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Article

Stochastic Behavior of a Two-Unit Parallel System with Dissimilar Units and Optional Vacations under Poisson Shocks

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Abstract: This article examines the impact of some system parameters on an industrial system composed of two dissimilar parallel units with one repairman. The active unit may fail due to essential factors like aging or deteriorating, or exterior phenomena such as Poisson shocks that occur at various time periods. Whenever the value of a shock is larger than the specified threshold of the active unit, the active unit will fail. The article assumes that the repairman has the right to take any of two decisions at the beginning of the system operation: either a takes a vacation if the two units work in a normal way, or stay in the system to monitor the system until the first system failure. In case of having a failure in any of the two units during the absence of the repairman, the failing unit will have to wait until the repairman is called back to work. We suppose that the value of every shock is assumed to be i.i.d. with some known distribution. The length of the repairman's vacation, repair time, and recall time are arbitrary distributions. Various reliability measures have been calculated by the supplementary variable technique and the Markov's vector process theory. At last, numerical computation and graphical analysis have been given for a particular case to validate the derived indices.

Keywords: Mean time to failure; Poisson shock; Steady-state availability; Steady-state frequency; Supplementary variable technique.

1. Introduction

The economic progress of any country depends to a great extent on developing the industrial and mechanical fields. This progress leads to the appearance of new technicalities that help solving problems that may occur in these complex industrial systems. But in spite of all that help, consumers hope to deal with low cost and high efficiency industrial systems. Therefore, system designers and researchers in this field face a great challenge to develop many systems and raise their efficiency, reliability, and safety.

Many researches have been made in the past to explain the concept of reliability and to raise the efficiency of many industrial systems, and to analyze the cost of the different redundant systems under the effect of some restrictions such as the periods of which the system is on or off, the different types of failures affecting that system, the different types of repairing these units, whether or not it's better for the repair man to take single or multiple vacations…etc.

In this context, we present some of the previous research work that deserves mention. The concept of vacation was first presented to the model analysis of the queuing system in article [1] as 2-standard vacation policies. These policies were defined as multiple vacations and single vacation. Some thorough and excellent studies from the modern results for a variety of vacation models, inclusive of some applications were presented by [2]. Many researchers, though, only studied different vacation models from a queuing theory viewpoint. Therefore, in article [3] the concept of vacation was introduced by the reliability theory viewpoint, and an n-component series system with multiple vacations of repairman was discussed. Since then, the researchers were interested in

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studying reliability models with the vacation. The reader can see e.g. [4, 5, 6, 7, 8, 9]. The 2-component repairable systems are one of the meaningful reliability applications. Therefore, many authors have studied various repairable systems and obtained many measures of reliability in those systems. For example, [10] used the Markov renewal theory to study the behavior of the system which consists of a 2- parallel component in the presence of the maintenance man. [11] applied the reliability theory on a two-component series repairable system. The system can be used to analyze the reliability measures of some automatic systems. The problem of repairing industrial equipment with two vacation policies (single vacation and multiple vacations) is studied in article [12] using queuing theory.

Most contemporary researchers focused on repairable systems under the effect of Poisson shocks. For example, [13] discussed the effect of Poisson Shocks on the behavior of a repairable system which consists of two units, one of them is working and the other is cold standby with a single repairman. [14] analyzed the reliability of the repairable system under the influence of two types of failure. One of them is external factors such as Poisson shocks and the other is fundamental factors such as aging or deterioration. The repairman has the right to take a vacation when the system is active. [15] displayed a two non-identical unit cold standby repairable system maintained by a single repairman. The proposed system suffers from one of two types of failures. One has to do with intrinsic factors, and the other is related to external shocks such as the stepwise Poisson process. [16] focuses on one unit under Poisson shocks with the assumption that the deterioration caused by a single shock may be ineffective, and the system fails only when the deterioration has accumulated to a specific level. The effect Poisson shocks has on the system behavior, which consists of two-non-symmetric parallel units, one of them is operative and the other is cold standby, with the presence of the imperfect key switch between these two units, was investigated by [17].

The reliability measurements of two dissimilar parallel units for the repairable systems are important to the industry field. Analyzing and elicitation of these measurements for the systems influenced by shocks can be more motivating and interesting as chocks take place from time to time in the world. This induces us to derive the reliability of two dissimilar parallel units for a repairable system. In this article, we assume the system might be subjected to shocks following a Poisson process and the repairman has the choice to either stay at the system to monitor the two units until one fails or takes a vacation if the two units work in a normal way. It should be noted that system behavior analysis is not an easy task when all system distributions are general distributions.

In this article, we present the supplementary variable method that helps in solving partial differential equations related to describe the dynamics of movement between the different states of the system. With the help of the ergodicity of the investigated process and supplementary variable method, we obtain explicit expressions of reliability metrics such as reliability function, steady-state availabilities, mean time to system failure, Steady-state probability that the repairman is on vacation, Steady-state probability that the system is waiting for repair, and Steady-state failure frequency.

This article is designed as follows:

Section 2 introduces more details about the system and assumptions of the system description. Section 3 deduces integro-differential equations which describe the movement between the different states of the system. The reliability measures for this system are calculated in section 4. Section 5 introduces a special case; the repairman stays in the system without taking any vacation. The numerical examples are presented in section 6. Conclusive remarks are offered in the final section.

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Notations

$h(t), H(t)$	p.d.f. and c.d.f. of the repair time,
$v_1(t), V_1(t)$	p.d.f. and c.d.f. of the vacation time of a repairman,
$v_2(t), V_2(t)$	p.d.f. and c.d.f. of the recall time of a repairman,
$P_i(t,x), Q_i(t,x)$	p.d.f. and c.d.f. of the system is in state i = 2,4 at period t and has an elapsed repair time of x for unit A,
$P_i(t,x), Q_i(t,x)$	p.d.f. and c.d.f. of the system is in state i = 1,8 at period t and has an elapsed repair time of y for unit B,
$P_0(t,u), Q_0(t,u)$	p.d.f. and c.d.f. of the system is in state S_0at period t and has an elapsed vacation time of u,
$P_i(t,z), Q_i(t,z)$	p.d.f. and c.d.f. of the system is in state i = 5,6,7 at period t and has recalling time of z,
p	the probabilities of calling the repairman from the vacation to repair the unit A when the two units are a failure, i.e. "the probability of moving from state 7 to state 4",
q	the probabilities of calling the repairman from the vacation to repair the unit B when the two units are a failure, i.e. "the probabilities of moving from state 7 to state 8",
$\phi(u)$	vacation time distribution function,
$\mu_1(x)$	the repair rate of unit A,
$\mu_2(y)$	the repair rate of unit B,
$\alpha(z)$	call time distribution function,
$h^*(s)$	LT (Laplace transform) $h(t)$,
$p_i(t)$	the probability of the system to be in state i at time t.

2. Description of the System and Assumptions

The system which consists of two dissimilar units and a single repairman under Poisson shock is subjected to the following assumptions:

A1: At the initial time, both units are working with high efficiency, and a repairman has the choice to either stay in the system or takes a vacation.

A2: The system is exposed to shocks continually. The arrival of shocks is considered as a Poisson process $\{S(t), t \geq 0\}$ with the strength$\lambda_i > 0$. The value of every shock is $\hat{Y}$, i.i.d random variable with distribution function F.

A3: When a shock occurs and the value of this shock overrides a threshold, the active unit will break down. The threshold of units (A and B) is a non-negative random variable τ_i with a distribution functionξ.

