Article

The impact of feedback information on dynamics performance of production-inventory control systems

Huthaifa AL-Khazraji[1], Colin Cole [2] and William Guo [2,*]

[1]   University of Technology, Iraq; 60141@uotechnology.edu.iq
[2]   Central Queensland University, Australia; c.cole@cqu.edu.au, w.guo@cqu.edu.au
*   Correspondence: w.guo@cqu.edu.au

Abstract: The aim of this paper is to examine the beneficial impact of feedback information in the dynamics of production-inventory control systems. Two production-inventory control system models are analyzed: APIOBPCS and 2APIOBPCS models. The simulation-based experiment designs were conducted by using the state-space equations of the two models. The bullwhip effect as measured by the variance ratio between the order rate and the consumption rate, and inventory responsiveness as measured by the Integral of Absolute Error between the actual and the target levels of inventory, are two metrics used to evaluate the performance of the production-inventory control system in response to a random customer demand. To ensure that both models work under optimal performance, multi-objective particle swarm optimization (MOPSO) is employed to address the problem of tuning the controller's parameters. The simulation results show the 2APIOBPCS model outperforms the APIOBPCS model at achieving the desired bullwhip effect and being able to provide better inventory responsiveness. The improvement in the inventory responsiveness becomes more significant when the system operates under mismatched lead time and/or an initial condition.

Keywords: Inventory Control; Modeling; Simulation; Optimization

1. Introduction

Achieving balance between conflicting objectives in a production-inventory control system is a challenging problem. Companies need to reduce their inventories to the minimum level while keeping the customer service within acceptable levels [1]. For example, Cisco encountered $2.2 billion in overstocked inventory due to an imbalance between supply and demand in May 2001 [2]. Sony Electronics faced an excessive production cost because of an over-anticipation of the demand for PlayStation®3 [3]. On the other hand, there is a bullwhip problem which is the scenario where orders to the suppliers tend to have larger fluctuations than sales to the buyers [4]. For instance, Holweg et al. [5] found that the actual demand signal from the customers in the supermarket for a soft drink is amplified many times before it reaches the soft drink supplier. Industry sometimes has to cope with real-world bullwhip, measured not just in terms of a 2:1 amplification which is bad enough, but a 20:1 amplification and even higher has been observed [6]. Production-inventory control systems are also subject to a variety of sources of uncertainties [7]. The combined impacts of various uncertainties have potentially severe impacts on the dynamic performance of production-inventory control systems. Therefore, studying the dynamic performance of production-inventory control systems under various uncertainties has been generally overlooked by academics in the literature. Ho [8] categorized them into two streams: environmental uncertainty and system uncertainty. In terms of environmental uncertainty, it could be demand uncertainty and

supply uncertainty. For example, demand uncertainty is a continuing problem that has a negative impact in terms of lower productivity and reduced customer satisfaction. System uncertainty refers to uncertainties within the production process, such as breakdown of the production system. As an example of this type of uncertainty, a major fire in a local Royal Philips Electronics plant in March 2000 halted the supply of microchips for Ericsson. Ericsson subsequently lost its market share leadership in the mobile phone market [9]. As a result, academic researchers have conducted several studies to assess whether new operation strategies within production-inventory control systems would improve the operational and financial performances of manufacturing companies [2].

Among various methods and tools that have been developed in order to improve the performance of production-inventory control systems, control theory with feedback mechanisms provides sufficient mathematical tools (such as Laplace transforms, Z-transforms, transfer functions, block diagrams and frequency analysis) to analyze and simulate production-inventory systems, based on dynamic models [10]. The first works in this field were developed as early as the 1950s. Simon [11] made the first attempt to use control theory, utilizing block diagram representation in applying continuous-time modeling theory to the production-inventory system problem. Vassian [12] extended Simon's work to apply discrete-time modeling theory based on Z-transforms to the production-inventory system problem. For a continuous time domain model, the Laplace transformation is used to convert differential equations into s-domain transfer functions. On the other hand, the z-transform is used to convert the difference equations into z-domain transfer functions in the discrete time domain model [13]. These close form transfer functions provide general guidelines for systems design such as the ability to guarantee stability of the system and tuning the controller parameters in order to make the system respond according to a specific behavior [14].

The Inventory and Order Based Production Control System (IOBPCS) model proposed by Towill [15] is well-recognized as a base framework for modeling the production-inventory control system [16]. John et al. [17] made an important extension of the IOBPCS model by including the work-in-process (WIP) feedback and proposed the Automatic Pipeline, Inventory and Order Based Production Control System (APIOBPCS). Mason-Jones et al. [18] showed that WIP feedback made a significant improvement in overall dynamic performance of a production-inventory control system. Motivated by this result of adding a WIP feedback element, AL-Khazraji et al. [19] introduced the Two Inventory and Order Based Production Control System (2APIOBPCS) model by incorporating a new feedback element, completion production rates, to the production-inventory control system. A comprehensive literature review on the applications of classical and modern control theory to production-inventory problems can be found in [20] [21] and [22]. Recently, Lin et al. [16] presented a systematic review on various aspects of IOBPCS models including how the IOBPCS archetypes have been developed, expanded and become suitable to study the dynamics behavior of production-inventory control systems.

In order to evaluate different ordering strategies on the dynamics performance of the production-inventory system, a number of comparative studies have been published. For example, Aggelogiannaki and Sarimveis [23] evaluated the efficiency of adaptive APIOBPCS (AAPIOBPCS) with two alternative control systems, namely APIOBPCS and Estimated Pipeline Variable Inventory and Order Based Production Control System (EPIOBPCS) that was introduced by [6].   Cannella and Ciancimino [24] investigated how different smoothing parameters of the APIOBPCS model impact on the performance of the system under progressive information sharing strategies. Tosetti et al. [25] assessed the performance of adding the Proportional, Integrative and Derivative (PID) controller to the APIOBPCS model. AL-Khazraji et al. [26] used simulation experiments to investigate the impact of applying two classical controller strategies on the performance of the production-inventory control system based on the state space approach. However, the majority of these studies used the recommended parameter settings that lead to 'opti-

mum' behavior. Only a few studies have used optimization techniques to select appropriate system parameters such as [9].

The purpose of this work is to examine the dynamic implications of two feedback control mechanisms to optimize inventory levels and reduce negative consequences such as order amplification of a single-product, single-stage, small-scale production-inventory system. Two models are analyzed: APIOBPCS and 2APIOBPCS models. To ensure a realistic scenario, the performance of the two models is investigated under different scenarios including mismatched lead time, production capacity constraints and initial condition. The methodology of conducting this study was addressed via control theory and simulation. The continuous closed-loop state space equations of the two models are used to design and perform the simulation-based experiments.

Simulation is a well-established methodology that has a wide range of applications in the manufacturing process [27]. To overcome the inconveniences and limitations of the analytic methods, simulation has been used in modeling and evaluating a wide range of different strategies in production-inventory control systems [28]. However, the trial-and-error of the tuning process in the simulation to obtain the desired performance is considered time consuming [29]. This limitation can be overcome by employing an optimization techniques such as particle swarm optimization (PSO) to perform the tuning process and search for the best system configuration. Therefore, the production-inventory problem was formulated in this research as a multiple objective optimization problem and a multi-objective particle swarm optimization (MOPSO) algorithm developed by AL-Khazraji et al. [30] was employed to tune the model's parameters to achieve the best performance of the system.

