The Entropy of Supermassive Black Holes during its Evaporation Time

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Abstract

Is it possible to quantify in General Relativity, GR, the entropy generated by supermassive black holes, BHs, during its evaporation time, since the intrinsic Hawking radiation in the infinity that, although insignificant, is important in the effects on the thermal quantum atmosphere? The purpose was to develop a formula that allows us to measure the entropy generated during the evaporation time of different types of BHs of: i. remnant BH of the binary black holes' merger, BBH: GW150914, GW151226 and LTV151012 detected by the Laser Interferometer Gravitational-Wave Observatory (LIGO), and ii. Schwarzschild, Reissner-Nordström, Kerr and Kerr-Newman, and thus quantify in GR the "insignificant" quantum effects involved, in order to contribute to the validity of the generalized second law (GSL) that directly links the laws of black hole mechanics to the ordinary laws of thermodynamics, as a starting point for unifying quantum effects with GR. This formula could have some relationship with the detection of the shadow's image of the event horizon of a BH. This formula was developed in dimensional analysis, using the constants of nature and the possible evaporation time of a black hole taking into account its distance to the Earth, to quantify the entropy generated during that time. The energy-stress tensor was calculated with the 4 metrics to obtain the material content and apply the proposed formula. The entropy of the evaporation time of BHs proved to be insignificant, its temperature is barely above absolute zero, however, the calculation of this type of entropy allows us to argue about the importance of the quantum effects of Hawking radiation mentioned by authors who have studied the quantum effects with arguments that are fundamentally based on the presence of the surrounding thermal atmosphere of the black hole.

Keywords: Entropy; Black hole physics; Radiation mechanism: thermal; Relativity; Methods: analytical; Astronomical databases: miscellaneous

1 Introduction

In the present work, a mathematical relationship is proposed using the constants of the nature and the evaporation time of BHs, in order to measure the entropy of the geometry of space-time that surrounds different types of black holes that were not formed due to the gravitational collapse of a star but to the extreme density of the Universe at the beginning of its expansion, according to the metrics of Schwarzschild, Reissner-Nordström, Kerr and Kerr-Newman [1, 2] taking the energy-impulse tensor as the material content coming from the metric as well as the remnant black hole of the binary black holes' merger, BBH: GW150914, GW151226 and LTV151012 detected by LIGO [3, 4]; and from the results obtained for the BHs, it could be shown that, in general, the lower the thermal Hawking radiation (just above absolute zero) and the greater the mass of the black hole, the greater the entropy generated by said radiation. Therefore, it is suggested, that the entropy measured by the proposed formula, represents the generalized entropy that, although the entropy and the area of a black hole decrease individually, the generalized entropy never decreases as in [5]. Said BHs radiation, included in the proposed entropy formula, makes it possible to show the quantum effects that other authors allude to [5, 6, 7, 8, 9, 10, 11], such as effects that could contribute to detect the image of this type of hole that, just on April 4, 2019,

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the Event Horizon Telescope (EHT) managed to reveal the first image of the M87 supermassive black hole [12, 13, 14]. Also, it is intended to introduce in the proposed formula for BHs, instead of the Planck constant, the energy and evaporation time to measure the entropy in low mass black holes. The Kerr and Reissner-Nordström black holes comply with the third law of thermodynamics and the natural law of cosmic censorship of [15]; with thermal Hawking radiation that manages to barely exceed absolute zero and surface gravity very close to zero.

From the obtained results, we can affirm that the measurement of quantum effects using the entropy formula, reveals the presence of BHs and therefore contributes to their detection.

2 Methods

This section summarizes the methodology used in this work that includes, for the determination of the entropy during the evaporation time formula and, for the determination of the material content of the BHs: i. dimensionless analysis and ii. general relativity by using the energy-impulse tensor through the four metrics used, respectively.

2.1 Dimensional Analysis

A system of units definitely plays a fundamental role in establishing the symbology that represents a physical phenomenon framed in cosmology. Said system of units has fundamental dimensions that define variables and constants involved in physical scenarios. The fundamental dimensions of two or more variables can form dimensionless parameters, which, together, give place to a dimensionless functional relationship useful to express the behavior of a physical system, in this case, of the entropy during the evaporation time in BHs. The methodology to form three dimensionless groups from the proposed formula, made use of four principles:

- The principle of geometric similarity to include the area of the black hole event horizon;
- The principle of kinematic similarity to include the fluxes of evaporation time, mass flow and Hawking temperature. This also includes the speed of light;
- The principle of dynamic similarity which includes the similarity of forces for the entropy fluxes during the evaporation time, the Planck constant and the gravitational constant;
- The principle of complete similarity. To complete it, we used the hierarchy of the dimensionless parameters involved in the physical phenomenon to be analyzed, given the geometric and kinematic similarities in all the relevant parameters, and the dynamic similarity in all the relevant independent variables parameters, obtaining complete dynamic similarity with the parameters of equal dependent variables.

The Π Buckingham Theorem [16] was used to obtain a dimensionally homogeneous entropy during evaporation time of BHs of the modeling section, this is, $S_{\mathrm{t_{evap}}}$ equation. This theorem states that:

Theorem 1 *Knowing that a physical system is governed by a dimensionally homogeneous relationship that comprises n dimensional parameters, namely*

$$w_1 = f(w_2, w_3, \cdots, w_n) \tag{1}$$

where the w are the dimensional variables, there is an equivalent relationship that contains a smaller number, $(n-k)$, of dimensionless parameters, such as

$$\Pi_1 = F\Big(\Pi_2, \Pi_3, \cdots, \Pi_{n\text{-}k}\Big) \tag{2}$$

where the Π are dimensionless numbers that are constructed from the w. The reduction k is generally equal to the number of fundamental dimensions contained in the w, but never greater than it.

The dimensionless parameters were constructed as products of powers of the dimensional parameters, fundamental dimensions; in this case, the physical scenario comprises 7 fundamental dimensions: the entropy during the evaporation time of BHs, $S_{\mathrm{t_{evap}}}$, the time of evaporation, t_{evap}, Planck constant, h, the speed of light, c, the gravitational constant, G, the mass, M, and, the thermal Hawking radiation at the Hawking temperature, T; 3 of them were chosen as repeated variables (c, G and M), and they were used with the other 4 dimensions ($S_{\mathrm{t_{evap}}}$, t_{evap}, h and T) to form the 4 dimensionless groups $\Pi(\Pi_1,\Pi_2,\Pi_3,\Pi_4)$. So

$$\Pi_1 = [S_{\mathrm{t_{evap}}}][c][G][M], \Pi_2 = [t_{\mathrm{evap}}][c][G][M], \Pi_3 = [h][c][G][M], \Pi_4 = [T][c][G][M]. \tag{3}$$

The appropriate dimensions of each variable were substituted, namely

$$\begin{aligned} \Pi_1 &= \left[\frac{N\cdot m}{K}\right]\left[\frac{m}{s}\right]\left[\frac{N\cdot m^2}{kg^2}\right]\left[kg\right], \\ \Pi_2 &= \left[s\right]\left[\frac{m}{s}\right]\left[\frac{N\cdot m^2}{kg^2}\right]\left[kg\right], \\ \Pi_3 &= \left[N\cdot m\cdot s\right]\left[\frac{m}{s}\right]\left[\frac{N\cdot m^2}{kg^2}\right]\left[kg\right], \\ \Pi_4 &= \left[K\right]\left[\frac{m}{s}\right]\left[\frac{N\cdot m^2}{kg^2}\right]\left[kg\right]. \end{aligned} \quad (4)$$

Each fundamental dimension was equated to the corresponding power, 1, a, b, c.