A4: When any unit fails with the existence of the repairman in the system, it will be repaired immediately. Once the repairman is done repairing the failed units, the repairman has the choice to either stay at the system or take a vacation, then he returns from vacation if at least one unit is failing. The repair rule is ''first-in-first-out''. If a unit fails while the other is being repaired, the recently failed unit must wait for repair, and the system has to stop working. To expedite the system operation when the repairman in a vacation and the system is a breakdown, the system requires repairing one of two units A or B, with probability p or q respectively.

A5: Shocks are the main reason for units to fail, and the system fails only if both the units fail.

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Based on the preceding assumptions, we can conclude that the conditional failure probability of unit A and unit B are random variables $\xi_i(\hat{Y})$ $(i = 1,2)$.
The probability distributions are:

$$K_i(y) = P(\xi_i(\hat{Y}) \le y) = P(\hat{Y} \le \xi_i^{-1}(y)) = F(\xi_i^{-1}(y)), 0 \le y \le 1. (i = 1,2).$$

The possibility of a single shock causes unit (A *or* B) to fail. According to the above assumptionsA_2, A_3, we get

$$\tau_i = P(\hat{Y} \le \tau_i) = \int_0^\infty P(\tau_i < \hat{y} | \hat{Y} = \hat{y})\, dP(\hat{Y} \le \hat{y}) = \int_0^\infty \xi_i(\hat{y})\, dF(\hat{y}), 0 \le y \le 1. (i = 1,2).$$

3. System Analysis

The states of this system $\Omega(t)$ at time t as a following:

S0: at any time t, unit A is active, unit B is active, and the repairman has the choice to either stay at the system or takes a vacation.

S1: at any time t, unit A is active unit B is being repaired, and the repairman still in the system, the system is working.

S2: at any time t unit A is being repaired, unit B is active, and the repairman still in the system, the system is working.

S3: at any time t, unit A is active, unit B is active, and the repairman chooses to take a vacation, the system is working.

S4: at any time t, unit A is still repaired from above S2, unit B is waiting for being repaired, and the system is down.

S5: at any time t, unit A is active, unit B is waiting for repair, the repairman in a vacation, and the system is working.

S6: at any time t, unit A is waiting for repair, unit B is active, and the repairman in a vacation, the system is working.

S7: at any time t, unit A is still waiting for repair from S6, unit B is waiting for repair and the repairman in a vacation either go to state 4 with probability p or going to state 8 with probability q and the system is down.

S8: at any time t, unit A is waiting for being repaired, unit B is under repair and the system is down.

The state space is$E = \{S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8\}$, where the working state is $U = \{S_0, S_1, S_2, S_3, S_5, S_6\}$and the down state is $D = \{S_4, S_7, S_8\}$. Accordance with the above assumptions, and $\{\Omega(t), t \ge 0\}$is not a Markov process we using the supplementary variables as a following:

$X_1(t)$: is time to make a vacation decision. $\Omega(t) = \{S_0\}$.

$X_2(t)$: is the elapsed vacation time. $\Omega(t) = \{S_5, S_6, S_7\}$.

$Y_1(t)$: is the elapsed repair time of unit A being repaired at time t. $\Omega(t) = \{S_2, S_4\}$.

$Y_2(t)$: is the elapsed repair time of unit B being repaired at time t. $\Omega(t) = \{S_1, S_8\}$.

State space is a following: $\Omega^* = \{(0,u), (1,y), (2,x), 3, (4,x), (5,z), (6,z), (7,z), (8,y)\}$ where $u, z, x$and $y$are explanatory values of $X_1(t), X_2(t), Y_1(t)$ and $Y_2(t)$, respectively. Indicate to:

$Q_0(t,u) = \rho(\Omega(t) = 0, X_1(t) \le u)$, $Q_i(t,z) = \rho(\Omega(t) = i, X_2(t) \le z), (i = 5,6,7)$,

$Q_i(t,x) = \rho(\Omega(t) = i, Y_1(t) \le x), (i = 2,4)$, $Q_i(t,y) = \rho(\Omega(t) = i, Y_2(t) \le y), (i = 1,8)$.

where $\rho(E)$ is probability of event E.

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$\rho_i(t,w) = \frac{d}{dw} Q_i(t,w), (i = 0,1,2,4,5,6,7,8).$

$\varphi_i(t) = \rho(s(t) = i), (i = 0,1,2,4,5,6,7,8).$

From the above, we can formulation the differential equations that represent this system by using the probability arguments and limiting transitions as following.

$$\rho_0(t + \Delta t, u + \Delta t) = \rho_0(t,u)(1 - (\lambda_1 r_1 + \lambda_2 r_2 + \phi(u))\Delta t) + o\Delta t$$

When Δt tend to zero, we get

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial u} + \lambda_1 r_1 + \lambda_2 r_2 + \phi(u)\right) \rho_0(t,u) = 0, \quad (1)$$

The same style, we get the following:

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \lambda_1 r_1 + \mu_2(y)\right) \rho_1(t,y) = 0, \quad (2)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_2 r_2 + \mu_1(x)\right) \rho_2(t,x) = 0, \quad (3)$$

$$\left(\frac{d}{dt} + \lambda_1 r_1 + \lambda_2 r_2\right) \varphi_3(t) = \int_0^\infty \rho_0(u,t)\, \phi(u) du, \quad (4)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_1(x)\right) \rho_4(t,x) = \lambda_2 r_2 \rho_2(t,x), \quad (5)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \lambda_1 r_1 + \alpha(z)\right) \rho_5(t,z) = 0, \quad (6)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \lambda_2 r_2 + \alpha(z)\right) \rho_6(t,z) = 0, \quad (7)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \alpha(z)\right) \rho_7(t,z) = \lambda_1 r_1 \rho_5(t,z) + \lambda_2 r_2 \rho_6(t,z), \quad (8)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \mu_2(y)\right) \rho_8(t,y) = \lambda_1 r_1 \rho_1(t,y), \quad (9)$$

The boundary conditions are:

$\rho_0(t,0) = \int_0^\infty \rho_1(t,y)\mu_2(y)dy + \int_0^\infty \rho_2(t,x)\mu_1(x)dx + \varepsilon(t),$ (10)

$\rho_1(t,0) = \int_0^\infty \lambda_2 r_2 \rho_0(t,u)du + \int_0^\infty \rho_4(t,x)\mu_1(x)dx + \int_0^\infty \rho_5(t,z)\alpha(z)dz,$ (11)

$\rho_2(t,0) = \int_0^\infty \lambda_1 r_1 \rho_0(t,u)du + \int_0^\infty \rho_8(t,y)\mu_2(y)dy + \int_0^\infty \rho_6(t,z)\alpha(z)dz,$ (12)

$\rho_5(t,0) = \lambda_2 r_2 \varphi_3(t),$ (13)

$\rho_6(t,0) = \lambda_1 r_1 \varphi_3(t),$ (14)

$\rho_7(t,0) = 0,$ (15)

$\rho_4(t,0) = \rho_8(t,0) = \int_0^\infty \rho_7(t,z)\alpha(z)dz,$ (16)

$\sum_{i=0}^{2} \int_0^\infty \rho_i(t,m)dm + \varphi_3(t) + \sum_{i=4}^{8} \int_0^\infty \rho_i(t,m)dm = 1, m = u,x,y,z$.

