Article

A population game model for the expansion of Airbnb in the city of Venice

Sophia Arbara [1,‡] and Roberto D'Autilia [2,‡,*]

[1]   Dipartimento Architettura, Università Roma Tre; sophia.armpara@uniroma3.it
[2]   Dipartimento di Matematica e Fisica, Università Roma Tre; roberto.dautilia@uniroma3.it
*   Correspondence: roberto.dautilia@uniroma3.it
‡   These authors contributed equally to this work.

Abstract: The emergence of Airbnb along with an increase in urban tourism has intensified the pressure on urban areas while adding a new dimension in the dynamics of the housing distribution especially in historic cities. These dynamics affect both local economies and alter significantly the characteristics of urban space arising the necessity to create not only policies that foster sustainable tourism development but also to advance urban models that explore the relation between Airbnb and the traditional rental and accommodation sector. Through the case of Venice, the present study sheds light on the potential evolution of Airbnb housing in comparison to the traditional rental and homeowner market. In particular, it seeks to understand whether a potential equilibrium between these uses exists and if so, at which point in regards to this equilibrium the historic center of Venice is now. To tackle this question, methods deriving from the field of game theory and specifically evolutionary game theory are used. With the agents (players) being the housing units, the designed theoretical model explores the population dynamics of the housing units in Venice given the three options of homeownership or long-term rental (residential), short term rental over Airbnb (airbnb) or no use (vacant). The findings of our theoretical population game model are validated and discussed against a dataset describing the use patterns in the city of Venice during the past 20 years. A verification of the outcome through further case studies could eventually provide insights on future behavior of tourism pressure in historic urban areas.

Keywords: tourism carrying capacity; game theory; population games; urban models

1. Introduction

According to UNWTO tourism is inherently linked to an urban area's development [1]. Playing a key role in many cities agendas, the tourism pressure is, in fact, capable of bringing new activities to cities, reshaping their urban environment and particularly central areas of historic cities [2]. And albeit urban tourism represents a driving force in the development of many cities [1], the unregulated expansion of this industry during the past decades has given rise to several negative externalities [3]. With overtourism principally occurring in urban areas [4], massive tourism flows put at risk especially the historic areas resulting to crowding phenomena which test not only the resilience of local economies but jeopardize the city's heritage and the quality of life of their residents [5][6].

The aforementioned pressure on urban areas increases through the emergence of the accommodation offered by peer to peer home-sharing platforms favoring short-stays and challenging the dynamics of the local housing market [2]. As these accommodation platforms compete with the traditional rental market [7], the offer of long-term rentals in neighborhoods is reduced, changing the urban populations of highly affected areas and encouraging phenomena of tourism-led gentrification [8]. The context of increasing pressure of touristification in urban areas along with the international

debate on assuring the cultural sustainability in cities demonstrate a growing need for a thorough reflection on a sustainable form of tourism [6].

This arises the necessity to create not only policies that foster sustainable tourism development but also to advance models that explore the dynamics of the growing tourism sector, test the limits of sustainable tourism development and eventually predict its future evolution in urban space taking into consideration different policy scenarios. Within this framework, the research focuses on understanding these dynamics by exploring the competition amongst the emergence in space of peer to peer accommodation rentals on one hand and the traditional rental and homeowner distribution on the other. A model is proposed to describe the tourist carrying capacity of urban areas to host peer to peer accommodation rentals and the potential existence of equilibria between housing units occupied by homeowners or long-term residents versus temporary visitors. To do so, we exploit ideas and methods from the field of game theory and in particular, population games and evolutionary dynamics [9].

Game theory has extensively been used to understand economic and biological systems [10][11], yet the literature in the field of urban management and planning is scarce, with the existing references focusing on the decision-making process amongst different actors in urban design [12]. Through a symmetric game given in normal form by a 3×3 payoff matrix, we explore the potential consequences for a population to transform their housing units over peer to peer accommodation platforms, to make it available for rent on a local rental platform (or ownership occupancy) or to leave it vacant, given the choices of the other players. It is important to stress at this point that while in evolutionary game models the agents range from biological organisms to economic institutions [13], in the given case the agents are the housing units per se. And as Airbnb is the leading marketplace of peer to peer accommodation for those seeking short-term housing options, the tourist pressure is measured in terms of houses transformed into Airbnb.

