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Modeling System Risk in the South African Insurance sector: A Dynamic Mixture Copula Approach

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Abstract: In this study, a dynamic mixture copula is used to estimate the marginal expected shortfall in the South African insurance sector. While other studies assumed non-linear dependence to be static over time, our model capture time-varying nonlinear dependence between institutions and the market. In order to capture time-varying nonlinear dependence, the generalized autoregressive score (GAS) is used to model the dynamic copula parameters. Furthermore, our study implements a ranking that expresses to what degree individual insurers are systemically important in South Africa. We use daily stock return of five South African insurers listed in the Johannesburg Stock Exchange (JSE) from November 13, 2007 to June 15, 2020. We find that Sanlam and Discovery contribute the most to systemic risk, while Santam is found to be the least contributor to the overall systemic risk in the South African insurance sector. Our findings would be of paramount importance for the South African regulators as they would be informed that not only banks are systemically important, but some insurers also are systemically important financial institutions. Hence, stricter regulation of these institutions in the form of higher capital and loss absorbency requirements could be required based on the individual business activities undertaken by the company.

Keywords: dynamic mixture copula, marginal expected shortfall, systemic risk, insurance sector

1. Introduction

The insurance sector (life and nonlife) performs important economic functions and are considered as big participants in the financial system. The European Central Bank (ECB, 2013), states that insurance companies are important for the stability of the financial system due to their investment ability, their growing links to banks, as well as their ability to safeguard the financial stability of households and firms by insuring their risks. In line with ECB, the National Treasury of South Africa (2011) argues that, the insurance sector is an important pillar of the South African financial system by being the guardian of the stability of the whole system. In addition, according to Acharya and Richardson (2014) the insurance sector is an important source for products to the economy and a source in the financing of credit-linked-activities. Thus, it is an essential part of the economic activity.

Given the role played by the insurance companies in the economy, the corollary is that a failure of one or more insurers could cause the insurance market to be disrupted, with negative implications for the financial system and the economy as a whole. The 2007-2009 financial crisis has revealed that the insurance sector was partly responsible of the crisis with American International Group (AIG) becoming the first example of an insurer in the U.S. that received federal assistance to prevent bankruptcy due to it being considered as systemically important. As a consequence, regulators such as Solvency II board, Financial Stability Board (FSB), the International Association of Insurance Supervisors (IAIS) have raised concern about identifying, monitoring, and mitigating systemic risk in the insurance

sector. For researchers, the main concern is how to measure and identify systemic risk in the insurance sector. Consequently, the literature on systemic risk in the insurance sector has gained momentum lately and several systemic risk measures have been proposed.

Grace (2011) uses granger-causality to assess systemic risk of 12 major insurance companies in the U.S. He claims that his technique is able to capture the interconnectedness among different insurance companies. Weiß and Muhlnickel (2014) apply Delta Conditional Value at Risk (ΔCoVaR), Marginal Expected Shortfall (MES), SRISK and Probit regression as systemic risk measures for 89 U.S. insurers in order to find whether the U.S. insurance sector contribute to systemic risk. Furthermore, Cummins and Weiss (2013) use SRISK and MES as a systemic risk measure to show that insurers' stock prices in the U.S. are severely negatively affected in times of crisis. Bierth et al (2015) compute ΔCoVaR , MES, and SRISK for 253 life and nonlife insurers worldwide and comparing the results with previous studies of banks. Drakos and Kouretas (2015) apply conditional value at risk (CoVaR) to measure systemic risk in the banking, insurance, and other financial services sectors for the United Kingdom between 2000 and 2012. Berdin and sottocornola (2015) apply linear Granger causality test, conditional value at risk (CoVaR) and MES to analyse systemic risk in the Eurozone banking, insurance, and non-financial sector.

Most of the methodologies used in the literature assumed nonlinear dependence between insurers and the market to be static over time. In addition, most of these studies have been conducted in developed countries. However, research on measuring systemic risk in the insurance sector and its consequences on the broader economy is still scarce in emerging economies. Therefore, our paper seeks to address this gap by using a model that captures time-varying nonlinear dependence between insurance companies and the insurance market in South Africa.

The main objective of this study is to implement a ranking that expresses to what degree individual insurer are systemically important in South Africa. In other words, which ones of the insurers would contribute the most to systemic risk in the South African insurance sector. Our result would inform regulators that not only banks are systemically important financial institutions, but some insurers could be as systemically risky as banks. Therefore, stricter regulation of these institutions in the form of higher capital and loss absorbency could be required based on the individual business activities undertaken by the company. To this end, we apply the Dynamic-Mixture-Copula to estimate the marginal expected (DMC-MES) proposed by Eckernkemper(2018). The DMC-MES is based on a dynamic two component mixture copula for the market and each insurance institution. The dynamic copula parameters are modeled by the Generalized Autoregressive Score (GAS) developed by Creal et al. (2013) which endows each parameter with its own dynamics. In addition, the DMC-MES is a flexible model because it accounts for symmetric and asymmetric dependencies with an overall consideration of time-varying aspects.

We analyse systemic risk using five insurers listed on the Johannesburg Stock Exchange (JSE) namely, Discovery Limited, Liberty Holdings Limited, Momentum Metropolitan Holdings, Sanlam Limited and Santam Limited. We cover the period November 2007 to June 2020. We use the DMC-MES to analyse the systemic risk contribution of the above-mentioned insurers. It turns out that Sanlam is the largest contributor to systemic risk followed by Discovery. However, Santam is the smallest contributor to systemic risk. The findings show that insurers present different threats to the insurance sector and the economy at large, hence specific measures such as macro prudential regulation are essential to ensure financial stability. The remainder of the paper is structured as follows: Section two discusses the existing literature while section three describes the methodology. Section four and five present the results and the conclusion, respectively.

2. Literature Review

In this section we discuss and review the existing literature on systemic risk in the insurance sector, a topic of great importance for researchers and regulators. There is a common agreement in the literature that non-traditional and non-insurance activities do appear to be relevant to systemic risk (Eling et al, 2016). In accordance with Eling et al, the International Association of Insurance Supervisors (IAIS, 2011) also reports that non-traditional and non-insurance activities such as securities lending businesses among others can contribute to systemic risk. Chen et al (2013) analyse the contribution of 40 U.S. insurers to systemic risk using four systemic risk measures namely, ΔCoVaR , modified ΔCoVaR , MES, and SRISK. Their results reveal that insurers can be systemically risky because financial risk measures for insurers peak in times of financial crisis, and life-health insurers respond more significantly to negative financial market conditions than do property-casualty insurers.

