wymore's system homomorphisms.

2.4.3 Applications of Category Theory in Systems Engineering, Analysis, and Design

Category Theory is gaining attention in recent years as a potential underlying formalism for systems engineering, as an important pillar in systems science, and as the basis for an open information modeling and analysis platform [45], with semantic roots in design theory [46]. The envisioned platform's robustness would allow for domain-agnostic cherry-picking and assembling of constructs and transforms. A roadmap for bridging the gap between theory and practice, including particular applications for systems engineering, was published by the US National Institute of Standards and Technology (NIST) [47]. State-of-the-art applications of Category Theory in systems analysis, engineering, and design are summarized in **Table 2**.

Table 2. Category Theory Applications in Systems Engineering, Analysis, and Design

Topic/ Title/Application	Description	References
Complex/ Cyber-Physical Systems Architecture and Design	Object-Process Networks, a language for System Architecting	[48, 49]
	Representing a system as a category with component types and inter-component relations as morphisms, with Ologs (knowledge representation structures that capture non-mathematical, freely-defined relations among objects as morphisms) to specify design constructs, current-state situations, constraints, and requirements	[50] [51]
	Iterative co-design of electro-mechanical functions in a cyber-physical system architecture	[52–55]
	hierarchical requirements engineering, gradually evolving a system architecture; formal and verifiable system design	[56–58]
	Structural and functional composition of system models	[59]
	Using operads—categorical structures that map multi-object compositions to a single object—for hierarchical decomposition, design synthesis, separation of syntax from semantics, and semantic reasoning about complex systems	[60, 61]
Model-Driven Software Engineering	Building mathematical foundations for model-driven software engineering (MDE), model management, model merging, bidirectional model updates, design patterns, and model transformations	[62, 63]
Computer-Aided Design	Representing, integrating, and coordinating computer-aided design (CAD) languages, models and simulations for the design of a difficult technical complex (DTC)	[64]
Categorical Formulations of Systems	Ontological analysis based on a category *ONT* with Ontologies as types and ontology mappings as morphisms	[13, 65]
	representing a system as a symmetric monoidal, compact closed category with block types and input-output morphisms	[66]
	algebraic formulation of open dynamical systems that can be characterized as resource sharers as a category of resource-sharing machines	[67]
Stochastic Models	domain-specific reasoning in stochastic systems, mapping causal probabilistic models the category $Stoch$, whose types are measurable spaces, and whose morphisms are Markov kernels between spaces	[68]
Scientific Models	Semantic modeling for extracting information from, reasoning about, augmenting, and composing computational models of complex systems and natural phenomena	[69–71]

As MBSE is becoming a common and preferred way of conducting systems engineering, it is essential to harness any applicable formalism to MBSE and to leverage MBSE through a holistic federation of such formalisms. This paper focuses on the appropriate representation and analysis of system models that already use formal modeling languages, without replacing the existing languages with a mathematical formalism. Instead, the existing models are referred to as given artifacts, and the focus is on model analysis, visualization, and delivery to various stakeholders.

A C-sets is a functor from some category C into Set, the category of sets and functions [72]. Thus, a C-set $F(C)$ maps every object $c \in C$ to a set $S_F(c)$, and every morphism $f_C : c \to c' \in C$, to a function $f_F(f_C) : X(c) \to X(c')$. If C is a theory or concept, then its C-set is an instance or model of that theory. If C is a programming language or ML, its C-set is a projection of programs or models into Set. One such set can be a set of tuples that represents a graph or hypergraph (with relations between more than two nodes).

Hypergraph categories are general mathematical models of hierarchy. They are applicable to various computational constructs including matrices, graphs, Markov chains, and Petri nets [72]. Hypergraph models can represent system structure, behavior, function, and interaction. Graphs of models are amenable to representation in the broader hypergraph category, an important enabler of model analysis, composition, comparison, and utilization [15].

2.5 *Transforming Models to Graphs*

Graph-centric model representation is not new [73]. MLs are inherently graphic, however graph-theoretic model analysis has not evolved alongside MLs. Recent interest can be attributed to the revival and maturation of graph technology, graph databases, and graph analytics, e.g., [74–76].

Mapping conceptual models to graphs remains a challenging task. Models are stored as logical data-structures, not straightforwardly as graphs. Graphs cannot directly capture advanced conceptual model notions, such as classification, nesting, logical relations, and overlaps. Transformation rules that apply directly for the data structure used by the ML can help. For instance, UML has a uniform interchange structure, XMI, an XML structure that facilitates UML and SysML model conversions among various modeling tools. SysML model XMI representations can be transformed into graphs, compared against pattern graphs, and searched for design patterns [14].

The Resource Description Framework (RDF) protocol for web ontology specification consists of a set of triplets and resembles GDS. GDS can be viewed as a set of RDF triplets, and vice versa. OPM models can be represented via RDF [77–79].

OPM's textual machine-generated specification is another representation that is easy to read and understand for both humans and computers [23]. The textual specification covers the OPM model's logical structure. Inferring the system architecture from the model's exported report is possible, but it is not possible to reconstruct the schematic layout of the diagrams. However, filtering out graphical layout bias can be beneficial because the layout may affect decision-makers (for instance, due to depicting entities as central or peripheral).

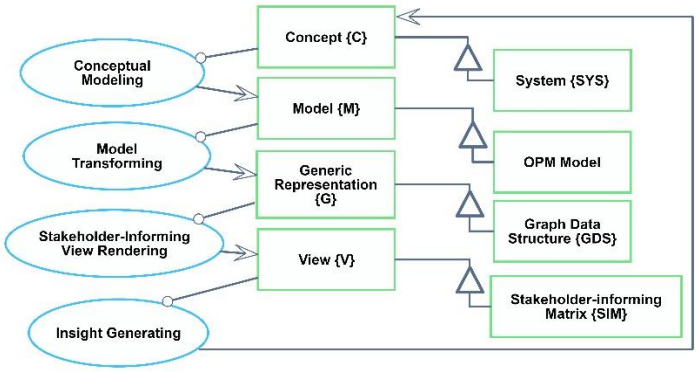
A transformation of a *systemigram*—a graph-like model representing a complex system—into a flat graph data structure has been used for assigning quantitative metrics to qualitative relations and applied for threat assessment in strategic warfare capability analysis [80].

3. Methods

This section describes a category-theoretical framework and methods for architecting and executing the CMGVC, which consists of four processes: a) Conceptual Modeling, which is essentially the transformation of concepts to models; b) Model Transforming, a process of converting models into generic, language-agnostic artifacts; c) Stakeholder-informing View Rendering, a process of transforming generic representations into insightful and usable visualizations or representations; and d) Reasoning, the cognitive process of rationalizing, understanding, perceiving, deciding, and revising the concept. This process is specified using an OPM model and illustrated in **Figure 5**.

The CMGVC framework consists of defining mappings between the conceptual and computational phases of the cycle: the concept phase (section 0), the model phase (section 3.2), the generic representation phase (section 3.3), the view phase (section 3.4), and the iteration (section 3.5).

While the CMGVC is universal, we focus on conceptual system architectures as an instance of C, on the OPM modeling language (M), on graph data structures as a global representation(G), and on stakeholder-informing matrices as views (V). This specialization is also shown in **Figure 5**.



1	**OPM Model** is a **Model, M.**
2	**Graph Data Structure, GDS,** is a **Generic Representation, G.**
3	**Stakeholder-informing Matrix, SIM,** is a **View, V.**
4	**System, SYS,** is a **Concept, C.**
5	**Model Transforming** requires **Model, M.**
6	**Model Transforming** yields **Generic Representation, G.**
7	**Insight Generating** requires **View, V.**
8	**Insight Generating** yields **Concept, C.**
9	**Conceptual Modeling** requires **Concept, C.**
10	**Conceptual Modeling** yields **Model, M.**
11	**Stakeholder-Informing View Rendering** requires **Generic Representation, G.**
12	**Stakeholder-Informing View Rendering** yields **View, V.**

Figure 5. The CMGVC, as a cognitive-computational cycle of Conceptual Modeling, Model Transforming, Stakeholder-informing View Rendering, and Reasoning. The System, OPM Model, GDS, and SIM are specializations of the respective generic artifacts of the cycle: Concept, Model, Generic Representation, and View.

We present a series of propositions regarding the possibility, validity, and superiority of creating views from models through a robust mechanism. Our main assertion in Proposition A is that a transform from any model M to any view V through an intermediate GDS representation G is superior to a set of direct transformations from all models M_1, .., M_M, to all views V_1, .., V_V.

Proposition A. Superiority of GDS-Based Transform to Direct Transforms

A graph-mediated transform M→G→V is superior to direct model-to-view transforms M→V: $\boldsymbol{MGV \succcurlyeq MV}$

Proposition B asserts that a transform from any model M to any view V through an intermediate GDS representation G is also superior to a transform from all models M_1, .., M_M through a ML-bound generic representation GR to all views V_1, .., V_V.

Proposition B. Superiority of GDS-Based Transform to Language-Specific Transform

A graph-mediated transformation M→G→V is superior to an ML-bound mediating representation-based transformation MRV: $\boldsymbol{MGV \succcurlyeq MRV.}$

MGV is a composition of M→G and G→V. Proposition C asserts that a transform M→G exists.

Proposition C. Existence of a Model-to-Graph Transform

Let ML be a modeling language and M a model in ML. There exists a formal, valid, and feasible model-to-graph transform MG from ML into GDS, MG: ML→GDS.

MGV is a composition of M→G and G→V. Proposition D asserts that a transform G→V exists.

Proposition D. Existence of Graph-to-View Transform

Let V be a view and G a set of GDS tuples. There exists a formal, valid, and feasible graph-to-view transform GV from GDS to V, GV: GDS→V.

Proposition E asserts specifically, with a focus on SIM views, that SIMs are valid views of GDS.

Proposition E. Existence of Graph-to-Matrix Transform

Let SIM be a stakeholder-informing view and G a set of GDS tuples. There exists a formal, valid, and feasible graph-to-SIM transform GV from GDS to SIM, GV: GDS→SIM.

