Source of the hot Universe

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Abstract

It is shown that the gravitational mass-energy defect could be the main cause of Big Bang energy and triggering inflation in the newborn Universe.

Keywords: Cosmology, Theory of gravitation.

1 Introduction

The most likely reason for the formation of a visible expanding universe is its spontaneous appearance as a point-like object as a result of the instability and fluctuations of the vacuum [1, 2, 3]. The possibility of this effect has been repeatedly demonstrated, first in [3a] and later, for example, using the Hawking-deWitt equation [4, 5, 6, 7]. However, its need remains unproven due to the ambiguous solution of the equation and arbitrary parameters. Moreover, the origin and magnitude of the enormous energy in the past cannot be considered reliably established.

In the present work, agreeing in principle with the idea [1, 2, 3], we demonstrate the possibility of a quantitative explanation of the origin and magnitude of the released energy and the exponential expansion of the visible Universe without introducing any arbitrary parameters. Essentially, the only parameter of model under consideration is the average density of the visible Universe.

Despite our insufficient understanding of the causes and circumstances of the appearance of the observable universe, we are faced with the fact that it appeared as a point-like object of the mass in order of magnitude of $M = 10^{56} \div 10^{57} g$. Therefore, a necessary condition for the appearance of such masses "from nothing" is the release of energy equal to $-Mc^2$. Only in this case, the fundamental law of conservation of energy will be fulfilled.

Apparently, Tryon himself understood this. However, a quantitative calculation of such an effect within the framework of the general theory of relativity (GR) is impossible due to the lack of a correct definition of the concept of energy. However, this is possible within the framework of a modified theory of gravitation, which is based on the correction of a theory that takes into account some mathematical fact unknown at the time of the creation of Einstein’s theory.

This work is organized as follows.
Section 2 discusses the problems that arise when trying to realize the idea of the Big Bang energy as a manifestation of the gravitational mass-energy defect and outlines ways to solve them.

In Section 3, based on [9, 10], a brief justification is given of the possibility of considering gravity in as the curvature of the Riemannian space and as a physical field in Minkowski space. This is necessary for the reader to understand the use of Minkowski space in the theory of gravitation.

In section 4, based on [9], bimetric projectively (geodesically) invariant equations of gravitation are briefly considered.

Section 5 gives the derivation of the expression for the gravitational field energy in the gravitational model under consideration, which is necessary to obtain a result.

In section 6, the main result of the work is obtained on the basis of everything considered above.

2 Problems and solutions

The fundamental law of conservation of energy prohibits the appearance of an observable matter in the Universe "from nothing"[1, 2, 3]. However, the appearance of the mass is accompanied by the appearance of gravitational binding energy $E_b$, so that an internal energy of the mass is $E = E_0 - E_b$ where $E_0$ is an inertial mass energy. Therefore, the appearance of a new mass is possible if $E = 0$ in this moment, i.e., $E_b = E_0$, and the energy $E_b$ will released from newborn matter in the form of radiation and kinetic energy.

In this case, the spontaneous appearance of a mass $M_0 = E/c^2$ causes the inevitable release of energy $E_b$, which is a necessary condition for the appearance of the mass $M_0$ as a real physical object.\(^1\)

For the visible mass of the Universe, $E_b$ is tremendous energy. Therefore, the question arises: Is this gravitational mass-energy defect a source of Big Bang energy and a cause of triggering inflation?

An answer to this question is not an easy task, as serious problems arise:

1. The idea that the total energy of the Universe can be zero is not new. In order to avoid conflict with the classical law of energy conservation, authors, for example, Hawking and Zeldovich, have expressed the hope that a correct gravitational binding energy will show that the internal energy of the Universe is zero.

The only, more or less rigorous basis for this hope is the result of Landau and Lifshitz\(^8\) [8], who, on the basis of the pseudo-tensor of the energy-momentum proves this fact within the framework of GR for a particular model and for a particular coordinate system. However, no one has proved that this result does not depend on the choice of the coordinate system, and therefore puts the conclusions into question.