Acharya and Richardson (2014) use the Credit Default Swaps Marginal Expected Shortfall (CDS-MES) as a measure of systemic risk for 20 insurance companies in the U.S. to provide a ranking for these 20 firms based on their CDS-MES. Genworth Financial Inc., AMBAC Financial Group Inc, MBIA Inc, and AIG are found to be the most systemically risky insurers, respectively. On the other hand, AETNA Inc; CIGNA Corp, and Marsh & McLennan Cos. Inc. are the least systematically risky firms, with CDS MESs being negative. They conclude that the insurance sector may pose a systemic risk as the industry is deviating from its traditional activities by offering financial product with non-diversifiable risk and it is more prone to run.

Dungey et al, (2014) measure systemic risk via interconnectedness of the banking, insurance, and real economy firms in the United States for 500 firms using eigenvector centrality measures. The results show that while the banks are the most consistently systemically risky financial institutions in the economy, insurers are becoming an identifiable group exhibiting substantial systemic risk via interconnectedness with the financial sector and the real economy. Bernal et al (2014) utilise ΔCoVaR and the Kolmogorov-Smirnov test to provide a ranking of commercial banks, insurance companies, and other financial services with respect to their contribution to systemic risk in the Eurozone and the United States during the period of the financial crisis and the European sovereign debt crisis. The results reveal that in the Eurozone, the other financial services sector contributes relatively the most to systemic risk at times of distress affecting this sector. In turn, the banking sector appears to contribute more to systemic risk than the insurance sector. By contrast, the insurance industry is the systemically riskiest financial sector in the United States for the same period, while the banking sector contributes the least to systemic risk in this area. In addition, they argue that beyond this ranking, the three financial sectors of interest are found to contribute significantly to systemic risk, both in the Eurozone and in the United States.

Wei and Muhlnickel (2014) examine systemic risk for 89 U.S. insurers for the period of the financial crisis using ΔCoVaR , MES, and SRISK as systemic risk measures. Their findings reveal that size is the main driver of insurers' exposure and contribution to systemic risk in the U.S., and the exposure to systemic risk additionally depends on non-traditional and non-insurance activities like CDS writing. Berdin and sottocornola (2015) apply linear Granger causality test, ΔCoVaR , MES to measure systemic for bank, insurance, and non-financial sectors in Europe. They find that the insurance industry shows a persistent systemic relevance over time and plays a subordinate role in causing systemic risk compared to banks. Moreover, their findings indicate that insurance companies which engage more in non-traditional and non-insurance activities tend to pose more systemic risk and size is also a significant driver of systemic risk in the banking sector and the insurance sector in Europe.

Furthermore, Bierth et al (2015) apply ΔCoVaR , MES, and SRISK as measures of systemic risk to study the exposure and contribution of 253 international life and non-life insurers to systemic risk between 2000 and 2012. The results indicate that the interconnectedness of large insurers with the insurance sector to be a significant driver of the insurers' exposure to systemic risk. However, the contribution of insurers to systemic risk appears to be principally driven by the insurers' leverage. Liu and Wang. (2016), utilise the DCC-GARCH-MES technique to measure systemic risk in China's insurance industry. They find that the largest insurers contribute the most to systemic risk in the Chinese's financial system. Kaserer and Klein (2019) use CDS-implied systemic risk measure to investigate how insurance companies contribute to systemic risk in the global financial system represented by 201 major banks and insurers over the period from 2004 through 2014. They find that the insurance sector contributes relatively little to the aggregate systemic risk. However, at the institution level, several multi-line and life insurers appear to be as systemically risky as the riskiest banks. They conclude that some insurers are systemically important and indicate that insurers' level of systemic risk varies by line of business.

3. Methodology

The first part of this section explains the Marginal Expected Shortfall (MES) which is used to measure systemic risk and how it is derived. The second part briefly discusses the Dynamic Mixture Copula Marginal Expected Shortfall (DMC-MES) employed to estimate our systemic risk measures. Thereafter, we detail the steps involved in estimating DMC-MES proposed by Eckernkemper (2018).

3.1. Marginal Expected Shortfall (MES)

The Marginal Expected Shortfall (MES) measures an institution's expected return under the condition that the return of the market is less or equal to a certain threshold over a given horizon. The MES is simple to compute and therefore easy for regulators to consider.

3.1.1. Expected Shortfall (ES)

Let us consider the bivariate time series process $\{r_{i,t}, r_{m,t}\}_{t=1}^T$ where $r_{m,t}$ denotes the return of the entire market m and $r_{i,t}$ the return of an institution i , both at time t . The market return $r_{m,t}$ is given by a linear combination of institution weights $w_{i,t}$ and institution returns $r_{i,t}$, that is $r_{m,t} = \sum_{i=1}^N w_{i,t} r_{i,t}$. The expected shortfall (ES) of the market is used to measure the risk of the Market. It is defined as $ES_{m,t} = E_{t-1}(r_{m,t} | r_{m,t} \leq c_t)$ with threshold c_t being the value at risk (VaR) of the market return. Plugging $r_{m,t} = \sum_{i=1}^N w_{i,t} r_{i,t}$ into $ES_{m,t}$ yields

$$ES_{m,t} = E_{t-1}\left(\sum_{i=1}^N w_{i,t} r_{i,t} | r_{m,t} \leq c_t\right) = \sum_{i=1}^N w_{i,t} E_{t-1}(r_{i,t} | r_{m,t} \leq c_t), \quad (1)$$

and partial differentiation with respect to weight $w_{i,t}$ gives the formal definition of the MES for institution i at time t :

$$MES_{i,t} = \frac{\partial ES_{m,t}}{\partial w_{i,t}} = E_{t-1} \left(r_{i,t} \mid r_{m,t} \leq c_t \right) \quad (2)$$

According to Eckernkemper(2018), the MES, besides measuring the expected return of institution i under the condition that the return of the market is less or equal to the threshold C_t , the MES measures the changes of the market's ES if the weight of institution i within the market portfolio will marginally increase. Thus, the MES measures the change of the market risk if institution i would have a higher weight on the market. In this way, it provides information on the risk contribution of an institution and its importance for the considered market.