The rest of this study is organized as follows. Section 2 provides an overview of the theory of the research in terms of mathematical modeling, performance metrics and the optimization procedure. Section 3 explains how the simulation based experiments is conducted. Section 4 discusses the results. Section 5 summarizes the main findings of this study.

2. Overview of the production and inventory control system

2.1. Mathematical modelling

The practical production-inventory control system models investigated in this study are the Automatic Pipeline Inventory and Order Based Production Control System (APIOPBCS) coined by [17] as shown in Figure 1 and the Two Automatic Pipeline Inventory and Order Based Production Control System (2APIOPBCS) introduced by [19] as shown in Figure 2. Disney et al. [31] listed some of the advantages of using the generic models that belong to the IOBPCS family. These include, for example, the ability to represent order-up-to (OUT) systems and the many variants of production planning strategies such as 'level scheduling' (i.e. lean production) right through to 'pure chase' (i.e. agile manufacture).

AL-Khazraji et al. [26] restructured the models that belong to the IOBPCS family as an integrated system consisting of three main components as shown in Figure 3 and these can be described as:

- Forecasting mechanism: the forward loop designed to provide an estimate of average sales.
- Lead time: represents the total time required between placing an order and receiving it as a finished item in the inventory.
- Controller strategy: represented by a controller strategy that utilizes the forward and feedback information to generate a sophisticated decision to place production orders for production-inventory systems.
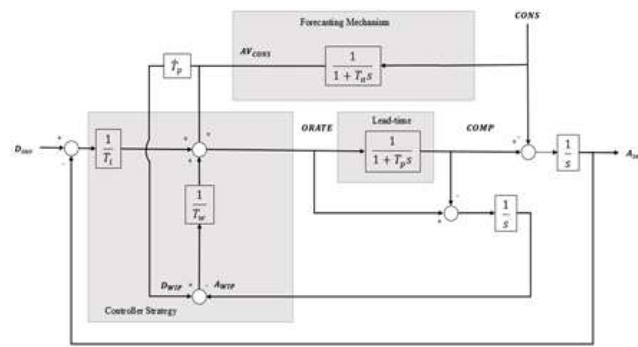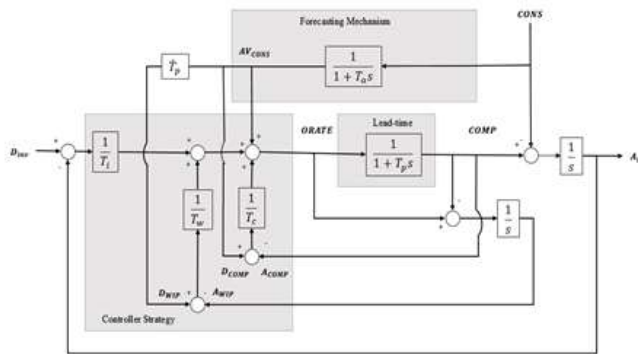
Figure 1. Block diagram of APIOBPCS model

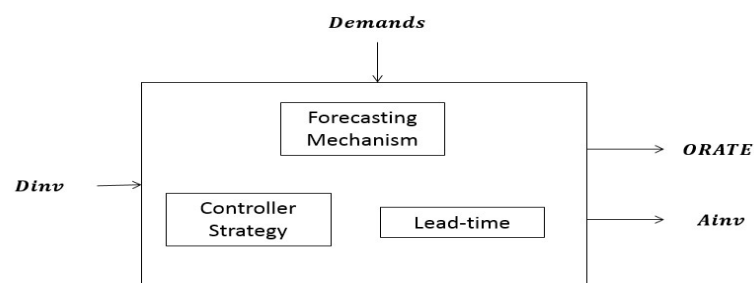**Figure 2.** Block diagram of 2APIOBPCS model

Figure 3. General structure of IOBPCS models family

Basically, the two models (APIOBPCS and 2APIOBPCS) can be described as an order-up-to (OUT) policy with a forecasting mechanism and proportional feedback controllers. In order to keep the models aligned with the paradigm of the IOBPCS family, the lead time delay for both models in this paper is modeled as an exponential lag where time constant T_p represents production lead-time. In the same way, the forecasting method is an exponential smoothing where time constant T_a represents the smoothing time constant [15] [17] [30]. The APIOBPCS model utilizes three policies (demand, inventory level, pipeline policies) to determine the Order Rate (ORATE) [25]. The average consumption rate AV_{CONS} based on exponential smoothing forecasts with time constant T_a is the forward control policy. The feedback consists of two control polices, the fraction $^1/_{T_i}$ of the difference between the desired inventory D_{inv} and the actual in-

ventory A_{inv}, and the fraction $1/T_w$ of the difference between the desired WIP D_{wip} and the actual WIP A_{wip}. An additional feedback, the fraction $1/T_c$ of the difference between the desired completion production rates D_{COMP} and the actual completion production rates A_{COMP} was added to the 2APIOBPCS model in comparison with the APIOBPCS model.

The state space equations required to model the two control systems in the simulations are given in Eqs. (1) and (2) for the 2APIOBPCS model and Eqs. (3) and (4) for the APIOBPCS model. The complete formulation and explanation of these state space equations is not presented in this study due to space restrictions but it can be found in [19]. The continuous closed-loop state space representation of the 2APIOBPCS model is given by:

$$
\dot{x} = \begin{bmatrix}
0 & \frac{1}{T_p} & 0 \\
-\frac{1}{T_i} & -\left(\frac{1}{T_w}+\frac{1}{T_p}+\frac{1}{T_pT_c}\right) & \left(\frac{1}{T_a}+\frac{\hat{T}_p}{T_wT_a}+\frac{1}{T_aT_c}\right) \\
0 & 0 & -\frac{1}{T_a}
\end{bmatrix} x + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \text{CONS} \quad (1)
$$

$$
\begin{pmatrix} \text{ORATE} \\ A_{inv} \end{pmatrix} = \begin{bmatrix}
-\frac{1}{T_i} & -\left(\frac{1}{T_w}+\frac{1}{T_pT_c}\right) & \left(\frac{1}{T_a}+\frac{\hat{T}_p}{T_wT_a}+\frac{1}{T_aT_c}\right) \\
1 & 0 & 0
\end{bmatrix} \quad (2)
$$

The continuous closed-loop state space representation of the APIOBPCS model is given by:

$$
\dot{x} = \begin{bmatrix}
0 & \frac{1}{T_p} & 0 \\
-\frac{1}{T_i} & -\left(\frac{1}{T_w}+\frac{1}{T_p}\right) & \left(\frac{1}{T_a}+\frac{\hat{T}_p}{T_wT_a}\right) \\
0 & 0 & -\frac{1}{T_a}
\end{bmatrix} x + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \text{CONS} \quad (3)
$$

$$
\begin{pmatrix} \text{ORATE} \\ A_{inv} \end{pmatrix} = \begin{bmatrix}
-\frac{1}{T_i} & -\frac{1}{T_w} & \left(\frac{1}{T_a}+\frac{\hat{T}_p}{T_wT_a}\right) \\
1 & 0 & 0
\end{bmatrix} \quad (4)
$$