The proposed model focuses on three possible choices for each agent: to convert the housing unit into an Airbnb rental (a), to either inhabit or rent it on the traditional rental market (r) or leave it vacant (v). Options such as hotel accommodation and bed and breakfast are not taken under the consideration. This is based on the hypothesis that Airbnb is an emerging sector that could follow the concept of disruptive innovation theory [7] and transform not only the tourism economy but by sharing the same resources as the traditional rental market, it could significantly alter the attributes and qualities of urban space. The main goal of the article is to use a population game model to understand whether this new sector will reach a state of equilibrium (i.e. the carrying capacity in terms of urban payoff) or whether the expansion of the Airbnb sector will take over the other strategies, leading to drastic changes in the city's socio-spatial profile.

To tackle these questions, a population game model is proposed [13] and discussed to understand if one or more Nash equilibrium states exist. If so, the spread of the current distribution pattern of housing units in the city of Venice is explored. Given a possible equilibrium, the question that follows is whether these could last over time without generating irreversible negative externalities for the studied area.

The case study chosen to test the model is the city of Venice and particularly the historic center, the area most affected by the tourism industry. Venice was selected as the suitable operational ground to conduct this study based on following two reasons; on one hand, the fact that it has been extensively impacted by the incoming number of visitors showing an increased discrepancy between temporary visitors and residents socio-economic needs. On the other hand, the fact that it is concentrated in a limited geographical area makes it a particularly interesting and compact case study which can serve as a baseline for understanding urban tourism pressure in other cities [14].

The data in the city of Venice are self-explanatory: the residents registered during the year 2018-2019 in the historic city of Venice were 52.996 [15], reporting a loss of 21.5% over the past 20 years [16]. At the same time, over the past 25 years, the number of visitor arrivals has increased 4-fold [17]. This reflected upon the housing accommodation sector, which showed impressive growth in tourism

accommodation. The bed places escalated from 8.249 in 2008 to 49.260 in 2019, showing a growth of 497% [14] with the emergence of the Airbnb rental accommodation platform playing an important role to this increase.

The paper is organized as follows; first, a brief introduction in regards to the concepts of evolutionary game theory is provided and subsequently the characteristics of the urban population model are described. In Sec.4 the dataset used to elaborate the case study model is explained. In chapter 5 the results of the proposed model are analysed and discussed against the dataset describing the use patterns in the city of Venice during the past 20 years. Finally, we consider limitations and future prospects of the game theory approach to urban planning.

2. The evolutionary game theory

As introduced above, the article explores the possibility of exploiting the population game and the evolutionary dynamics approach in urban management and planning. In particular, a model based on the framework of evolutionary game theory is developed, a field of game theory that originated in biological contexts and has become of interest amongst a broader spectrum of social and economic sciences [18] [19]. The choice derives from the fact that "evolution" is not limited to biological systems but can be a powerful paradigm to understand social systems [9] or complex systems such as the urban context. Moreover, evolutionary game theory has become an important tool to model the interaction amongst large populations of agents where one individual agent is competing not against another individual agent but the distribution of the strategies of the whole population [10].

Following the main results of population games and evolutionary dynamics [13] the theory is reinterpreted in urban terms modelling the behavior of large stakeholders of strategically interacting agents who occasionally reevaluate their choices in light of current urban quality. The main difference from economic models of evolutionary games is that here the payoff of each strategy, as well as the urban quality, is not considered in mere economic terms, but is the result of a synthetic element which concerns the morphological and environmental aspects of urban space, its intangible qualities (social and historic values) as well as the economic values of these.

The payoff is a quantity determined not only on the agent's strategy, but it depends on the strategies chosen by the other agents. In other words, the game is between one agent and a population.In our case a housing unit (the agent) can adopt a strategy amongst "resident", "vacant" and "airbnb" (r, v, a) but the payoff of this decision depends on how the different strategies are distributed on the overall population. Once the sets of strategies and the payoff of the agents are defined, we analyse the agent's possible collective behaviors.

The dominant idea in game theory is the Nash equilibrium, a state in which no agent can change strategy without decreasing their payoff, and each currently selected strategy is the best response to the strategy of the other players [20][21]. In general, the Nash equilibrium is neither unique, nor a Pareto optimum, and always exists for strategies played with a given probability (mixed strategies) [9].

While the traditional game theory has been primarily occupied with identifying the equilibrium states, in evolutionary game theory with a large population of agents the focus is on the dynamics of the equilibrium approaching [22]. Nash proposed a "mass-action" interpretation of the equilibrium in 1949 [23], introducing the concept of population games. This aspect, according to [24] remained unexplored for nearly 45 years.

In urban modelling, the need to switch from static models towards dynamic ones has often been stressed, and the population game approach can add to the exploration of dynamic urban models [12][25] and [26]. In the case of tourism-led gentrification, a dynamic evolutionary game approach where the housing units act as agents that choose amongst the three available strategies, can become a useful way to understand the urban space's reaction in pressing issues and obtain indicators for future scenarios.