3.2. Dynamic Mixture Copula-Marginal Expected Shortfall (DMC-MES)

This section presents the concepts of the DMC-MES. First, we give a quick review on copulas. Second, the framework and model setup of the DMC-MES is presented. Finally, the model components and an estimation procedure of the DMC-MES are provided.

3.2.1. Copulas

A copula function is a multivariate distribution for which the marginal distribution of each variable is uniform. It couples the marginal distribution together to form a joint distribution. Copulas are used to describe the dependence between random variables. Let us consider two random variables $(\varepsilon_{i,t}, \varepsilon_{m,t})$ which are respectively the insurer and market innovation. These two variables are assumed to be individually *iid* and H_t represents the joint distribution of the two random variables $(\varepsilon_{i,t}, \varepsilon_{m,t})$. The marginal distributions are denoted by G_i and G_m . Sklar's theorem (1959) states that any joint distribution can be written in terms of univariate marginal distribution, G_i and G_m , linked via a copula function C_t , that is,

$$p(\varepsilon_{i,t} \leq \varepsilon_i, \varepsilon_{m,t} \leq \varepsilon_m) = H_t(\varepsilon_i, \varepsilon_m) = C_t \left(\underbrace{G_i(\varepsilon_i)}_{=v_i}, \underbrace{G_m(\varepsilon_m)}_{=v_m}; \theta_t \right) \quad (3)$$

Equation (3) can be solved for

$$C_t(v_i, v_m; \theta_t) = H_t(G_i^{-1}(\varepsilon_i), G_m^{-1}(\varepsilon_m)) \quad (4)$$

with $v_i, v_m \in [0,1]$, $\varepsilon_i, \varepsilon_m \in \mathbb{R}$, θ_t , the dynamic copula parameter and quantile function G_m^{-1} and G_i^{-1} . The benefit of using copula is that it allows to model a large range of joint distribution functions with specific behaviors of the marginal and a certain dependence structure advantage. However, Chollete et al (2009) argue that it is judicious to use a mixture of copula to model asymmetric dependencies as it can join different dependence patterns which stimulates a higher flexibility relative to a single-component copula presented by equation (4). A mixture of K copula functions is defined as

$$C_t(v_i, v_m; \theta_t) = \sum_{k=1}^K w_k C_{k,t}(v_i, v_m; \theta_{k,t}),$$

$$\sum_{k=1}^K w_k = 1 \quad (5)$$

where the k th copula is denoted by $C_{k,t}$ with the weight $w_k \in [0,1]$ and dynamic parameter $\theta_{k,t}$. By choosing different copula families with different properties, it is possible to combine their individual properties within the copula function C_t .

Finally, a conditional expectation of the form in Equation (2) as a function of the copula C_t obtain as

$$E_{t-1}(\varepsilon_{i,t} | \varepsilon_{m,t} \leq k) = \frac{1}{\alpha} \int_0^1 G_i^{-1}(v_i) \cdot \frac{\partial C_t(v_i, \alpha; \theta_t)}{\partial v_i} dv_i \quad (6)$$

and for the mixture copula as given by Equation(5) as

$$E_{t-1}(\varepsilon_{i,t} | \varepsilon_{m,t} \leq k) = \sum_{k=1}^K w_k \frac{1}{\alpha} \int_0^1 G_i^{-1}(v_i) \cdot \frac{\partial C_{k,t}(v_i, \alpha; \theta_{k,t})}{\partial v_i} dv_i \quad (7)$$

where $\alpha = G_m(k)$. The conditional expectation in Equation (7) represents the basis of the DMC-MES presented in the following section and will be discussed in more details.

3.2.2 Framework of the DMC-MES

The framework of the DMC-MES allows to model volatility, correlation, nonlinear dependence over time, and it allows for dynamic effects in the nonlinear dependence structure. The conditional marginal distributions of the institution and market return processes are modeled as

$$r_{m,t} = \sigma_{m,t} \varepsilon_{m,t} \quad \text{and} \quad r_{i,t} = \sigma_{i,t} \varepsilon_{i,t} \quad (8)$$

where $\sigma_{m,t}$ and $\sigma_{i,t}$ represent the volatility of the market and the institution, respectively. The conditional standard deviations follow the GJR-GARCH model developed by Glosten, et al. (1993). For the mutual dependence of the institution and market innovations, we use a dynamic copula function with high degree of flexibility for modeling dependencies.

Overall, the assumptions are as follows:

1. $\varepsilon_{m,t}$ and $\varepsilon_{i,t}$ are each *iid* with unspecified, static distribution G_m and G_i
2. $E(\varepsilon_{m,t}) = E(\varepsilon_{i,t}) = 0$, $Var(\varepsilon_{m,t}) = Var(\varepsilon_{i,t}) = 1$ and
3. $p(\varepsilon_{i,t} \leq \varepsilon_i, \varepsilon_{m,t} \leq \varepsilon_m) = C_t \left(\underbrace{G_i(\varepsilon_i)}_{v_i}, \underbrace{G_m(\varepsilon_m)}_{v_m}, \theta_t \right)$ with dynamic copula parameter θ_t .

Plugging $r_{m,t} = \sigma_{m,t}\varepsilon_{m,t}$ and $r_{i,t} = \sigma_{i,t}\varepsilon_{i,t}$ in Equation (2) yields

$$MES_{i,t} = \sigma_{i,t} E_{t-1}(\varepsilon_{i,t} | \varepsilon_{m,t} \leq k) \quad (9)$$

with $k = \frac{C_t}{\sigma_{m,t}}$. Using Equation (6), the MES in equation (9) obtains as

$$DC - MES_{i,t} = \frac{\sigma_{i,t}}{\alpha} \int_0^1 G_i^{-1}(v_i) \cdot \frac{\partial C_t(v_i, \alpha; \theta_t)}{\partial v_i} dv_i \quad (10)$$

with quantile function G_i^{-1} , copula of institution and market innovation C_t and dynamic copula parameter θ_t . The