However, how to find this gravitational energy, if this concept in General Relativity cannot be considered correct?

\(^1\)This scenario does not depend on the causes of the appearance of mass, and, therefore, is also true for any mass formed as the result of a gravitational condensation of matter in the Universe.
2. What is the quantitative meaning of the equality \( E = E_0 - E_b \) if we do not know true dimensions of the Universe, which, moreover, is probably spatially infinite?

3. Is it possible to find confirmation of the above assumption about the energy source of the Big Bang in the observations?

A positive answer to these questions follows from bimetric projectively invariant gravitational equations\([9, 10, 15, 16, 14, 13]\).

The first problem can be solved with the modified theory of gravitation, according to which gravity can be considered both as the curvature of space-time and as a physical field in Minkowski space\([9, 10]\).

This statement is not trivial. This should not be confused with the Rosen bimetrics~\([11]\). This idea follows from the fact that only the complex “space-time geometry + properties of measuring instruments” has a physical experimentally verified meaning which is based on Poincaré ideas~\([10]\).

It follows from the above equations of gravity in flat space-time that it is possible to give a correct expression for the energy-momentum tensor of the gravitational field, which is impossible in the classical Einstein’s theory.

The results obtained show that the appearance of the Universe “from nothing” in the form of a point mass does not contradict the law of conservation of energy.

The second problem can be solved due to the fact that, according to the gravitational equations under consideration, the gravitational field of a point mass does not have a singularity and an event horizon. The force of gravity tends to zero when approaching the source.

Such properties of gravity that do not contradict observations lead to an unexpected conclusion that the effect of the Universe gravity on distant objects is limited mainly by value \( R_g = c \left( \frac{3}{8\pi G \rho} \right)^{1/2} \) where \( \rho \) is the average density of matter, \( G \) is the gravitational constant and \( c \) is the speed of light. This value is the Schwarzschild radius of the matter inside the spherical volume of radius \( R \). Only the mass of matter contained in the interior of the sphere of radius \( R_g \) (\( \sim 10^{28} \div 10^{29} \text{ cm} \)), has a physical meaning for studying the history of the expanding Universe as the gravitational influence of the more distant matter quickly disappears. In fact, this is a gravitational event horizon.

The third problem can be solved using the fact that, knowing the gravitational energy density, we can find the radiation energy density as a function of the radius of expanding matter contained within \( R_g \). Comparison of this quantity with the observed cosmic microwave radiation density shows that the gravitational mass-energy defect the emerging point-like universe can indeed be a source of Big Bang energy.

Moreover, we do not need to introduce hypothetical fields to explain the appearance of an exponential expansion of the Universe. The appearance of inflationary expansion of the universe some time after the appearance of the universe as a point masses are clearly seen from particle motion graphs.
3 Projectively invariant bimetric equation of gravitation

3.1 General Properties

The need to modify the Einstein equations follows from the fact that mathematicians
discovered many years after the appearance of the classical Einstein equations.

All Christoffel symbols \( \Gamma_{\alpha \beta \gamma}(x) \) obtained by the transformations
\[
\Gamma_{\alpha \beta \gamma}(x) = \Gamma_{\alpha \beta \gamma}(x) + \delta_{\alpha}^{\alpha} \phi_{\gamma}(x) + \delta_{\gamma}^{\alpha} \phi_{\beta}(x),
\]
where \( \phi_{\beta}(x) \) are an arbitrary differentiable vector-function, describe the same gravitational field because the geodesic equations remain invariant under this transformation[13].

This is most easily seen if the geodesic equation is written as
\[
\ddot{x}^{\alpha} + (\Gamma_{\beta \gamma}^{\alpha} - c^{-1} \Gamma_{\beta \gamma}^{0} \dot{x}^{\alpha}) \dot{x}^{\beta} \dot{x}^{\gamma} = 0
\]
were points denote differentiation with respect to \( t = x^{0}/c, c \) is speed of light.