3. The urban population game

In order to describe the dynamic evolution of a system of agents in terms of population game it is necessary to have a large population and at the same time assume that the action of each individual agent has a very small impact on the distribution of strategies in the overall population. The participants in the game are in our case the proportions between inhabited, vacant and Airbnb housing units which form part of the total population of Venice's units, here represented as $x_r(t)$, $x_v(t)$ and $x_a(t)$ respectively. In general, these quantities tend to change over time giving rise to a state dynamics.

The state $x(t)$ of the population of Venice's housing units is therefore given by the distribution vector of the three possible uses of these units.

$$x(t) = \left(x_r(t), x_v(t), x_a(t)\right) \quad \forall t \in [t_0, \infty) \tag{1}$$

A population game is the set of functions that describe the temporal evolution of this state. In the given case these functions describe a housing unit's transformation of use that adapts to the landscape generated by the distribution of other uses, and, unlike the "classic" formulation of game theory, they allow us to study the system far from the equilibrium. It is not known if the pressure of Airbnb in Venice has reached a state of equilibrium or not, but we do know that understanding the dynamics beyond equilibrium is decisive from an urbanistic point of view as it will allow us to design urban policy interventions which would shift the state of equilibrium.

The questions that can be answered by analyzing the population game is whether there are equilibrium states for our system based on the three urban uses (r, v, a) and what the current state of the city is in regards to these equilibria. This information is summarized by the vector field determined by the population game equations on the simplex

$$\{(x_r, x_v, x_a) \in \mathbb{R}^3 : \quad x_i \geq 0, \forall i \in \{r, v, a\}, \quad x_r + x_v + x_a = 1\} \tag{2}$$

which represents the set of all possible states of the considered population.

Each agent has a finite strategy set $\mathcal{S} = (r, v, a)$ consisting of the following three options: to rent the housing unit over the traditional rental market or leave it available for homeowner accommodation (r), to leave it vacant (v) or to rent over the shared rental platform of Airbnb (a).

The payoff of an agent is given by the matrix

$$\Pi = \begin{pmatrix} \pi_{rr} & \pi_{rv} & \pi_{ra} \\ \pi_{vr} & \pi_{vv} & \pi_{va} \\ \pi_{ar} & \pi_{av} & \pi_{aa} \end{pmatrix} \tag{3}$$

where π_{ij} is the payoff of the agent adopting the strategy i when the opponent plays j. The matrix Π represents a symmetrical game with two players and three strategies and describes the payoff of a strategy depending on the strategy chosen by its opponent. It is supposed that π_{ji} is the payoff of the opponent so that the game is symmetric.

The payoff of a strategy depends on what other agents do, and every agent does not need to have a clear vision nor a global understanding of the game. In fact, it is assumed that an agent is chosen randomly from the population and plays with an opponent always chosen randomly as well. In other words, a housing unit decides to adopt the state (r, v, a) but the payoff of its choice depends on how they are distributed (x_r, x_v, x_a) in the overall population.

The urban constraints of the matrix can be easily understood if we assume, according to Zeeman [27] that *"the growth rate of those playing each strategy is proportional to the advantage of that strategy"*, and therefore the prevalence of the strategy is the difference between the payoff from the strategy and the payoff of the population. An inhabited housing unit, being an important element in urban space, includes values from an economic, social, environmental and intangible point of view [28]. For example, it might be economically convenient and feasible in the short term to rent all the houses over

Airbnb yet not socially sustainable on a long term, as this would lead to a museumization putting at risk the viability of the city. The values of the elements of the matrix Π are therefore a balance between economic, social, environmental and intangible payoffs. It is important to mention that the current payoff matrix is designed taking into account urban areas where tourism is considered to be one of the core activities of the city. If an agent is currently playing strategy $i \in \mathcal{S}$ and the opponent is also playing the same strategy, then it can be assumed that $\pi_{rr} = \pi_{vv} = \pi_{aa} = 0$, considering that when the agents are using the same strategy their behavior does not affect the evolution of the overall population. When the agent plays the strategy r against an agent playing v, the payoff π_{rv} is positive because we assume that in a touristic urban area empty units stimulate the expansion of r, but if the opponent is a then the π_{ra} is negative as in a touristic city where Airbnb is operating, the expansion of r is hindered and eventually decreased. As stated in [29] "... *while the total supply of housing is not affected by the entry of Airbnb, Airbnb listings increase the supply of short-term rental units and decrease the supply of long-term rental units*".