DC-MES depends on (a) individual institution effects through $\sigma_{i,t}$ and G_i^{-1} as well as (b) interdependencies

between market and institution through the copula function C_t . As mentioned above, recent studies show that a copula function, that only depends on a single copula parameter is often unable to fully capture the observed dependence structure (Eckernkemper, 2018). Therefore, we use different copula functions by choosing a mixture of two dynamic copulas with static weights. Using Equation (7) for $K=2$, the MES according to Equation (9) can be written as

$$\begin{aligned} DMC - MES_{i,t} &= \tilde{w} \frac{\sigma_{i,t}}{\alpha} \int_0^1 G_i^{-1}(v_i) \cdot \frac{\partial C_{1,t}(v_i, \alpha; \theta_{1,t})}{\partial v_i} dv_i \\ &+ (1 - \tilde{w}) \frac{\sigma_{i,t}}{\alpha} \int_0^1 G_i^{-1}(v_i) \cdot \frac{\partial C_{2,t}(v_i, \alpha; \theta_{2,t})}{\partial v_i} dv_i \end{aligned} \quad (11)$$

where \tilde{w} denotes the copula weight. The DMC-MES is free in the choice of copula parameterization and it divides the dependence structure of the MES into the contribution of $C_{1,t}$ and $C_{2,t}$. To create a flexible dependence

structure, we chose two asymmetric copulas with different properties for $C_{1,t}$ and $C_{2,t}$, namely, a mixture of

Rotated Clayton and Clayton copula (RC&C) and a mixture of Rotated Gumbel and Gumbel Copula (RG&G). First, we use the DMC-MES because it accounts for asymmetric dependencies with a clear separation of nonlinear dependence structures into lower and upper tail dependence as the Clayton and rotated Gumbel copula allow for lower tail dependence, whereas the Gumbel and rotated Clayton copula allow for upper tail dependence. The Clayton and Gumbel copula differ by their respective degree of asymmetry. Second, the DMC-MES allows for symmetric dependencies for the parameter setting $\tilde{w}=0.5$ and $\theta_{1,t} = \theta_{2,t}$. In this way, symmetric dependence structures are nested within the proposed framework and it is possible to model asymmetric and symmetric dependencies together in one framework. Hence, the DMC-MES approach provides high flexibility as it is able to capture asymmetric and symmetric dependencies with an overall consideration of time-varying aspects.

3.3 DMC-MES Estimation Procedure

In the following, we discuss the model components and the parameterization of the DMC-MES, as well as its estimation procedure. We will focus on the parameterization of the two-component mixture copula in Equation (11).

Step 1

To allow for the model to capture conditional heteroskedasticity and autocorrelation of each series, we begin by running a group of GARCH models as candidates. We found out that the market and insurers return processes follow independent AR(1)- GJR-GARCH(1,1) specifications. The particularity of this model is that it accounts for leverage effect, that is, bad news about an insurer increases its future volatility more than good news. The model for the market and insurer return $r_t = (r_{m,t}, r_{i,t})'$ is as follows:

$$r_t = u_t + \sigma_t * \varepsilon_t \quad (12)$$

$$u_t = (u_{m,t}, u_{i,t})', \sigma_t = (\sigma_{m,t}, \sigma_{i,t})', \varepsilon_t = (\varepsilon_{m,t}, \varepsilon_{i,t})' \quad (13)$$

$$\varepsilon_{m,t} \sim iid G_m(\varepsilon_m), \varepsilon_{i,t} \sim iid G_i(\varepsilon_i), \quad (14)$$

the j th elements of u_t and σ_t are given by

$$u_t = \alpha_0 + \beta_0 r_{t-1} \quad (15)$$

$$\sigma_t^2 = \omega + (\alpha_1 + \gamma I_{t-1}) \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (16)$$

Step 2

We use the GAS model in order to model the dynamic copula parameters $\theta_{1,t}$ and $\theta_{2,t}$. The GAS specification is an observation-driven model where the score function acts as an observation driven part. It allows to model more than one dynamic parameter simultaneously and independently of each other because the score function provide information for each time-varying parameter. The specification of is as follows:

The joint cdf for $\varepsilon_{m,t}$ and $\varepsilon_{i,t}$ is specified as

$$H_t(\varepsilon_m, \varepsilon_i) = \tilde{w} * C_{1,t}(v_m, v_i; \theta_{1,t}(\psi_{1,t})) + (1 - \tilde{w}) * C_{2,t}(v_m, v_i; \theta_{2,t}(\psi_{2,t})) \quad (17)$$

$$\psi_{t+1} = \omega + A s_t + B \psi_t, \psi_t = (\psi_{1,t}, \psi_{2,t})', \quad (18)$$

$$s_t = S_t \cdot \nabla_t \quad (19)$$

$$\nabla_t = \frac{\partial \log c_t(v_{i,t}, v_{m,t}; \theta_t(\psi_t), w | f_{t-1})}{\partial \psi_t} \quad (20)$$

$$\log c_t(v_{i,t}, v_{m,t}; \theta_t(\psi_t), w | f_{t-1}) = \log \left[\tilde{w} c_{1,t}(v_{i,t}, v_{m,t}; \theta_{1,t}(\psi_{1,t})) + (1 - \tilde{w}) c_{2,t}(v_{i,t}, v_{m,t}; \theta_{2,t}(\psi_{2,t})) \right] \quad (21)$$

The information set at period $t-1$ is $f_{t-1} = \{v_m^{t-1}, v_i^{t-1}, \Theta^{t-1}\}$. The specifications of the GAS(1,1) model follow

Creal et al (2013) where S_t is the score function, S_t a scaling matrix, ∇_t the gradient, ω a vector of constants and A , B parameter matrices. The matrices A and B are assumed to be diagonal. For more details on the GAS model, one can read Blasques et al (2014a) and Blasques et al (2014b).

Step 3

To link the copula parameters to the framework, the transformation $\theta_{1,t} = 1 + \exp(\psi_{1,t})$ and $\theta_{2,t} = 1 + \exp(\psi_{2,t})$ are used to restrict the range of the Gumbel and rotated Gumbel parameters $\theta_{1,t}, \theta_{2,t} \in [1, \infty)$ and the transformations $\theta_{1,t} = \exp(\psi_{1,t})$ and $\theta_{2,t} = \exp(\psi_{2,t})$ are used to restrict the range of the Clayton and rotated Clayton parameters $\theta_{1,t}, \theta_{2,t} \in (0, \infty)$. Finally, the unknown parameters in ω , A , B and the copula weight W are estimated by maximum likelihood. The maximization problem is

$$(\hat{\omega}, \hat{A}, \hat{B}, \hat{W}) = \arg \max \sum_{t=1}^T \log c_t(v_{i,t}, v_{m,t}; \theta_t(\psi_t), \tilde{w} | f_{t-1}) \quad (22)$$

with implementation of the parameter updating in Equation (18).