3.1 *The Conceptual System Architecture as a Category*

Wymore's definition of a system [44], in conjunction with the conventional representation of system architecture as a combination of structure and behavior [3], gives rise to a system category, SYS, with the following types: Structure, Input-Output (or interchangeably, Onput [81]), Resource, and State. Onputs and Resources can be referred to as Operands. The morphisms include the Behaviors, which transforms Operands, and Relations that map the types to themselves and to each other. A morphism $f: Op \otimes St \rightarrow Op \otimes St$ is a behavior of the system at any level, which transforms a combination (denoted as a Cartesian product) of operands (Op) and the system's state (St) to another combination of operands and state. The morphism f is a composition of input→state and state→output :

$$f_{in}: Op \otimes St \rightarrow St \ ; \qquad f_{out}: St \rightarrow Op \otimes St \ ; \qquad f = f_{out} \circ f_{in}$$

A system sys is an instantiation of the category SYS. Therefore, Wymore's system homomorphism is a *functor*, denoted as $F_{Wymore}: SYS \rightarrow SYS$.

Cyber-physical systems (CPS) reside concurrently in the physical and cybernetic spaces, and constitute the primary concern of systems engineering. In our categorization, CPS behaviors/functions are morphisms, while system components (sensors, processors, actuators, etc.) and operands (Currency, Data, Energy, and Matter – CDEM) are types. CPS components carry out functions that convert input operands to output operands. Some examples are shown in **Table 3**.

Table 3. Examples of Cyber-Physical System Morphisms

CPS Component	CPS Morphism	Input	Output	Formulation
Engine	Energy Generating	Matter	Energy	$f_{Engine}: Matter \rightarrow Energy$
Sensor	Sensing	Energy	Data	$f_{Sensor}: Energy \rightarrow Data$
Actuator	Actuating	Data	Energy	$f_{Actuator}: Data \rightarrow Energy$

Conceptual architecting usually specifies the function before the form, while physical architecting might work in the opposite direction. The attribution of a functional-behavioral morphism f_C to a structural entity C implies or entails a relational morphism $r_{f \rightarrow C}: f_C \rightarrow C$ that assigns a function to a function-performing system component, and a dual $r_{C \rightarrow f}: C \rightarrow f_C$ that allocates a component to perform a function. This mapping of morphism to type and type to morphism implies a type-morphism duality of system behavior. CPS functions can be composed to create higher-level functionalities, for example: sensing, then actuating: $Matter \rightarrow Energy \rightarrow Data \rightarrow Energy$. We can also define functions as Cartesian products, e.g. a plant's function is:

$$f_{Plant}: Data \otimes Matter \otimes Energy \rightarrow Data \otimes Matter \otimes Energy \ .$$

Input data (commands), matter (raw material), and energy (e.g., electrical power) are converted to output data (statuses, reports), matter (finished goods, byproducts), and energy (e.g., heat). Some sequences may not be valid within particular system domains. For instance, in logistics, manufacturing a device (an actuator or sensor) is possible, but in an operational system such as an aircraft or autonomous vehicles, manufacturing a new part may not make sense. On the other hand, sensor "manufacturing" of images from signals may be valid.

Our categorical formulation views systems as behavior-enabling structures, which affect operands. Operands are also types. In fact, a system can be an operand, and an operand can be a system. Imagine a vehicle exiting the production line as an operand, and emerging as a system, or software delivered as a file, i.e., as an operand, and emerging as an executable, i.e. a system.

3.2 *The Modeling Language as a Category*

Modeling languages are categories, since MLs are essentially programming languages [82], and programming languages are categories [20, 21]. Moreover, programs are also equivalent to theories [22], and likewise, models are equivalent to system concepts, which can be thought of as theories about how a system works or might work. System models are instantiations of MLs, much like software programs are instantiations of programming languages. Similarly, representation languages for creating graphs, matrices, trees, or animations may also be thought of as categories. Transitions across representations are essential for gaining system understanding, and for implementing the system or parts of the system. For example, transforming functional models into a visual animations, hardware and software designs may clarify how a system works and how to implement it.

The assertion that a ML is a category must be backed up by a valid categorical representation. MLs like OPM and SysML [83] capture structural, behavioral, and relational entities syntactically. The concept set constitutes a syntactic domain-agnostic ontology that accommodates a wide range of instantiations. The *Block*, for instance, is a fundamental SysML concept. SysML Blocks are modeled in Block Definition Diagrams (BDDs) and Internal Block Diagrams (IBDs). BDDs capture relations among blocks, while IBDs capture interactions among blocks and their internal structure. A block may be both structural and functional, which can be both useful and confusing. For example, a block can specify both the sensor and its sensing function, f_{Sensor}.

In SysML and its predecessor, the Unified Modeling Language (UML) [84], capturing a component's behavior, rather than its structure or function, requires behavioral notation such as Activity Diagram or State Chart. In the Activity Diagram notation activities and actions are 'boxes' (types), while control or data flows are 'arrows' (morphisms). In the State Chart notation states are *boxes* and state-transitions are *arrows*. The different semantics applied to boxes and arrows in SysML diagrams can be confusing, although experienced analysts know which notation to use to interpret various diagrams to make sense. Transforming SysML models through categorical specifications into a unified notation requires tremendous effort. We defer this endeavor to future research.

Conversely, OPM models use a minimal ontology of objects, processes, and relations, in which representation ambiguity and redundancy are eliminated or minimized. Since OPM's fundamental building blocks are objects that represent structure and processes that represent behavior, with a relatively small set of relations among them, a category of the OPM language is likely to be small and handy. The challenge is in correctly representing the language domain. One apparent categorical specification could define an OPM category with objects as types and processes as morphisms.

Types in one category can be morphisms in another category. OPM makes no syntactic distinction between component-representing objects and operand-representing objects. In fact, the same object can be both an operand of one system and the operator in another. Therefore, objects can be morphisms if they perform processes, and operands if they feed into or out of processes. Processes and relations represent system morphisms but are captured as graphical shapes, which may be puzzling. Processes and relations also have attributes of their own (durations, cardinalities, etc.).

Any model element is a type, including those that might appear to have operational semantics, such as processes, functions, transitions, etc. This also makes sense from a visual perspective: each shape on the canvas must be explicitly defined, regardless of its geometry or semantics. Thus, both 'boxes' and 'arrows' must be captured in MLs as types.

If operational entities are types, rather than morphisms – then what are the morphisms? Every model has a logical layer that underlies the visual layer. Therefore, every model element must be captured logically, otherwise it is no more than a meaningless sketch. It follows that there exists a hidden type, Specification (Spec) that captures the logical representation of the visual model. Thus, ***specifying*** – mapping visual elements to Spec – is a morphism.

However, there is another kind of morphism: ***modeling*** – which leads us through model creation or transformation. As modelers, we can take various modeling steps based on the state of our mode: a) start with a blank diagram; b) add an object, or process – transforming our diagram into a diagram with an object or a process – Hence the morphisms Diagram→Object and Diagram→Process exist; c) create a new diagram to specify the details of an object or process – hence the morphisms

Object→Diagram and Process→Diagram exist; d) add a state to an object: Object→State; or e) add a relation between an entity (object, process, or state) and another entity, which results in a two-step morphism: Entity→Relation and Relation→Entity. OPM does not permit creating a Diagram from a State or Relation – although such an extension can be theoretically valid and possible. Challenging the existence or absence of a modeling morphism may help enhance the ML.

Computational commands implement manual modeling operations on the screen or automated commands via a programming interface. This allows for automated modeling and for creating models based on other formal representations. To the best of our knowledge OPM does not have a programming interface, but if provided in the future, it can adhere to our categorical formulation.

Reconstructing a model from its specifications gives rise to a third morphism to transform Spec statements into visual geometries or textual/logical reports: ***rendering***. The pair *specifying—rendering* is isomorphic if a Spec can facilitate complete reconstruction (rendering) of the model. OPML also renders part of the Spec as a textual OPL paragraph – OPM's textual modality.

The notion that *modeling*, *specifying*, and *rendering* are morphisms in a ML category is illustrated in **Figure 6** for the OPM ML Category, *OPML*. The Spec is a critical type, as it enables reconstructing, composing, transforming, and comparing models. Rendering the Spec as a processable mediating Report artifact that represents the model substantiates mapping to another representation – a functor that maps the ML to another category. OPCloud's Exported Report is a mediating artifact.

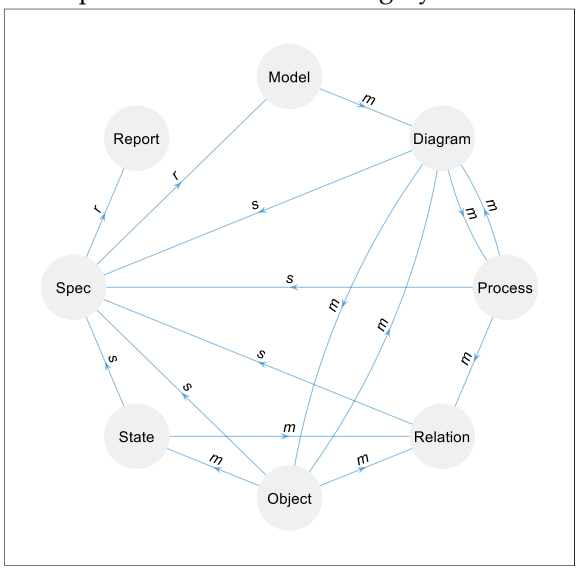


Figure 6. The OPM modeling language, OPML, as a category. Modeling (the morphism *m*) is like hopping through the instantiation of category types based on existing artifacts. For example: a) creating a new Diagram D1 in a Model M, b) creating an Object Ob1 in D1, c) creating a Process P1 in D1, d) creating a RelationRr1(Ob1,P1) from Ob1 to P1, e) creating a new Diagram D2 from P1, f) adding States St11, St12 to Ob1 in D2, and so on. Every instantiation of a type also affects the Spec, and therefore all graphical elements have a mapping into Spec (the morphism *s*). Spec also feeds into the Rendering morphism *r*, which renders a Report from the Spec, currently in the form of a PDF file. Diagram created using MATLAB 2019b digraph plotting capabilities [85].

Model entities are its interfaces to the System category, *SYS*. Model, Diagram, and Spec are not real System concepts – they merely help us manage system complexity. This System-to-Model mapping constitutes the C→M portion of the CMGVC. The OPML→GDS functor maps all the types in OPML to GDS tuples. This C→M→G segment of the CMGVC is illustrated in **Table 4**.