However, Einstein’s equations are not invariant with respect to such transformations[14] because, for example, the Ricci tensor is transformed under (1) as follows
\[
\bar{R}_{\alpha \beta} = R_{\alpha \beta} - \phi_{\alpha \beta},
\]
where \( \phi_{\alpha \beta} = \phi_{\alpha ; \beta} - \phi_{\alpha} \phi_{\beta} \), and \( \phi_{\alpha ; \beta} \) is a covariant derivative of \( \phi_{\alpha} \) with respect to \( x^{\beta} \).

Transformation (1) induces some mapping \( g_{\alpha \beta} \rightarrow \bar{g}_{\alpha \beta} \) of the metric tensor \( g_{\alpha \beta} \) [13]. Consequently, such transformations should be some gauge transformations in any theory based on the hypothesis of motion of free test particles alone geodesic lines. Only objects that are projectively (geodesically) invariant can have physical meaning in such theory.

In paper [9] a theory in which geodesic mappings play a role of gauge transformations is considered.

The only projectively invariant object which can be created by Chrissoffel symbols is the Thomas symbols
\[
\Pi_{\alpha \beta}^{\gamma} = \Gamma_{\alpha \beta}^{\gamma} - (n + 1)^{-1} \left[ \delta_{\alpha}^{\gamma} \Gamma_{\beta \gamma}^{\alpha} + \delta_{\beta}^{\gamma} \Gamma_{\alpha \gamma}^{\alpha} \right]
\]
where \( \Gamma_{\alpha}^{\alpha} = \Gamma_{\alpha \beta}^{\beta} \).

However, \( \Pi_{\alpha \beta}^{\gamma} \) is not a tensor. A tensor object can be formed only within the framework of a bimetric theory. It is of the form [9]:
\[
B_{\beta \gamma}^{\alpha} = \Pi_{\beta \gamma}^{\alpha} - \hat{\Pi}_{\beta \gamma}^{\alpha},
\]
where
\[
\hat{\Pi}_{\alpha \beta}^{\gamma} = \Gamma_{\alpha \beta}^{\gamma} - (n + 1)^{-1} \left[ \delta_{\alpha}^{\gamma} \hat{\Gamma}_{\beta \gamma}^{\alpha} + \delta_{\beta}^{\gamma} \hat{\Gamma}_{\alpha \gamma}^{\alpha} \right]
\]
and \( \hat{\Gamma}_{\alpha}^{\alpha} = \Gamma_{\alpha \beta}^{\beta} \), are the Thomas symbols in Minkowski space in a coordinate system used.
The simplest bimetric geodesic-invariant generalization of Einstein’s vacuum equations is \[ \nabla_\alpha B^\alpha_{\beta\gamma} = B^\epsilon_{\beta\delta} B^\delta_{\epsilon\gamma}. \] \( (7) \)

The symbol \( \nabla_\alpha \) denotes a covariant derivative in Minkowski space with respect to \( x^\alpha \).

These equations have similarities to vacuum equations of classical electrodynamics in Minkowski space:

\[ F^\gamma_\alpha = 0, \quad F_{\alpha\beta} = \partial_\alpha A_\beta(x) - \partial_\beta A_\alpha(x), \]

which are invariant relative to mappings of 4-potentials: \( A_\alpha \rightarrow A_\alpha + \phi(x)_\alpha \). However, eqs. (7) contains the right-hand side which expresses self-interaction of gravitation.

When we select the covariant gauge conditions in the form \( Q_\alpha = \Gamma^\beta_{\alpha\beta} - \hat{\Gamma}^\beta_{\alpha\beta} = 0 \), equations (7) coincide with the classical vacuum Einstein equations \( R_{\alpha\beta} = 0 \). In other words, the classic vacuum Einstein’s equations are equations (7) at the gauge conditions \( Q_\alpha = 0 \).