3.4 Robustness Test

In this section we use the bivariate Clayton and Gumbel copula to double check the dependence structure between the insurer with the highest contribution to systemic risk and the rest of the insurers as well as the market. Finally, in order to explain the relationship among the insurers we use the Vector Autoregressive (VAR) model through the impulse response function (IRF) and give an overview of the relationship of the South African's insurance sector.

3.4.1 Clayton Copula

The Clayton copula is mostly used to model asymmetric dependence and correlated risks because of their ability to capture lower tail dependence. The closed form of the bivariate Clayton copula is given by:

$$C^{cl}(u_1, u_2, \theta) = (u_1^{-\theta} + u_2^{-\theta} - 1)^{-1/\theta} \quad (23)$$

Where θ is the copula parameter restricted on the interval $(0, \infty)$. If $\theta = 0$ then the marginal distributions become independent; when $\theta \mapsto \infty$ the Clayton copula approximates the Fréchet-Hoeffding upper bound.

3.4.2 Gumbel Copula

As the Clayton copula, the Gumbel copula is used to model asymmetric dependence. However, it is famous for its ability to capture strong upper tail dependence. If outcomes are expected to be strongly correlated at high values

but less correlated at low values, then the Gumbel copula is an appropriate choice. The bivariate Gumbel copula is given by:

$$C^{Gu}(u_1, u_2, \theta) = \exp\left(-\left[(-\log u_1)^\theta + (-\log u_2)^\theta\right]^{1/\theta}\right)$$

(24)

where θ is the copula parameter restricted on the interval $[1, \infty)$. When θ approaches 1, the marginals become independent and when θ goes to infinity the Gumbel copula approaches the Fréchet-Hoeffding upper bound.

3.4.3 Vector Autoregressive Model and Impulse Responses

Vector autoregression (VAR) is an econometric model which represents the correlations among a set of variables, they are often used to analyze certain aspects of the relationships between the variables of interest. It is a multi-equation system where all the variables are treated as endogenous (dependent). The VAR(p) model is given by:

$$Y_t = a + A_1Y_{t-1} + A_2Y_{t-2} + \dots + A_pY_{t-p} + \varepsilon_t$$

(25)

where:

- $Y_t = (y_{1t}, y_{2t}, \dots, y_{nt})'$: an $(n \times 1)$ vector of time series variables
- a : an $(n \times 1)$ vector of intercepts
- $A_i (i = 1, 2, \dots, p)$: an $(n \times n)$ coefficient matrices
- ε_t : an $(n \times 1)$ vector of unobservable *i.i.d* zero mean error term.

The Impulse Response Function enables us to know the response of one variable to an impulse in another variable in a system that involves a number of further variables as well.

4. Empirical Analysis

4.1. Data

While there are 10 insurance companies listed in the Johannesburg Stock Exchange (JSE), due to data availability, our analysis is based on 5 insurers which have data covering our entire sample period. The dataset contains daily returns with 3144 observations from November 13, 2007 to June 15, 2020. The stock prices are taken from Bloomberg and converted to log returns multiplied by 100. Table 1 provides the list of the insurers listed in the JSE that constitute the South African insurance sector.

TABLE 1. List of the South African Insurance Companies.

Company	Symbol	Sector
Conduit Capital Limited	CND	Nonlife Insurance
Clientele Limited	CLI	Life Insurance
Discovery Limited	DSY	Life Insurance
Indequity Group Limited	IDQ	Nonlife Insurance
Liberty Holdings Limited	LBH	Life Insurance
Momentum Metropolitan Holdings	MTM	Life Insurance

Old Mutual Plc	OML	Insurance
Rand Merchant Investment	RMI	Life Insurance
Sanlam Limited	SLM	Life Insurance
Santam Limited	SNT	Nonlife Insurance

4.2 Estimation Results

The skewness parameters of each institution and the market are nonzero with the majority of the institutions having negative skewness parameters indicating left-skewed distributions except for Momentum Metropolitan which is right-skewed distributions. In addition, the kurtosis is significantly greater than 3 for all five insurers and the market, which is an evidence of fat-tailed distribution. Using the Ljung-Box Q-test, the null hypothesis of no autocorrelation is rejected at lag 10 for all institutions and market. Overall, the descriptive statistics show, asymmetry, autocorrelation, and heteroscedasticity of equity returns. According to Sklar's theorem (1959), before modelling the joint distribution of returns, the first step is to select a suitable model for the marginal return distribution, because misspecification of the univariate model probably results in biased copula parameter estimates. To allow for the heteroskedasticity of each series, we considered a group of GARCH models as candidates and found that the GJR-GARCH model developed by Glosten, et al. (1993) is preferred to the others based on their likelihood values. The particularity of this model is that it accounts for leverage effect, that is, bad news about an institution increases its future volatility more than good news. We then consider the GJR-GARCH (1,1). The model parameters are estimated by using maximum likelihood estimation (MLE) and the results of AR (1) and GJR-GARCH (1,1) estimations are presented in Table 2.

Table 2. Estimation results of marginal distributions

	AR (1)		GJR-GARCH (1,1)			Kurtosis	Skewness	
	α_0	$\beta_0 \quad \omega$	α_1	β_1	γ			
Discovery	0.0476 (0.0252)	-0.0058 (0.0183)	0.0385** (0.0115)	0.0517** (0.0126)	0.9058** (0.0133)	0.0669** (0.0182)	4.9419** (1.571)	-0.2145** (0.0259)
Liberty	0.0159 (0.0251)	-0.0664** (0.0165)	0.0366** (0.0054)	0.0001 (0.0036)	0.9573** (0.0024)	0.0603** (0.0024)	12.32** (0.330)	-0.0801** (0.0246)
MTM	0.0321 (0.0263)	-0.0683** (0.0181)	0.0567** (0.0214)	0.0564** (0.0173)	0.9138** (0.0191)	0.0318* (0.0171)	8.3628** (0.9165)	0.0101** (0.0027)
Sanlam	0.0221 (0.0259)	-0.0690** (0.0180)	0.0526** (0.0154)	0.0353** (0.0123)	0.9098** (0.0138)	0.0849** (0.0182)	4.7317** (1.3719)	-0.1677** (0.0153)
Santam	0.0299 (0.0242)	-0.1172** (0.0182)	0.4707** (0.1510)	0.2065** (0.0485)	0.6849** (0.0737)	-0.0292** (0.0462)	16.038** (2.195)	-0.6265** (0.249)

Note: The reported numbers are the parameter estimates and standard errors (in parentheses) of the univariate AR (1) and GJR-GARCH (1,1). Statistical significance at the 10%, 5% levels are denoted by * and ** respectively. Time period from November 13, 2007 to June 15, 2020.