Table 4. Mapping the category of Systems, ***SYS*** to a category of OPM models, ***OPML***, and to a category of Graph Data Structures, ***GDS***

Category	**Systems, *SYS***	**C→M**	**Object-Process Modeling Language, *OPML***	**M→G**	**Graph Data Structure, *GDS***
Types			Model, Diagram, Spec		
	Structure	→	Object		
	Relation	→	Process, Relation		
	State	→	State		
	Operand	→	Object		
			Report	→	Tuple set
Morphisms	Behavior	→	Process		

Category	Systems, *SYS*	C→M	Object-Process Modeling Language, *OPML*	M→G	Graph Data Structure, *GDS*
	N/A		modeling, specifying, rendering		tuple operation

3.3 *Transforming Models to Graphs*

The MGV transform is a composition of a transform from a model into a uniform GDS representation (M→G), with a second transform from GDS into view (G→V). Transforms into GDS require mapping each ML construct to type in the GDS category. Graphs have only two elements: *nodes* and *edges*. GDS has only one element: *tuple* (an abstraction and extension of a node-edge-node triplet). A graph is merely a view on GDS. Such transforms are possible on graphic MLs, although graphics do not always map to simple graphs – for instance, when shapes are contained inside other shapes or overlap with other shapes (as in Venn diagrams). OPCloud's Exported Report includes the textual OPL specification and lists of all the objects, processes, and relations in the model. This helps in mapping all model entities and relations and translating them to GDS tuples. The final product G is a set of tuples, which includes a relation R, a source node S, a target node T, a unique identifier U, and a valuator V, denoted as the RSTUV tuple. V is useful for various applications such as quantification, verification, validation, versioning, and configuration control.

Let us revisit the lane keeping system model in **Figure 2** and study its transformation into a GDS. We illustrate the same model as a graph, as shown in **Figure 7**.

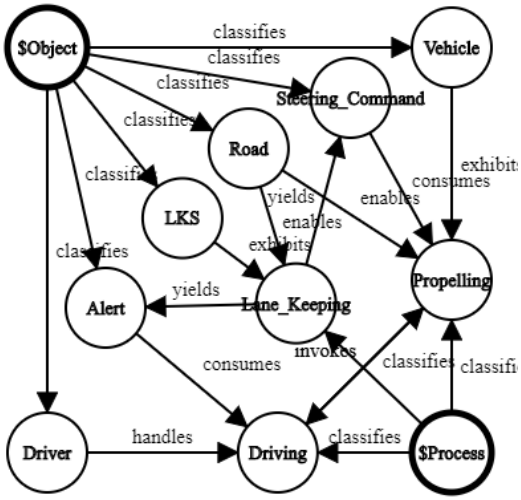


Figure 7. A graph representation of a subset of the Lane Keeping System (LKS) model in Figure 2. The graph maps the OPM-specifiable terms "$Object", and "$Process" to Domain entities: Object classifies Vehicle, Driver, and LKS; Process classifies Driving, Propelling and Lane_Keeping. Domain-internal relations include the mutual invocation between Driving and Propelling, output of Alert and Steering_Command to Driving and Propelling, respectively, and the exhibition of Lane_Keeping by LKS. The graph is a visualization of a subset of the GDS that represents the model. It is clearly already difficult to comprehend, and is illustrated for intuition about the possible mapping of a model to a graph.

RSTUV is sufficient for representing model data. It also captures aspects that are not specifically illustrated in the model. For example, it specifies the model's inclusion of the diagrams, as well as each diagram's inclusion of visualized objects and processes. The set of RSTUV tuples is obtained by executing the following functorial rules from *OPML* to *GDS* (also summarized in **Table 5**):

0. ML types are defined as RSTUV identity tuples. This step need not execute for each model.
1. Entities are mapped to their OPM entity type (model, diagram, object, process, etc.) using a Classification relation.
2. Relations, such that E_i and E_j are entities connected by relation R, are mapped as is.
3. Entities E_j are mapped to each diagram D_i that includes them by an Inclusion relation.
4. Relations are mapped to each diagram that includes them through an Inclusion relation.
5. Classifications of the affiliation of an entity (as either systemic or environmental) [4].
6. Classification of the essence of an entity (as either physical or informatical) [4].

According to Category Theory, each object must be connected to itself through an identity morphism [50]. Thus two more mappings are required:

7. Entities are mapped to universally-unique identification numbers (UUIDs) as an Identity relation that constitutes the identity morphism of each entity onto itself.

8. Relations are mapped to UUIDs through an Identity relation that constitutes the identity morphism of each relation onto itself.

Table 5. Mapping rules from OPML to GDS

Rule	R	S	T
0.	Identity	$Type_t$	UUID($Type_t$)
1.	Classification	E_i	OPM.Classification(E_i)
2.	OPM.Relation(E_i, E_j)	E_i	E_j
3.	OPM.Inclusion(D_i, E_j)	D_i	E_j
4.	OPM.Inclusion(D_i, Relation(E_j, E_k)	D_i	UUID(Relation(E_j, E_k))
5.	OPM. Affiliation	E_i	OPM.Affiliation(E_i)
6.	OPM.Essence	E_i	OPM.Essence(E_i)
7.	Identity	E_i	UUID(E_i)
8.	Identity	R_k	UUID(R_k)

The OPML→RSTUV functor that executes the above mappings is illustrated in **Figure 8**. The functor extracts the OPM elements from the report generated by OPCloud (`OPM.ExportedReport`). `ExportedReport` provides most of the information needed for analyzing an OPM model (It does not include shape positions in the diagrams, and therefore, it is not possible to reconstruct the diagrams with the shapes' exact positioning, but only up to the participation of elements in each diagram).

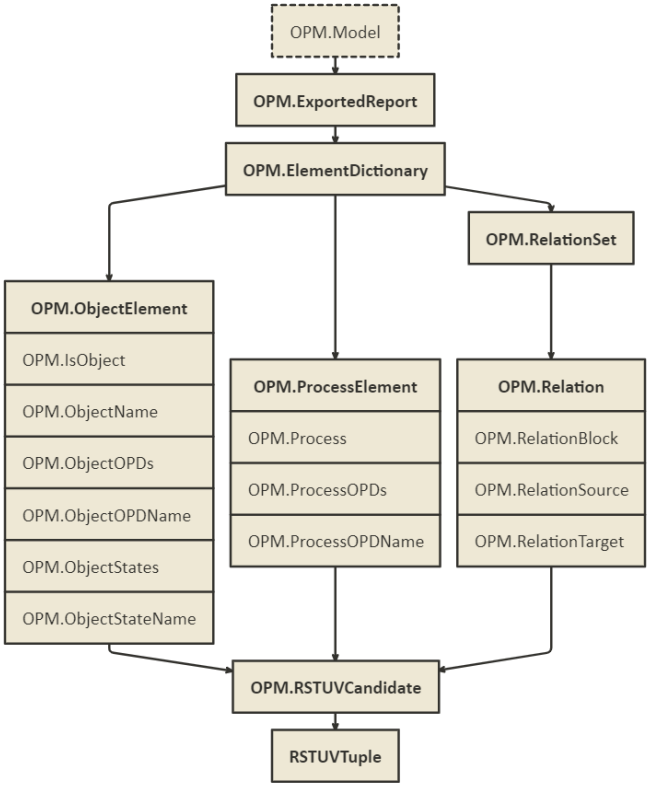


Figure 8. An ontology mapping functor schema from OPML to GDS RSTUV tuples extracts an `ElementDictionary` from the model's `ExportedReport`. and transforms `ElementDictionary` into three sets: `ObjectElement`, `ProcessElement`, and `RelationSet`. `RelationSet` is decomposed into a set of Relations. The functor computes attributes for each entity type from the report. For example, it lists the `ObjectOPDs` in which each Object is defined, and extracts the Source object/ process and the Target object/process for each Relation. Each entity is then transformed into a set of `RSTUVCandidates`. Duplicate candidates are removed before the set of `RSTUVTuple` is compiled. Illustration created using the www.NomnoML.com tool, a candidate for future MGV transformation.

From `ExportedReport` we generate two interim categories: `ElementDictionary` and `OPLSpec`. We refer to both objects as sets, but due to the complicated and heterogenous structure of set members, we need further decomposition and transformation of set members to their appropriate data structures. For example, we need to find the OPDs in which each Object and Process appear.

`ElementDictionary` does not specify appearance of relations in OPDs, but the statements referring to the relations are specified in the OPL text accompanying each OPD. To extract this information, we need to analyze `OPLSpec`, identify the sentence that specifies each relation under the diagrams in which it is shown, and match the OPD→`OPLStatement` relation to the right Relation.

`OPLStatements` are compound, i.e., they aggregate multiple target entities having the same relation with a source entity into one `OPLStatement`. In fact, the OPL is a textual representation of the model that provides some information about the model. For example, the sentence "**Lane Keeping System** exhibits **Lane Keeping**, as well as **Improved Safety**" refers to two separate Exhibition relations shown in **Figure 2**: **Lane Keeping System → Lane Keeping**, **Lane Keeping System → Improved Safety**. This mandates additional text parsing, including the following steps:

1. Identify the position of the keyword " exhibits " in the `OPLStatement`;
2. Parse the left part of the sentence before the keyword position to extract the Source;
3. Parse the right part of the sentence after the keyword end to obtain a list of Targets;
4. Extract target entities from Targets by detecting the separators ", " and ", as well as ";
5. Create a mapping {R=Exhibition, S=Source, T=Targets(t), U=uniqueID} for each t in Targets;

For each exhibition link in `ElementDictionary`, if the RST sub-tuple is found in the OPD, create a mapping {R =Inclusion, S = OPD, T = U of the relation}.

`OntologyMapping` is a generic transformation function that converts or extends ontological terms and execute all parts of the functor illustrated in **Figure 8**. A prototype of `OntologyMapping` has been implemented using MATLAB 2019b. The protoype can read a set of mappings for any ML and apply them to an input source that represents a model in that ML. The prototype supports reading model representations from various input sources, including Microsoft Excel, PDF files, and JSON (.json), and storing the results to a Microsoft SQL Server database or Microsoft Excel file, text file, `OntologyMapping`. The evolving prototype architecture will be discussed in future publications.

`OntologyMapping` starts from the raw input representing external models and other sources of information (e.g., `OPM.ExportedReport`). It continues **recursively** to either create new items or extend existing items. The `ExportedReport`→`ElementDictionary` transform, followed by the three transforms `ElementDictionary`→{`ObjectElement`, `ProcessElement`, `RelationSet`} create new items sets. `RelationSet` is further transformed into a new set of `Relation` items. This additional step is due to OPCloud's exported report clustering of all relations of the same type in groups.