These equations lead to the same results that Einstein’s equations at the distances \( r \) from a gravity center which is much larger of the Schwarzschild radius \( r_g \) of the central mass [15]. However, gravity properties become quite different when the distance approaches \( r_g \). Instead of the classic Newtonian an expression for force \( F \) acting on a rest test mass \( m \) at rest we obtain:

\[ F = -\frac{mMG}{r^2} \left(1 - \frac{r_g}{r}\right) \] \( (8) \)

where \( r_g = 2GM/c^2 \) is the Schwarzschild radius of a dot mass \( M \).

A singularity in the center is missing in this model. The gravitational force decreases within the Schwarzschild radius, and eventually tends to zero along with the distance from the center.

The weakness of the gravitational field inside the Schwarzschild radius leads to the possibility of the existence of cold stable supermassive objects without an event horizon, which are an alternative to supermassive black holes in the centers of galaxies[14, 13, 15]. This modification of the classic Newtonian law is also consistent with recent observations. Observation of gravitational waves during the merging of supermassive objects do not allow us to identify the nature of the objects.

The existence of supermassive objects without event horizon also does not contradict the result obtained with Event Horizon Telescope (EHT) collaboration...
observations [19] since the radius of the observed dark object is several times larger than their Schwarzschild radius. This remarkable result demonstrates the existence of supermassive objects in the centers of galaxies, but it still does not make it possible to uniquely establish their nature. Greater accuracy is needed to establish the object mass[26] and which theory of gravity better describes the motion of light and particles in the vicinity of the Schwarzschild radius. The motion of light in the vicinity of the Schwarzschild radius in the alternative theory was considered in [15].

It should be noted that an important result of the theory is also a quantitative explanation of the acceleration of the expansion of the Universe, which is impossible in the framework of General relativity. This will be shown briefly in this paper later.

3.2 Gravitational event horizon

In this paper, we start from a standard cosmological principle. However, the physical meaning of the existence of the Minkowski space along with the Riemannian space in equations (7) requires serious justification.

This problem is considered in [10] in detail, and in Section 6 briefly. In present Section, we simply postulate that Riemannian and Minkowskian geometry are, at least locally, two physically equivalent possibilities.

According to this, we can consider the motion of a test particle in Minkowski space by a Lagrangian

$$L = mc \left( A(r) \dot{r}^2 + B(r) (\dot{\theta}^2 + \sin^2 \theta \dot{\varphi}^2) - c^2 C(r) \right)^{1/2}$$

which is connection with $L$ by relation

$$ds = -(mc)^{-1} dS(x, dx)$$

where $S$ is the action $S=\int L(x, \dot{x})dt$.

The most important consequence of the equations of gravitation under consideration for cosmology is that for any observer, the influence of the Universe gravity to remote objects is mainly limited by some distance $R_g$.

The Lagrangian, which is invariant with respect to mapping $t \rightarrow -t$ and describes the motion of test particles in the the spherically symmetric field of a mass $M$ in Minkowski space has the form:

$$L = mc[A(r)\dot{r}^2 + B(r)(\dot{\theta}^2 + \sin^2 \theta \dot{\varphi}^2) - c^2 C(r)]^{1/2}$$

(9)

The solution of the equations (5) at the conditions

$$\lim_{r \rightarrow \infty} A(r) = 1, \quad \lim_{r \rightarrow \infty} (B(r)/r^2) = 1, \quad \lim_{r \rightarrow \infty} C(r) = 1$$

(10)

is given by:

$$C = 1 - r_g/f, \quad f = (r^3 + r_g^3)^{1/3}, \quad B = f^2, \quad A = r^4/f^4 C, \quad r_g = 2GM/c^2$$

(11)