We set the net exponent of any dimension on the right hand side to zero, which provided a set of simultaneous equations, which allowed us to find the power to which each fundamental dimension must be raised to dimensionless each Π. they must be dimensionless, that is

$$\begin{aligned} \Pi_1 &= \left[\frac{N\cdot m}{K}\right]^1\left[\frac{m}{s}\right]^a\left[\frac{N\cdot m^2}{kg^2}\right]^b\left[kg\right]^c, \\ \Pi_2 &= \left[s\right]^1\left[\frac{m}{s}\right]^a\left[\frac{N\cdot m^2}{kg^2}\right]^b\left[kg\right]^c, \\ \Pi_3 &= \left[N\cdot m\cdot s\right]^1\left[\frac{m}{s}\right]^a\left[\frac{N\cdot m^2}{kg^2}\right]^b\left[kg\right]^c, \\ \Pi_4 &= \left[K\right]^1\left[\frac{m}{s}\right]^a\left[\frac{N\cdot m^2}{kg^2}\right]^b\left[kg\right]^c. \end{aligned} \quad (5)$$

Finally, all the results are brought together in a dimensionless functional relationship, namely:

$$\Pi_1 = F\Big(\Pi_2, \Pi_3, \Pi_4\Big) \quad (6)$$

Where the fundamental dependent parameter is Π_1 and, the fundamental independent parameters are Π_2, Π_3 and Π_4

The fundamental dependent parameter was not chosen as a repeated variable. No parameter was chosen whose influence on the problem was questionable or only important for a time. Parameters were chosen whose dimensions were as close as possible to "pure", for example, the pure dimension of temperature is Kelvin degrees, density is mass and velocity is time.

The dimensionless expression that describes the phenomenon was determined with 7 dimensional parameters, 3 fundamental dimensions and 4 dimensionless parameters, obtaining 2 dimensionless parameters (Π_2 and Π_3) and Π_1 and Π_4 were combined to achieve the principle of complete similarity.

Given the $\Pi_1 = \left[\frac{N\cdot m}{K}\right]^1\left[\frac{m}{s}\right]^a\left[\frac{N\cdot m^2}{kg^2}\right]^b\left[kg\right]^c$, with

$$N^{1+b}K^{-1}m^{1+a+2b}s^{-a}kg^{-2b+c} \quad (7)$$

the following simultaneous equations are obtained to determine the powers

$$1+b=0, b=-1; 1+a+2b=0, a=1; -2b+c=0, c=-2. \quad (8)$$

and

$$\Pi_1 = \left[\frac{N\cdot m}{K}\right]^1\left[\frac{m}{s}\right]^1\left[\frac{N\cdot m^2}{kg^2}\right]^{-1}\left[kg\right]^{-2} = \frac{S_{\mathrm{tevap}}c}{GM^2} \quad (9)$$

In the same way,

$$\Pi_2 = \left[s\right]^1\left[\frac{m}{s}\right]^3\left[\frac{N\cdot m^2}{kg^2}\right]^{-1}\left[kg\right]^{-1} = \frac{t_{\mathrm{evap}}c^3}{GM}, \quad (10)$$

$$\Pi_3 = \left[N\cdot m\cdot s\right]^1\left[\frac{m}{s}\right]^1\left[\frac{N\cdot m^2}{kg^2}\right]^{-1}\left[kg\right]^{-2} = \frac{hc}{GM^2}, \quad (11)$$

$$\Pi_4 = \Big[K\Big]^1 \Big[\frac{m}{s}\Big]^0 \Big[\frac{N \cdot m^2}{kg^2}\Big]^0 \Big[kg\Big]^0 = T. \tag{12}$$

Π_2 and Π_3 are dimensionless. Sometimes matching two or more independent similarity parameters is not practical. In such cases, a series of model tests and theoretical calculations must be made to obtain the database that makes up the information. For this, it is useful to know the hierarchy of the dimensionless parameters involved in the physical phenomenon to be analyzed, in this case, the procedure described was carried out.

Multiplying Π_1, Π_4 and t_{evap}, we obtained

$$\Pi_1 \cdot \Pi_4 \cdot T \cdot t_{\text{evap}} = \frac{S_{t_{\text{evap}}} T c t_{\text{evap}}}{GM^2} \tag{13}$$

With this, the complete dimensional analysis of the proposed formula was obtained. Agree with Eq. (6),the groups were reordered

$$\Pi_1 \cdot \Pi_4 \cdot T \cdot t_{\text{evap}} = F\Big(\Pi_2, \Pi_3\Big), \frac{S_{t_{\text{evap}}} T c t_{\text{evap}}}{GM^2} = F\Big(\frac{t_{\text{evap}} c^3}{GM}, \frac{hc}{GM^2}\Big). \tag{14}$$

With this, three dimensionless numbers for the entropy of supermassive black holes were obtained.

2.2 General Relativity

The energy-stress tensors of general relativity were calculated with the equation of Albert Einstein [17], using metrics from Schwarzschild, Reissner-Nordström, Kerr and Kerr-Newman

$$T_{11} = -\frac{1}{8\pi G}(R_{11} - \frac{1}{2}R g_{11} + \Lambda g_{11}) \tag{15}$$

where, R_{11} is the Ricci tensor, R the Ricci scalar, g_{11} the metric tensor and Λ the cosmological constant.

T_{11} of Schwarzschild was obtained from Eq. (15), as

$$T_{11} = \Big(\frac{e^u}{8}\Big)\Bigg(\Big(\frac{-1}{4\rho}\Big)\Big(4\frac{du}{d\rho} - 2\rho\frac{d^2v}{d\rho^2} - \rho(\frac{dv}{d\rho})^2 + \rho\frac{du}{d\rho}\frac{dv}{d\rho}\Big)$$

$$-\Big(\frac{1}{4\rho^2 e^u}\Big)\Big(4\rho\frac{du}{d\rho} - 2\rho^2\frac{d^2v}{d\rho^2} - \rho^2(\frac{dv}{d\rho})^2 + \rho^2\frac{du}{d\rho}\frac{dv}{d\rho} - 4 + 4e^u - 4\rho\frac{dv}{d\rho}\Big) - \Lambda e^u\Bigg).$$

(16)

Where, ρ is the radial coordinate and u and v are unknown functions of ρ to be determined as

$$u(\rho) = -\ln\Big(\frac{-C2 + C3\rho}{\rho}\Big) + C1, v(\rho) = -\ln\Big(\frac{-C2 + C3\rho}{\rho}\Big), \rho = \frac{2GM}{c^2}. \tag{17}$$

Where, $C1$, $C2$ and $C3$ are unknown constants to be determined.

While T_{11} obtained with the Reissner-Nordström metric, it is given by

$$T_{11} = \frac{-1}{8\pi G}\frac{e^2(2Mr - r^2 - e^2)}{r^6} + \Big(-1 + \frac{2M}{r} - \frac{e^2}{r^2}\Big)\ \Big(\Lambda - \frac{R}{2}\Big), \tag{18}$$

where, r is the radial coordinate and e is the electric charge.