`OntologyMapping` extends `ObjectElement` with attributes such as `IsObject`, `ObjectName`, and `ObjectStates`. It similarly extends `ProcessElement` with the attributes `IsProcess`, `ProcessOPDs` etc., and `RelationSet` with the attributes `RelationBlock`, `RelationSource`, and `RelationTarget`.

The strength and robustness of `OntologyMapping` allows it to recursively search for additional mappings needed due to the creation of new items, but it would first make sure all the extended attributes are computed, because those attributes might be needed for creating new items. For instance, to create `RSTUVCandidates` that cover all the relevant relations pertaining to the `Process` item, we need to know which OPDs the process is in, and create a separate `RSTUVCandidate` with R=Inclusion, S=ProcessElement(p).ProcessOPD, and T= ProcessElement(p).Process, where p is a member of the ProcessElement set, p=1..,|`ProcessElement`|.

`OntologyMapping` can receive an extensible JavaScript Object Notation (JSON) data structure that defines required conversions for any item. An example of the extensible JSON configuration for mapping `ProcessElement` to `RSTUVCandidate` is shown in **Figure 9**. The JSON object can define:

- Identity Attributes (`identityVars`): attributes whose values are converted to the Identity relation and the Classification relation.
  - The Identity relation maps each entry to a universally unique ID (UUID).
  - The Classification relation maps each entry to its type, which is the attribute name. For example, each item in the `OPM.Diagram` column is mapped to an `RSTUVTuple` with R=Classification, S=OPM.Diagram, and T=Item.UUID.
- Pairs of Source and Target attributes with a specified relation (`sourceTargetPairs`): each pair of attribute values is transformed into an RST tuple with a specified relation. For example, the `OPM.Object` and `OPM.ObjectState` attributes are mapped to a set of RST tuples with R=StateSpecification, S={items in OPM.Object – one object per ObjectElement}, and T={items in OPM.ObjectState – one or more names of states}

- Triplets of Source, Target, and Relation (`relationSourceTargetTriplets`): each triplet maps directly into an RSTUV tuple. For example: under the `OPM.Relation` block, R=Relation, S={one object, process, or state}, T={ one object, process, or state}.

The mapping function combinatorically searches for all valid pair and triplet permutations and create a set of unique RSTUV candidates. An `RSTUVCandidate` referring to the same relation, source, and target entities might be created from multiple blocks. For example, identity tuples for `OPM.Diagram` can be generated for both `ObjectElement` and `ProcessElement`. OPDs may have two `RSTUVCandidates`: one due to including an object, and one due to including a process. This is because diagram names are not defined separately in `ElementDictionary`, only indirectly through the listing of OPDs in which each object and process are visualized. We only keep the first appearance of similar `RSTUVCandidates` with the same RST.

```
{ "keyVar": "OPM.ProcessElement",
  "outputMode":4,
  "identityVars": [
    "OPM.Diagram",
    "OPM.Process",
    "ModelView",
    "ModelID:Model" ],
  "sourceTargetPairs": [
  ["OPM.Inclusion","ModelID","ModelView"],
  ["OPM.Inclusion","ModelView","OPM.Process"],
  ["OPM.Inclusion","ModelView","OPM.Diagram"],
  ["OPM.Inclusion","OPM.Diagram","OPM.Process"] ]
}
```

Figure 9. A JSON structure defining the required transformations of `ProcessElement` attributes to `RSTUVCandidate` items. This example does not include `relationSourceTargetTriplets`.

3.4 *Transforming Graphs to Views and SIMs*

The next step in the process is to define informative views on the model's GDS (set of RSTUV tuples), such as the SIMs discussed in section 2.3. As shown in **Figure 3** the essential graph, RST tuple set, and adjacency matrix are equivalent representations. Therefore, any relational pattern in the model, captured in the GDS, can be represented in a matrix. Furthermore, mappings via selected relations or subsets of relations that make sense or helps in reasoning about a problem, support this reasoning process as a stakeholder-informing view, such as a matrix based on the GDS and thus on the model.

A SIM captures a mapping of row items to column items. We can specify a subset of relations and get a mapping of row items to column items according to the selected relations. We use SQL as a robust querying language, to reconstruct threads and spans of relations over the GDS, with the RSTUV tuples stored in a relational SQL Server database. Sequential SQL join queries make up functors from the raw GDS to matrix or tensor datasets [86], which visualization and analysis tools can render as matrices. A common spreadsheet tool like Microsoft Excel can import the query results from the database management system, construct a pivot table on top of the raw data, and render the appropriate visualization. Pivot tables have a flexible structure that allows the analyst to adjust, transpose, and organize the matrix hierarchically for various visualization needs. Readily available data analysis tools like pivot tables simplify the analysis and minimize tool dependency.

Additional analysis can include tallies, sums, minima, maxima, subtotals on indications, or associated/converted matrix cells values. For example, if the number of allowed relations in each intersection must positive, we can quickly find the discrepancies (both visually and computationally).

3.5 *Transforming Views into Concepts, and Concepts back into Models*

The CMGVC's cognitive segment, V→C→M consists of reasoning, decision-making, and action-taking. During this process, we study the views, mentally fuse the information with knowledge and beliefs, and create or revise a mental model, or concept. Based on the concepts we have in mind, we make decisions that alter the concept, or alter reality to match the revised concept. These cognitive view-to-concept and concept-to-model mappings close the loop by returning to the category of conceptual systems, *SYS*. A model of an existing system explains hypotheses, theories, or facts about the system. A model of a future system serves to inform system that will suit their concept. Stakeholders may later take action to execute decisions and adjust the model developers about the system's expected structure and behavior to fulfill the concept that the system was meant to realize.

The conceptual transformation is a cognitive process that closes the loop and allows us to re-iterate through conceptual models of described or prescribed systems. Rigorous formulation of these transforms requires substantial grounding in cognitive psychology, and is beyond our current scope.

4. Assessment

The CMGVC offers a significant departure from direct generation of views on models, and a more robust alternative to indirect generation of views based on DSML-specific representations. In section 1 we have defined five propositions for this research:

A. M→G→V (a GDS-mediated transformation from model to view) is superior to M→V (a direct transformation from model to view)
B. M→G→V is superior to M→R→V (mediated by a language-bound representation)
C. M→G is a feasible and valid transformation.
D. G→V is a feasible and valid transformation.
E. G→SIM is a feasible and valid transformation.

We have presented a categorical framework for converting conceptual system models from one ML, OPM, into a GDS, and deriving SIM views from the GDS. We have thus managed to validate Propositions C, D, and E regarding the existence of the building blocks of a composed transform M→G→V based on $GV \circ MG$, where $MG: M \rightarrow G$, $GV: G \rightarrow V$.

To prove Propositions A and B we must compare the benefits and limitations of the CMGVC approach vis-à-vis the two alternative approaches:

- MV: Direct generation of views from a model, that we denote V(M) or MV
- MRV: Indirect generation of views from a model via a common DSML-specific representation, that we would demote $V(R_{DSML}(M_{DSML}))$ or MRV

We define four lower-is-better (LIB) criteria for comparison, which reflect stakeholders needs for efficiency (C1), flexibility (C2), robustness (C3), and resilience (C4):

- **C1- Efficiency** is measured by **the number of required transformations** of M models to V views. For MGV, MRV: sum of model-to-graph transformations (M) and graph-to-view transformations (V); For MV: product of models by views ($M \cdot V$).
- **C2- Flexibility** is measured by the **effort of creating new views for existing models**. For MGV, MRV: one effort unit per view (graph-to-view or representation-to-view); For M: M effort units (models-to-view).
- **C3- Robustness** is measured by the **effort of creating existing views for new models**: For MGV: a single effort unit (model-to-graph); for MV: V effort units (model-to-views); for MRV: V+1 effort units (model-to-representation and representation-to-views)
- **C4- Resilience** is measured by the **dependency on DSML updates**: for MGV, MRV: a single effort unit for updates (model-to-graph or model-to-representation); for MV: V effort units (model-to-views)

Since all criteria are LIB, the total score is also LIB. **Table 6** defines metrics for each criterion. Total scores for MGV, MV, and MRV are defined in Eq. (1), (2), and (3).

Table 6. Metrics for comparing approaches to generate views from models.

Criterion	Weight	MGV	MV	MRV
C1-Efficiency	W1	M+V	MV	M+V
C2-Flexibility	W2	1	M	1
C3-Robustness	W3	1	V	V+1
C4-Resilience	W4	1	V	1
Total	1	Eq. (1)	Eq. (2)	Eq. (3)

$$S_{MGV} = W_1(M + V) + W_2 + W_3 + W_4 \quad (1)$$

$$s_{MV} = W_1(MV) + W_2M + W_3V + W_4V \quad (2)$$

$$S_{MRV} = W_1(M + V) + W_2M + W_3(V + 1) + W_4 \quad (3)$$

MGV is superior to MRV (Proposition B) if $S_{MGV} \leq S_{MRV}$:

$$W_1(M+V) + W_2 + W_3 + W_4 \leq W_1(M+V) + W_2M + W_3(V+1) + W_4 \ .$$

Cancelling equal terms, we obtain: $W_2 + W_3 \leq W_2M + W_3(V+1)$.

Homogenizing we obtain: $0 \leq W_2(M-1) + W_3(V)$.

This inequality always holds, and therefore $S_{MGV} \leq S_{MRV}$, and MGV is superior to any MRV with a DSML-specific representation:

$$S_{MGV} \leq S_{MRV} \Leftrightarrow \boldsymbol{MGV \succcurlyeq MRV} \quad (4)$$

Conditions for MGV superiority to MV:

$$S_{MGV} \leq S_{MV} \Leftrightarrow W_1(M+V) + W_2 + W_3 + W_4 \leq W_1(MV) + W_2M + W_3V + W_4V \ .$$

Homogenizing we obtain the following:

$$S_{MGV} \leq S_{MV} \ \Leftrightarrow \ W_1(MV - M - V) + W_2(M-1) + W_3(V-1) + W_4(V-1) \geq 0 \quad (5)$$

We try Eq. (5) with combinations of V=1,V=2,V>2 with M=1,M=2,M>2, as summarized in **Table 7**. For all practical purposes, multiple MLs $M \geq 2$, and at least 2 views (we already have 3),

$$M \geq 2, V \geq 2 \ \Leftrightarrow S_{MGV} \leq S_{MV} \ \Leftrightarrow \ \boldsymbol{MGV \succcurlyeq MV} \quad (6)$$

Table 7. Preference Relations of MGV over MV depending on the number of MLs (M) and views (V). MV is preferred when M=V=1; MGV is preferred when $M \geq 2$ & $V \geq 2$. When $M = 1$ & $V \geq 2$ or $M \geq 2$ & $V = 1$ preference rotates based on weighting. MGV is preferable when weights are equal.