The sum of the GJR-GARCH parameters α_1 and β_1 is close to unity for all the insurers indicating highly persistent volatilities. Moreover, we find the leverage effect parameter γ to be significant and positive at 5% confidence interval for Discovery, Liberty, Sanlam, and Santam except for MTM where it is negative and significant at 10%. A positive leverage effect implies that a negative return on the series increases volatility more than a positive return with the same magnitude. Hence, in general, the diagnostics provide evidences that our marginal distribution models are well-specified and therefore, we can reliably use the combination of AR (1), GJR-GARCH (1,1) and the Skew Student t distribution along with the dynamic mixture copula to model the dependence structure.

Table 3. Estimation Results of the Dynamic Mixture Copula of the Rotated Clayton and Clayton (RC&C): between Market and Insurers

	GAS (1,1)						
	Weight		w_2	A_{11}	A_{22}	B_{11}	B_{22}
	\tilde{w}	w_1					
Discovery	0.392 (0.031)	0.109 (0.078)	0.002 (0.0001)	0.233 (0.118)	0.012 (0.004)	0.849 (0.107)	0.998 (0.002)
Liberty	0.485 (0.039)	0.0005 (0.00001)	0.0206 (0.021)	0.036 (0.009)	0.136 (0.097)	0.998 (0.004)	0.959 (0.045)
MTM	0.463 (0.065)	-0.003 (0.0001)	0.007 (0.0001)	0.067 (0.104)	0.023 (0.0012)	1.003 (0.0001)	0.989 (0.0004)
Sanlam	0.347 (0.029)	-0.002 (0.0001)	-0.001 (0.0005)	0.011 (0.012)	0.044 (0.008)	1.0014 (0.0001)	1.0013 (0.0003)
Santam	0.423 (0.057)	-0.002 (0.004)	0.0001	0.094 (0.09)	0.034 (0.041)	0.995 (0.011)	0.998 (0.003)
			(0.0004)				

Note: The reported numbers are the parameter estimates and standard errors (in parentheses) of the dynamic mixture copula of rotated Clayton and Clayton (RC&C). Time period from November 13, 2007 to June 15, 2020.

Secondly, the parameters of the copulas between the South African insurance sector and each insurer are estimated. The estimation results for the dynamic mixture copula are reported in table 3 and 4 for the Rotated Clayton and Clayton (RC&C) and Rotated Gumbel and Gumbel (RG&G), respectively. The estimates show that the copula weights are between 0.392 and 0.485 for the dynamic mixture of RC&C (see table 3) and between 0.555 and 0.696 for the dynamic mixture of RG&G (see table 4). This provides strong evidence for an asymmetric dependence structure between insurers and the market.

Table 4. Estimation Results of Dynamic Mixture Copula of the Rotated Gumbel and Gumbel (RG&G): between Markets and Insurers.

	Weight		GAS (1,1)				
	\tilde{w}	w_1	w_2	A_{11}	A_{22}	B_{11}	B_{22}

	\tilde{w}	w_1	w_2	A_{11}	A_{22}	B_{11}	B_{22}
Discovery	0.639 (0.044)	-0.006 (0.026)	0.082 (0.113)	0.144 (0.093)	0.175 (0.235)	0.659 (0.221)	0.539 (0.606)
Liberty	0.555 (0.049)	-0.132 (0.105)	-0.327 (0.241)	0.229 (0.159)	-0.175 (0.227)	0.497 (0.329)	-0.394 (0.641)
MTM	0.567 (0.0009)	-0.0006 (0.00001)	-0.002 (0.0001)	-0.017 (0.0001)	0.048 (0.003)	1.0004 (0.0001)	0.991 (0.0004)
Sanlam	0.696 (0.045)	0.019 (0.009)	0.039 (0.029)	0.032 (0.012)	-0.154 (0.114)	0.982 (0.009)	0.961 (0.029)
Santam	0.643 (0.086)	-0.003 (0.006)	-1.212 (0.524)	0.042 (0.047)	2.159 (1.528)	0.996 (0.007)	0.048 (0.316)

Note: The reported numbers are the parameter estimates and standard errors (in parentheses) of the dynamic mixture copula of rotated Gumbel and Gumbel (RG&G). Time period from November 13, 2007 to June 15, 2020.

4.3 Risk Analysis Based on DMC-MES

This section illustrates how DMC-MES can be used for a detailed risk analysis for each institution of the South African insurance sector under study. The aim is to establish a ranking based on the contribution to systemic risk of the different insurers and identify possible systemically important insurers in South Africa.

Table 5: Maximum Likelihood Estimates

Dynamic Mixture Copula	RC&C	RG&G
Log Likelihood Estimates	LLE1	LLE2
Discovery	-1093.4	-1117.8
Liberty	-762.9	-778.4
Momentum Metro	-903.3	-925.6
Sanlam	-2693.7	-2774.4
Santam	-305.9	-307.5

Table 5 presents the results for the log likelihood estimates for the mixture of rotated Clayton and Clayton and the mixture of rotated Gumbel and Gumbel. It can be noticed that the mixture of rotated Gumbel and Gumbel outperforms the mixture of rotated Clayton and Clayton as it has the lowest log likelihood estimates for all insurers. Hence, table 6 exhibits the average DMC-MES for the insurers based on RG&G.