For the Kerr metric, T_{11} is

$$T_{11} = -\frac{1}{8\pi G}\frac{-2a^2M(18M^3r^6 - 12M^2r^7 + 3Mr^8 + 3Ma^2r^6)}{12r^{10}M^2a^2 - 2r^{13}M + r^{12}a^2 + r^{14}} \tag{19}$$
$$+\Big(-1 + \frac{2M}{r}\Big)\ \Big(\Lambda - \frac{R}{2}\Big),$$

Finally, for T_{11} of Kerr-Newman with Eq. (15), it was used

$$g_{11} = 1 - \frac{2Mr - Q^2}{r^2}, \tag{20}$$

$$R = \frac{2\Big(-1.12\times 10^{162}r + 1.19\times 10^{131}r^8 \ldots - 2.62\times 10^{163}r^3 + 7.89\times 10^{127}\Big)}{2.25\times 10^{160}r - 2.23\times 10^{195}r^8 \ldots + 1.83\times 10^{33}r^{13} - r^{14} - 1.15\times 10^{126}}, \tag{21}$$

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$$R_{11} = \frac{(-80M^3ra^2cos^2(\theta) + 56Q^2M^2a^2cos^2(\theta)\ldots r^4Q^2 + 2r^2Q^4 + Q^6)}{-12r^6M^2 + 8r^3M^3 - 12r^4Q^2M^2 \ldots + 6MrQ^4 - Q^6 - r^6 - 3r^4Q^2}. \tag{22}$$

with $\theta = \frac{\pi}{2}$ for planar orbits.

3 Modeling

The formula of entropy generated during the evaporation time of BHs that we propose here, is given by

$$S_{t_{\text{evap}}} = \frac{t_{\text{evap}} hc^6}{G^2M^2T} \tag{23}$$

where [18],

$$T = \frac{1}{4}\frac{h\kappa}{\pi^2 ck}, \tag{24}$$

$$\kappa = \frac{4\pi}{A}\sqrt{G^2M^2 - GQ^2 - c^2a^2}, \tag{25}$$

$$A = \frac{4G\pi}{c^4}(2GM^2 - Q^2 + 2M\sqrt{G^2M^2 - GQ^2 - c^2a^2}). \tag{26}$$

Meanwhile,

$$S_{t_{\text{evap}}} = S_{\text{GSL}} = S + S_{\text{BH}} = S + \frac{\pi c^3 kA}{2hG}, \tag{27}$$

where, S_{GSL} the Bekenstein's generalized second law of thermodynamics, S_{BH} the Bekenstein-Hawking entropy, A the event horizon area, k the Boltzmann constant, a the spin, Q the electric charge and κ the surface gravity.

Also, it is proposed to measure the entropy during the evaporation time of low mass black holes S_{lmbh}, changing h of the Eq. (23) by energy multiplied by time (Et_{evap}), $E = Mc^2$, such as

$$S_{\text{lmbh}} = E\frac{(t_{\text{evap}})^2 c^6}{G^2M^2T} \tag{28}$$

where [18] ,

$$T = \frac{hc^3}{16\pi^2 GMk}. \tag{29}$$

$S_{t_{\text{evap}}}$ and S_{lmbh}, are inversely proportional to h, as expected when there are quantum effects. The respective areas of the Eqs. (23) and (28), are

$$A_{S_{t_{\text{evap}}}} = 2\frac{S_{t_{\text{evap}}} hG}{c^3 k\pi}, \tag{30}$$

$$A_{S_{\text{lmbh}}} = 2\frac{S_{\text{lmbh}} hG}{c^3 k\pi}. \tag{31}$$

4 Results

According to the laws of black hole mechanics from Stephen Hawking's Physics, a black hole has a mass, a total spin and a total charge when compressed, therefore, calculations were made of different types of black holes with different parameters of mass, charge and angular momentum that define them according to the metrics used. In this section, partial results of the calculations of thermophysical properties are presented. Figures 1 to 3 show the entropy and the thermal Hawking radiation at the Hawking temperature, moreover, Tables 1 to 3 show the surface gravity, the evaporation time and the event horizon area of BHs. In Tables A1-A8 of Appendix A, some additional results of the calculations performed with LIGO databases of [3, 4] and three metrics are presented. Figure 1 shows the curves of $S_{t_{\text{evap}}}$ vs T of the remnant BH of BBH, detected by LIGO in [3, 4]. The same type of curves are shown in Figures 2 and 3 for Kerr and Reissner-Nordström BHs, respectively.

Calculations obtained with T_{11} (for $\theta = \frac{\pi}{2}$ and $\Lambda = 10^{-51}m^{-2}$) of the Kerr-Newman metric, for a mass of $M = 4.5 \times 10^{10}kg$, a spin of $a = 0.7$ and electrical charges of $Q = 0.2$ and $Q = M$, yield an evaporation time, $t_{\text{evap}} = 2.4 \times 10^3$ years, which perhaps corresponds to a primordial black hole. In this critical case, the relationship $G^2M^2 < GQ^2 + a^2c^2$, indicates that the two event horizons disappear, but

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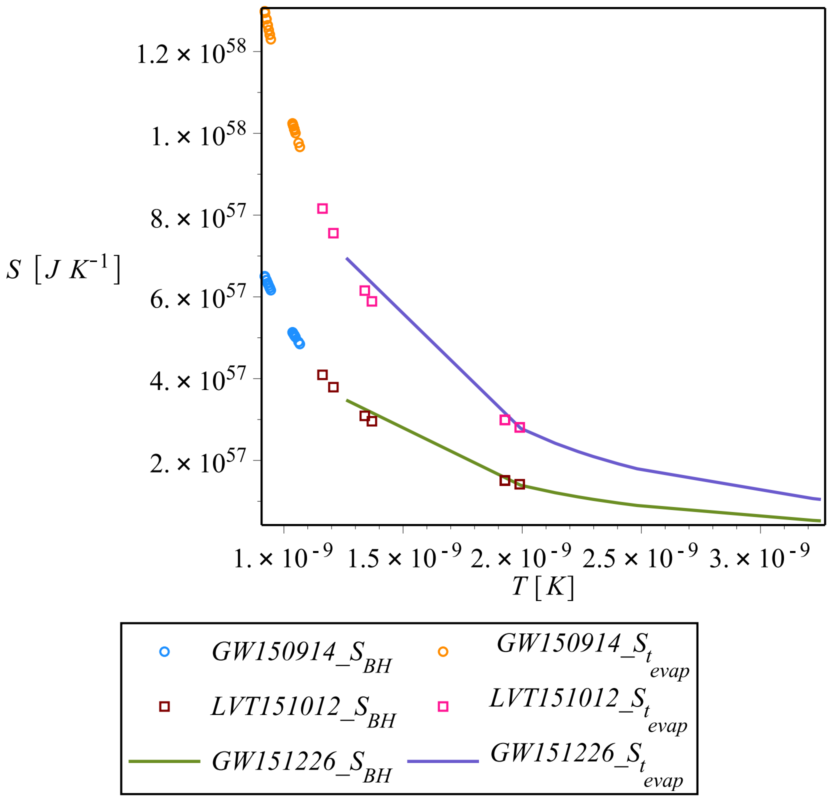

Figure 1: Entropy versus thermal Hawking radiation at the Hawking temperature of the remnant black hole of the BBH: GW150914, GW151226 and LTV151012 detected by LIGO.

non-zero surface gravity ($\kappa \neq 0$) indicates that they are not destroyed and therefore do not leave the naked singularity, according to the cosmic censorship conjecture [15].