In [9, 10] was shown that the equations of motion of a test particle resulting from this solution do not change if $A, B, C$ are functions not only of $r$, but also of time $t$. Since
the equations of motion determine the properties of the field, this means that spherically
symmetric solution is static. This proves the analog of the Birkhoff theorem. However,
this is not surprising since Einstein’s vacuum equations are equations (7) at the gauge
defined by the condition \( Q_\alpha = 0 \). In other words, in the gauge used, these equations are
equivalent to the system

\[
R_{\alpha\beta} = 0
\]

\[Q_\alpha = 0.\]

The solution (11) is valid for any spherically symmetric substance, therefore it is
correct inside the spherical hollow shell as well. Gravitational field is absent here since
\( M(r) = 0 \), and the space-time is pseudo-Euclidean. Consequently, the matter of the
spherical shell does not affect the gravity inside the shell. This important conclusion will
be used hereafter.

A projectively invariant generalization of Einstein’s equations in the presence of mat-
ter can also be obtained [9]. We do not use them, since their validity by comparison with
observations is difficult to verify.

It is easy to see that the main peculiarity of this solution is that functions \( A, B, C \) tend to their values in the Minkowski space \( (A = 1, B = r, C = 1) \) at \( r/r_g \to 0 \), i.e.,
when the matter radius is much less then the Schwarzschild radius.

Consider when this condition can be satisfied.

For this, we must take into account
that the radius of a homogeneous sphere,
which is equal to the Schwarzschild radius
of the matter contained in it, is

\[
R_g = \sqrt[3]{\frac{3}{8\pi G\rho}} \tag{12}
\]

where \( \rho \) is the density.

Radius \( R \) of an mass becomes less than
\( R_g \) in two cases:

1. This happens at densities of the order of \( 10^4 \div 10^5 \, g/cm^3 \). For example, for
density of \( 10^5 g/cm^3 \), \( R_g \) is equal in order
to the radius of the Sun. In such cases, due to the weakness of gravity inside \( R_g \), the
substance may have a huge mass [14, 15, 16, 13]. Such configurations are candidates for
supermassive objects in the centers of galaxies.

2. At a very low average density of the visible Universe, the distance to the object
becomes less than the Schwarzschild radius of the matter contained in it already at
\( R > 10^{29} cm \).

![Figure 2: \( r_g(R) \) vs. the distance \( R \)](image-url)
More information can be obtained if we consider the force $F = m\ddot{r}$ (or acceleration $\ddot{r}$) acting on a test particle of mass $m$ as a function of the distance $r$ from the observer. For a free radial motion of a test particle Lagrangian (9) is: $L = -mc\left(c^2C - A\dot{r}^2\right)^{1/2}$. It has the energy integral
\[ \frac{cC}{(c^2C - A\dot{r}^2)^{1/2}} = \bar{E} \] (13)
where $\dot{r} = dr/dt$, $\bar{E} = E/mc^2$ and $E$ is the energy of the particle. This equation gives the velocity $v = \dot{r}$ as a function of $r$:
\[ v^2 = c^2\frac{C^4f^4}{r^4}\left(1 - \frac{C}{\bar{E}^2}\right) \] (14)

For parameter $\bar{E}$ three variants are possible: $\bar{E} > 1$, $\bar{E} < 1$ and $\bar{E} = 1$. However, only $\bar{E} = 1$ is consistent with observations.

Indeed, the distance to nearby galaxies satisfies inequality $r \gg r_g$ where $r_g = 2GM/c^2$ and $M = \frac{4}{3}\pi\rho r^3$. At this condition, $f \approx r$, and $C = 1 - r_g/r$. Consequently, we obtain from (14) that
\[ v = Hr, \]
where
\[ H = \sqrt{\frac{8}{3}\pi G\rho} = 1.8 \cdot 10^{-18} \text{s}^{-1} \]
at $\rho = 6 \cdot 10^{-30} \text{g/cm}^3$. This is consistent with the observations.