2.2. Performance metrics

According to Neely et al. [32], performance measurement could be described as a systematic process of effectively and efficiently quantifying an action. In general, the performance metrics that are used to evaluate the performance of production-inventory systems should have implications on total costs (inventory related costs and productions related costs) and customer service level [33]. In this study, the production-inventory control system performance is evaluated by the variance ratio (Var_O) between the order rate and the consumption as given in Eq. (5) and the Integral of Absolute Error (IAE) between the actual and the target levels of inventory as given in Eq. (6). The Var_O index is used as a metric to measure bullwhip effect, whereas the IAE index is used as a metric to measure inventory responsiveness. The bullwhip effect and inventory responsiveness are two objectives that have direct impacts on the nature of the basic trade-offs between maintaining the order rates at the optimal performance, in order to avoid the impact of high amplification of orders, and maintaining stocks at a desired level to improve the Customer Service Level (CSL).

$$
\text{Var}_O = \frac{\sigma_{ORATE}^2}{\sigma_{CONS}^2} \quad (5)
$$

where σ_{ORATE}^2 refers to variance of the orders placed to the manufacturer and σ_{CONS}^2 represents the consumption variance. In this criterion, there is zero bullwhip if $\text{Var}_O = 1$; the system is amplified if $\text{Var}_O > 1$; the system is smoothed when $\text{Var}_O < 1$.

$$IAE = \int_0^t |E| \; dt \tag{6}$$

where t denotes the period and E refers to the error in the inventory levels measured as the deviation of the A_{inv} level from the D_{inv} level. The IAE measures positive and negative errors equally. The minimum IAE indicates that the system has a better CSL.

2.3. Optimisation procedure

Optimizing methods in the area of production-inventory systems fall into two categories: cost-based and non-cost based approaches. Cost-based approaches use the costs of inventory versus production costs in order to optimize the system. In contrast, non-cost based approaches optimize the dynamic performance of the system. The optimization procedure selected here is the non-cost approach and is based on optimizing the dynamic performance of the system. It is well-known that the common problem in production-inventory systems is the trade-off between the customer service level and cost. Therefore, the production-inventory problem could be formulated as a multiple objective optimization problem. PSO has been applied successfully into multi-objective problems. In order to solve the multi-objective problem, a Pareto optimal concept is used. A Pareto optimal solution can be described as all the solutions that improve one particular objective without disadvantaging any of the other objectives [34].
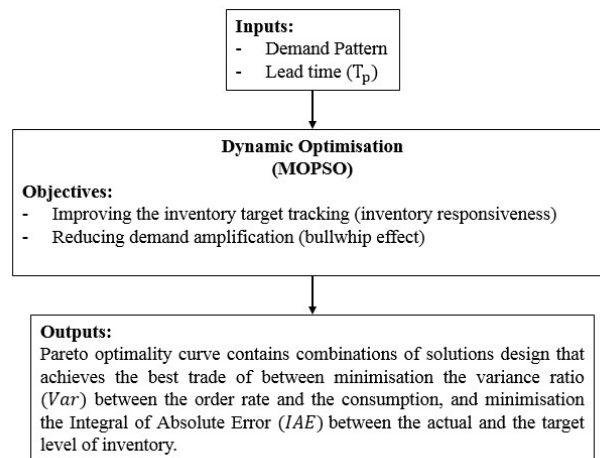

Figure 4. Optimization procedure

AL-Khazraji et al. [30] recently developed a multi-objective particle swarm optimization (MOPSO) algorithm by incorporating the Pareto optimality into PSO. Figure 4 summarizes the optimization procedure. The inputs to the algorithm are the lead time required for the orders and the demand pattern. The outputs of the dynamic optimization are combinations of solutions representing the best choice of the system's configuration that provides optimal performance in terms of improving the inventory target tracking (system responsiveness) and reducing demand amplification (bullwhip effect). More details about this optimization approach can be found in [30].

3. Simulation experiment

This section illustrates how the simulation based experiments were conducted. The major assumptions in all simulations are:
- The demand rate was generated by a sum of $\sin$ and $\cos$ terms with different frequencies as shown in Figure 5.

- Single product, single-retailer and single-factory operations with small production scale are considered.
- The paper concentrates on the case where the physical production/distribution lead time period is four units of time ($T_p = 4$).
- Backorders (negative inventory) are permitted.
- The desired inventory is set to zero ($D_{inv} = 0$).
- Day is the basic time unit in the model.
- The simulation was run for 180 days for each scenario.
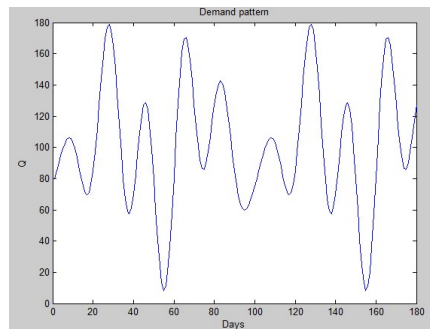- The production process can only produce a single unit at a time.

Figure 5. Demand pattern

The simulations were designed in MATLAB software. Simulation based experiments were developed based on the state space equations of the 2APIOBPCS and the APIOBPCS models. The MOPSO was utilized to choose system parameters that reduce the bullwhip effect and improve the inventory responsiveness. The optimization process calculates Var_O and IAE. The multi-objective optimization problem for the production-inventory control system was defined as:

$$\text{minimise } \{Var_O(p), IAE(p)\} \tag{7}$$

where Var_O and IAE are the two conflicting objective functions that need to be minimised. The decision vector is $p = \{ T_a, T_i \text{ and } T_w\}$ for the APIOBPCS model, whereas it is $p = \{ T_a, T_i, T_w \text{ and } T_c\}$ for the 2APIOBPCS model. A heuristic method (such as PSO) does not ensure finding a global optimal solution [35] and different results can occur depending on the randomness of the parameters chosen. The MOPSO algorithm parameters used in the simulation were:

- The maximum number of iterations (t) is set 100.
- The number of particles in the swarm (N) is set to 50.
- The learning coefficients for local (c_1) and global (c_1) search are both set to 2.
- The inertia weight (θ) is set as 0.6.
- The size of the archive (A) is set to 20.

The performance of the two models was examined by simulating them by the demand pattern under a normal scenario (matched lead time, no capacity constraint and no initial conditions) first. Three operation designs were selected to plot the Pareto curve of each model. The selection was performed by assuming the target of Var_O in the range of 0.8 to 1.3 and calculating the corresponding IAE achieved by each model. The Var_O in the range of 0.8 to 1.3 represents three different responses:

- Design1: Smoothing is when $0.8 < Var_O < 1$.
- Design 2: Bullwhip avoidance is when $Var_O = 1$.
- Design3: Small bullwhip is when $1 < Var_O < 1.3$.