For the entropy calculations from the metrics, the material content of the respective energy-stress tensor (T_{11}) was used.

Table 1. BHs's thermophysical properties calculated from M, a and $Q = 0$.†.

Black hole type	T_{11} $(M(kg))$	a	A (m^2)	κ (ms^{-2})	T (K)
GW151226	9.8×10^{31}	0.78	2.6×10^{11}	3.1×10^{11}	1.3×10^{-9}
	3.8×10^{31}	0.56	3.9×10^{10}	8.0×10^{11}	3.3×10^{-9}
Schwarzschild	9.9×10^{30}	0	2.7×10^{9}	3.1×10^{12}	1.2×10^{-8}
R-Nordström	5.0×10^{95}	0	6.9×10^{138}	6.1×10^{-53}	2.5×10^{-73}
Kerr	7.1×10^{91}	0.7	1.4×10^{131}	4.3×10^{-49}	1.7×10^{-69}
	1.3×10^{32}	0.7	4.8×10^{11}	2.3×10^{11}	9.4×10^{-10}
Messier 87	1.2×10^{40}	0.95	4.2×10^{27}	2.5×10^{3}	9.9×10^{-18}

† Except in Reissner-Nordström, where $Q = 0.2$.

In Tables 1 and 2 it can be seen that, for a Schwarzschild black hole of a solar mass, the evaporation time in years turns out to be of the order of 10^{64}! Much older than the age of the Universe, with a temperature of the order of $10^{-8}K$, a temperature at which it is difficult to detect radiation from the astronomical object. For the M87 black hole of approximately six thousand two hundred million solar masses or $1.23 \times 10^{40} kg$ and a temperature of the order of $10^{-18}K$, the EHT obtained the first image of the shadow of its event horizon, which means that, for the Schwarschild, Kerr-Newman and Kerr $(1.2 \times 10^{32} kg)$ BHs studied here, their detection is also possible since the thermal Hawking radiation is "mostly significant" compared to that of the M87 BHs.

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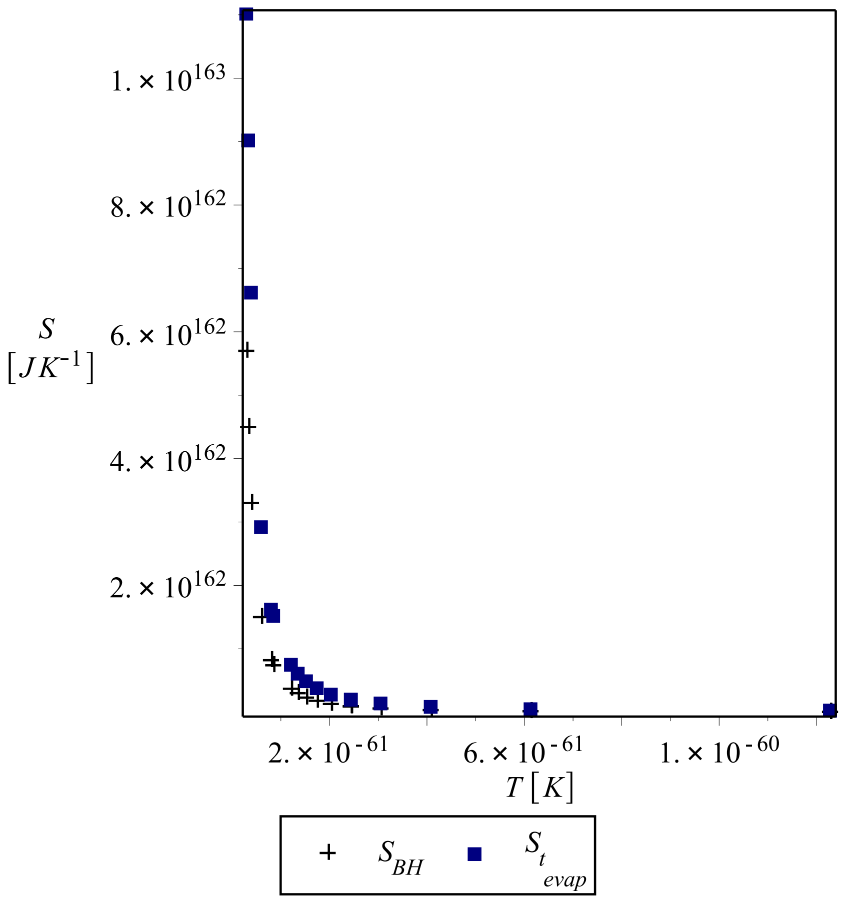

Figure 2: Entropy versus thermal Hawking radiation at the Hawking temperature of Kerr supermassive black holes.

Table 2. BHs's thermophysical properties calculated from Table 1.

Black hole type	$S_{\mathbf{BH}}$ (JK^{-1})	$t_{\mathbf{evap}}(years)$	$S_{\mathbf{t_{evap}}}(JK^{-1})$	$A_{\mathbf{S_{t_{evap}}}}(m^2)$
GW151226	3.48×10^{57}	2.44×10^{67}	6.95×10^{57}	5.25×10^{11}
	5.23×10^{56}	1.42×10^{66}	1.05×10^{57}	7.9×10^{10}
Schwarzschild	3.5×10^{55}	2.55×10^{64}	7.16×10^{55}	5.41×10^{9}
R-Nordström	9.2×10^{184}	3.17×10^{258}	1.8×10^{185}	1.4×10^{139}
Kerr	1.8×10^{177}	9.20×10^{246}	3.7×10^{177}	2.8×10^{131}
	6.31×10^{57}	5.96×10^{67}	1.26×10^{58}	9.53×10^{11}
Messier 87	5.57×10^{73}	5.07×10^{91}	1.11×10^{74}	8.41×10^{27}

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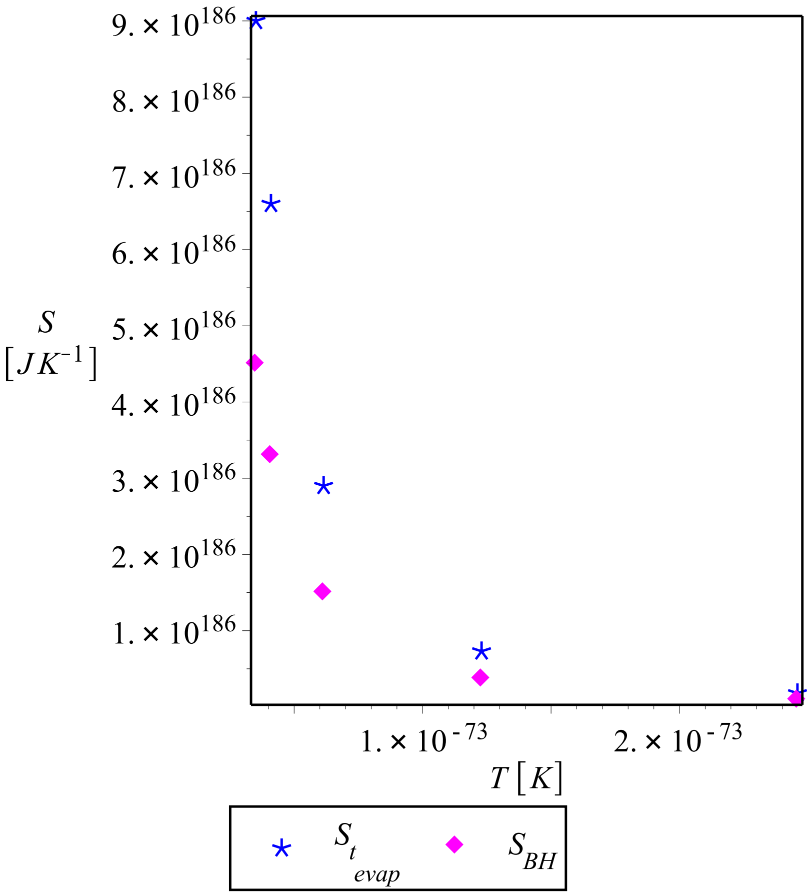

Figure 3: Entropy versus thermal Hawking radiation at the Hawking temperature of Reissner-Nordström supermassive black holes.