If $\bar{E} \neq 1$, then equation (14) does not lead to the Hubble law since $v$ does not tend to zero when $r \to 0$. For this reason, we set $\bar{E} = 1$.

With eq. (14) we can obtain a graph of the acceleration $a = v\ddot{v}/dr$ as a function of the distance to a test particle.

It is very useful to compare the acceleration of a free test particle close a point mass $M$ and this value in the Universe close distance of $10^{28} \text{cm}$, which are given in fig 3 and 4.

A comparison of graphs 3 and 4 shows that they have similar features, since in both cases, accelerations are considered in the area where the equality $r/r_g = 1$ holds.

Two conclusions can be made from figure 4.

1. The Schwarzschild radius $R_g$ becomes more than $R$ at $R > 1.5 \cdot 10^{28} \text{cm}$. Before this, at $R = 6 \cdot 10^{27} \text{cm}$, the acceleration change sign, in analogy with the particle free falling to a point mass (fig 2). If $R > 6 \cdot 10^{27} \text{cm}$, the acceleration is positive. Hence, for sufficiently large radius $R$ the gravitational force gives rise to an acceleration of remote galaxies.

This is a natural explanation of the observed acceleration of the expanding Universe, which is confirmed by the satisfactory coincidence of the theoretical and the real Hubble law.

\[ \text{In this paper, we assume that the average density of the visible Universe is equal to } 6 \cdot 10^{-28} \text{ since this value leads to satisfactory agreement between our theoretical Hubble diagram and the observed one[9].} \]
diagram [9]. Besides, in the next section, additional justification for this statement provides by calculating the deceleration parameter.

2. At $R > 10^{28} \text{cm}$, gravitational force $F = m\ddot{R}$ affecting the particles, quickly tends to zero when $R$ tends to the infinity. The reason for the fact is that the ratio $R/R_g$ tends to zero when $R$ tends to infinity. Consequently, the gravitational influence on galaxies at large distance $R$ caused mainly by the matter insider of the sphere of the radius $R_g$ which can be called “a gravitational event horizon”.

Thus, the solution of the gravitational equations under consideration are valid in the infinite Universe due to the peculiarity of the gravitational force and the fact that the functions $A$, $B$, and $C$ tend to their values in the Minkowski space at distances shorter than the Schwarzschild radius.

For this reason, only the mass of the Universe within a radius of $R_g$ has a physical meaning for studying the history of the Universe.

### 3.2.1 Deceleration parameter calculation of the deceleration parameter is an important method for checking the relativistic expression used above for the speed and acceleration of a test particle in the expanding Universe.

The acceleration as a function of the distance is $g = v'(r)v(r)$ where the velocity $v$ is given by (14). It allows us to find the value of the deceleration parameter:

$$q = -\frac{g \cdot r}{v^2}.$$  

The fig. 5 shows $q$ as a function of the distance of an object from the observer.

This graph shows that the observed value of present-day $q_0 \approx -0.55$ [18, 20] corresponds to the geometrical distance of the remote objects of about $10^{28} \text{cm}$. It is consistent
4 Gravity in inertial and proper reference frame.

In Einstein’s classical theory, coordinates play a twofold and difficultly compatible role. On the one hand, coordinates are only a way to parameterize events, that is, points of space-time. From this point of view, they are completely arbitrary. On the other hand, they play the role of gauge transformations.

The projectively invariant generalization of Einstein’s equations considered in Section 3 seems to be possible only within the framework of some bimetric theory of gravity. Rosen [11] was the first to recognize the need for introducing Minkowski space into theory. The possibility of considering Einstein’s equations in flat space was also considered by some authors after the work of Tirring[12].

The physical meaning of bimetrism used in the present paper is based on ideas going back to Poincaré , who realized that there is a strange situation: In order to characterize the properties of the geometry of space, we must know the properties of measuring
in order to characterize the properties of instruments, we must know the geometric space properties. In the modern interpretation, this can be summarized as follows: The physical, operational sense has only the aggregate "space-time geometry + properties of measuring instruments" [10]. In this form, this fact has never been realized in physics.