The performance of the 2APIOBPCS model was then compared with the performance of the APIOBPCS model under three different scenarios as illustrated in Figure 6. The three different cases are explained below:

- Lead Time: In the matched scenario, the actual lead time and the estimated lead time were assumed to be matched through the operation. On the other hand, in the mismatch scenario it was considered that the mismatch may come from two different sources: production-side (i.e. machine breakdown), and supplier-side (i.e. material shortage). In such situations, the lead time of the system is increased, resulting in a mismatch with the estimated lead time. These simulation-based experiments in the mismatched scenario evaluate the robustness of the two control strategies by measuring how the systems recover from such disruptions and get back to the normal level. In these simulations, the lead time starts at the nominal value ($T_p = 4$), but the lead time value suddenly changes to ($T_p = 6$) for a period of time and goes back to the normal ($T_p = 4$).
- Production Capacity: In the flexible capacity strategy, overtime working and extra resources were assumed to be available when the production required is above the capacity limit. In the inflexible capacity strategy, when the production capacity in a period is insufficient to complete production for an order, then the next period's capacity is used to continue the production of this order. The order in the time period t is given as:

$$ORATE_t = \begin{cases} ORATE_t, & ORATE_t < C \\ C, & \text{Othwise} \end{cases} \qquad (8)$$

where C refers to production capacity that was set equal to 110 items per day.
- Initial Condition: It was assumed in the first case that the simulation runs with no initial condition, which means that no inventory existed before the operation. In the second case, it was assumed that there is an initial condition and it was considered the system starts with 1000 items as initial stock.
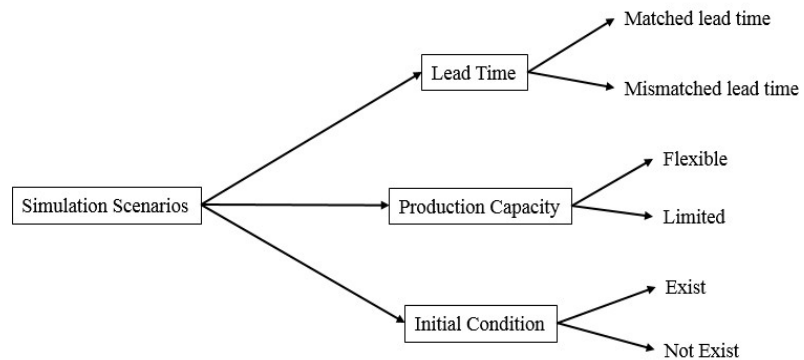

Figure 6. Different cases assumed in the simulation

4. Simulation results and discussion

The results of each of the eight cases are discussed below. Note the optimal parameters for the simulation are resulted from MOPSO under the known demand pattern and lead time (T_P) that are the same for all cases simulated in this study. As a result, the values for these parameters are the same for all eight cases simulated. To make comparisons between close cases easier, we group the figures and tables of the two cases under the same Capacity and Initial Condition but different in lead time together in our discussion.

4.1. Case 1: Lead Time: Matched, Capacity: Flexible, Initial Condition: No

Figure 7a plots the Pareto optimality curves of the two models under matched lead time scenarios. In this case, it can be seen from Figure 7a that the Pareto curve of the 2APIOBPCS model is located below the Pareto curve of the APIOBPCS model, which

means that the 2APIOBPCS model is able to achieve less IAE than the APIOBPCS model while being able to achieve the same Var_O. Table 1a gives the optimal system configuration with the corresponding performance of both models under matched lead time scenarios. It can also be seen form Table 1a that the improvement becomes more significant if the desired Var_O was more than 1. For example, as shown in Table 1a, the percentage of improving inventory responsiveness (IR) is increased from 3% to 9% if the desired Var_O changes from 1 to 1.25. The simulations of Design 2 (bullwhip avoidance scenario) of the two models under matched lead time are shown in Figure 8a.
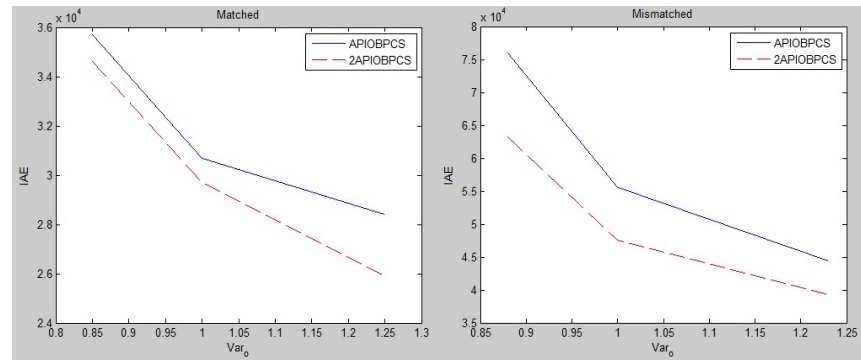

Figure 7. Pareto optimality curves for flexible capacity without initial condition of the two models

a . Matched lead time   b. Mismatched lead time

Table 1 Optimal system configuration with the corresponding performance for flexible capacity without initial condition of the two models under matched and mismatched lead time

a.	Operation: Matched, Capacity: Flexible, Initial Condition: No										
	APIOBPCS						**2APIOBPCS**				
Ti	**Tw**	**Ta**	**Var**	**IAE**	**Ti**	**Tw**	**Ta**	**Tc**	**Var**	**IAE**	**IR**
3.91	0.55	6.71	0.85	3.57E+04	2.65	0.53	7.8	10	0.85	3.46E+04	3%
5	1.13	5.66	1	3.07E+04	1.14	0.37	8.94	21.9	1	2.97E+04	3%
3.5	1.2	6	1.25	2.84E+04	1.2	0.5	7.3	2	1.25	2.59E+04	9%
b.	Operation: Mismatched, Capacity: Flexible, Initial Condition: No										
	APIOBPCS						**2APIOBPCS**				
Ti	**Tw**	**Ta**	**Var**	**IAE**	**Ti**	**Tw**	**Ta**	**Tc**	**Var**	**IAE**	**IR**
3.91	0.55	6.71	0.85	7.60E+04	2.65	0.53	7.8	10	0.85	6.32E+04	17%
5	1.13	5.66	1	5.56E+04	1.14	0.37	8.94	21.9	1	4.76E+04	14%
3.5	1.2	6	1.23	4.44E+04	1.2	0.5	7.3	2	1.23	3.93E+04	12%

4.2. Case 2: Lead Time: Mismatched, Capacity: Flexible, Initial Condition: No

Figure 7b plots the Pareto optimality curves of the two models under mismatched lead time scenarios. Table 1b gives the optimal system configuration with the corresponding performance of both models under mismatched lead time scenarios. In this case, it is seen from Table 1b (mismatched case) in comparison to Table 1a (matched case) that Var_O for both models is not affected, whereas IAE for both models is increased. This means that, under mismatched lead time, the inventory responsiveness for both models is reduced. In spite of that, it can be seen from Figure 7b that the Pareto curve of the 2APIOBPCS model is located below the Pareto curve of the APIOBPCS model, and this indicates that the 2APIOBPCS model is able to achieve less IAE than the APIOBPCS model at the same Var_O under uncertainties. It can also be observed from Table 1a in