If an agent adopts the strategy v and the opponent the strategy r, the payoff π_{vr} is negative, but bigger than π_{ra}. This is based on the assumption that the strategy of a population of Airbnbs over an agent currently under residential use, tends to take over residential uses in a more "aggressive" way than the residential uses tend to take over the vacant ones hindering their expansion. However, if an agent is currently adopting the strategy v and the opponent chooses a, then the payoff π_{va} is again negative, but bigger than π_{vr} and π_{ra}.

$$\pi_{ra} \leq \pi_{vr} \leq \pi_{va} < 0 \qquad (4)$$

The hypothesis made here is that although use patterns of Airbnb would hinder the expansion of vacant units, it is less likely that a vacant unit will be directly transformed into an Airbnb but more likely to shift to homeownership or long-term rental before becoming an Airbnb. As per [30] it is more probable that Airbnb will shift to long-term rentals than consider to sell a housing unit which could mean shifting to any of the three strategies. This is reflected in the payoff of our strategies with π_{ar} being the most "aggressive" one.

If the agent is playing a and the opponent plays r, the π_{ar} is negative as in a population of r an expansion of Airbnb can be hindered, but if the opponent plays v the payoff is positive; in a touristic city, Airbnb's tend to expand when there are empty housing units.

Overall, the positive numbers on the matrix represent a strategy where it is convenient for the opponent to change its strategy and adopt that of the agent. Negative numbers, on the other hand, represent strategies where an agent benefits from changing its strategy and adopting the one of the opponent. The payoff of each couple of strategies refers to the advantage/growth rate obtained by adopting each strategy given each condition. For example, the advantage in urban space of having a unit inhabited by residents over a vacant one π_{rv} is the most significant one, as social, economic and environmental value is being added by this move.

Since the agents are the housing units of Venice distributed in the three classes of strategies $\mathcal{S} = \{r, v, a\}$ and the total number of houses is large enough then $x_i \geq 0$, with $i \in \mathcal{S}$, can be considered in an equivalent way as the proportion of houses that adopt a given strategy or the probability that the strategy is adopted and therefore $x_r(t) + x_v(t) + x_a(t) = 1$ for all t. If this point of view is considered, then, making time dependence implicit for notational simplicity, the i strategy adopted by the population in the state x has the average payoff

$$\Pi_i = \sum_{j=r,v,a} \pi_{ij} x_j = (Ax)_i \qquad i \in \{r, v, a\} \qquad (5)$$

and if its average value is taking over all the strategies, the result is the average payoff of the entire population

$$\overline{\Pi} = \sum_{i=r,v,a} (Ax)_i = xAx \qquad (6)$$

If it is assumed that a strategy spreads proportionally to its convenience $\Pi_i - \overline{\Pi}$, the replication equation can be written [31]

$$\frac{dx_i}{dt}(t) = x_i(t)\left(\Pi_i - \overline{\Pi}\right) = x_i(t)\left((Ax)_i(t) - x(t)Ax(t)\right) \qquad i \in \mathcal{S} = \{r, v, a\} \tag{7}$$

which describes the evolution over time of the state of the population.

Therefore, it can be assumed that the dynamics of this distribution are determined by the social and economic dynamics defined by the behavior of the inhabitants. This dynamic is complex and is conditioned by individual choices, local needs, irrationality or chance, but the result is only the change in the use of housing, which in our model is a kind of revision protocol [13]. If we assume this point of view we can adopt the theoretical framework of population games in which the interaction between agents is repeated over and over again. Thus, unlike game theory, which is focused on determining the possible Nash equilibria, the dynamics of the three populations can be studied (r, v, a) in the configurational landscape determined by these equilibria.

Finally, the research introduces a coefficient of randomness $\mu \in [0,1]$ which represents the probability that an agent will choose his strategy randomly.
The equation (7) then becomes [13][32]

$$\frac{dx_i}{dt}(t) = (1 - \mu)x_i(t)\left(\Pi_i - \overline{\Pi}\right)$$
$$= x_i(t)\left((Ax)_i(t) - x(t)Ax(t)\right) - \mu\left(\frac{1}{3} - x_i(t)\right) \qquad i \in \mathcal{S} = \{r, v, a\} \tag{8}$$

so that we have dynamics simultaneously determined by the maximization of a payoff and by random elements. The solutions of this system of differential equations describe the possible behaviors of the city and, as demonstrated, the probability μ of choosing a random strategy is a control element for the possible states of equilibrium of the system and the Airbnb carrying capacity of the city.