However, a step in its implementation can be made if we notice that it is the reference frame used by the observer that is the measuring tool that is necessary to establish the geometric properties of space-time. Therefore, it should be assumed that the following statement is true: Only the combination "space-time geometry + properties of the reference frame used" has physical meaning.

Of course, by reference frame we mean here not a coordinate system, but a physical device consisting of a reference body and a clock attached to it.

Remembering all the above, we now consider a classical field \( \mathcal{F} \) in an inertial reference system (\( IRF \)), where space-time according to experience is Minkowski space. The world lines of the particles of mass \( m \) moving under the action of the field \( \mathcal{F} \) form the reference body of a non-inertial reference which can be named the proper reference frame (\( PRF \)) of the field \( \mathcal{F} \).

If an observer in a \( PRF \) of the field \( \mathcal{F} \) is at rest, his world line coincides with the world line of some point of the reference body. It is obvious for the observer that the accelerations of the point masses forming his reference body are equal to zero in non-relativistic and relativistic meaning. That is, if the line element of space-time in an inertial reference frame is denoted by \( d\sigma \) and \( u^\alpha = dx^\alpha/d\sigma \) is the field of 4-velocities of the point masses forming the reference body, then the absolute derivative of \( u^\alpha \) is equal to zero: \(^3\)

\[
Du^\alpha/d\sigma = 0.
\] (15)

(We mean that an arbitrary coordinate system is used.)

The same should occur in the \( PRF \) used. That is, if the line element of space-time in the \( PRF \) is denoted by \( ds \), the 4-velocity vector \( \zeta^\alpha = dx^\alpha/ds \) of the point-masses forming the reference body of the \( PRF \) should satisfy the equation

\[
D\zeta^\alpha/ds = 0
\] (16)

The equation (16) uniquely determines the fundamental metric form in \( PRF \)s.

Indeed, the differential equations of these world lines are at the same time the Lagrange equations describing, in Minkowski space, the motion of the point masses forming the reference bodies of the \( PRF \). The eq. (16) can be derived from a Lagrange action \( S \) by the principle of the least action. Therefore, the equations of the geodesic lines can be obtained from a line element \( ds = k \, dS \), where \( k \) is a constant, \( dS = \mathcal{L}(x, \dot{x})dt \), and \( \mathcal{L}(x, \dot{x}) \) is a Lagrange function describing, in Minkowski space, the motion of identical point masses \( m \) forming the body reference of the \( PRF \). The constant \( k \) is equal to \(- (mc)^{-1} \), as follows from the analysis of the case when the frame of reference is inertial, when \( \mathcal{L}(x, \dot{x}) = -mc \, ds \) at the signature \((+ - - -)\).

\(^3\)We use notations and definitions, following the Landau and Lifshitz book [8]
Thus, if we proceed from relativity of space and time in the Berkeley-Leibniz-Mach-Poincaré (BLMP) sense, then the line element of space-time in PRFs can be expected to have the following form \[9\]

\[\text{ds} = -(mc)^{-1} \, dS(x, dx). \tag{17}\]

For example, if the field \(F\) is electromagnetic, then the space-time in such reference frames is Finslerian\[9, 10\]. And the space-time in the reference frames comoving to an isentropic ideal fluid is conformal to the Minkowski space.