5 Discussion

From the calculations for M87's BH, the thermal Hawking radiation is so low that it makes the BHs "undetectable". Neverteless, the mass of the BHs can be detected, and the quantum effects measured by the entropy formula, which could be the reason why, the EHT was able to get the first M87's BH photograph in [12, 13, 14] despite the "insignificant" quantum effects. Then, the BHs can also be detected.

For the Schwarzschild, Kerr, Reissner-Nordström BHs and the remnant BH from BBH merge of GW151226 in Table 1, when the mass decreases during the evaporation process, S_{tevap}, $A_{S_{\mathrm{tevap}}}$, t_{evap}, S_{BH} and A, decrease (Tables 1 and 2), while κ and T increase (Table 1). The same, respectively for the entropy and thermal Hawking radiation at the Hawking temperature of supermassive black holes of GW150914, GW151226 and LVT151012 of Figure 1, of Kerr of Figure 2 and of Reissner-Nordström of Figure 3. See Tables A1-A8 of Appendix A.

For supermassive black holes, BHs, at the limit, when the thermal Hawking radiation tends to absolute zero (see Table 1), the rate of change of the entropy generated during its time of evaporation (Table 2) and eventual disappearance with respect to T is: $\lim \frac{dS_{\mathrm{tevap}}}{dT}_{T\to 0} = -Float(\infty)$. Said rate of change with respect to time is $Float(undefined)$.

The BHs of Kerr and Reissner-Nordström, as well as, the remnant BH from BBH merge of GW150914, GW151226, and LVT151012 have temperatures that tend to absolute zero, especially the former ones, where S_{tevap} and its change with respect to t_{evap} tend to $Float(undefined)$. On the other hand, if $T \longrightarrow \infty$, S_{tevap} and its change with respect to T tend to cancel, as it could have happened at the Big Bang. The quantity $Float(\infty)$ represents a floating-point infinity. This value is used to indicate a floating-point value that is too large to be otherwise represented. While, the quantity $Float(undefined)$ represents a non-numeric object in the floating-point system. This value can be returned by a function

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or operation if the input operands are not in the domain of the function or operand.

In Tables 1 and A7 of Appendix A for Reissner-Nordström and Kerr BHs: i. $\kappa \rightarrow 0$ and $T \rightarrow 0$: the 3rd law of black hole mechanics is satisfied because the extreme black hole state cannot be reached (1st law of black hole mechanics: the effective temperature of a BH is absolute zero as stated in [6]). ii. the 2nd law of thermodynamics because thermal Hawking radiation increases as the mass evaporates, generating an increase in ordinary entropy outside the BH, according to authors such as [6, 7, 8, 9, 10, 11], which makes that the total entropy increases. iii. GSL according with [7] for a region containing a BH in which, the BH's entropy plus the entropy in the BH's exterior never decreases.

From the calculations obtained considering the rotating black hole of M87 with a spin $a = 0.90 \mp 0.05$ of the [19] calculation, a thermalized Hawking radiation of $9.97 \times 10^{-18} K$ and a $t_{\mathrm{evap}} = 4.91 \times 10^{91}$ years are obtained; much higher than the age of the Universe. These results allow us to compare the BHs studied here using the energy-stress tensor with the 4 metrics (especially Schwarzschild, Kerr-Newman and Kerr) and with the data from the remnants BH of the binary black holes' merger detected by LIGO, in order to argue that, although in both cases, the thermal Hawking radiation is so low that it makes the BHs "undetectable", there are quantum effects that we measure by the entropy of the BHs and the entropy generated during their evaporation time, which provides us with results that allow us to argue that its detection is possible, through telescopes of the size of the EHT. So, the mass of the BHs can be detected, and the quantum effects measured by the entropy formula, could be the reason why, the EHT was able to get the first M87 BHs photograph in [12, 13, 14] despite the "insignificant" quantum effect of $9.97 \times 10^{-18} K$ obtained of calculations (see Table 1).

The results of the T_{11} calculations with the Kerr y Reissner-Nordström metrics, some of which are shown in Tables 1 and 2 and additionally in Tables A7 and A8 of Appendix A, with masses much higher than the mass of the M87 black hole, show a surface gravity approaching nullity and thermal Hawking radiation just above absolute zero, however, since κ is different from zero, the relationships $G^2M^2 \gg GQ^2 + a^2c^2$ and $t_{\mathrm{evap}} \gg \frac{h}{Mc^2}$, are indicative, respectively, that the process does not produce naked singularity according to the cosmic censorship of [15] and that, apparently, the lifetime of the analyzed mass particles does not correspond to the lifetime of virtual particles, which means that they can be detected by a highly sensitive detector such as the EHT.

The entropy formula, Eq. (23), manages to join three different fields of physics: Thermodynamics, General Relativity, and Quantum Mechanics, containing three dimensionless numbers, Eq. (14); it satisfies the second law of thermodynamics and contains the quantum information by which the image of the BHs could be detected. In all cases, thermal Hawking radiation increases as the mass evaporates, generating an increase in ordinary entropy outside the black hole, according to authors such as [6, 7, 8, 9, 10, 11], which makes that the total entropy increases.

For Kerr and Reissner-Nordström BHs in Table 1 and Tables A7 and A8 of Appendix A, the third law of black hole mechanics is satisfied that states that the extreme black hole state cannot be reached, the boundary between a black hole and the naked singularity as in [6], since $T \neq 0\ K$ (barely exceeds absolute zero) and $\kappa \neq 0\ ms^{-2}$ (very close to zero). In fact, by the first law of black hole mechanics, the effective temperature of a BH is absolute zero as stated in [6].

With this, the objective of giving an approach that contrasts with the GSL is fulfilled, which is validated by authors such as [9, 10], with arguments that are fundamentally based on the presence of the thermal atmosphere that surrounds the black hole. Other related arguments for the validity of the GSL have been given in [11] where it is argued that the generalized entropy of the state of the region outside the black hole should increase under the assumption that it experiences an autonomous evolution [20], which reinforces the results obtained for $S_{t_{\mathrm{evap}}}$.

The results allow us to argue that, the area theorem is not violated since, as the black hole evaporates, decreasing its mass, both the entropy generated during its evaporation and eventual disappearance, and the surface area of the event horizon, they decrease individually, but the total entropy always increases, according to [6].