4. The Venice Data

The dataset considered in our analysis focuses on the timeframe of the past 20 years. This makes it possible to observe the impact and evolution pattern of the Airbnb shared accommodation in the city of Venice as the platform started operating only after 2008 [33]. In order to study the model and the population's selections in regards to the three strategies (r, v, a), the number r_u of housing units inhabited by residents (homeowners or tenants), the vacant housing units v_u and the housing units rented over the Airbnb platform a_u are measured. The number N_u represents the non-vacant housing units, which include units occupied by both residents r_u and Airbnb tenants a_u.
T_u refers to the total number of available housing units in the historic center of Venice

$$T_u = r_u + v_u + a_u \tag{9}$$

The non-vacant housing units N_u is the sum between residentially occupied ones and the Airbnb ones $N_u = a_u + r_u$ so that we have

$$T_u - v_u = N_u = a_u + r_u \tag{10}$$

Data from two sources were used to estimate the population of agents choosing among the three different strategies for the time period 2001-2020. The dataset regarding the total available housing units T_u and the vacant units v_u were obtained from the independent research studio "Scenari Immobiliari" for the years 2001, 2010 and 2019 (https://www.scenari-immobiliari.it). For the units rented over the platform of Airbnb a_u the data for each area was obtained from the independent online platform "Inside Airbnb" (http://insideairbnb.com) available for the city of Venice from 2015 and onwards.

The initial datasets provided partial information for the period 2001-2020 to measure the behavior of agents choosing one of the three main strategies (r, v, a). In order to estimate the missing data for each month of these years, the datasets of the vacant housing units v_u, the Airbnb a_u and the total available housing units T_u were interpolated by means of polynomial functions.

Based on the known and estimated data, the non-vacant units, $N_u = T_u - v_u$ and the residential ones $r_u = N_u - a_u$ were computed. Once completed the dataset, the normalized data x_r, x_v and x_a for the whole historic center for each month i.e. the proportions of the city's population playing the strategies $i \in \mathcal{S}$ were computed.

The normalization process made it possible to, on one hand, bring all the values on a common scale and on the other, obtain the distribution of the population's subgroups according to the three strategies.
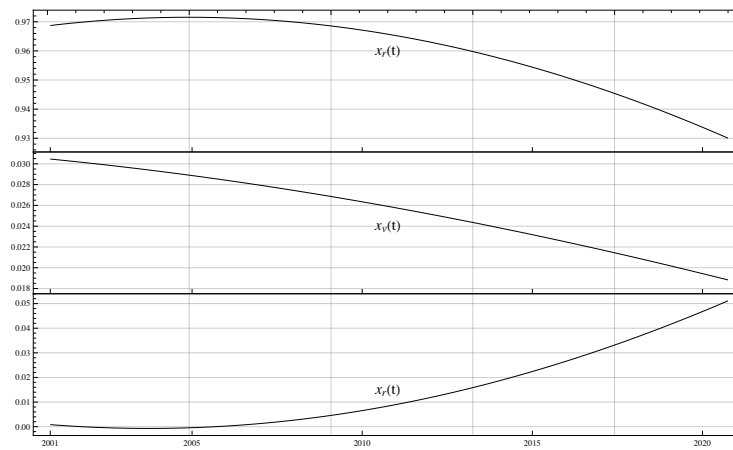

Figure 1. The time evolution of $(x_r(t), x_v(t), x_a(t))$ for the historic center of Venice with $t \in [2001, 2020]$.

When elaborating the data following assumption was made; the housing units offering a room and not the entire unit over Airbnb were calculated under the Airbnb category a_u and not under the category of residential units r_u. This assumption was made on the basis of "testing" the scenario where a maximum number of Airbnb rentals is considered. If a housing unit appeared more than once in Airbnb, meaning that the owner would rent the rooms into different visitors, these duplicates were removed. That way every housing unit in Airbnb appears only once in our dataset independent of the various rooms that might be available to different groups of visitors.

5. Results

The equation (8) determines the time evolution of the urban state vector $x(t)$. The dynamic we have used is the replication dynamics, which is a sort of "survival of the fittest" [31] [34]. Replication dynamics do not allow for new strategies to emerge. However, in the given case the a strategy appears for the first time in 2008. Therefore, it can be assumed that the replication dynamic is valid from 2008 and onwards analysing how the a strategy invaded the population previously formed only by r and v.