	V=1	V=2	V>2
M=1	$\nexists\, W_1(-1) \geq 0$ $\Rightarrow S_{MV} \leq S_{MGV} \Rightarrow \boldsymbol{MGV \preccurlyeq MV}$	$W_3 + W_4 \leq W_1$ $\Rightarrow S_{MV} \leq S_{MGV} \Rightarrow \boldsymbol{MGV \preccurlyeq MV}$	$(W_3 + W_4) \leq \frac{W_1}{3}$ $\Rightarrow S_{MV} \leq S_{MGV} \Rightarrow \boldsymbol{MGV \preccurlyeq MV}$
		$W_3 + W_4 \geq W_1$ $\Rightarrow S_{MGV} \leq S_{MV} \Rightarrow \boldsymbol{MGV \succcurlyeq MV}$	$(W_3 + W_4) \geq \frac{W_1}{3}$ $\Rightarrow S_{MGV} \leq S_{MV} \Rightarrow \boldsymbol{MGV \succcurlyeq MV}$
M=2	$W_1 \geq W_2$ $\Rightarrow S_{MV} \leq S_{MGV} \Rightarrow \boldsymbol{MGV \preccurlyeq MV}$	$W_2 + W_3 + W_4 \geq 0$ $\Rightarrow S_{MGV} \leq S_{MV} \Rightarrow \boldsymbol{MGV \succcurlyeq MV}$	$mW_1 + W_2 + n(W_3 + W_4) \geq 0$ $m,n \in \mathbb{N}$ $\Rightarrow S_{MGV} \leq S_{MV} \Rightarrow \boldsymbol{MGV \succcurlyeq MV}$
	$W_2 \geq W_1$ $\Rightarrow S_{MGV} \leq S_{MV} \Rightarrow \boldsymbol{MGV \succcurlyeq MV}$		
M>2	$W_2 \leq \frac{W_1}{M-1}$ $\Rightarrow S_{MV} \leq S_{MGV} \Rightarrow \boldsymbol{MGV \preccurlyeq MV}$	$mW_1 + nW_2 + W_3 + W_4 \geq 0$ $m,n \in \mathbb{N}$ $\Rightarrow S_{MGV} \leq S_{MV} \Rightarrow \boldsymbol{MGV \succcurlyeq MV}$	$pW_1 + mW_2 + n(W_3 + W_4) \geq 0$ $m,n,p \in \mathbb{N}$ $\Rightarrow S_{MGV} \leq S_{MV} \Rightarrow \boldsymbol{MGV \succcurlyeq MV}$
	$W_2 \geq \frac{W_1}{M-1}$ $\Rightarrow S_{MGV} \leq S_{MV} \Rightarrow \boldsymbol{MGV \succcurlyeq MV}$		

With this analysis, we have managed to corroborate Proposition A and Proposition B.

5. Application

5.1 *Representation and Analysis of Process-to-Process Input-Output Exchange*

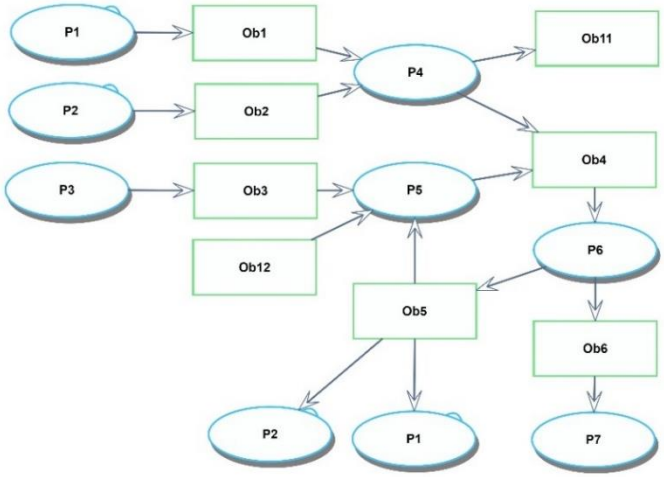
Table 1 illustrates two examples of SAMs that map various system architecture aspects to each other. Both SAMs include a mapping of processes to each other, with **Table 1**-a being more explicit about the mapping of process to process via operands. The operands serve as indirect process mediators: the output of one process is the input of another. However, there is no explicit mapping in the model in the form <P,O,P>. Therefore, this mapping must be composed. This composition is defined in the following manner, using SQL queries to retrieve and cross the data from the raw set of RSTUV tuples:

1. Find all the processes, i.e., targets in tuples with S='OPM.Process' and R= 'Classification'.
2. Find all outputs, i.e., targets in tuples where R= 'Result', and retrieve the source process item.
3. Find all tuples in which the above outputs are sources in a Consumption relation (i.e., inputs), and retrieve the target process items.
4. Cross the process-output set with the input-process set such that output = input.

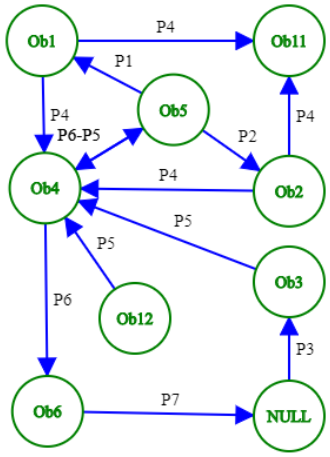
5. Cross process set (1) with output-generating process set (2). Keep all processes including those that are generating no output.
6. Cross process set (1) with input-receiving process set (3). Keep all processes including those that are receiving no input.
7. Layout a matrix with output-generating processes as rows, input-receiving processes as columns, and identity of matching output-input (*onput* [81]) item as the cell value.

Figure 10 illustrates the MGV transform a generic OPM model (**Figure 10**-a) with seven processes and eight objects into three views that the processes generate or consume:

- A visual graph with onputs on nodes and processes on edges that highlights onput transformations, the systemic morphisms of onputs through processes (**Figure 10-**b)
- A visual graph with processes on nodes and onputs on edges – the dual graph of Figure 10-b – highlights process flow and functional compositions through shared onputs, process threads, functional boundaries and interfaces (**Figure 10**-c).
- A process-operand-process (POP) exchange matrix maps processes to each other via shared/exchange onputs, including indications for partial exchanges such as inputs without source processes, processes without input or output, outputs without sinks, etc. (**Figure 10**-d).



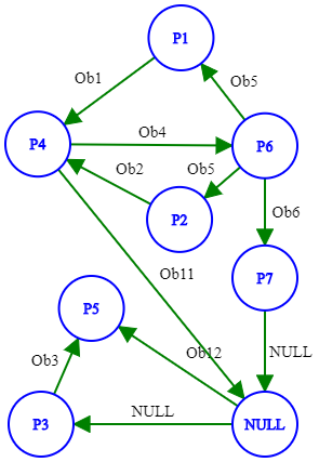
(a) Object-process model with processes that generate outputs and consume inputs. An onput is concurrently an output of one process and input to another



(b) Graph visualization of the model, with onputs as nodes and processes as edges

Onputs	Input Receiving Process							
Output Providing Process	NULL	P1	P2	P3	P4	P5	P6	P7
NULL				NULL		Ob12		
P1					Ob1			
P2					Ob2			
P3						Ob3		
P4	Ob11						Ob4	
P5							Ob4	
P6		Ob5	Ob5			Ob5		Ob6
P7	NULL							

(d) Process-Operand-Process (POP) Matrix of the model: the row process generates the cell onput, which is received or used by the column process



(c) Graph visualization of the model, with processes as nodes and onputs as edges

Figure 10. Transforming an OPM model with processes and onputs (a) through the MGV transform into three renditions: b) an onput-on-node/process-on-edge graph, c) a process-on-node/onput-on-edge graph, and d) a process-to-process onput exchange matrix, which is based on the same query as the process-on-node graph. NULL processes and onputs (in all renditions) serve as placeholders for missing or partial relations, e.g., an input without a source process (e.g. Ob12), a process without an

input (e.g. P3) or without an output (e.g. P7), and an output without a target process (e.g. Ob11). Both graph visualizations were rendered using https://csacademy.com/app/graph_editor/.

5.2 *The Lane Keeping System Revisited*

Our lane keeping system (LKS) running example can now be analyzed according to the CMGVC. An in-zoomed diagram of the LKS, specifying the functions performed by the system and their interactions, is illustrated in **Figure 11**. This is an intentionally partial diagram, with some critical things missing. As we study this diagram we should notice, for instance, that the Lane Crossing object has no source, and that the Road is consumed by the Imaging Road process, rather than remain an instrument as it was specified in the topmost diagram. It is difficult to ensure continuity and consistency this way, and the problem intensifies as the model grows bigger with more objects, processes, and diagrams.