comparison to Table 1b that the percentage of IR increases from 3% in the case of matched lead time to 14% in the case of mismatched lead time to achieve a desired $Var_O = 1$. This means that the 2APIOBPCS model offers better inventory responsiveness under uncertainties. The simulations of Design 2 (bullwhip avoidance scenario) of the two models under mismatched lead time are shown in Figure 8b.
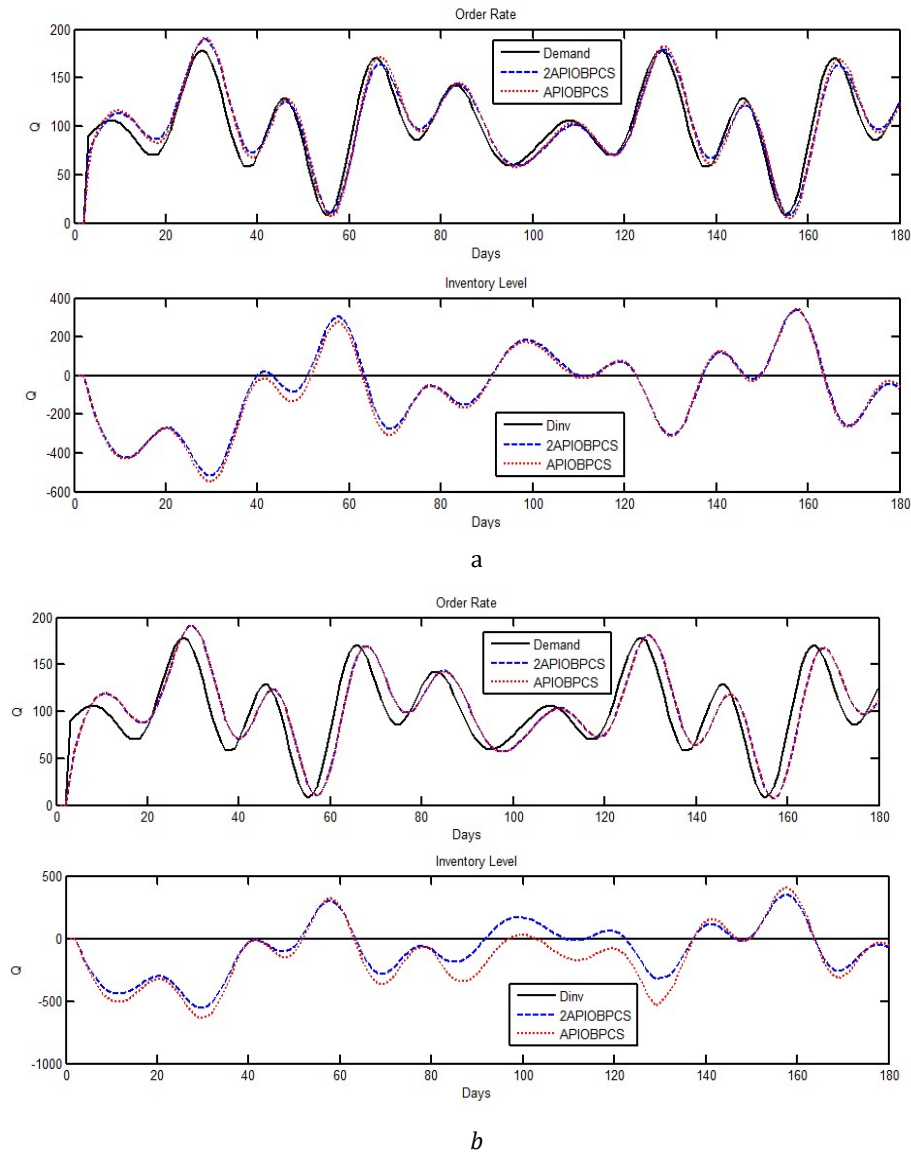

a

b

Figure 8. Simulation for flexible capacity without initial condition of the two models
a . Matched lead time   b. Mismatched lead time

4.3. Case 3: Lead Time: Matched, Capacity: Inflexible, Initial Condition: No

Figure 9a plots the Pareto optimality curves of the two models under matched lead time scenarios. Table 2a gives the optimal system configuration with the corresponding performance of both models under matched lead time scenarios. In general, it can be seen from Table 2a in comparison to Table 1a that, after introducing the capacity constraints, the order amplification Var_O for both models is reduced. For example, in the case of matched lead time under flexible capacity, the system configuration for Design 2 for the APIOBPCS and 2APIOBPCS models that generates a bullwhip avoidance response ($Var_O = 1$) becomes a smoothing response with $Var_O = 0.51$ for both models when the

capacity constraint is introduced. The second observation is that the inventory respon-
siveness for both models does not change. This finding is aligned with the conclusions of
[24] that showed that capacity constraints lead to reduced bullwhip effect, but this does
not mean improved inventory responsiveness. The last observation is that no superiority
is seen for either model in regard to this case. The simulations of Design 2 (bullwhip
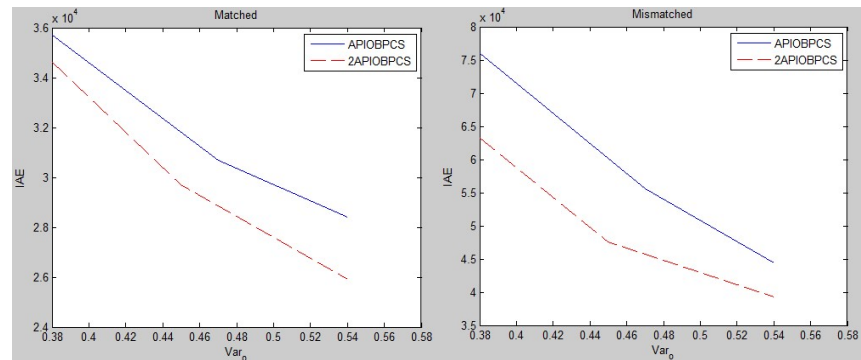avoidance scenario) of the two models under matched lead time are shown in Figure 10a.

Figure 9. Pareto optimality curves for inflexible capacity without initial condition of the two models
a . Matched lead time   b. Mismatched lead time

Table 2 Optimal system configuration with the corresponding performance for inflexible capacity without initial condition of the
two models under matched and mismatched lead time

		a.	Operation: Matched, Capacity: Inflexible, Initial Condition: No							
		APIOBPCS						**2APIOBPCS**			
Ti	**Tw**	**Ta**	**Var**	**IAE**	**Ti**	**Tw**	**Ta**	**Tc**	**Var**	**IAE**	**IR**
3.91	0.55	6.71	0.38	3.57E+04	2.65	0.53	7.8	10	0.38	3.46E+04	3%
5	1.13	5.66	0.47	3.07E+04	1.14	0.37	8.94	21.9	0.45	2.97E+04	3%
3.5	1.2	6	0.54	2.84E+04	1.2	0.5	7.3	2	0.54	2.59E+04	9%
		b.	Operation: Mismatched, Capacity: Inflexible, Initial Condition: No							
		APIOBPCS						**2APIOBPCS**			
Ti	**Tw**	**Ta**	**Var**	**IAE**	**Ti**	**Tw**	**Ta**	**Tc**	**Var**	**IAE**	**IR**
3.91	0.55	6.71	0.38	7.60E+04	2.65	0.53	7.8	10	0.38	6.32E+04	17%
5	1.13	5.66	0.47	5.56E+04	1.14	0.37	8.94	21.9	0.45	4.76E+04	14%
3.5	1.2	6	0.54	4.44E+04	1.2	0.5	7.3	2	0.54	3.93E+04	12%