(17)

The initial conditions are:

$$\rho_0(0,u) = \varepsilon(u) = \begin{cases} 1 & u = 0 \\ 0 & u \neq 0 \end{cases},$$

$\rho_0(0,m) = 0, \quad m \neq 0, m = u,x,y,z. i = 1,\ldots,8. \varphi_3(0) = 0.$

Laplace transform can be defined as a following:

$$k^*(s) = L\{k(x)\} = \int_0^\infty k(x)e^{-sx}dx, s > 0.$$

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The ergodicity of the investigation process guarantees the presence of the following steady-probability: $\varphi_i = lim_{t\to\infty} \quad \varphi_i(t), i = 1,\ldots,8.$ $\theta_i(m) = lim_{t\to\infty}\rho_i(t,m),\ i = 1,2,4,5,6,7,8.$ which follows the following relations: $\varphi_i = \int_0^\infty \theta_i(m)\,dm, \quad i = 1,2,4,5,6,7,8.$

Taking the limit $t\to\infty$ in the equations (1) – (17), the following equations are obtained:

$$\left(\frac{d}{du} + \lambda_1 r_1 + \lambda_2 r_2 + \phi(u)\right)\theta_0(u) = 0, \qquad (18)$$

$$\left(\frac{d}{dy} + \lambda_1 r_1 + \mu_2(y)\right)\theta_1(y) = 0, \qquad (19)$$

$$\left(\frac{\partial}{\partial x} + \lambda_2 r_2 + \mu_1(x)\right)\theta_2(x) = 0, \qquad (20)$$

$$(\lambda_1 r_1 + \lambda_2 r_2)\varphi_3 = \int_0^\infty \theta_0(u)\,\phi(u)du, \qquad (21)$$

$$\left(\frac{d}{dx} + \mu_1(x)\right)\theta_4(x) = \lambda_2 r_2 \theta_2(x), \qquad (22)$$

$$\left(\frac{d}{dz} + \lambda_1 r_1 + \alpha(z)\right)\theta_5(z) = 0, \qquad (23)$$

$$\left(\frac{\partial}{\partial z} + \lambda_2 r_2 + \alpha(z)\right)\theta_6(z) = 0, \qquad (24)$$

$$\left(\frac{d}{dz} + \alpha(z)\right)\theta_7(z) = \lambda_1 r_1 \theta_5(z) + \lambda_2 r_2 \theta_6(z), \qquad (25)$$

$$\left(\frac{d}{dy} + \mu_2(y)\right)\theta_8(y) = \lambda_1 r_1 \theta_1(y), \qquad (26)$$

$$\theta_0(0) = \int_0^\infty \theta_1(y)\mu_2(y)dy + \int_0^\infty \theta_2(x)\mu_1(x)dx, \qquad (27)$$

$$\theta_1(0) = \int_0^\infty \lambda_2 r_2 \theta_0(u)du + \int_0^\infty \theta_4(x)\mu_1(x)dx + \int_0^\infty \theta_5(z)\alpha(z)dz, \qquad (28)$$

$$\theta_2(0) = \int_0^\infty \lambda_1 r_1 \theta_0(u)du + \int_0^\infty \theta_8(y)\mu_2(y)dy + \int_0^\infty \theta_6(z)\alpha(z)dz, \qquad (29)$$

$$\theta_5(0) = \lambda_2 r_2 \varphi_3, \qquad (30)$$

$$\theta_6(0) = \lambda_1 r_1 \varphi_3, \qquad (31)$$

$$\theta_7(0) = 0, \qquad (32)$$

$$\theta_4(0) = \theta_8(0) = \int_0^\infty \theta_7(z)\alpha(z)dz, \qquad (33)$$

$$\sum_{i=0}^{2}\int_0^\infty \theta_i(m)dm + \varphi_3 + \sum_{i=4}^{8}\int_0^\infty \theta_i(m)dm = 1, m = u, x, y, z.$$

(34)

4. Reliability Characteristics

According to the results derived from the analysis of the system in the previous section, the reliability index of the system is obtained as follows:

4.1. Steady-state availability is

$$Av(\infty) = \sum_{i=0}^{3}\varphi_i + \sum_{i=5}^{6}\varphi_i \quad .$$

(35)

4.2. Steady-state probability that the repairman is on vacation is

$$\varphi_{rep.V} = \varphi_3 + \sum_{i=5}^{7}\varphi_i \quad .$$

(36)

4.3. Steady-state probability that the system is waiting for repair is

$$\varphi_{rep.W} = \varphi_7.$$

(37)

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4.4. Steady-state failure frequency is

$$f.f = \lambda_1 r_1(\varphi_1 + \varphi_5) + \lambda_2 r_2(\varphi_2 + \varphi_6). \quad (38)$$

where,

The steady-state probabilities can be obtained as follows from $\varphi_i = \int_0^\infty \theta_i(m)\, dm$, for all $i = 1,2,4,5,6,7,8$.

$$\varphi_0 = C_0 \bar{V}_1^*(\lambda_1 r_1 + \lambda_2 r_2),$$

$$\varphi_1 = \{C_0 \bar{H}_2^*(r_1\lambda_1)(v_1^*(\lambda_1 r_1 + \lambda_2 r_2)(\lambda_2 r_2(p + q + q h_1^*(\lambda_2 r_2)(v_2^*(\lambda_1 r_1) - 1) - (p + q - 1) v_2^*(\lambda_1 r_1))$$

$$+\lambda_1 r_1(p + q - (p + q - 1) v_2^*(\lambda_2 r_2) + h_1^*(\lambda_2 r_2)((q-1) v_2^*(\lambda_2 r_2) - q))) - (\lambda_1 r_1 + \lambda_2 r_2)(\lambda_1 r_1 - \lambda_2 r_2$$

$$\frac{(h_1^*(\lambda_2 r_2) - 1))\bar{V}_1^*(\lambda_1 r_1 + \lambda_2 r_2)))}{((\lambda_1 r_1 + \lambda_2 r_2)(h_2^*(\lambda_1 r_1) - h_1^*(\lambda_2 r_2)(h_2^*(\lambda_1 r_1) - 1)))},$$

$$\phi_2 = -\{C_0 \bar{H}_1^*(r_2\lambda_2)(v_1^*(\lambda_1 r_1 + \lambda_2 r_2)(\lambda_2 r_2(p + q - (p + q - 1) v_2^*(\lambda_1 r_1) + h_2^*(\lambda_1 r_1)((p-1) v_2^*(\lambda_1 r_1) - p))$$

$$+\lambda_1 r_1(p + q + p h_2^*(\lambda_1 r_1)(v_2^*(\lambda_2 r_2) - 1) - (p + q - 1) v_2^*(\lambda_2 r_2))) + (\lambda_1 r_1 + \lambda_2 r_2)(\lambda_1 r_1 - \lambda_2 r_2$$

$$\frac{(h_1^*(\lambda_1 r_1) - 1))\bar{V}_1^*(\lambda_1 r_1 + \lambda_2 r_2)))}{((\lambda_1 r_1 + \lambda_2 r_2)(h_2^*(\lambda_2 r_2)(h_2^*(\lambda_1 r_1) - 1) - h_2^*(\lambda_1 r_1)))},$$

$$\varphi_3 = \frac{C_0 v_1^*(\lambda_1 r_1 + \lambda_2 r_2)}{(\lambda_1 r_1 + \lambda_2 r_2)},$$