5.1 Forthcoming Research

The time of life of the material content of the BHs would have to be analyzed from the quantum point of view of the particles that make it up, to verify that these are particles that can be detected by a highly sensitive detector such as EHT, since the calculations showed that $t_{\mathrm{evap}} \gg \frac{h}{Mc^2}$.

6 Conclusions

The proposed formula to measure the entropy generated during the evaporation time of BHs, which we propose to equate with Bekenstein's generalized second law of thermodynamics, allows us to fulfill the

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objective of elucidating that the intrinsic quantum effects in said formula, occupy a central role in the compliance with physical laws.

This formula that links thermodynamics, general relativity and quantum mechanics and that includes three dimensionless numbers, allows us to argue that the following laws are satisfied: the second law of thermodynamics because it is verified that total entropy is always increasing; the third law of thermodynamics and the law of cosmic censorship through calculations that yield results of thermal Hawking radiation at the Hawking temperature just above absolute zero and surface gravity very close to zero for BHs by Kerr and Reissner-Nordström mainly, but, in general for BHs, which allow us to argue that extreme black hole status cannot be achieved according to laws of black hole mechanics. With this, the importance of the unification of the quantum-classical effects that give rise to a single theory that allows them to be measured theoretically is highlighted.

In conclusion, the calculations allow us to argue that BHs can be detected despite the fact that the Hawking effect, which reveals their presence, is not significant in this type of holes; however, the proposed formula to measure quantum effects through entropy, allows us to affirm that it can contribute to the detection of the image of these BHs, since the detection of this type of black hole is directly linked to the "insignificance" of the intrinsic Hawking effect in obtaining the shadow image of the event horizon of the BHs of M87 achieved by the EHT.

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A Thermophysical properties of supermassive black holes

Table A1. Determination of thermophysical properties of BHs with LIGO Astronomical Databases of Remnant BH of BBH Mergers adapted from [3, 4].

GW150914

$M_{\mathbf{Sun}}$	$M\ (kg)$	a	$A\ (m^2)$	$\kappa\ (ms^{-2})$	$T\ (K)$
$62.3^{+3.7+0.9}$	1.33×10^{32}	$0.68^{+0.05+0.01}$	4.89×10^{11}	2.28×10^{11}	9.24×10^{-10}
$62^{+4.1+0.7}$	1.33×10^{32}	$0.67^{+0.05+0.01}$	4.88×10^{11}	2.28×10^{11}	9.25×10^{-10}
$62.5^{+3.9}$	1.32×10^{32}	$0.68^{+0.05}$	4.82×10^{11}	2.30×10^{11}	9.31×10^{-10}
$62^{+4.4}$	1.32×10^{32}	$0.67^{+0.06}$	4.82×10^{11}	2.30×10^{11}	9.31×10^{-10}
62^{+4}	1.31×10^{32}	$0.67^{-0.04}$	4.76×10^{11}	2.31×10^{11}	9.37×10^{-10}
$62.3^{+3.7}$	1.31×10^{32}	$0.68^{+0.05}$	4.76×10^{11}	2.31×10^{11}	9.37×10^{-10}
$62^{+3.7}$	1.31×10^{32}	$0.66^{+0.04}$	4.72×10^{11}	2.32×10^{11}	9.41×10^{-10}
$62.1^{+3.3}$	1.30×10^{32}	$0.68^{+0.06}$	4.68×10^{11}	2.33×10^{11}	9.45×10^{-10}
$62_{+4.1-0.7}$	1.30×10^{32}	$0.67_{+0.05-0.01}$	4.68×10^{11}	2.33×10^{11}	9.45×10^{-10}
$62.3_{+3.7-0.9}$	1.30×10^{32}	$0.68_{+0.05-0.01}$	4.63×10^{11}	2.34×10^{11}	9.50×10^{-10}
$62.3^{-3.1+0.2}$	1.18×10^{32}	$0.68^{+0.06+0.02}$	3.86×10^{11}	2.57×10^{11}	1.04×10^{-09}
$62.1_{-2.8}$	1.18×10^{32}	$0.68_{-0.05}$	3.85×10^{11}	2.57×10^{11}	1.04×10^{-09}
$62.3_{-3.1}$	1.18×10^{32}	$0.68_{-0.06}$	3.83×10^{11}	2.58×10^{11}	1.04×10^{-09}
$62.5_{-3.5}$	1.17×10^{32}	$0.68_{-0.07}$	3.81×10^{11}	2.59×10^{11}	1.05×10^{-09}
$62_{-0.3}$	1.17×10^{32}	$0.67_{-0.05}$	3.81×10^{11}	2.59×10^{11}	1.05×10^{-09}
$62.3_{-3.1-0.2}$	1.17×10^{32}	$0.68_{-0.06-0.02}$	3.81×10^{11}	2.59×10^{11}	1.05×10^{-09}
$67^{-3.7+0.6}$	1.17×10^{32}	$0.68^{-0.07+0.02}$	3.79×10^{11}	2.59×10^{11}	1.05×10^{-09}
$62_{-3.3}$	1.17×10^{32}	$0.66_{-0.06}$	3.77×10^{11}	2.60×10^{11}	1.05×10^{-09}
62_{-4}	1.15×10^{32}	$0.67_{-0.08}$	3.68×10^{11}	2.63×10^{11}	1.07×10^{-09}
$62_{-3.7-0.6}$	1.15×10^{32}	$0.67_{-0.07-0.02}$	3.64×10^{11}	2.64×10^{11}	1.07×10^{-09}

Table A2. Determination of thermophysical properties of BHs with LIGO Astronomical Databases of Remnant BH of BBH Mergers adapted from [3, 4]. Continuation of Table A1.