Table 6: Summary Statistics for DMC-MES for all insurers based on RG&G

	mean	st.dev	min	max	Ranking
Discovery	0.983	7.709	0.0551	334.3986	2

Liberty	0.583	5.748	0.0375	216.4342	4
Momentum Metro	0.805	5.580	0.0618	223.8966	3
Sanlam	1.942	18.135	0.1128	894.1638	1
Santam	-0.007	0.100	-2.349	4.3039	5

Table 6 provides the ranking of these 5 insurers based on their average rotated Gumbel and Gumbel DMC-MES. At the top of the list is Sanlam the largest insurer in South Africa in size whose systemic risk measure is as high as 1.942, making Sanlam the largest contributor to systemic risk in the South African insurance sector. The second largest insurer in South Africa in size, Discovery is the second largest contributor to systemic risk with an average DMC-MES of 0.983. According to the literature, this might be due to the fact that Sanlam and Discovery are deeply engaged in business activities outside the traditional insurance sector such the issuance of Credit Default Swaps and securities lending businesses. Moreover, our result is in line with the Too Big To Fail (TBTF) thinking as financial institutions tend to be as systemically important as they are large, and this is the case of Sanlam and Discovery. Momentum Metropolitan and Liberty are next with a contribution to systemic risk of 0.805, 0.583. On the other hand, Santam is the least systemically risky insurer in South Africa, with a DMC-MES being negative, -0.007. This indicates that Santam is relatively resilient when the insurance market is in distress. This might be explained by the fact that Santam is probably involved in traditional insurance business activities.

4.4 Robustness Test

4.4.1 Bivariate Copula

After filtering the data using AR(1)-GJRARCH(1,1) models, the obtained pair of innovations (standardized residuals) was transformed to uniforms using the estimated skew student-t distributions. The uniform series will be used as input for the bivariate Clayton and Gumbel copula in order to get an insight of the dependence structure between the largest systemically insurer in South Africa, Sanlam, the insurance market, and the other insurers.

Table 7. Parameters for the copulas and their 95% confidence intervals for sanlam and the insurance market; and Sanlam and the other insurers.

Copula	parameters	95% CI	AIC	BIC
Gumbel(MKT)	3.4309	[3.343 3.519]	5160	5170
Clayton(MKT)	3.5677	[3.462 3.673]	4510	4520
Gumbel(DSY)	1.4503	[1.412 1.487]	780.35	786.4
Clayton(DSY)	0.7812	[0.726 0.837]	802.08	808.13
Gumbel(LBH)	1.3842	[1.349 1.419]	617.59	623.65
Clayton(LBH)	0.6360	[0.582 0.689]	594.01	600.06
Gumbel(MTM)	1.4551	[1.418 1.492]	791.17	797.24
Clayton(MTM)	0.7528	[0.697 0.808]	765.28	771.33
Gumbel(STN)	1.775	[1.149 1.206]	175.5	181.56
Clayton(STN)	0.3540	[0.305 0.403]	233.89	239.95

The estimated parameters corresponding to each copula, the confidence interval (CI), the AIC, and the BIC values are reported in Table 7.

First it can be seen that that the Gumbel copula provides the best fit to the data between Sanlam and Discovery; and Sanlam and Santam since it has the lowest values for the AIC criteria. This indicates a greater dependence in the positive tail than in the negative. In other words, large gains from Sanlam and Discover; and Sanlam and Santam are more likely to occur simultaneously than large losses.

On the other hand, according to the AIC criteria the Clayton copula provides the best fit to the data between Sanlam and Liberty; Sanlam and Momentum Metropolitan; and Sanlam and the insurance market, exhibiting greater dependence in the negative tail than in the positive. This means that large losses from Sanlam and Liberty; Sanlam and Momentum Metropolitan; and Sanlam and the insurance market are more likely to happen simultaneously than large gain. This is in line with our result as we expect a left tail dependence between the largest systemically insurer in South Africa and the insurance market.

4.4.2 Impulse Response Function

The aim of this section is to estimate a bivariate VAR model to determine the effect of the biggest contributor to systemic risk in the insurance sector, Sanlam to the insurance market and the insurers understudy; and the effect of the insurance market to the insurers. Following Giglio et al (2016), we use the Cholesky decomposition to orthogonalise the innovation shocks.

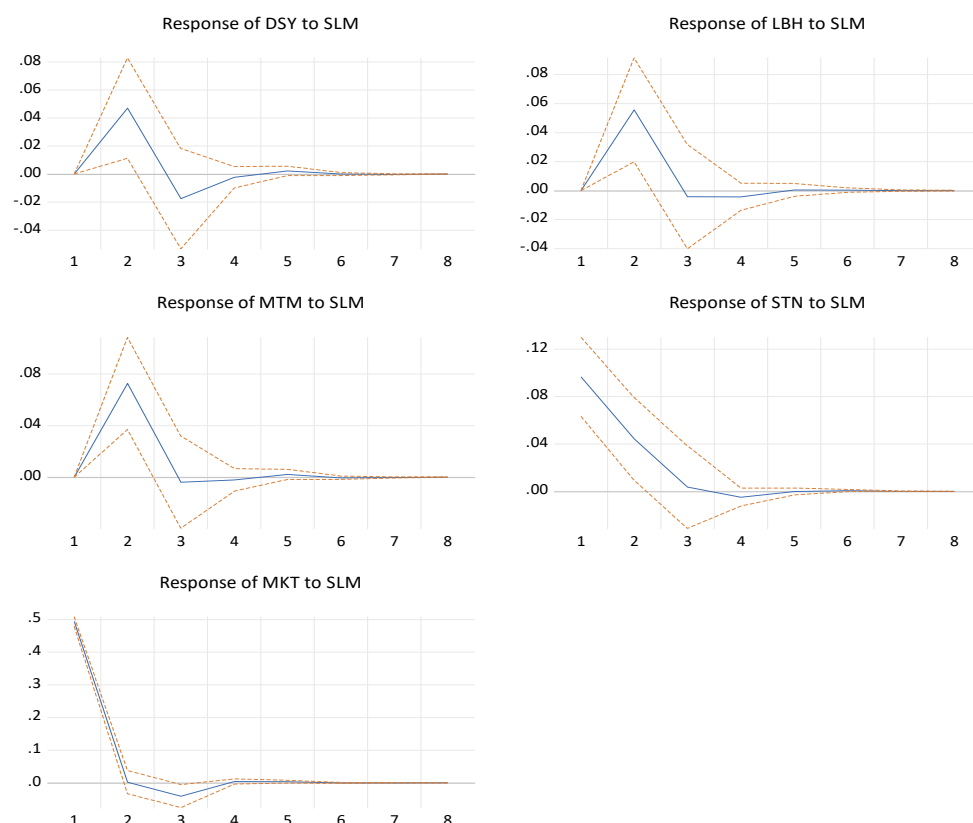


Figure 1: Impulse response of Sanlam to the insurers and the insurance market.