$$\varphi_4 = \frac{C_0(-p v_1^*(\lambda_1 r_1 + \lambda_2 r_2)(\lambda_2 r_2(v_2^*(\lambda_1 r_1) - 1) + \lambda_1 r_1(v_2^*(\lambda_2 r_2) - 1))}{(\lambda_1 r_1 + \lambda_2 r_2)\mu_1} + \frac{1}{h_1^*(\lambda_2 r_2)(h_2^*(\lambda_1 r_1) - 1) - h_2^*(\lambda_1 r_1)}$$

$$(\mu_1 \bar{H}_1^*(\lambda_2 r_2) - 1)(v_1^*(\lambda_1 r_1 + \lambda_2 r_2)(\lambda_2 r_2(p + q - (p + q - 1) v_2^*(\lambda_1 r_1) - h_2^*(\lambda_1 r_1)(q v_2^*(\lambda_1 r_1) + p))$$

$$+\lambda_1 r_1(p + q + p h_2^*(\lambda_1 r_1)(v_2^*(\lambda_2 r_2) - 1) - (p + q - 1) v_2^*(\lambda_2 r_2))) + (\lambda_1 r_1 + \lambda_2 r_2)(\lambda_1 r_1 - \lambda_2 r_2$$

$$(h_2^*(\lambda_1 r_1) - 1))\bar{V}_1^*(\lambda_1 r_1 + \lambda_2 r_2))),$$

$$\varphi_5 = \frac{C_0 \lambda_2 r_2 v_1^*(\lambda_1 r_1 + \lambda_2 r_2)\bar{V}_2^*(\lambda_1 r_1)}{(\lambda_1 r_1 + \lambda_2 r_2)},$$

$$\varphi_6 = \frac{C_0 \lambda_1 r_1 v_1^*(\lambda_1 r_1 + \lambda_2 r_2)\bar{V}_2^*(\lambda_2 r_2)}{(\lambda_1 r_1 + \lambda_2 r_2)},$$

$$\varphi_7 = \frac{C_0 v_1^*(\lambda_1 r_1 + \lambda_2 r_2)(\lambda_2 r_2(1 - \alpha\bar{V}_2^*(\lambda_1 r_1)) + \lambda_1 r_1(1 - \alpha\bar{V}_2^*(\lambda_2 r_2)))}{\alpha(\lambda_1 r_1 + \lambda_2 r_2)},$$

$$\varphi_8 = \frac{C_0}{(\lambda_1 r_1 + \lambda_2 r_2)}\Bigg\{\frac{-q v_1^*(\lambda_1 r_1 + \lambda_2 r_2)(\lambda_2 r_2(v_1^*(\lambda_1 r_1) - 1) + \lambda_1 r_1(v_2^*(\lambda_2 r_2) - 1))}{\mu_2} +$$

$$\frac{\left(\frac{1}{\mu_2} - \bar{H}_2^*(\lambda_1 r_1)\right)}{h_1^*(\lambda_2 r_2)(1 - h_2^*(\lambda_1 r_1)) + h_2^*(\lambda_1 r_1)}(v_1^*(\lambda_1 r_1 + \lambda_2 r_2)(\lambda_2 r_2(p + q + q h_1^*(\lambda_2 r_2)((v_2^*(\lambda_1 r_1) - 1))$$

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$$-(p+q-1)v_2^*(\lambda_1 r_1))$$

$$+\lambda_1 r_1\Big(p+q-(p+q-1)v_2^*(\lambda_2 r_2)+h_1^*(\lambda_2 r_2)\big(-q+(q-1)v_2^*(\lambda_2 r_2)\big)\Big)\Big)$$

$$-(\lambda_1 r_1+\lambda_2 r_2)\big(-\lambda_2 r_2+\lambda_1 r_1(h_1^*(\lambda_2 r_2)-1)\big)\bar{V}_1^*(\lambda_1 r_1+\lambda_2 r_2)\Big),$$

$$C_0=\Big\{(\lambda_1 r_1+\lambda_2 r_2)\mu_1\mu_2\Big(h_1^*(\lambda_2 r_2)\big(h_2^*(\lambda_1 r_1)-1\big)-h_2^*(\lambda_1 r_1)\Big)\Big\}\Big/\Big\{\frac{1}{\alpha}v_1^*(\lambda_1 r_1+\lambda_2 r_2)\big(\alpha\mu_1\mu_2 h_1^*(\lambda_2 r_2)$$

$$(h_2^*(\lambda_1 r_1)-1)-h_2^*(\lambda_1 r_1)\big)+\lambda_2 r_2(-(p+q)\alpha(\mu_1+\mu_2)-(p\alpha+\mu_1)\mu_2 h_1^*(\lambda_2 r_2)+(\mu_1(q\alpha+\mu_2)$$

$$(h_1^*(\lambda_2 r_2)-1)+p\alpha\mu_2 h_1^*(\lambda_2 r_2)\big)h_2^*(\lambda_1 r_1)+\alpha\big(\mu_2\big(-1+p+q-ph_1^*(\lambda_2 r_2)(h_2^*(\lambda_1 r_1)-1)\big)+$$

$$\mu_1\big(-1+p+q+h_1^*(\lambda_2 r_2)-q(h_1^*(\lambda_2 r_2)-1)h_2^*(\lambda_1 r_1)\big)\Big)v_2^*(\lambda_2 r_2)\Big)\Big)$$

$$+(\lambda_1 r_1+\lambda_2 r_2)(\lambda_1 r_1(-\mu_2+$$

$$\mu_1\big(-1+h_1^*(\lambda_2 r_2)\big)\big)-\lambda_2 r_2\big(\mu_1+\mu_2-\mu_2 h_2^*(\lambda_1 r_1)\big)+\mu_1\mu_2\big(-h_1^*(\lambda_2 r_2)+\big(h_1^*(\lambda_2 r_2)-1\big)h_2^*(\lambda_2 r_2)\big)$$

$$V_1^*(\lambda_1 r_1+\lambda_2 r_2)\}.$$

5. Mean Time to the First Failure (MTTFF)

In this section, we deduce the mean time to the first failure (MTTFF) of the system. We assumed that t be the time to the first failure of the system, therefore the reliability function of this system is calculated as follows$R(t)=P(\tau>t)$. To obtain the reliability function, we consider the failure states {4, 7, 8} of the system are absorbing states.

Let:

$L_0(t,u)=\frac{d}{du}\rho[\tilde{S}(t)=0,\tilde{X}_1(t)\le u]$, $\quad L_i(t,u)=\frac{d}{dz}\rho[\tilde{S}(t)=i,\tilde{X}_2(t)\le z], i=5,6.$

$L_2(t,x)=\frac{d}{dx}\rho[\tilde{S}(t)=2,\tilde{Y}_1(t)\le x]$, $\quad L_1(t,y)=\frac{d}{dy}\rho[\tilde{S}(t)=1,\tilde{Y}_2(t)\le y]$, $\quad L_3(t)=\rho[\tilde{S}(t)=3]$.