GW150914

$M_{\mathbf{Sun}}$	a	$S_{\mathbf{BH}}\ (JK^{-1})$	$t_{\mathbf{evap}}(years)$	$S_{t_{\mathbf{evap}}}(JK)^{-1})$	$A_{S_{t_{\mathbf{evap}}}}\ (\mathbf{m^2})$
$62.3^{+3.7+0.9}$	$0.68^{+0.05+0.01}$	6.48×10^{57}	1.96×10^{75}	1.30×10^{58}	9.79×10^{11}
$62^{+4.1+0.7}$	$0.67^{+0.05+0.01}$	6.46×10^{57}	1.95×10^{75}	1.29×10^{58}	9.76×10^{11}
$62.5^{+3.9}$	$0.68^{+0.05}$	6.38×10^{57}	1.91×10^{75}	1.28×10^{58}	9.64×10^{11}
$62^{+4.4}$	$0.67^{+0.06}$	6.38×10^{57}	1.91×10^{75}	1.28×10^{58}	9.64×10^{11}
62^{+4}	$0.67^{-0.04}$	6.31×10^{57}	1.88×10^{75}	1.26×10^{58}	9.53×10^{11}
$62.3^{+3.7}$	$0.68^{+0.05}$	6.31×10^{57}	1.88×10^{75}	1.26×10^{58}	9.53×10^{11}
$62^{+3.7}$	0.66+0.04	6.25×10^{57}	1.85×10^{75}	1.25×10^{58}	9.44×10^{11}
$62.1^{+3.3}$	$0.68^{+0.06}$	6.19×10^{57}	1.83×10^{75}	1.24×10^{58}	9.35×10^{11}
$62_{+4.1-0.7}$	$0.67_{+0.05-0.01}$	6.19×10^{57}	1.83×10^{75}	1.24×10^{58}	9.35×10^{11}
$62.3_{+3.7-0.9}$	$0.68_{+0.05-0.01}$	6.14×10^{57}	1.80×10^{75}	1.23×10^{58}	9.27×10^{11}
$62.3^{-3.1+0.2}$	$0.68^{+0.06+0.02}$	5.11×10^{57}	1.37×10^{75}	1.02×10^{58}	7.72×10^{11}
$62.1_{-2.8}$	$0.68_{-0.05}$	5.09×10^{57}	1.36×10^{75}	1.02×10^{58}	7.69×10^{11}
$62.3_{-3.1}$	$0.68_{-0.06}$	5.07×10^{57}	1.35×10^{75}	1.01×10^{58}	7.66×10^{11}
$62.5_{-3.5}$	$0.68_{-0.07}$	5.04×10^{57}	1.34×10^{75}	1.01×10^{58}	7.61×10^{11}
$62_{-0.3}$	$0.67_{-0.05}$	5.04×10^{57}	1.34×10^{75}	1.01×10^{58}	7.61×10^{11}
$62.3_{-3.1-0.2}$	$0.68_{-0.06-0.02}$	5.04×10^{57}	1.34×10^{75}	1.01×10^{58}	7.61×10^{11}
$67^{-3.7+0.6}$	$0.68^{-0.07+0.02}$	5.02×10^{57}	1.33×10^{75}	1.00×10^{58}	7.59×10^{11}
$62_{-3.3}$	$0.66_{-0.06}$	4.99×10^{57}	1.32×10^{75}	9.98×10^{57}	7.54×10^{11}
62_{-4}	$0.67_{-0.08}$	4.87×10^{57}	1.27×10^{75}	9.74×10^{57}	7.36×10^{11}
$62_{-3.7-0.6}$	$0.67_{-0.07-0.02}$	4.82×10^{57}	1.25×10^{75}	9.64×10^{57}	7.28×10^{11}

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Table A3. Determination of thermophysical properties of BHs with LIGO Astronomical Databases of Remnant BH of BBH Mergers adapted from [3, 4].

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$M_{\mathbf{Sun}}$	M (kg)	a	A (m^2)	κ (ms^{-2})	T (K)
35^{+14}	9.75×10^{31}	$0.66^{+0.09}$	2.63×10^{11}	3.11×10^{11}	1.26×10^{-09}
35_{-4}	6.17×10^{31}	$0.66_{-0.01}$	1.05×10^{11}	4.92×10^{11}	1.99×10^{-09}
$20.8^{+6.1+2}$	5.75×10^{31}	$0.74^{+0.06+0.03}$	9.13×10^{10}	5.28×10^{11}	2.14×10^{-09}
$20.06^{+7.6}$	5.50×10^{31}	$0.73^{+0.05}$	8.37×10^{10}	5.52×10^{11}	2.23×10^{-09}
$20.8^{+6.1}$	5.35×10^{31}	$0.74^{+0.06}$	7.91×10^{10}	5.67×10^{11}	2.30×10^{-09}
$20.9^{+4.8}$	5.11×10^{31}	$0.75^{+0.07}$	7.22×10^{10}	5.94×10^{11}	2.41×10^{-09}
$20.8_{+6.1-2}$	4.95×10^{31}	$0.74_{+0.06+0.03}$	6.78×10^{10}	6.13×10^{11}	2.48×10^{-09}
$20.8_{-1.7-0.1}$	3.82×10^{31}	$0.74_{-0.06-0.03}$	4.03×10^{10}	7.95×10^{11}	3.22×10^{-09}
$20.8_{-1.7}$	3.80×10^{31}	$0.74_{-0.06}$	3.99×10^{10}	7.99×10^{11}	3.24×10^{-09}
$20.9_{-1.8}$	3.80×10^{31}	$0.75_{-0.05}$	3.99×10^{10}	7.99×10^{11}	3.24×10^{-09}
$20.6^{-1.6}$	3.78×10^{31}	$0.73_{-0.06}$	3.95×10^{10}	8.03×10^{11}	3.25×10^{-09}
$20.8^{-1.7+0.1}$	3.78×10^{31}	$0.74^{-0.06+0.03}$	3.95×10^{10}	8.03×10^{11}	3.25×10^{-09}

Table A4. Determination of thermophysical properties of BHs with LIGO Astronomical Databases of Remnant BH of BBH Mergers adapted from [3, 4]. Continuation of Table A3.

GW151226

$M_{\mathbf{Sun}}$	a	$S_{\mathbf{BH}}$ (JK^{-1})	$t_{\mathbf{evap}}(years)$	$S_{\mathbf{tevap}}(JK)^{-1})$	$A_{S_{\mathbf{tevap}}}$ $(\mathbf{m^2})$
35^{+14}	$0.66^{+0.09}$	3.48×10^{57}	7.68×10^{74}	6.95×10^{57}	5.25×10^{11}
35_{-4}	$0.66_{-0.01}$	1.39×10^{57}	1.95×10^{74}	2.78×10^{57}	2.10×10^{11}
$20.8^{+6.1+2}$	$0.74^{+0.06+0.03}$	1.21×10^{57}	1.58×10^{74}	2.42×10^{57}	1.83×10^{11}
$20.06^{+7.6}$	$0.73^{+0.05}$	1.11×10^{57}	1.38×10^{74}	2.22×10^{57}	1.67×10^{11}
$20.8^{+6.1}$	$0.74^{+0.06}$	1.05×10^{57}	1.27×10^{74}	2.10×10^{57}	1.58×10^{11}
$20.9^{+4.8}$	$0.75^{+0.07}$	9.56×10^{56}	1.11×10^{74}	1.91×10^{57}	1.44×10^{11}
$20.8_{+6.1-2}$	$0.74_{+0.06+0.03}$	8.98×10^{56}	1.01×10^{74}	1.80×10^{57}	1.36×10^{11}
$20.8_{-1.7-0.1}$	$0.74_{-0.06-0.03}$	5.34×10^{56}	4.62×10^{73}	1.07×10^{57}	8.06×10^{10}
$20.8_{-1.7}$	$0.74_{-0.06}$	5.28×10^{56}	4.55×10^{73}	1.06×10^{57}	7.98×10^{10}
$20.9_{-1.8}$	$0.75_{-0.05}$	5.28×10^{56}	4.55×10^{73}	1.06×10^{57}	7.98×10^{10}
$20.6^{-1.6}$	$0.73^{-0.06}$	5.23×10^{56}	4.48×10^{73}	1.05×10^{57}	7.90×10^{10}
$20.8^{-1.7+0.1}$	$0.74^{-0.06+0.03}$	5.23×10^{56}	4.48×10^{73}	1.05×10^{57}	7.90×10^{10}

Table A5. Determination of thermophysical properties of BHs with LIGO Astronomical Databases of Remnant BH of BBH Mergers adapted from [3, 4].