A possible form of the matrix (3) is given by

$$\Pi = \begin{pmatrix} 0 & 6 & -4 \\ -3 & 0 & -1 \\ -1 & 3 & 0 \end{pmatrix} \tag{11}$$

which is a modified version of Zeeman's game, introduced to prove the existence of equilibria which are not Evolutionary Stable Strategies [27] and belong to a class of models studied in biology in order to model collective behaviors ranging from prebiotic evolution to animal behavior. The numeric values

are only indicative because the matrix represents a whole class of games [9]. An agent playing one of the three strategies (r, v, a) is being matched with a population consisting of a population of agents playing the strategies with distribution (x_r, x_v, x_a).

The dynamics of Airbnb in Venice seem to be well described by this class of models, as can be observed below. Based on the interaction amongst the agent and the population as well as the possibility of random choice, the agents are able to reevaluate their strategy. To test the evolutionary game model, the notebook EvoDyn-3s, designed to analyse the dynamics and equilibria of 3×3 matrix games is used [32].

5.1. The case $\mu = 0$.

When $\mu = 0$ the system has four equilibrium states, three pure states composed of only residents, vacant or Airbnb (the three vertices of the triangle in Fig.2) and a non-stable one, consisting of 80% of resident homes and 20% of Airbnb. The equilibrium represented by $x_v = 1$ is not stable and the state of the "empty" city tends to move towards a city of residents or Airbnb.
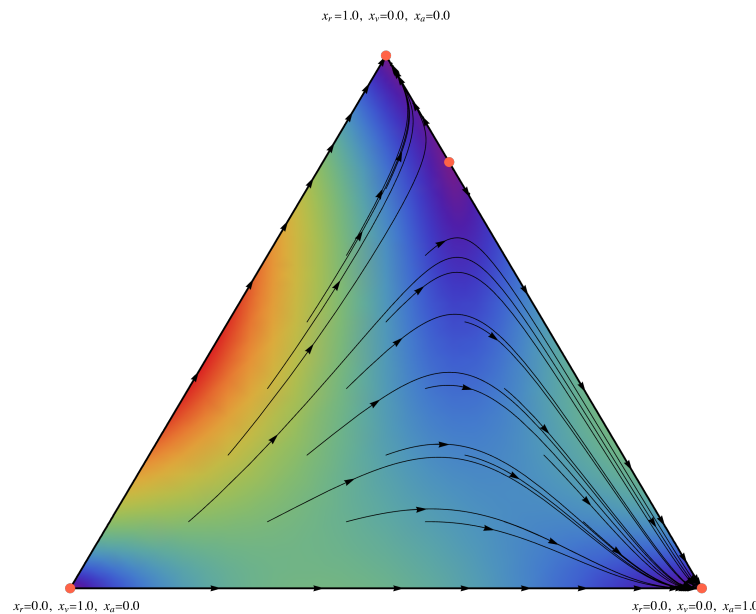

$x_r = 1.0, \ x_v = 0.0, \ x_a = 0.0$

$x_r = 0.0, \ x_v = 1.0, \ x_a = 0.0$ $x_r = 0.0, \ x_v = 0.0, \ x_a = 1.0$

Figure 2. The geometrical representation of dynamics and the equilibrium states with $\mu = 0.0$

The two states $x_r = 1$ and $x_a = 1$ are the only two equilibria based on which the urban dynamics can converge, because the point $(0.8, 0.0, 0.2)$ of Fig.2 is non-stable. This means that a replication dynamics with the payoff matrix given by (11) can only lead to a city of residents or Airbnb. This is the general landscape from which one can move by increasing the value of μ, that is, introducing the factor of randomness in the choice of the housing unit's use.

5.2. The case $\mu \neq 0$.

Starting from $\mu = 0$, the value of μ is being progressively increased until obtaining the minimum distance between the theoretical curves and the data. The meaning of μ is that behind the dynamics of the strategies (r, v, a) there is a complex social and urban dynamic that can give rise to non-rational, or at least non-optimal behaviors.

We found that for $\mu = 0.154$, the solution starting on the segment $r - v$ with initial condition $(x_r(t_0) = 0.971, x_v(t_0) = 0.029, x_a(t_0) = 0.0)$ minimizes the total variation distance [35] between the theoretical curve and the data, with mean 0.005 and variance $4.3 \cdot 10^{-6}$. These dynamics, represented