In the same manner as previously mentioned in Section 4, we conclude reliability function as following:

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial u}+\lambda_1 r_1+\lambda_2 r_2+\phi(u)\right)L_0(t,u)=0, \tag{39}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\lambda_1 r_1+\mu_2(y)\right)L_1(t,y)=0, \tag{40}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_2 r_2+\mu_1(x)\right)L_2(t,x)=0, \tag{41}$$

$$\left(\frac{d}{dt}+\lambda_1 r_1+\lambda_2 r_2\right)L_3(t)=\int_0^\infty L_0(u,t)\,\phi(u)du, \tag{42}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\lambda_1 r_1+\alpha(z)\right)L_5(t,z)=0, \tag{43}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\lambda_2 r_2+\alpha(z)\right)L_6(t,z)=0, \tag{44}$$

The boundary conditions are:

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$L_0(t,0) = \int_0^\infty L_1(t,y)\mu_2(y)dy + \int_0^\infty L_2(t,x)\mu_1(x)dx + \varepsilon(t),$ (45)

$L_1(t,0) = \int_0^\infty \lambda_2 r_2 L_0(t,u)du + \int_0^\infty L_5(t,z)\alpha(z)dz,$ (46)

$L_2(t,0) = \int_0^\infty \lambda_1 r_1 L_0(t,u)du + \int_0^\infty L_6(t,z)\alpha(z)dz,$ (47)

$L_5(t,0) = \lambda_2 r_2 L_3(t),$ (48)

$L_6(t,0) = \lambda_1 r_1 L_3(t),$ (49)

The initial conditions are:

$$R_0(0,u) = \varepsilon(u) = \begin{cases} 1 & u = 0 \\ 0 & u \neq 0 \end{cases}'$$

Taking the Laplace transform of the equations (39-49), as well as initial conditions, we have:

$\left(\frac{d}{du} + s + \lambda_1 r_1 + \lambda_2 r_2 + \phi(u)\right) L_0^*(s,u) = 0,$ (50)

$\left(\frac{d}{dy} + s + \lambda_1 r_1 + \mu_2(y)\right) L_1^*(s,y) = 0,$ (51)

$\left(\frac{\partial}{\partial x} + s + \lambda_2 r_2 + \mu_1(x)\right) L_2^*(s,x) = 0,$ (52)

$(s + \lambda_1 r_1 + \lambda_2 r_2) L_3^*(s) = \int_0^\infty L_0^*(u,s)\, \phi(u) du,$ (53)

$\left(\frac{d}{dz} + s + \lambda_1 r_1 + \alpha(z)\right) L_5^*(s,z) = 0,$ (54)

$\left(\frac{d}{dz} + s + \lambda_2 r_2 + \alpha(z)\right) L_6^*(s,z) = 0,$ (55)

$L_0^*(s,0) = \int_0^\infty L_1^*(s,y)\mu_2(y)dy + \int_0^\infty L_2^*(s,x)\mu_1(x)dx + \varepsilon(t),$ (56)

$L_1^*(s,0) = \int_0^\infty \lambda_2 r_2 L_0^*(s,u)du + \int_0^\infty L_5^*(s,z)\alpha(z)dz,$ (57)

$L_2^*(s,0) = \int_0^\infty \lambda_1 r_1 L_0^*(s,u)du,$ (58)

$L_5^*(s,0) = \lambda_2 r_2 L_3^*(s),$ (59)

$L_6^*(s,0) = \lambda_1 r_1 L_3^*(s).$ (60)

From previous equations, we defined the reliability function as follows:

$R^*(s) = \int_0^\infty L_0^*(s,u)\, du + \int_0^\infty L_1^*(s,y)\, dy + \int_0^\infty L_2^*(s,x)\, dx + \sum_{i=5}^{6} \int_0^\infty L_i^*(s,z)\, dz + L_3^*(s).$

(61)

and the mean time to the first failure of the system (*MTTFF*) is given by

$MTTFF = \lim_{s \to 0} R^*(s).$ (62)

where,

$L_0^*(s) = \int_0^\infty L_0^*(s,u)\, du = L_0^*(s,0)\bar{V}_1^*(s + r_1\lambda_1 + r_2\lambda_2),\ L_1^*(s) = \int_0^\infty L_1^*(s,y)\, dy = L_1^*(s,0)\bar{H}_2^*(s + r_1\lambda_1),\ L_2^*(s) = \int_0^\infty L_2^*(s,x)\, dx = L_2^*(s,0)\bar{H}_1^*(s + r_2\lambda_2),\ L_3^*(s) = \frac{L_0^*(s,0)v_1^*(s+r_1\lambda_1+r_2\lambda_2)}{s+r_1\lambda_1+r_2\lambda_2},$

$L_5^*(s) = \int_0^\infty L_5^*(s,z)\, dz = L_5^*(s,0)\bar{V}_2^*(s + r_1\lambda_1),\ L_6^*(s) = \int_0^\infty L_6^*(s,z)\, dz = L_6^*(s,0)\bar{V}_2(s + r_2\lambda_2),$

$L_0^*(s,0) = \frac{-\{\varepsilon^*(s)(s+r_1\lambda_1+r_2\lambda_2)\}}{\left\{-s+r_2\lambda_2\left(-1+h_2^*(s+r_1\lambda_1)v_1^*(s+r_1\lambda_1+r_2\lambda_2)v_2^*(s+r_1\lambda_1)\right)\right.} + r_1\lambda_1\left(-1 + h_1^*(s + r_2\lambda_2)v_1^*(s + r_1\lambda_1 + r_2\lambda_2)v_2^*(s + r_2\lambda_2)\right) + (s + r_1\lambda_1 + r_2\lambda_2)(r_1\lambda_1 h_1^*(s + r_2\lambda_2),$

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$+r_2\lambda_2 h_2^*(s+r_1\lambda_1)\big)\bar{V}_1^*(s+r_1\lambda_1+r_2\lambda_2)\Big\}$, $L_1^*(s,0) = r_2\lambda_2 L_0^*(s,0)\Big\{\bar{V}_1^*(s+r_1\lambda_1+r_2\lambda_2)+$

$\left\{\frac{v_1^*(s+r_1\lambda_1+r_2\lambda_2)v_2^*(s+r_2\lambda_2)}{(s+r_1\lambda_1+r_2\lambda_2)}\right\}\Big\}$,

$L_2^*(s,0)=r_1\lambda_1 L_0^*(s,0)\Big\{\bar{V}_1^*(s+r_1\lambda_1+r_2\lambda_2)+\big\{v_1^*(s+r_1\lambda_1+r_2\lambda_2)v_2^*(s+r_2\lambda_2)/(s+r_1\lambda_1+r_2\lambda_2)\big\}\Big\}$

$L_5^*(s,0) = \frac{\{r_2\lambda_2 L_0^*(s,0)v_1^*(s+r_1\lambda_1+r_2\lambda_2)\}}{(s+r_1\lambda_1+r_2\lambda_2)}$,

$L_6^*(s,0) = \frac{\{r_1\lambda_1 L_0^*(s,0)v_1^*(s+r_1\lambda_1+r_2\lambda_2)\}}{(s+r_1\lambda_1+r_2\lambda_2)}$.

6. Special Case

In this section, we present the following special cases, which confirm the results of the previous sections.

Case 1: $r_1 = r_2, \phi(u) = 1$, then it means any shock will cause the active units to fail and the repairman is in the system.

Case 2: $r_1 > r_2, \phi(u) = 1$, then it means the unit A failure is faster than unit B failure and the repairman is in the system.

Case 3: $r_2 > r_1, \phi(u) = 1$, then it means the unit B failure is faster than unit A failure and the repairman is in the system.

Corresponding results can easily get for the previous particular cases.