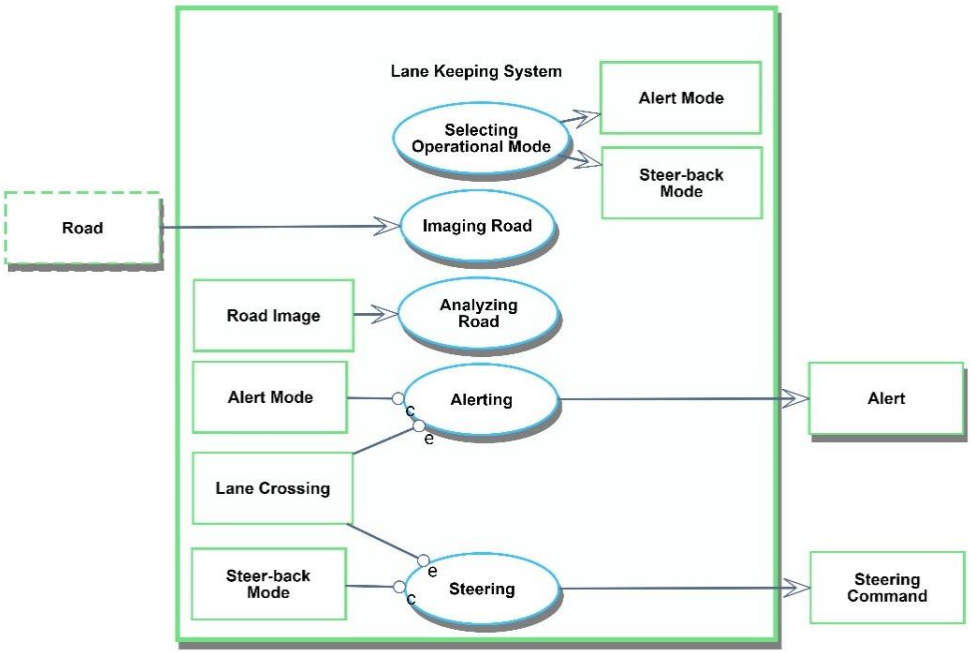


Figure 11. **Lane Keeping System** zooms into **Alert Mode**, **Steer-back Mode**, **Road Image**, **Alert Mode**, **Lane Crossing**, and **Steer-back Mode**, in that vertical sequence, as well as **Alerting**, **Analyzing Road**, **Imaging Road**, **Selecting Operational Mode**, and **Steering** (This caption is in fact a formal OPL sentence that specifies the in-zooming of Lane Keeping System in a new diagram).

Analyzing this model with a POP matrix could help detect some discrepancies in the model. **Table 8** is a POP matrix that was derived from the model through the same SQL query defined in section 5.1 for the POP matrix-supporting records, and rendered as a matrix using a MS Excel pivot table on the query results. The Lane Keeping System model utilizes additional procedural relations besides the result and consumption: the instrument relation is used for non-consumable resources, and the invocation relation is used for specifying process activation by another process, implying an unspecified interface object. Additional metrics were added to the POP matrix, including the number of inputs and number of outputs per process (calculated on the columns and rows, respectively).

The POP matrix highlights several issues in the model, which could be helpful for reviews by various stakeholders – customers, system architects, process or function owners, etc. We summarize proposed corrections by entity and issue in **Table 9**. It would be easy to apply these to the model and reiterate to obtain the revised POP matrix. As details are added to the model, the POP matrix is likely to grow and detect new issues, in a continuous concept revision and improvement cycle.

Table 8. POP matrix of LKS model maps the modeled processes to each other via onput exchange, instrument providing, and invocation. The matrix uses color coding to highlight discrepancies and potential issue. Red cells with a value of 0 indicate processes without inputs or outputs, while lightly-colored cells indicate that further attention is needed.

Target Process / Source Process	(Null)	Alerting	Analyzing Road	Imaging Road	Lane Keeping	Selecting Operational Mode	Steering	Driving	Propelling	Outputs per Process
(Null)		Lane Crossing	Road Image	Road	Road		Lane Crossing		Road	6
Lane Keeping	Improved Safety							Alert	Steering Command	3
Selecting Operational Mode		Alert Mode					Steer-back Mode			2
Imaging Road										0
Analyzing Road										0
Alerting								Alert		1
Driving									Invocation	1
Steering									Steering Command	1
Propelling								Invocation		1
Inputs per Process	1	2	1	1	1	0	2	3	4	

Table 9. Issues identified in LKS model POP matrix and possible corrections in the model. Critical issues are marked in red.

Entity	Issue	Correction
Lane Keeping	no functional decomposition	Unfold **Lane Keeping** in a separate diagram and link it to the internal functions in **Lane Keeping System** to create functional decomposition of top level system function to lower-level functions
Selecting Operational Mode	no input	Add input from Driver
Imaging Road	no output	Link through a Result relation to **Road Image**
Analyzing Road	no output	Link through a Result relation to **Lane Crossing**
Improved Safety	no receiver	Specify as attribute of **Lane Keeping System.** Since this object represents emergent value, it is not recommended to associate it with a specific function
Road Image	no provider	Link **Imaging Road** through a Result relation
Lane Crossing	no provider	Link **Analyzing Road** through a Result relation
Road	no provider	Add Effect Link from Propelling, indicating that vehicle movement affects the immediate portion of interest of the road with which the vehicle interacts
Driving invokes **Propelling**	no explicit onput	Consider replacing invocation by driver command gestures (turning, signaling, braking, etc.)
Propelling invokes **Driving**	no explicit onput	Consider replacing invocation by vehicle response to driver action

6. Discussion

This paper has several results and outcomes to discuss at the methodological and empirical levels. The primary result of this study is the formulation of the CMGVC using robust category-theoretical foundations. In addition to the overarching framework that emerges from this study, we have results for each phase of the cycle at varying levels of specification.

The emergence of the CMGVC is a promising direction for MBSE, as also indicated by both colleagues and the anonymous referees. Systems engineering is undergoing a digital transformation, compounded with the constant growth of systems complexity and interconnectedness. Conceptual modeling is becoming more common for representing complex systems, and more critical for generating deliverables that impact the digital value chain [10]. Considering the challenge and opportunity associated with these trends, robust foundations for interoperability and collaboration across digital enterprises are likely to be significant enablers of digital systems engineering.

The assertion and demonstration that modeling languages are categories is an important contribution to the body of knowledge, as it extends the Curry-Howard-Lambek Correspondence from programing languages to modeling languages. We have used categorical structures to map system concepts to formal models, models to robust graph data structures (GDS), and GDS to stakeholder-informing views – graphs and matrices. With these representations defined as categories, a new category-theory-driven MBSE paradigm can emerge. Many more extensions of this paradigm are possible, including the definition of additional modeling languages as categories, and the definition of additional functorial mappings of GDS to views, including model-to-model translation.

Stakeholder-informing visualizations, particularly matrices, are enhanced by this study with rigorous and robust foundations for generating, processing, and analyzing model-based data. The two graph renditions we generated are in fact duals: a process-on-node/onput-on-edge graph and an onput-on-node/process-on-edge graph. Both dual graph renditions have uses and advantages. Being able to generate both graphs from the same data structure is an important benefit, which also ensures consistency and complementarity. The SysML Internal Block Diagram (IBD) uses an object-on-node graph, while the Activity Diagram uses a process-on-node graph, and State Charts use a state-on-node graph. These three aspects can be generated from the same GDS of an OPM model. Therefore, GDS facilitates modeling language translation, a promising direction for future research.

The process-operand-process exchange matrix is an important architectural analysis tool. It captures a non-trivial mapping relation that we could build by composing relation segments in the GDS through robust data retrieval and integration queries. Obtaining and maintaining such a matrix manually is a significant cognitive effort. Dynamically visualizing the matrix using a pivot table is another strong benefit, due to its accessibility and ability to accommodate evolving needs. We have also shown how detected relation anomalies drive concept and model revisions. While such analyses can be based on a manually-constructed or otherwise-generated matrices, our approach provides additional confidence since the data comes out of the model and lends itself to any necessary composition or aggregation that can yield beneficial visualization. Our approach also sets the stage for a broad range of visual and digital representations. The latter can inform digital ecosystem agents and greatly enhance interoperability and coordination across and among enterprises.

7. Conclusion

In this paper, we have explored the potential of Category Theory to serve as an underlying formalism for systems engineering, particularly in the context of MBSE. Category Theory is an appropriate holistic paradigm, a state of mind, and a formal foundation for the model transformation and reasoning pipeline, which is essential for a smooth, rational, and reliable MBSE cycle that constantly improves, corrects, and refines system architecture specifications.

The Concept-Model-Graph-View-Concept cycle (CMGVC) facilitates the transformation of conceptual models to stakeholder-informing and decision-supporting views. We have shown that it would be imperative to include an intermediate generic representation in the form of a GDS, which serves as a common outlet to all MLs, and a common basis for all views, visualizations, and reports. We have proven the superiority of our approach to direct and ML-bound mappings.

The CMGVC has several advantages:

1. Using the CMGVC, **stakeholders and decision-makers will be able to derive critical information and insight** regarding system development and operation from the system model, rather than through a disparate information gathering and presentation channel, which is the common practice nowadays.
2. The preferential dominance that we have proven in section 4 facilitates **efficiency in model analytics**, and thus encourages further adoption.
3. The transition through GDS **enhances system understanding by adding another modality: graphs**, which map concepts and relations through one common substantial representation.
4. The simple-yet-robust GDS can be a prime facilitator of **MBSE interoperability and collaboration** across digital value chains.
5. Subject matter experts will be able to leverage the CMGVC via semantic and ontological frameworks to **better represent emerging patterns and concepts**.

We have demonstrated how a model can be represented in multiple ways - two visual graph renditions and a process-to-process operand exchange matrix – that are all based on its single GDS.

We plan to demonstrate that the CMGVC can be robustly suited for various other MLs and views. Particularly, we plan to explore the transformation of SysML and Simulink models via the CMGVC pipeline and to create meaningful source-agnostic views, including SIMs, sub-graphs, state spaces, and specifications in other MLs. Isomorphic mapping out of and back into the same ML to support round-trip engineering also pose a major challenge [87].

Reconciling MBSE and Discrete Event Simulation (DEVS) paradigms is a recently trending effort [88]. Ongoing collaboration between the MBSE and DEVS communities has been institutionalized in a joint workgroup of the International Council on Systems Engineering (INCOSE) and International Association for Engineering Modeling and Simulation (NAFEMS) [89]. Our framework can facilitate interoperability across MBSE and DEVS platforms, tools, and models. It alleviates efforts to find and apply an interoperability standard that would appeal to both MBSE and DEVS. The reason is that graph-theoretic DEVS algorithms are abundant, and graph-representable problems have been studied with DEVS technology and methodology for decades, e.g., as part of the Agent-Based Modeling & Simulation (ABMS) paradigm [90, 91]. This direction should be further explored.

One limitation in our approach is its scalability. We plan to build a service-oriented software platform that will provide robust transformation capabilities as web services to MBSE practitioners and researchers, who will be able to upload or plug-in live models and generate useful visualizations. MBSE experts would be able to define and manage mappings and sets of views to run on specific models. These will provide continuous visualization or integration with external systems across the digital enterprise. We plan to employ state-of-the-art graph database and information visualization techniques to provide stakeholders with flexible, dynamic, and elegant decision-supporting views.

Author Contributions: Conceptualization, Y.M., E.C., J.F.; methodology, Y.M, J.F., E.C.; software, Y.M.; validation, Y.M, J.F.; formal analysis, Y.M.; writing, Y.M., J.F.; visualization, Y.M., J.F.; supervision, E.C.; funding acquisition, Y.M, J.F., E.C. All authors have read and agreed to the published version of the manuscript.