4.4. Case 4: Lead Time: Mismatched, Capacity: Inflexible, Initial Condition: No

Figure 9b plots the Pareto optimality carves of the two models under mismatched
lead time scenarios. Table 2b gives the optimal system configuration with the corre-
sponding performance of both models under mismatched lead time scenarios. In this
case, it is seen from Table 2b in comparison to Table 2a that Var_0 for both models is the
same which is the same finding as in the case of flexible capacity. However, the Var_0 is
less in comparison to the flexible capacity case as shown in Table 1b due to the capacity
constraint. For instance, the system configuration of Design 2 for APIOBPCS and
2APIOBPCS models that generates a bullwhip avoidance response ($Var_0 = 1$) becomes a
smoothing response ($Var_0 = 0.47$) for both models when the capacity constraint is in-
troduced. The same findings with regard to the matched lead time was obtained re-
garding inventory responsiveness, and IAE was the same for both flexible and inflexible
strategies for both models. In spite of that, it can be seen from Figure 9b that the Pareto
curve of the 2APIOBPCS model is located below the Pareto curve of the APIOBPCS
model, which indicates that the 2APIOBPCS model is still able to achieve less IAE than

the APIOBPCS model at the same Var_O under uncertainties and capacity constraint. The simulations of Design 2 (bullwhip avoidance scenario) of the two models under mismatched lead time are shown in Figure 10b.
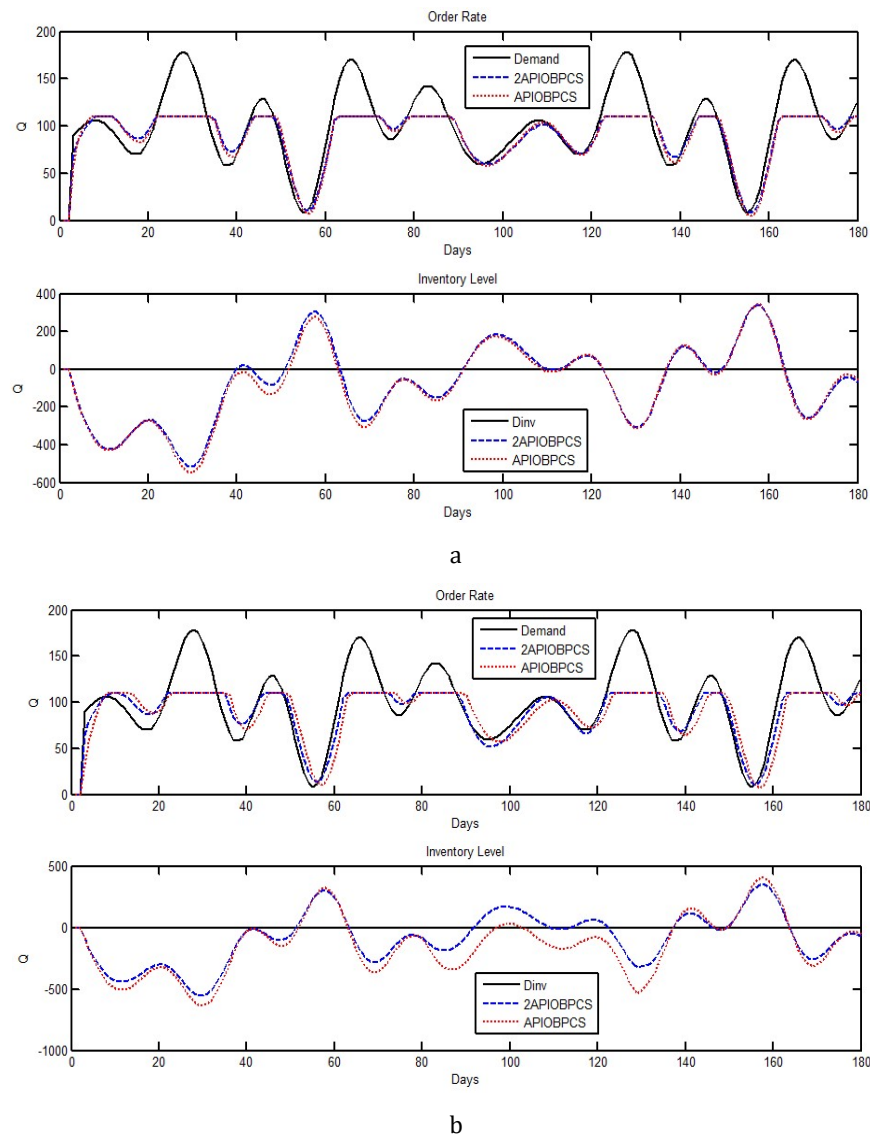

Figure 10. Simulation for inflexible capacity without initial condition of the two models
a . Matched lead time   b. Mismatched lead time

4.5. Case 5: Lead Time: Matched, Capacity: Flexible, Initial Condition: Yes

Figure 11a plots the Pareto optimality curves of the two models under matched lead time scenarios. Table 3a gives the optimal system configuration with the corresponding performance of both models under matched lead time scenarios. In general, it can be seen from Table 3a in comparison to Table 1a that, after introducing the initial condition, the order amplification Var_O for both models is increased. For example, in the case of matched lead time and flexible capacity, the system configuration Design 2 for the APIOBPCS and 2APIOBPCS models that generates a bullwhip avoidance response ($Var_O = 1$) becomes amplified responses with $Var_O = 1.23$ for the APIOBPCS model and $Var_O = 1.16$ for the 2APIOBPCS model after the initial condition is introduced. It can also be seen that the 2APIOBPCS model is less affected. The second observation is that the inventory responsiveness IAE of both models is reduced which is reasonable because

of the initial stock that was added. It can be observed by comparing Table 1a and Table 3a that the percentage of IR increases from 3% in the case of no initial condition to 12% in the case of the initial condition for system configuration Design 2. Therefore, the superiority of the 2APIOBPCS model in comparison to the APIOBPCS model is that the inventory responsiveness is increased with the initial condition. The simulations of Design 2 (bullwhip avoidance scenario) of the two models under matched lead time are shown in Figure 12a.
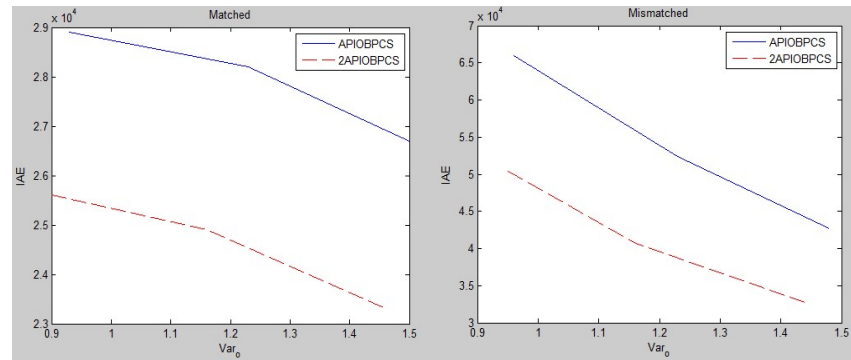

Figure 11. Pareto optimality curves for flexible capacity with initial condition of the two models
a . Matched lead time   b. Mismatched lead time