7. Numerical Illustration

This section shows the usefulness of the proposed system by examining the impact of the repairman and other parameters on the system through the following numerical illustrations taking into consideration that:

$V_1(t) = \begin{cases}1-e^{-\phi} & t>0\\ 0 & t\le 0\end{cases}$, $V_2(t) = \begin{cases}1-e^{-\alpha} & t>0\\ 0 & t\le 0\end{cases}$, $H_i(t) = \begin{cases}1-e^{-\mu_i t} & t>0\\ 0 & t\le 0\end{cases}$, $\forall i =$ 1,2,

At first, we show Illustrative numerical examples comparing the reliability metrics for the above special cases when the repairman is present in the system. Figs (1-3) we can observe the effect of α when it has the values of $\{1,7,14,21,28\}$ on steady-state availability when $\lambda_1 \in [0,1]$. From the curves of Figs (1-3), we conclude that the steady-state availability for this system increases very slowly when $\alpha > 7$. Fig.4 illustrates the effect of both r_1 and r_2 on steady-state availability when $\lambda_1 \in [0,1]$. In curve of Fig.4, we deduce that the steady-state availability is increasing when $r_1 > r_2$. The idea is also clear when examining Figs. (5-7). In these Figs.(5-7) the impact of α when it has the values of $\{1,7,14,21,28\}$on mean time to system failure when $\lambda_1 \in [0,1]$. The curves of Figs.(5-7) show that the mean time to system failure is more stable when $\alpha > 7$. The curve of Fig.8 shows that the mean time to system failure increases when $r_1 < r_2$ in interval $\lambda_1 \in [0,0.35]$as it is increases also when $r_1 > r_2$ in interval $\lambda_1 \in [0.35,1]$.

The steady-state availability and the mean time to system failure are examined, when λ_1 and α change, as shown in Tables (1-6). We vary the values of λ_1

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and $\alpha$and note their cross-impact on the steady-state availability and the mean time to system failure. It shows that increasing λ_1 can greatly decrease the steady-state availability and the mean time to system failure; however, increasing α seldom affects the values of steady-state availability and the mean time to system failure.

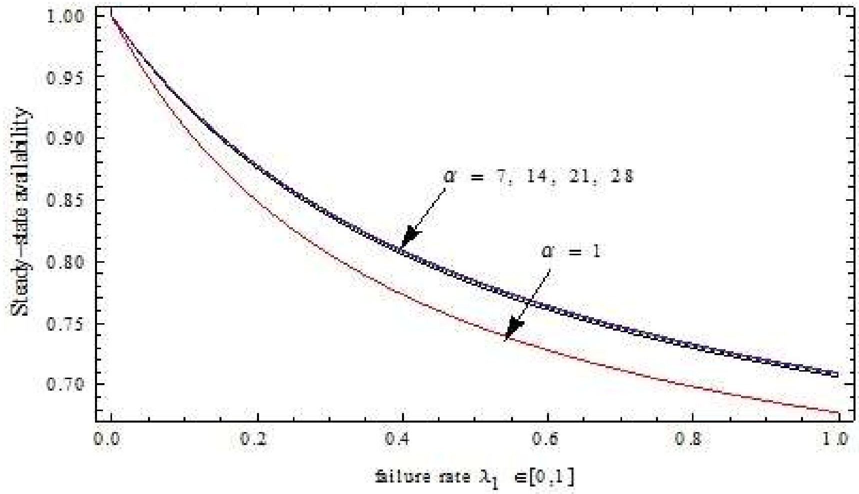

Fig. 1. Steady-state availability versus rate $\lambda_1$when $r_1 = r_2$and parameter$\alpha = 1,7,14,21,28$

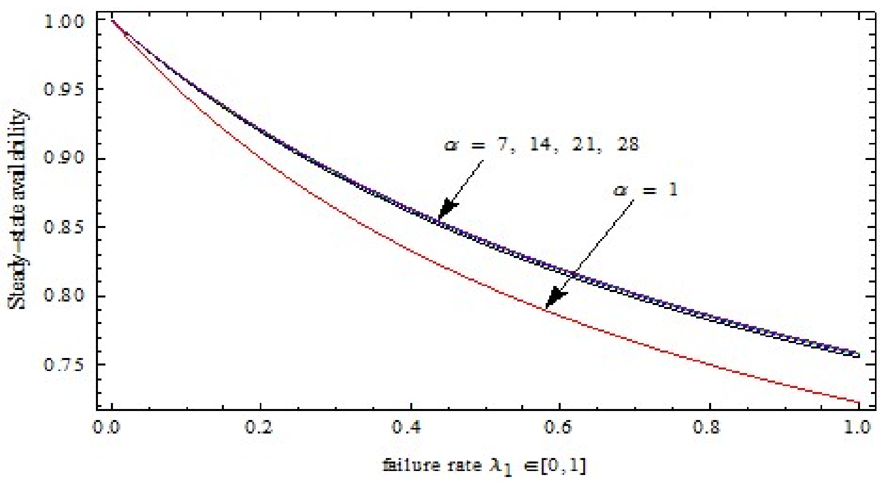

Fig. 2. Steady-state availability versus rate $\lambda_1$when $r_1 < r_2$and parameter$\alpha = 1,7,14,21,28$

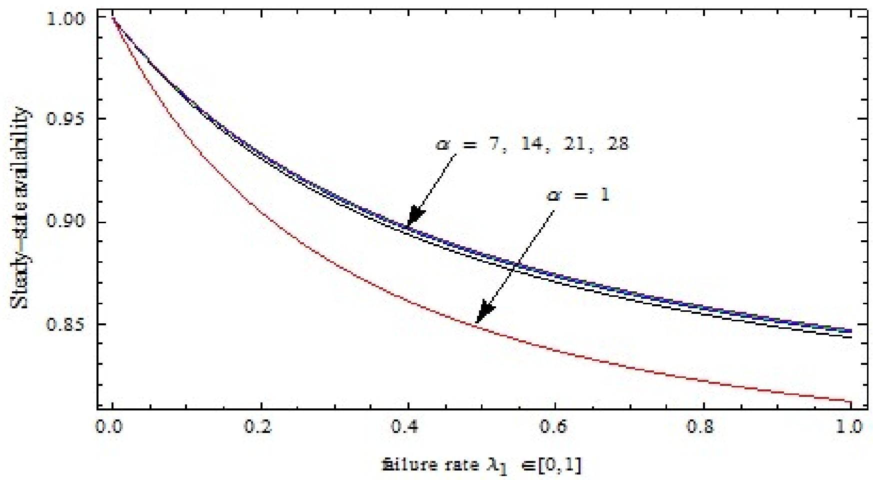

Fig 3. Steady-state availability versus rate $\lambda_1$when $r_1 > r_2$and parameter$\alpha = 1,7,14,21,28$