First, we can notice an initial increase from period 1 to 2 of Discovery, Liberty, and Momentum Metropolitan to a one standard deviation shock (innovation) to Sanlam. Afterward between period 2 and 3, there is a sharp decline in their responses, they become negative and increase from the negative region to the positive region until they hit their steady state. Conversely, we can see a sharp decline of Santam and the insurance market to a one standard deviation shock (innovation) to Sanlam. They become negative and increase gradually until they reach their steady state.

In conclusion, this result is in line with our finding as we expect a shock to Sanlam to have a negative impact on the insurance market and the insurers in a short run.

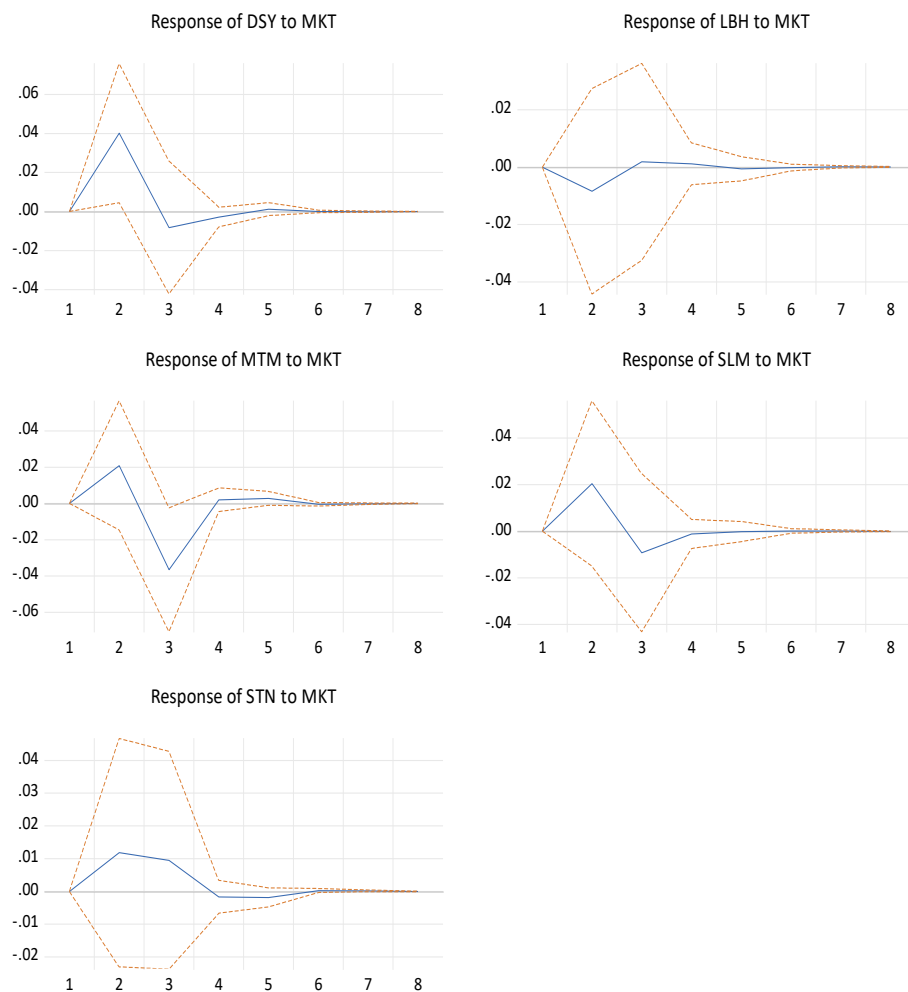


Figure 2: Impulse response of the insurance market to the insurers

First, we can notice an initial increase from period 1 to 2 of Discovery, Momentum Metropolitan, Sanlam and Santam to a one standard deviation shock (innovation) to the insurance market. Afterward between period 2 and 3, there is a sharp decline in their responses, they become negative and increases from the negative region to the positive region until they hit their steady state, except for Santam where between period 2 to 3, there is a slight decline in the positive region and from period 3 to 4 it declines to the negative region. Afterward, it reaches the steady state between period 4 and 5, and gradually increases and hit again the steady state in the long run. Finally, we can see a decline of Liberty to the negative region from period 1 to 2 to a one standard deviation shock (innovation) to the insurance market. It gradually increases between period 2 and 3 until the positive region where it reaches the steady state in the long run.

In conclusion, this result is in line with our finding as we expect a shock to the insurance market to have a negative impact on the insurance companies in a short run except for Santam where a shock in the market does not really have a negative effect on her.

5. Conclusion

The role played by the insurance sector in the 2007-2009 financial crisis has led researchers and regulators to intensify research on systemic risk in the insurance sector with the goal of identifying and managing systemic risk in the above-mentioned sector. In this paper, we empirically investigate systemic risk in the South African insurance sector using DMC-MES proposed by Eckernkemper (2018). The main objective of the study is to identify the most systemically insurers in South Africa over our sample period. While there is an agreement in the literature that traditional insurance activities contribute less to systemic risk, the majority of peer-reviewed papers and regulators' reports argue that some non-traditional insurance activities appear to be relevant to systemic risk and hence contribute to it.

Using stock return data collected for the period 13 November 2007 to 15 June 2020, we find that Sanlam is the largest contributor to systemic risk in the South African insurance sector, followed by Discovery. According to the existing literature on systemic risk in the insurance sector, this may imply that Sanlam and Discovery are more likely to be involved in non-traditional insurance activities such as financial securities, Credit Default Swap, derivatives, and many others.

Additionally, our results show that the contribution of insurers to systemic risk is linked to the size of the insurer, as Sanlam and Discovery are respectively the largest and second largest insurers in South Africa in size. This may indicate that instability in the insurance sector is more likely to occur when one of these two insurers are in financial difficulty. Therefore, our results are in line with the "theory" that "no financial institution should be too big to fail (TBTF)".

Finally, our findings would be useful for South African regulators as they can put stricter supervision on Sanlam and Discovery. For instance, they can ask more capital requirement from these two insurers and more transparency in their business activities in order to ensure stability in the South African insurance sector and the financial system as whole. However, the existing literature highlights the fact that there is plenty of room for further research on this topic. One could be interested to know which of the current systemic risk measures are adequate to measure an insurer's contribution to systemic risk or its vulnerability to impairments of the financial system. One, could also be interested to conduct more research on non-core activities on insurers in South Africa and their systemic impact.

Supplementary Materials:

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