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$M_{\mathbf{Sun}}$	M (kg)	a	A (m^2)	κ (ms^{-2})	T (K)
35^{+14+4}	1.05×10^{32}	$0.66^{+0.09+0}$	3.07×10^{11}	2.88×10^{11}	1.17×10^{-09}
36^{+15}	1.01×10^{32}	$0.65_{+0.09}$	2.84×10^{11}	2.99×10^{11}	1.21×10^{-09}
35^{+11}	9.15×10^{31}	$0.66^{+0.08}$	2.31×10^{11}	3.32×10^{11}	1.34×10^{-09}
35_{+14-4}	8.95×10^{31}	$0.66_{+0.09-0}$	2.21×10^{11}	3.39×10^{11}	1.37×10^{-09}
36_{-4}	6.36×10^{31}	$0.65_{-0.01}$	1.12×10^{11}	4.77×10^{11}	1.93×10^{-09}
35_{-3}	6.36×10^{31}	$0.66_{-0.01}$	1.12×10^{11}	4.77×10^{11}	1.93×10^{-09}
35^{-4+0}	6.17×10^{31}	$0.66^{+0.10+0.02}$	1.05×10^{11}	4.92×10^{11}	1.99×10^{-09}
35_{-4+0}	6.17×10^{31}	$0.66_{-0.10-0.02}$	1.05×10^{11}	4.92×10^{11}	1.99×10^{-09}

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Table A6. Determination of thermophysical properties of BHs with LIGO Astronomical Databases of Remnant BH of BBH Mergers adapted from [3, 4]. Continuation of Table A5.

		LVT151012			
$M_{\mathbf{Sun}}$	a	$S_{\mathbf{BH}}\ (JK^{-1})$	$t_{\mathbf{evap}}(s)$	$S_{\mathbf{tevap}}(JK)^{-1})$	$A_{S_{\mathbf{tevap}}}\ (m^{\mathbf{2}})$
35^{+14+4}	$0.66^{+0.09+0}$	4.07×10^{57}	9.72×10^{74}	8.13×10^{57}	6.14×10^{11}
36^{+15}	$0.65_{+0.09}$	3.77×10^{57}	8.66×10^{74}	7.53×10^{57}	5.69×10^{11}
35^{+11}	$0.66^{+0.08}$	3.06×10^{57}	6.36×10^{74}	6.13×10^{57}	4.63×10^{11}
35_{+14-4}	$0.66_{+0.09-0}$	2.93×10^{57}	5.95×10^{74}	5.86×10^{57}	4.43×10^{11}
36_{-4}	$0.65_{-0.01}$	1.48×10^{57}	2.14×10^{74}	2.97×10^{57}	2.24×10^{11}
35_{-3}	$0.66_{-0.01}$	1.48×10^{57}	2.14×10^{74}	2.97×10^{57}	2.24×10^{11}
35_{-4+0}	$0.66^{+0.10+0.02}$	1.39×10^{57}	1.95×10^{74}	2.78×10^{57}	2.10×10^{11}
35_{-4+0}	$0.66_{-0.10-0.02}$	1.39×10^{57}	1.95×10^{74}	2.78×10^{57}	2.10×10^{11}

Table A7. BHs's thermophysical properties calculated from M, a and Q with 3 metrics.

Black hole type	$T_{\mathbf{11}}\ (M(kg))$	a	Q	$A\ (m^2)$	$\kappa\ (ms^{\mathbf{-2}})$	$T\ (K)$
Schwarzschild	9.89×10^{30}	0	0	2.7×10^{9}	3.07×10^{12}	1.24×10^{-8}
	6×10^{16}	0	0	1.0×10^{-19}	5.06×10^{26}	2.05×10^{6}
	4×10^{16}	0	0	4.4×10^{-20}	7.59×10^{26}	3.07×10^{6}
	2×10^{16}	0	0	1.0×10^{-20}	1.52×10^{27}	6.15×10^{6}
	1×10^{11}	0	0	2.8×10^{-31}	3.03×10^{32}	1.23×10^{12}
	2.7×10^{8}	0	0	2.0×10^{-36}	1.12×10^{35}	4.55×10^{14}
	2×10^{8}	0	0	1.1×10^{-36}	1.52×10^{35}	6.15×10^{14}
	1×10^{8}	0	0	2.8×10^{-37}	3.03×10^{35}	1.23×10^{15}
Reissner-Nordström	3.5×10^{96}	0	0.2	3×10^{140}	8.67×10^{-54}	3.51×10^{-74}
	3×10^{96}	0	0.2	2×10^{140}	1.01×10^{-53}	4.1×10^{-74}
	2×10^{96}	0	0.2	1×10^{140}	1.52×10^{-53}	6.15×10^{-74}
	1×10^{96}	0	0.2	3×10^{139}	3.03×10^{-53}	1.23×10^{-73}
	5×10^{95}	0	0.2	7×10^{138}	6.07×10^{-53}	2.46×10^{-73}
Kerr	7.07×10^{91}	0.7	0	1.4×10^{131}	4.29×10^{-49}	1.74×10^{-69}
	2.50×10^{86}	0.7	0	1.7×10^{120}	1.21×10^{-43}	4.92×10^{-64}
	3.94×10^{84}	0.7	0	4.3×10^{116}	7.7×10^{-42}	3.12×10^{-62}
	1.42×10^{84}	0.7	0	5.6×10^{115}	2.14×10^{-41}	8.66×10^{-62}
	1.31×10^{32}	0.7	0	4.76×10^{11}	2.31×10^{11}	9.37×10^{-10}

Table A8. BHs's thermophysical properties with the same M, a and Q from Table A7.

Black hole type	$S_{\mathbf{BH}}\ (JK^{-1})$	$t_{\mathbf{evap}}(years)$	$S_{\mathbf{tevap}}(JK)^{-1})$	$A_{S_{\mathbf{tevap}}}$
Schwarzschild	3.58×10^{55}	2.55×10^{64}	7.16×10^{55}	5.41×10^{9}
	1.32×10^{27}	5.68×10^{21}	2.64×10^{27}	1.99×10^{-19}
	5.86×10^{26}	1.68×10^{21}	1.17×10^{27}	8.85×10^{-20}
	1.46×10^{26}	2.11×10^{20}	2.93×10^{26}	2.21×10^{-20}
	3.66×10^{15}	2.63×10^{4}	7.32×10^{15}	5.53×10^{-31}
	2.67×10^{10}	5.18×10^{-4}	5.34×10^{10}	4.03×10^{-36}
	1.46×10^{10}	2.11×10^{-4}	2.93×10^{10}	2.21×10^{-36}
	3.66×10^{9}	2.63×10^{-5}	7.32×10^{9}	5.53×10^{-37}
Reissner-Nordström	4.5×10^{186}	1.1×10^{261}	9×10^{186}	6.8×10^{140}
	3.3×10^{186}	7.1×10^{260}	6.6×10^{186}	5.0×10^{140}
	1.5×10^{186}	2.1×10^{260}	2.9×10^{186}	2.2×10^{140}
	3.7×10^{185}	2.6×10^{259}	7.3×10^{185}	5.5×10^{139}
	9.2×10^{184}	3.3×10^{258}	1.8×10^{185}	1.4×10^{139}
Kerr	1.8×10^{177}	9.3×10^{246}	3.7×10^{177}	1.8×10^{177}
	2.3×10^{166}	4.1×10^{230}	4.6×10^{166}	2.3×10^{166}
	5.7×10^{162}	1.6×10^{225}	1.1×10^{163}	5.7×10^{162}
	7.4×10^{161}	7.5×10^{223}	1.5×10^{162}	7.4×10^{161}
	6.31×10^{57}	5.9×10^{67}	1.26×10^{58}	6.31×10^{57}

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