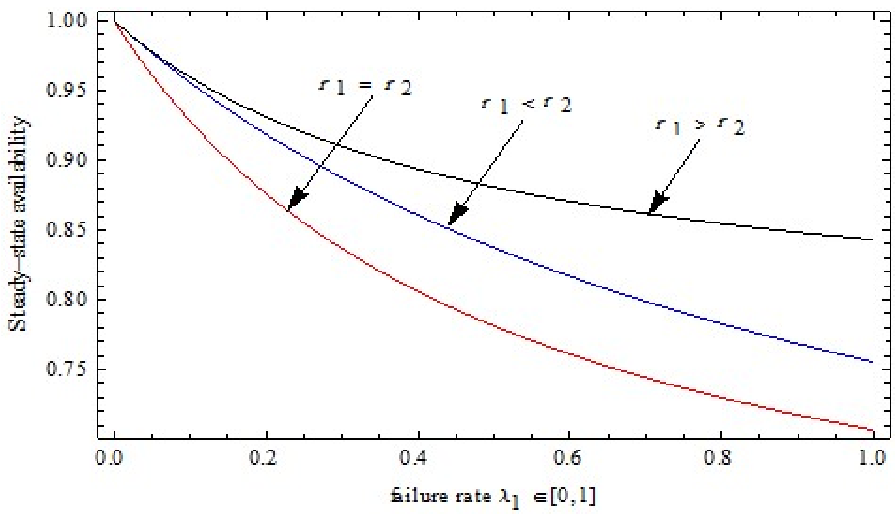

Fig 4. Steady-state availability versus rate $\lambda_1$when $\alpha = 7$

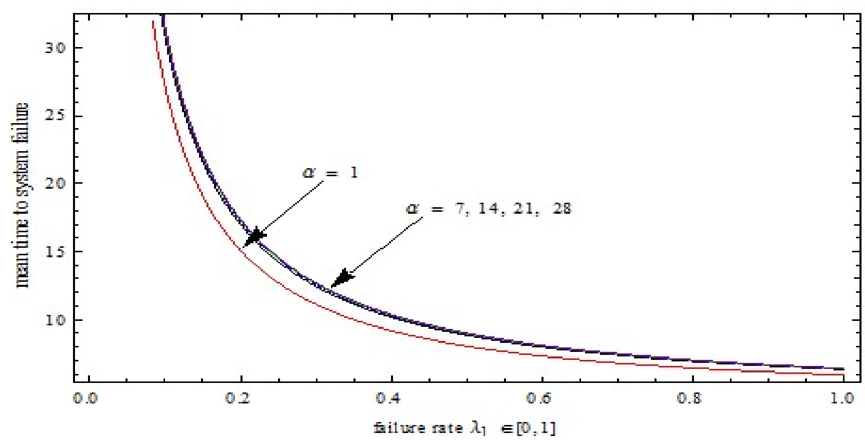

Fig. 5. Mean time to the first failure versus rate $\lambda_1$when $r_1 = r_2$and parameter$\alpha = 1,7,14,21,28$

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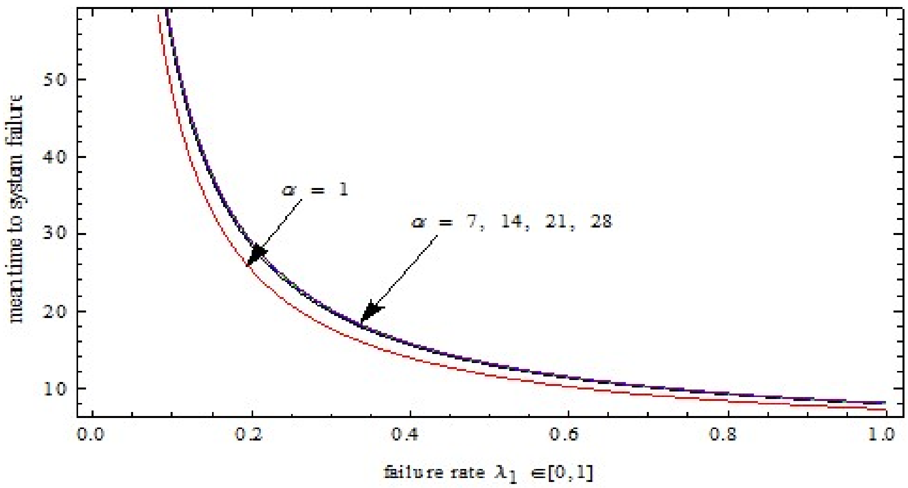

Fig. 6. Mean time to the first failure versus rate λ_1 when $r_1 < r_2$ and parameter $\alpha = 1,7,14,21,28$

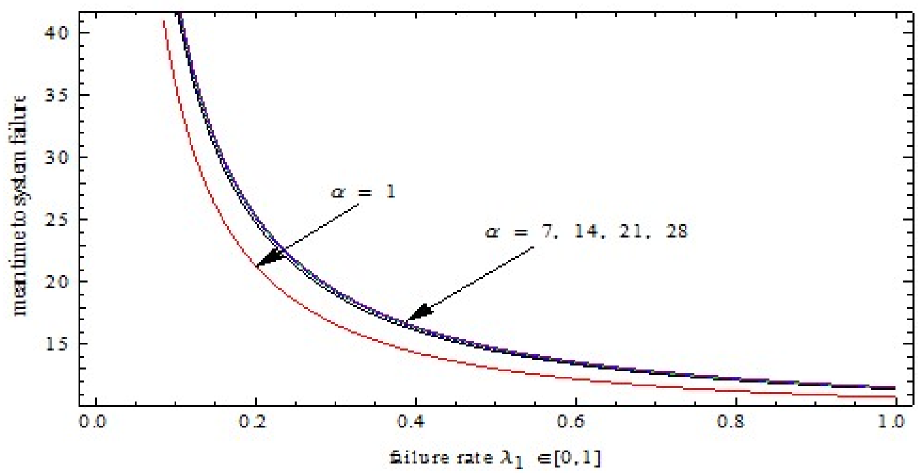

Fig. 7. Mean time to the first failure versus rate λ_1 when $r_1 > r_2$ and parameter $\alpha = 1,7,14,21,28$.

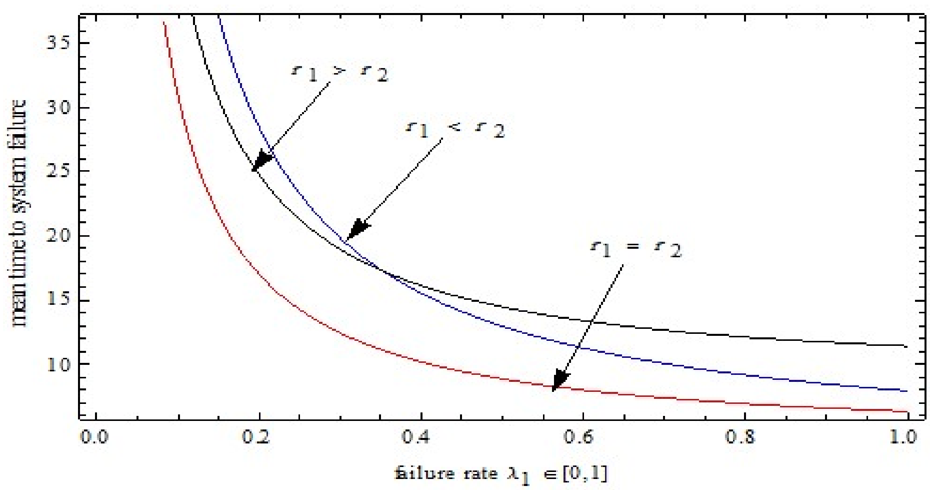

Fig 8. Mean time to the first failure versus rate λ_1 when $\alpha = 7$

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Table 1. Steady-state availability for different a and $r_1 = 0.3 < r_2 = 0.7$when $\lambda_2 = 0.4, p = 0.3, q = 0.7, \mu_2 = 0.4, \mu_1 = 0.5$

λ_1	$\alpha = 1$	$\alpha = 7$	$\alpha = 14$	$\alpha = 21$	$\alpha = 28$	$\alpha = 35$	$\alpha = 42$	$\alpha = 49$
0.1	0.945163	0.955857	0.956584	0.956819	0.956935	0.957004	0.95705	0.957083
0.2	0.900995	0.918948	0.92018	0.920579	0.920776	0.920893	0.920971	0.921026
0.3	0.864701	0.887634	0.889219	0.889732	0.889986	0.890137	0.890237	0.890309
0.4	0.834379	0.860735	0.862568	0.863161	0.863454	0.863629	0.863745	0.863827
0.5	0.808691	0.83738	0.839385	0.840034	0.840355	0.840546	0.840673	0.840763
0.6	0.786666	0.816913	0.819037	0.819724	0.820064	0.820266	0.820401	0.820497
0.7	0.767587	0.798832	0.801034	0.801747	0.802099	0.802309	0.802449	0.802548
0.8	0.75091	0.782743	0.784994	0.785723	0.786083	0.786297	0.78644	0.786541