Table 3 Optimal system configuration with the corresponding performance for flexible capacity with initial condition of the two models under matched and mismatched lead time

a.	Operation: Matched, Capacity: Flexible, Initial Condition: Yes										
	APIOBPCS					2APIOBPCS					
Ti	Tw	Ta	Var	IAE	Ti	Tw	Ta	Tc	Var	IAE	IR
3.91	0.55	6.71	0.93	2.89E+04	2.65	0.53	7.8	10	0.90	2.56E+04	11%
5	1.13	5.66	1.23	2.82E+04	1.14	0.37	8.94	21.9	1.16	2.49E+04	12%
3.5	1.2	6	1.5	2.67E+04	1.2	0.5	7.3	2	1.46	2.33E+04	13%

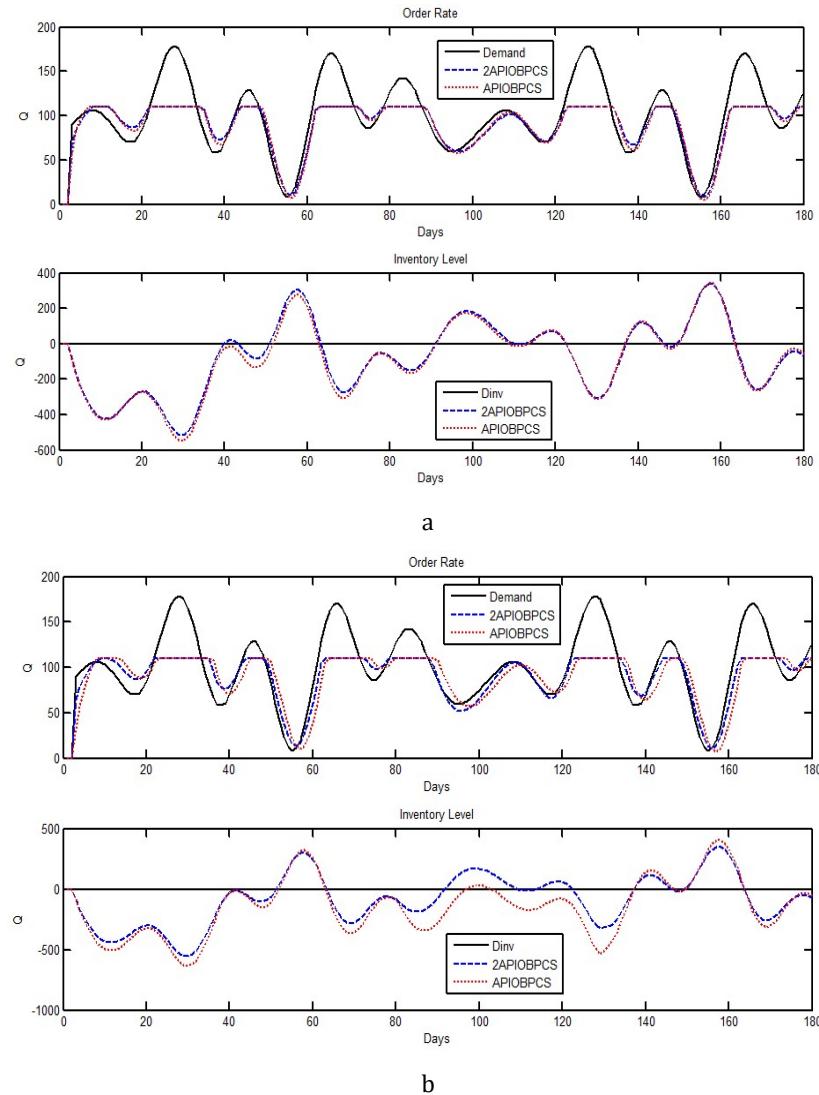| b.	Operation: Mismatched, Capacity: Flexible, Initial Condition: Yes										
	APIOBPCS					2APIOBPCS					
Ti	Tw	Ta	Var	IAE	Ti	Tw	Ta	Tc	Var	IAE	IR
3.91	0.55	6.71	0.96	6.60E+04	2.65	0.53	7.8	10	0.95	5.04E+04	24%
5	1.13	5.66	1.23	5.24E+04	1.14	0.37	8.94	21.9	1.16	4.07E+04	22%
3.5	1.2	6	1.48	4.27E+04	1.2	0.5	7.3	2	1.44	3.28E+04	23%

Figure 12. Simulation for inflexible capacity without initial condition of the two models
a . Matched lead time   b. Mismatched lead time

4.6. Case 6: Lead Time: Mismatched, Capacity: Flexible, Initial Condition: Yes

Figure 11b plots the Pareto optimality curves of the two models under mismatched lead time scenarios. Table 3b gives the optimal system configuration with the corresponding performance of both models under mismatched lead time. In this case, it is seen from results in Table 3b that the Var_O for both models is not significantly affected in comparison to the results of the matched scenario in Table 3a, whereas IAE for both models is increased. This means that, under mismatched lead time, the inventory responsiveness for both models is reduced. In spite of that, it can be seen from Figure 11b that the Pareto curve of the 2APIOBPCS model is located below the Pareto curve of the APIOBPCS model, and this means that the 2APIOBPCS model is able to achieve less IAE than the APIOBPCS model at the same Var_O under uncertainties and with the initial condition. Also in this case, the percentage of IR for system configuration Design 2 becomes 22% which is more than the percentage of IR in the case of mismatched lead time with no initial condition as given in Table 1b. Therefore, the superiority of the 2APIOBPCS model in comparison to the APIOBPCS model in terms of inventory responsiveness is considerably increased if considering the initial condition under the

mismatched scenario. The simulations of Design 2 (bullwhip avoidance scenario) of the two models of each scenario for mismatched lead time are shown in Figure 12b.

4.7. Case 7: Lead Time: Matched, Capacity: Inflexible, Initial Condition: Yes

Figure 13a plots the Pareto optimality curves of the two models under matched lead time scenarios. Table 4a gives the optimal system configuration with the corresponding performance of both models under matched lead time scenarios. The combination of the capacity constraint (decrease Var_0) with the initial condition (increase Var_0) cancels each other out in terms of Var_0 with the advantage of improving IAE. For example, by comparing Table 4a with Table 3a, it can be seen that introducing the capacity constraint reduces the Var. For more illustration to this case, the system configuration for Design 2 for the APIOBPCS and 2APIOBPCS models that generates a bullwhip avoidance response ($Var_0 = 1$) becomes a smoothing response with $Var_0 = 0.62$ for the APIOBPCS model and $Var_0 = 0.59$ for the 2APIOBPCS as given in Table 4a. However, by comparing Table 4a with Table 2a, it can be seen that with initial condition there is slightly increase in the Var_0. The inventory responsiveness in this case is also better than the inventory responsiveness in the case with inflexible capacity with no initial condition. The simulations for design 2 (bullwhip avoidance scenario) of the two models under matched lead time are shown in Figure 14a.
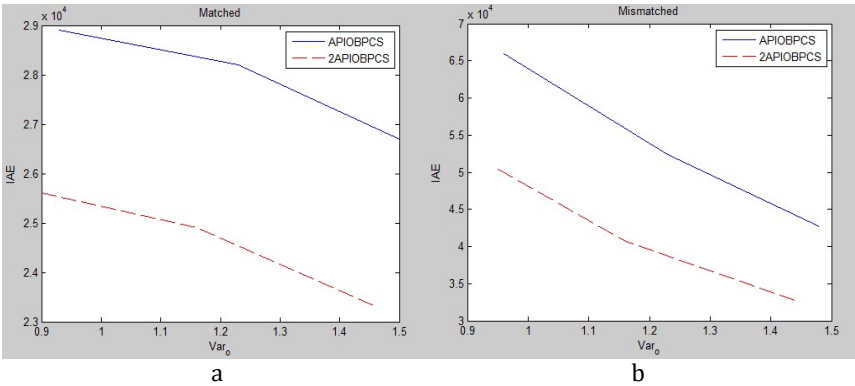