in Fig.3 correspond to the 15.4% of the population choosing a random strategy, or rather a strategy determined by aspects extraneous to the dynamics of the population game.
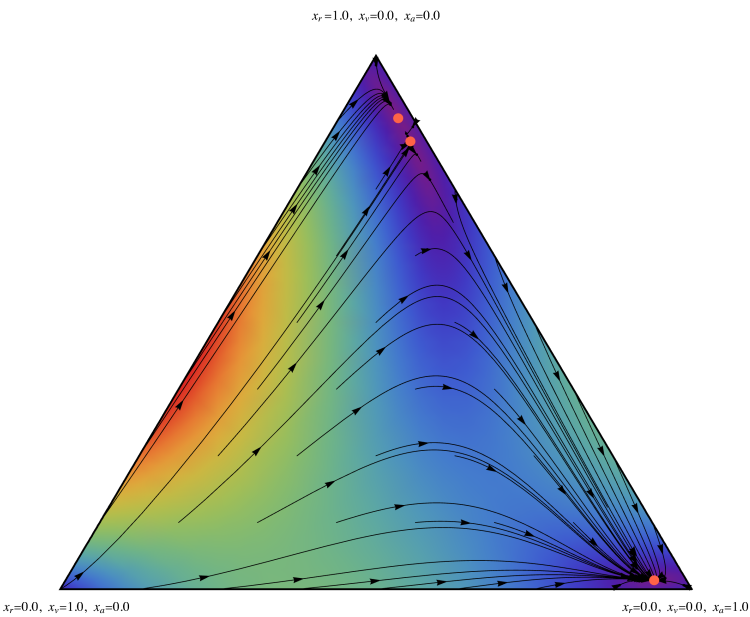

Figure 3. The dynamics on the simplex for $\mu = 0.154$

This dynamic shows the three equilibrium points reported in the table (1) together with the corresponding eigenvalues.

Table 1. The three equilibrium points for $\mu = 0.154$ together with the corresponding eigenvalues

x_r	x_v	x_a	λ_1	λ_2
0.0167	0.0512	0.932	-2.95	-1.03
0.839	0.0259	0.135	-1.99	0.142
0.883	0.0236	0.0938	-2.19	-0.149

The point $(x_r = 0.839, x_v = 0.0259, x_a = 0.135)$ has the second eigenvalue with positive real part and it is not stable. The closest theoretical equilibrium to the state of the population in 2020, which is $x_{2020} = (x_r = 0.935, x_v = 0.018, x_a = 0.045)$, is the point $(x_r = 0.883, x_v = 0.0236, x_a = 0.0938)$. Therefore it seems that, in this model, the state of the housing units in Venice is converging towards an equilibrium where less of 1% of the houses will be transformed into Airbnb and the 0.23% will be uninhabited, as can be seen in Fig.4, which is an enlargement of Fig.3. A further equilibrium is at the state $(x_r = 0.0167, x_v = 0.0512, x_a = 0.932)$ which corresponds to the city transformed almost entirely into Airbnb.
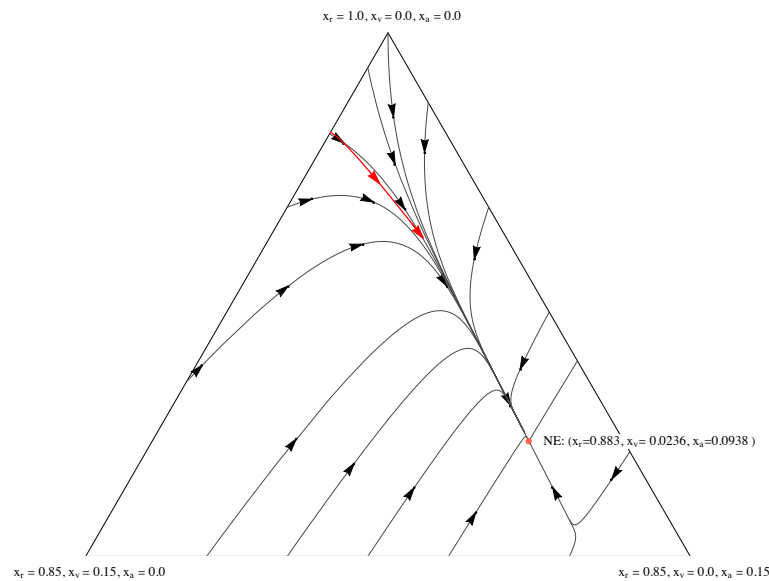
Figure 4. The $\mu = 0.154$ dynamics for the simplex subset $x_r \geq 0.85$. The real trajectory of the $(x_r(t), x_v(t), x_a(t))$ dynamics for the city of Venice from 2008 to 2020 is represented in red.

The stationary points establish the topology of the space in which the population x evolves, but the trajectories on this space are also the objects of urban interest.