Table 2. Steady-state availability for different a and $r_1 = 0.7 > r_2 = 0.3$when $\lambda_2 = 0.4, p = 0.7, q = 0.3, \mu_2 = 0.4, \mu_1 = 0.5$

λ_1	$\alpha = 1$	$\alpha = 7$	$\alpha = 14$	$\alpha = 21$	$\alpha = 28$	$\alpha = 35$	$\alpha = 42$	$\alpha = 49$
0.1	0.94198	0.959523	0.960715	0.961101	0.961292	0.961405	0.961481	0.961535
0.2	0.904875	0.931008	0.932854	0.933453	0.93375	0.933927	0.934045	0.934129
0.3	0.879432	0.909885	0.912099	0.912822	0.91318	0.913393	0.913535	0.913637
0.4	0.861099	0.893637	0.89606	0.896853	0.897246	0.897481	0.897638	0.897749
0.5	0.847388	0.88077	0.883306	0.884137	0.88455	0.884797	0.884962	0.885079
0.6	0.836832	0.87034	0.872928	0.873779	0.874202	0.874455	0.874623	0.874743
0.7	0.82851	0.861724	0.864325	0.865182	0.865608	0.865864	0.866034	0.866155
0.8	0.821822	0.854491	0.85708	0.857935	0.858361	0.858617	0.858786	0.858908

Table 3. Steady-state availability for different a and $r_1 = r_2 = 0.6$when $\lambda_2 = 0.4, p = 0.5, q = 0.5, \mu_2 = 0.4, \mu_1 = 0.5$

λ_1	$\alpha = 1$	$\alpha = 7$	$\alpha = 14$	$\alpha = 21$	$\alpha = 28$	$\alpha = 35$	$\alpha = 42$	$\alpha = 49$
0.1	0.909832	0.927946	0.929189	0.929592	0.92979	0.929909	0.929988	0.930044
0.2	0.849187	0.875919	0.877782	0.878386	0.878684	0.878862	0.87898	0.879064
0.3	0.805794	0.836605	0.838777	0.839481	0.839829	0.840037	0.840174	0.840272
0.4	0.773317	0.805862	0.808177	0.808927	0.809298	0.809519	0.809666	0.809771
0.5	0.748162	0.781169	0.783533	0.7843	0.784679	0.784904	0.785055	0.785161
0.6	0.728147	0.760904	0.763264	0.764029	0.764408	0.764634	0.764784	0.76489
0.7	0.711866	0.743977	0.746302	0.747056	0.747429	0.747652	0.747799	0.747905
0.8	0.698382	0.729627	0.731899	0.732637	0.733002	0.733219	0.733364	0.733467

Table 4. Mean time to the first failure for different a and $r_1 = 0.3 < r_2 = 0.7$when $\lambda_2 = 0.4, p = 0.5, q = 0.5, \mu_2 = 0.4, \mu_1 = 0.5$

λ_1	$\alpha = 1$	$\alpha = 7$	$\alpha = 14$	$\alpha = 21$	$\alpha = 28$	$\alpha = 35$	$\alpha = 42$	$\alpha = 49$
0.1	48.3086	54.3999	55.1436	55.4034	55.5357	55.6158	55.6696	55.7081
0.2	25.3126	28.4357	28.8173	28.9506	29.0184	29.0596	29.0871	29.1069
0.3	17.6815	19.8073	20.0674	20.1584	20.2047	20.2327	20.2515	20.265
0.4	13.8895	15.5115	15.7104	15.78	15.8154	15.8369	15.8513	15.8616
0.5	11.6318	12.9476	13.1095	13.1662	13.195	13.2125	13.2242	13.2327
0.6	10.14	11.249	11.386	11.4339	11.4583	11.4732	11.4831	11.4902

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0.7	9.08501	10.0442	10.1631	10.2048	10.2261	10.2389	10.2476	10.2538
0.8	8.30235	9.1475	9.25282	9.28974	9.30855	9.31996	9.32762	9.33311

Table 5. Mean time to the first failure for different a and $r_1 = 0.7 > r_2 = 0.3$when $\lambda_2 = 0.4, p = 0.7, q = 0.3, \mu_2 = 0.4, \mu_1 = 0.5$

λ_1	$\alpha = 1$	$\alpha = 7$	$\alpha = 14$	$\alpha = 21$	$\alpha = 28$	$\alpha = 35$	$\alpha = 42$	$\alpha = 49$
0.1	35.8646	42.5894	43.3861	43.6633	43.8042	43.8894	43.9466	43.9876
0.2	21.3104	24.7677	25.181	25.3249	25.3982	25.4425	25.4722	25.4935
0.3	16.6047	18.9434	19.2262	19.3249	19.3752	19.4056	19.426	19.4406
0.4	14.3401	16.104	16.3201	16.3957	16.4342	16.4575	16.4732	16.4844
0.5	13.0387	14.4495	14.6247	14.6861	14.7174	14.7363	14.7491	14.7582
0.6	12.2105	13.3813	13.5286	13.5804	13.6068	13.6228	13.6335	13.6412
0.7	11.6469	12.6439	12.771	12.8157	12.8385	12.8524	12.8617	12.8684
0.8	11.2448	12.1102	12.2218	12.2612	12.2813	12.2935	12.3017	12.3076

Table 6. Mean time to the first failure for different a and $r_1 = r_2 = 0.6$when $\lambda_2 = 0.4, p = 0.5, q = 0.5, \mu_2 = 0.4, \mu_1 = 0.5$

λ_1	$\alpha = 1$	$\alpha = 7$	$\alpha = 14$	$\alpha = 21$	$\alpha = 28$	$\alpha = 35$	$\alpha = 42$	$\alpha = 49$
0.1	27.171	30.8908	31.341	31.4981	31.5781	31.6265	31.659	31.6822
0.2	15.074	17.0007	17.235	17.3169	17.3585	17.3837	17.4007	17.4128
0.3	11.1095	12.4238	12.5849	12.6412	12.6698	12.6872	12.6989	12.7072
0.4	9.17023	10.1699	10.2935	10.3368	10.3589	10.3722	10.3812	10.3876
0.5	8.03594	8.84161	8.9423	8.97761	8.99561	9.00652	9.01384	9.0191
0.6	7.30062	7.97369	8.05874	8.08862	8.10386	8.1131	8.11931	8.12376
0.7	6.79076	7.36705	7.4407	7.46662	7.47985	7.48788	7.49327	7.49714
0.8	6.42001	6.9224	6.98732	7.01021	7.02191	7.02901	7.03378	7.0372

8. Conclusion

In this article, we deduced the reliability measurements of a system consisting of two dissimilar parallel units and a single repairman. The repairman might take a vacation or not at the beginning of the system operation and the active units might be attacked from successive shocks. Such a system can be considered as an evolution of a general repairable Industrial system and is also difficult to theoretically analyze the existence of many random variables with general distributions. The numerical illustration explains the relationship between the derived reliability measurements and system parameters.

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