Funding: Y.M.'s research was conducted as part of a research fellowship, funded by the MIT-Technion Post-Doctoral Research Fellowship Program. J.F.'s research was funded by DARPA under agreements HR00112090067 and W911NF2010292.

Acknowledgments: We thank David Spivak from MIT's Department of Mathematics for planting the seeds for the collaboration that resulted in this research, for inspiring it, and for very useful comments. We thank Prof. Dov Dori from Technion and Prof. Yoram Reich from Tel-Aviv University for their useful comments. Finally, we thank the anonymous referees who helped improve this paper with constructive suggestions.

Conflicts of Interest: The authors declare no conflict of interest. The funders had no role in designing the study; collecting, analysing, or interpreting data; writing the manuscript, or in deciding to publish the results.

8. References

1. Partridge, C., Gonzalez-Perez, C., Henderson-Sellers, B.: Are conceptual models concept models? Lect. Notes Comput. Sci. (including Subser. Lect. Notes Artif. Intell. Lect. Notes Bioinformatics). 8217 LNCS, 96–105 (2013). https://doi.org/10.1007/978-3-642-41924-9_9.
2. Bailer-Jones, D.M.: Scientific models in philosophy of science. (2009). https://doi.org/10.1080/02698595.2010.543353.
3. Soderborg, N.R., Crawley, E.F., Dori, D.: System Function and Architecture: OPM-Based Definitions and Operational Templates. Commun. ACM. 46, 67–72 (2003). https://doi.org/10.1145/944217.944241.
4. Dori, D.: Model-Based Systems Engineering with OPM and SysML. Springer, New York (2016). https://doi.org/10.1007/978-1-4939-3295-5.
5. McDermott, T.A., Hutchinson, N., Clifford, M., Van Aken, E., Slado, A., Henderson, K.: Benchmarking the Benefits and Current Maturity of Model-Based Systems Engineering across the Enterprise. Systems Engineering Research Center (SERC) (2020).
6. Bondar, S., Hsu, J.C., Pfouga, A., Stjepandić, J.: Agile digital transformation of System-of-Systems architecture models using Zachman framework. J. Ind. Inf. Integr. 7, 33–43 (2017). https://doi.org/10.1016/j.jii.2017.03.001.
7. Hale, J.P., Zimmerman, P., Kukkala, G., Guerrero, J., Kobryn, P., Puchek, B., Bisconti, M., Baldwin, C., Mulpuri, M.: Digital Model-based Engineering: Expectations, Prerequisites, and Challenges of Infusion. NASA (2017).
8. Bone, M., Blackburn, M., Kruse, B., Dzielski, J., Hagedorn, T., Grosse, I.: Toward an Interoperability and Integration Framework to Enable Digital Thread. Systems. 6, 46 (2018). https://doi.org/10.3390/systems6040046.
9. Hagedorn, T., Bone, M., Kruse, B., Grosse, I., Blackburn, M.: Knowledge Representation with Ontologies and Semantic Web Technologies to Promote Augmented and Artificial Intelligence in Systems Engineering. Insight. 23, 15–20 (2020). https://doi.org/10.1002/inst.12279.
10. Mordecai, Y., de Weck, O.L., Crawley, E.F.: Towards an Enterprise Architecture for a Digital Systems Engineering Ecosystem. Conf. Syst. Eng. Res. (2020).
11. Subrahmanian, E., Levy, S.N., Westerberg, A.W., Monarch, I., Konda, S.L., Reich, Y.: Equations aren't enough: Informal modeling in design. Artif. Intell. Eng. Des. Anal. Manuf. 7, 257–274 (1993). https://doi.org/10.1017/S0890060400000354.
12. Sharpe, J.E.E., Bracewell, R.H.: Application of bond graph methodology to concurrent conceptual design of interdisciplinary systems. In: IEEE Systems Man and Cybernetics Conference - SMC (1993). https://doi.org/https://doi.org/10.1109/ICSMC.1993.384711.
13. Ernadote, D.: An ontology mindset for system engineering. 1st IEEE Int. Symp. Syst. Eng. ISSE 2015 - Proc. 454–460 (2015). https://doi.org/10.1109/SysEng.2015.7302797.
14. Cotter, M., Hadjimichael, M., Markina-khusid, A., York, B.: Automated Detection of Architecture Patterns in MBSE Models. In: Conference on Systems Engineering Research (CSER) (2020).
15. Reich, Y., Konda, S., Subrahmanian, E., Cunningham, D., Dutoit, A., Patrick, R., Thomas, M., Westerberg, A.W.: Building agility for developing agile design information systems. Res. Eng. Des. - Theory, Appl. Concurr. Eng. 11, 67–83 (1999). https://doi.org/10.1007/PL00003884.
16. Dennis, A.R., Hayes, G.S., Daniels, R.M.J.: Re-engineering business process modeling. HICSS '94 Proc. Hawaii Int. Conf. Syst. Sci. 244–253 (1994).
17. Object Management Group (OMG): OMG Unified Modeling Language (OMG UML). OMG (2015).

https://doi.org/10.1007/s002870050092.

18. United States Department of Defense (DoD): The DoDAF Architecture Framework Version 2.02, https://dodcio.defense.gov/Library/DoD-Architecture-Framework/, last accessed 2020/12/03.
19. Fong, B., Spivak, D.I.: Seven Sketches in Compositionality: An Invitation to Applied Category Theory. (2018).
20. Chakraborty, S.: Curry-Howard-Lambek Correspondence. (2011).
21. Hamilton, M.: Category Theory and the Curry-Howard-Lambek Correspondence. (2016).
22. Naur, P.: Programming as theory building, (1985). https://doi.org/10.1016/0165-6074(85)90032-8.
23. ISO/TC 184: ISO 19450 Automation systems and integration — Object-Process Methodology. International Organization for Standardization (ISO), Geneva, Switzerland (2015).
24. Crawley, E., Cameron, B., Selva, D.: Systems Architecture: Strategy and Product Development for Complex Systems. Prentice Hall (2015).
25. Mordecai, Y., Dori, D.: Model-Based Operational-Functional Unified Specification for Mission Systems. In: 10th Annual IEEE International Systems Conference (SysCon). IEEE, Orlando FL, USA (2016). https://doi.org/10.1109/SYSCON.2016.7490662.
26. Mordecai, Y., Dori, D.: Model-based requirements engineering: Architecting for system requirements with stakeholders in mind. In: IEEE International Symposium on Systems Engineering, ISSE (2017). https://doi.org/10.1109/SysEng.2017.8088273.
27. Osorio, C.A., Dori, D., Sussman, J.: COIM: An Object-Process Based Method for Analyzing Architectures of Complex, Interconnected, Large-Scale Socio-Technical Systems. Syst. Eng. 14, (2011). https://doi.org/DOI 10.1002/sys.20185.
28. Dori, D., Jbara, A., Levi, N., Wengrowicz, N.: Object-Process Methodology, OPM ISO 19450 – OPCloud and the Evolution of OPM Modeling Tools, https://www.ppi-int.com/wp-content/uploads/2018/01/SyEN_61.pdf, (2018).
29. Levi-Soskin, N., Shaoul, R., Kohen, H., Jbara, A., Dori, D.: Model-Based Diagnosis with FTTell: Assessing the Potential for Pediatric Failure to Thrive (FTT) During the Perinatal Stage. In: Wrycza, S. and Maślankowski, J. (eds.) SIGSAND/PLAIS, LNBIP 359. pp. 37–47. Springer Nature Switzerland AG (2019). https://doi.org/10.1007/978-3-030-29608-7_4.
30. Dori, D., Kohen, H., Jbara, A., Wengrowicz, N., Lavi, R., Levi-Soskin, N., Bernstein, K., Shani, U.: OPCloud: An OPM Integrated Conceptual-Executable Modeling Environment for Industry 4.0. In: Kenett, R.S., Swarz, R.S., and Zonnenshain, A. (eds.) Systems Engineering in the Fourth Industrial Revolution: Big Data, Novel Technologies, and Modern Systems Engineering. Wiley (2020).
31. Mordecai, Y., James, N.K., Crawley, E.F.: Object-Process Model-Based Operational Viewpoint Specification for Aerospace Architectures. In: IEEE Aerospace Conference. IEEE (2020). https://doi.org/10.1109/AERO47225.2020.9172685.
32. Ford Motor Company: Lane-Keeping System, https://www.youtube.com/watch?v=8O3u20MBmsE, last accessed 2020/11/20.
33. Object Management Group: Unified Architecture Framework Profile (UAFP). (2019).
34. Browning, T.R.: Design Structure Matrix Extensions and Innovations: A Survey and New Opportunities. IEEE Trans. Eng. Manag. 63, 27–52 (2016). https://doi.org/10.1109/TEM.2015.2491283.
35. Sharon, A., Dori, D., de Weck, O.: Model-Based Design Structure Matrix: Deriving a DSM from an Object-Process Model. In: Second International Symposium on Engineering Systems (2009). https://doi.org/10.1002/sys.