The equilibrium points depend on μ. If $\mu \in [0.0, 0.16)$ the system has two stable equilibria, separated by a non-stable which gradually approaches the upper stable one as μ increases. When $\mu = 0.16$ the system collapses and the only equilibrium given by $(x_r = 0.0176, x_v = 0.0532, x_a = 0.929)$ is a city made up of almost only Airbnbs where residents nearly disappear. By increasing the value of μ, this unique equilibrium moves towards the center of the symplex, and with $\mu = 1$, the strategies become entirely random and the system has a unique equilibrium state $(0.333, 0.333, 0.333)$. The upper equilibrium state $\overline{x}(\mu)$ as function of μ is plotted in Fig.5 where the collapse of the state for $\mu = 0.16$ is evident.
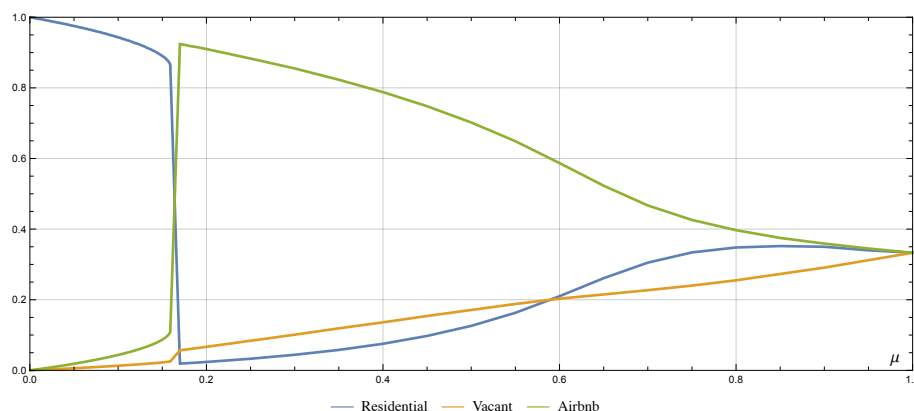

Figure 5. The population equilibrium $(\overline{x}_r, \overline{x}_v, \overline{x}_a)$ as function of the μ parameter

These equilibria can be interpreted as the tourist carrying capacity of the city, i.e. the number of Airbnb that Venice can sustain given μ and the payoff (11). In this framework, if the payoffs of the matrix Π represent a balance between costs and benefits in terms of quality of life and economics, then if $\mu < 0.16$ the city can "contain" the expansion of Airbnb.

The μ parameter can be controlled by an urban planning policy that reduces the randomness rate in strategic choices. This means that although the randomness is governed by personal situations that

can lead to non-optimal choices, an urban management policy that "stabilizes" the quality of life of the inhabitants of Venice can reduce the value of μ and consequently limit the tourist pressure. This means that in order to control the growth of Airbnb, it is not necessary to impede such activities in urban areas but to provide incentives that would support local inhabitants. Adding to that, finding ways to minimize the factor of uncertainty for the agents choices could help mitigating phenomena such as the uncontrolled expansion of Airbnb.

6. Conclusions

The main purpose of this work is to explore the possibility to use some of the ideas of population dynamics and game theory to model the urban dynamics in regards to the Airbnb pressure. Using the houses as agents, it is possible to determine a dynamic that reproduces the qualitative behavior of the real data and offers an overall picture of the possible states of equilibrium.

The model remains rather abstract and could not be considered realistic in the sense that the μ parameter should be calculated through a much more detailed dataset than the one we had the opportunity to examine. However, the theoretical implications of this simple model are important as they represent possible consequences of a competitive mechanism in urban dynamics that takes into account not only the economic value of the housing units but also their urban use.

In order to further explore its capacity, the proposed class of models could be compared with the dynamics of other cities of tourist interest to verify if it captures the essential characteristics of the tourist pressure on urban space. Furthermore, the model can be extended to include other forms of tourism pressure, such as the purchase of houses by the non-resident population, although this would require a very detailed dataset.

An analysis regarding the kind of stability of the equilibrium points, which involves the use of techniques derived from the theory of differential equations goes beyond the scope of this work; so does the possibility that a concept of Evolutionarily Stable Strategy can be introduced in urban planning.

These could become subjects for future exploration along with the possibility of a correlated game, in which an external intervention such as possible incentives for residents, could modify the equilibrium points of the system by reducing the value of μ. Furthermore, a functional form for the elements of Π which make explicit the possibly non-linear functional relationship between the economic aspects and the urban payoff are subjects to be explored. Finally, the possibility of having more detailed and exhaustive datasets regarding the use of the housing units could allow us to compare the present model with the urban dynamics of other touristic cities.

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