Figure 13. Pareto optimality curves for flexible capacity with initial condition of the two models
a . Matched lead time   b. Mismatched lead time

4.8. Case 8: Lead Time: Mismatched, Capacity: Inflexible, Initial Condition: Yes

Figure 13b plots the Pareto optimality curves of the two models under mismatched lead time scenarios. Table 4b gives the optimal system configuration with the corresponding performance of both models under mismatched lead time scenarios. In this case, it is seen from the results in Table 4b that Var_0 for both models is not affected in comparison to the results from the matched scenario in Table 4a, whereas IAE for both models is increased which means that, under mismatched lead time, the inventory responsiveness for both models is reduced. In spite of that, it can be seen from Figure 13b that the Pareto curve of the 2APIOBPCS model is located below the Pareto curve of the APIOBPCS model, and this indicates that the 2APIOBPCS is able to achieve less IAE than the APIOBPCS model at the same Var_0 under uncertainties associated with capacity constraint and initial condition. No superiority for either model is seen regarding this case. The simulations for Design 2 (bullwhip avoidance scenario) of the two models under mismatched lead time are shown in Figure 14b.

Table 4 Optimal system configuration with the corresponding performance for flexible capacity with initial condition of the two models under matched and mismatched lead time

a.   Operation: Matched, Capacity: Inflexible, Initial Condition: Yes

APIOBPCS					2APIOBPCS						
Ti	Tw	Ta	Var	IAE	Ti	Tw	Ta	Tc	Var	IAE	IR
3.91	0.55	6.71	0.48	2.89E+04	2.65	0.53	7.8	10	0.45	2.56E+04	11%
5	1.13	5.66	0.62	2.82E+04	1.14	0.37	8.94	21.9	0.59	2.49E+04	12%
3.5	1.2	6	0.76	2.67E+04	1.2	0.5	7.3	2	0.72	2.33E+04	13%

b.　Operation: Mismatched, Capacity: Inflexible, Initial Condition: Yes

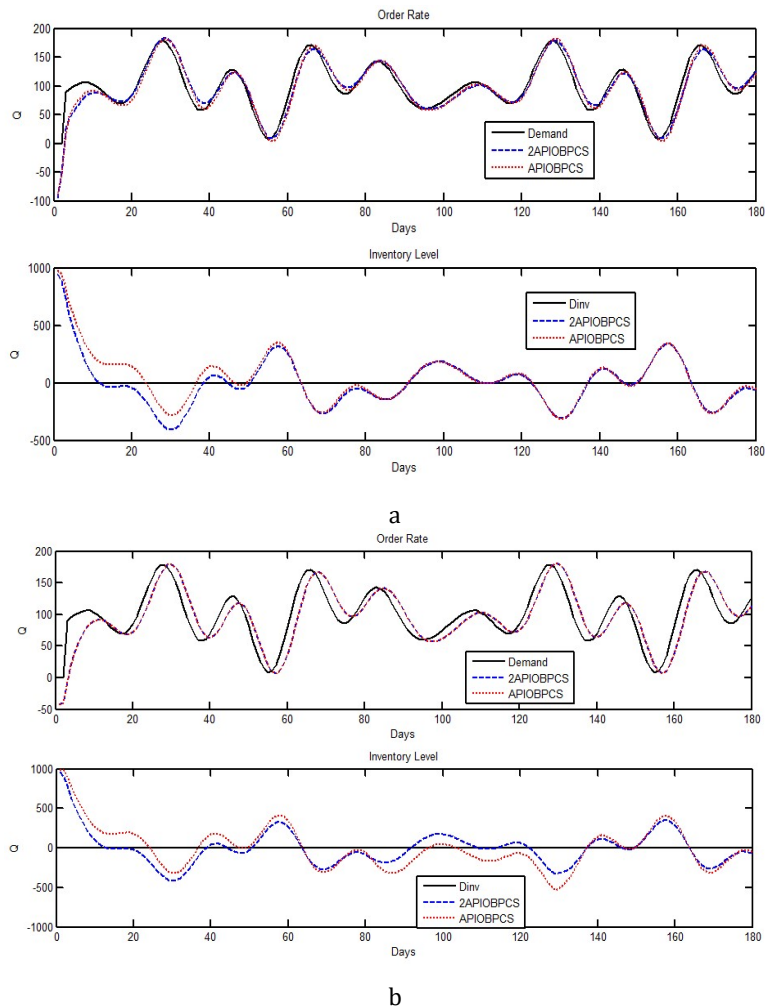| APIOBPCS					2APIOBPCS						
Ti	Tw	Ta	Var	IAE	Ti	Tw	Ta	Tc	Var	IAE	IR
3.91	0.55	6.71	0.48	6.60E+04	2.65	0.53	7.8	10	0.45	5.04E+04	24%
5	1.13	5.66	0.62	5.24E+04	1.14	0.37	8.94	21.9	0.59	4.07E+04	22%
3.5	1.2	6	0.76	4.27E+04	1.2	0.5	7.3	2	0.72	3.28E+04	23%

a

b

Figure 14. Simulation for flexible capacity with initial condition of the two models
a . Matched lead time 　 b. Mismatched lead time

4. Conclusions

In this paper, the advantages of adopting a new feedback element in production-inventory control systems are examined when the system is subjected to a random customer demand. The state space equations of the 2APIOBPCS and APIOBPCS models were used to model a single stage single product production-inventory control system.

The dynamic performance of the two models is investigated in eight cases under different conditions for assessing the performance of the systems in terms of reducing bullwhip effect and improving inventory responsiveness.

In order to ensure that the models operate under optimal configurations, MOPSO was used. The conclusions of the paper can be summarized as follows:

- In the normal operation condition, the 2APIOBPCCS model was able to improve inventory responsiveness at the same bullwhip.
- Under mismatched lead time operation, the inventory responsiveness for both models was affected negatively. However, the 2APIOBPCS model was able to offer a better inventory responsiveness to achieve the same bullwhip.
- Under production capacity constraint, the bullwhip effect for both models was reduced, but the inventory responsiveness was the same. Neither models show any superiority.
- Under an initial condition (inventory stock available upon start up), the bullwhip effect for both models was increased, and the inventory responsiveness was improved. The 2APIOBPCS model was able to provide a better inventory responsiveness.
- Under combinations of the capacity constraint and initial condition, the effect of the capacity constraint (decrease bullwhip) cancelled out the effect of the initial condition (increase bullwhip) in terms of bullwhip but with the advantage of improving inventory responsiveness. If the system works under mismatched lead time operation, the inventory responsiveness for both models was affected negatively. However, the 2APIOBPCS model was able to offer a better inventory responsiveness to achieve the same bullwhip.

The superior production-inventory control system exhibited by the 2APIOBPCCS model, and its ability to provide a systematically better inventory responsiveness performance to achieve a desired bullwhip, make the 2APIOPBCS a good choice for companies that place an emphasis on inventory on costs and service levels.

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