36. Sharon, A., Dori, D.: A Project–Product Model–Based Approach to Planning Work Breakdown Structures of Complex System Projects. Systems. (2014). https://doi.org/10.1109/JSYST.2013.2297491.
37. Do, S., Weck, O. De: A Grammar for Encoding and Synthesizing Life Support. In: 44th International Conference on Environmental Systems (2014).
38. Wilschut, T., Etman, L.F.P., Rooda, J.E., Vogel, J.A.: Generation of a function-component-parameter multi-domain matrix from structured textual function specifications. Res. Eng. Des. 29, 531–546 (2018). https://doi.org/10.1007/s00163-018-0284-9.
39. Knippenberg, S.C.M., Etman, L.F.P., Wilschut, T., van de Mortel-Fronczak, J.A.: Specifying Process Activities for Multi-Domain Matrix Analysis Using a Structured Textual Format. Proc. Des. Soc. Int. Conf. Eng. Des. 1, 1613–1622 (2019). https://doi.org/10.1017/dsi.2019.167.
40. Object Management Group: OMG Systems Modeling Language (OMG SysML™) Version 1.4. Object Management Group (OMG) (2015). https://doi.org/10.1002/j.2334-5837.2009.tb01046.x.
41. Au-Yong-Oliveira, M., Moutinho, R., Ferreira, J.J.P., Ramos, A.L.: Present and future languages – How innovation has changed us. J. Technol. Manag. Innov. 10, 166–182 (2015). https://doi.org/10.4067/S0718-27242015000200012.
42. Johnson, J.M.: Analysis of Mission Effectiveness: Modern System Architectures Tools For Project Developers, (2017).
43. Kong, P.O.: Spreadsheet-Based Graphical User Interface For Modeling Of Products Using The Systems Engineering Process, (2014).
44. Wymore, W.A.: Model-Based Systems Engineering. CRC Press (2000).
45. Breiner, S., Subrahmanian, E., Jones, A.: Categorical foundations for system engineering. Discip. Converg. Syst. Eng. Res. 449–463 (2017). https://doi.org/10.1007/978-3-319-62217-0.
46. Subrahmanian, E., Reich, Y., Krishnan, S.: The Story of n-Dim. In: We Are Not Users: Dialogues, Diversity, and Design. pp. 181–196. MIT Press (2020). https://doi.org/10.1525/9780520938342-009.
47. Breiner, S., Pollard, B., Subrahmanian, E.: Workshop on Applied Category Theory: Bridging Theory and Practice. (2020). https://doi.org/https://doi.org/10.6028/NIST.SP.1249.
48. Koo, H.-Y.B.: A Meta-language for Systems Architecting, (2005).
49. Koo, B., Hurd, A., Loda, D., Dori, D., Crawley, E.F.: Architecting Systems Under Uncertainty with Object-Process Networks. In: International Conference on Complex Systems (ICCS'04). , Boston, MA, USA (2004).
50. Spivak, D.I., Kent, R.E.: Ologs: A categorical framework for knowledge representation. PLoS One. 7, (2012). https://doi.org/10.1371/journal.pone.0024274.
51. Mabrok, M.A., Ryan, M.J.: Category theory as a formal mathematical foundation for model-based systems engineering. Appl. Math. Inf. Sci. 11, 43–51 (2017). https://doi.org/10.18576/amis/110106.
52. Censi, A.: Uncertainty in Monotone Codesign Problems. IEEE Robot. Autom. Lett. 2, 1556–1563 (2017). https://doi.org/10.1109/LRA.2017.2674970.
53. Censi, A.: A Class of Co-Design Problems with Cyclic Constraints and Their Solution. IEEE Robot. Autom. Lett. 2, 96–103 (2017). https://doi.org/10.1109/LRA.2016.2535127.
54. Legatiuk, D., Dragos, K., Smarsly, K.: Modeling and evaluation of cyber-physical systems in civil engineering. Proc. Appl. Math. Mech. 17, (2017). https://doi.org/10.1002/pamm.201710371.
55. Bakirtzis, G., Vasilakopoulou, C., Fleming, C.H.: Compositional Cyber-Physical Systems Modeling. Proc. ACT 2020. 333, 125–138 (2020). https://doi.org/10.4204/eptcs.333.9.
56. Kibret, N., Edmonson, W., Gebreyohannes, S.: Category theoretic based formalization of the verifiable

design process. 13th Annu. IEEE Int. Syst. Conf. (2019). https://doi.org/10.1109/SYSCON.2019.8836804.

57. Gebreyohannes, S., Edmonson, W., Esterline, A.: Formal behavioral requirements management. IEEE Syst. J. 12, 3006–3017 (2018). https://doi.org/10.1109/JSYST.2017.2775740.
58. Breiner, S., Sriram, R.D., Subrahmanian, E.: Compositional Models for Complex Systems. (2019). https://doi.org/10.1016/b978-0-12-817636-8.00013-2.
59. Kovalyov, S.P.: Leveraging category theory in model based enterprise. Adv. Syst. Sci. Appl. 20, 50–65 (2020). https://doi.org/10.25728/assa.2020.20.1.781.
60. Foley, J.D., Breiner, S., Subrahmanian, E., Dusel, J.M.: Operads for complex system design specification, analysis and synthesis. 1–34 (2021).
61. Breiner, S., Pollard, B., Subrahmanian, E., Marie-Rose, O.: Modeling hierarchical system with operads. Proc. ACT 2020. (2020). https://doi.org/10.4204/EPTCS.323.5.
62. Diskin, Z., Maibaum, T.: Category theory and model-driven engineering: From formal semantics to design patterns and beyond. Proc. Seventh ACCAT Work. Appl. Comput. Categ. Theory. (2012). https://doi.org/10.4204/EPTCS.93.1.
63. Diskin, Z., Gómez, A., Cabot, J.: Traceability mappings as a fundamental instrument in model transformations. In: Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics) (2017). https://doi.org/10.1007/978-3-662-54494-5_14.
64. Korobeynikov, A.G., Fedosovsky, M.E., Gurjanov, A.V., Zharinov, I.O., Shukalov, A.V.: Development of conceptual modeling method to solve the tasks of computer-aided design of difficult technical complexes on the basis of category theory. Int. J. Appl. Eng. Res. 12, 1114–1122 (2017). https://doi.org/10.6025/jism/2018/8/2/45-54.
65. Cafezeiro, I., Haeusler, E.H.: Semantic interoperability via category theory. 26th Int. Conf. Concept. Model. (2007).
66. Luzeaux, D.: A Formal Foundation of Systems Engineering. In: F. Boulanger et al. (eds.) (ed.) Complex Systems Design & Management. Springer International Publishing Switzerland (2015). https://doi.org/10.1007/978-3-319-11617-4_10.
67. Libkind, S.: An Algebra of Resource Sharing Machines. (2020).
68. Baez, J.C., Fong, B., Pollard, B.S.: A compositional framework for Markov processes. J. Math. Phys. 57, (2016). https://doi.org/10.1063/1.4941578.
69. Halter, M., Herlihy, C., Fairbanks, J.: A Compositional Framework for Scientific Model Augmentation. (2019).
70. Herlihy, C., Cao, K., Reparti, S., Briscoe, E., Fairbanks, J.: Semantic Program Analysis for Scientific Model Augmentation. In: Modeling the World's Systems (2019).
71. Halter, M., Patterson, E., Baas, A., Fairbanks, J.P.: Compositional Scientific Computing with Catlab and SemanticModels. arXiv. 1–3 (2020).
72. Patterson, E.: Hausdorff and Wasserstein metrics on graphs and other structured data. In: American Matehmatical Society Sectional Meeting on Category Theory (2019).
73. Reich, Y., Konda, S.L., Levy, S.N., Monarch, I.A., Subrahmanian, E.: New roles for machine learning in design. Artif. Intell. Eng. 8, 165–181 (1993). https://doi.org/10.1016/0954-1810(93)90003-X.
74. Needham, M., Hodler, A.E.: Graph Algorithms - Practical Examples in Apache Spark & Neo4j. O'Reilly Media, Inc. (2019).
75. Jim Webber, Bruggen, R. Van: Graph Databases. John Wiley & Sons, Inc. (2020).
76. Medvedev, D., Shani, U., Dori, D.: Gaining Insights into Conceptual Models: A Graph-Theoretic

Querying Approach. Appl. Sci. 11, 765 (2021). https://doi.org/10.3390/app11020765.

77. Dori, D.: ViSWeb - The Visual Semantic Web: Unifying human and machine knowledge representations with Object-Process Methodology. VLDB J. 13, 120–147 (2004). https://doi.org/10.1007/s00778-004-0120-x.
78. Shani, U., Jacobs, S., Wengrowicz, N., Dori, D.: Engaging ontologies to break MBSE tools boundaries through semantic mediation. In: Conference on Systems Engineering Research. INCOSE (2016).
79. Zhu, Y., Wan, J., Zhou, Z., Chen, L., Qiu, L., Zhang, W., Jiang, X., Yu, Y.: Triple-to-text: Converting RDF triples into high-quality natural languages via optimizing an inverse KL divergence. SIGIR 2019 - Proc. 42nd Int. ACM SIGIR Conf. Res. Dev. Inf. Retr. 1, 455–464 (2019). https://doi.org/10.1145/3331184.3331232.
80. Nadolski, M., Fairbanks, J.: Complex systems analysis of hybrid warfare. Procedia Comput. Sci. 153, 210–217 (2019). https://doi.org/10.1016/j.procs.2019.05.072.
81. Mordecai, Y., Orhof, O., Dori, D.: Model-Based Interoperability Engineering in Systems-of-Systems and Civil Aviation. IEEE Trans. Syst. Man Cybern. Syst. 48, 637–648 (2018). https://doi.org/10.1109/TSMC.2016.2602543.
82. Thalheim, B.: The Theory of Conceptual Models, the Theory of Conceptual Modelling and Foundations of Conceptual Modelling. In: Embley, D.W. and Thalheim, B. (eds.) Handbook of Conceptual Modeling. pp. 543–577. Springer-Verlag Berlin Heidelberg (2011). https://doi.org/10.1007/978-3-642-15865-0.
83. Object Management Group: OMG Systems Modeling Language Version 1.6. (2019).
84. Booch, G., Rumbaugh, J., Jacobson, I.: The Unified Modeling Language User Guide. (1998).
85. MathWorks: Graph with directed edges - MATLAB, https://www.mathworks.com/help/matlab/ref/digraph.html, last accessed 2020/11/26.
86. Spivak, D.I.: Databases are Categories. (2010).
87. Bucchiarone, A., Cabot, J., Paige, R.F., Pierantonio, A.: Grand challenges in model-driven engineering: an analysis of the state of the research. Softw. Syst. Model. 19, 5–13 (2020). https://doi.org/10.1007/s10270-019-00773-6.
88. Zeigler, B., Mittal, S., Traore, M.: MBSE with/out Simulation: State of the Art and Way Forward. Systems. 6, 40 (2018). https://doi.org/10.3390/systems6040040.
89. NAFEMS, INCOSE: What Is Systems Modeling and Simulation? (2019). https://doi.org/10.1002/9780470403563.ch1.
90. Mittal, S., Zeigler, B.P., Martin, J.: Implementation of formal standard for interoperability in M&S/systems of systems integration with DEVS/SOA. … Command Control …. (2009).
91. Abar, S., Theodoropoulos, G.K., Lemarinier, P., O'Hare, G.M.P.: Agent Based Modelling and Simulation tools: A review of the state-of-art software. Comput. Sci. Rev. 24, 13–33 (2017). https://doi.org/10.1016/j.cosrev.2017.03.001.