In the case of gravity, we proceed from Thirring’s assumption that gravity is described by a tensor field \(\psi_{\alpha\beta}(x)\) and the Lagrangian describing the motion of test particles has the form

\[L = -mc\left[g_{\alpha\beta}(\psi) \, \dot{x}^\alpha \, \dot{x}^\beta\right]^{1/2}.\]

Then it is obvious from (1) that space-time in the PRF is Riemannian with a linear element of the form

\[\text{ds} = \left(g(x)\, dx^\alpha \, dx^\beta\right)^{1/2}\]

This conclusion leads to the possibility of a double interpretation of gravity. Gravity can be considered as physical field \(\psi\) in inertial reference frames and manifests itself as curvature in its own reference frames (PRFs) of this field.\[4\]

**Example 1. Electromagnetic field**

The reference body consists of noninteracting electric charges in an electromagnetic field. In a Cartesian coordinate system the action describing the motion of the particles can be written as follows [8]:

\[S = \int \left(-mc^2(1 - v^2/c^2)^{1/2} - \frac{e}{c} A_\alpha(x) \, dx^\alpha / dt\right) \, dt, \tag{18}\]

where \(A_\alpha\) is the 4-potential, \(e\) is charge of the particles.\[5\]

For the given reference frame

\[\text{ds} = d\sigma + \frac{e}{mc} A_\alpha \, dx^\alpha, \tag{19}\]

where \(d\sigma\) is the line element of space-time in the IRF. It is a Finslerian metric.

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4) The question: "Where is this inertial reference frame?" does not make sense, since, as noted above the properties of the reference system do not have a physical meaning in themselves. In the concrete, we can imagine that we are in an inertial reference frame and consider the gravity of the Universe as a physical field in Minkowski space, but we can also assume that we are in the reference frame comoving to the radial flow of galaxies and consider space-time as Riemannian.

2) Are both descriptions of gravity fully equivalent or just locally? This question remains open, but this plays no role for problem under consideration.

5) As usual, in this paper Greek letters run from 0 to 3, and Latin - from 1 to 3.
Of course, such a frame of reference is not similar to the accelerated reference frame formed by neutral particles. However, this does not prevent to its theoretical analysis, assuming that the reference body is formed by identical ions. They can be regarded as an atomic clocks that are almost unaffected by accelerations.

Based on the fact that the clocks measure the length of its own world line, one can find the time interval in such PRFs.

Evidently,

\[ dT = 0 \frac{dT}{d\tau} + e \frac{mc^2}{\alpha} d\dot{x}^{\alpha} dt, \]

where \( dT = ds/c \) and \( 0 \frac{dT}{d\tau} = d\sigma/c \) are proper time intervals in the PRF and IRF, correspondingly.

The following two cases are of interest:

**A**. The reference body consists of noninteracting electric charges in a constant homogeneous electric field \( E \) directed along the axis \( x \). According to (19)

\[ \frac{dT}{dt} = 1 - \frac{e}{mc^2} \varphi = 1 + \frac{e}{mc^2} Ex, \tag{20} \]

where \( \varphi = A_0 \), and \( E \) is the electric field strength. Because the electric force \( eE = mw \), where \( w \) is the acceleration with respect to the IFR, this result is equivalent to the well known one:

\[ \frac{dT}{dt} = 1 + \frac{wx}{c^2}. \tag{21} \]

This result shows that difference between clock in IRF and PRF is not a kinematic effect, and is caused by a force field.

The same result is obtained for the PRF of the homogeneous field of the Earth, reference body of which is formed by particles free falling in the field. The reason is that the replacement \( e\varphi \) by the gravitational potential leads to the same equation of the motion of test particles as the equations for charges.

**B**. The reference body consists of non-interacting electric charges in a constant homogeneous magnetic field \( H \) directed along the axis \( z \). The Lagrangian describing the motion of the particles can be written as follows [8]:

\[ L = -mc^2 \left(1 - v^2/c^2\right)^{1/2} - \left( m\Omega_0/2 \right)(\dot{x}y - xy) \], \tag{22} \]

where \( \dot{x} = dx/dt, \dot{y} = dx/dt, \) and \( \Omega_0 = eH/2mc. \)

The points of such a system rotate in the plane \( xy \) around the axis \( z \) with the angular frequency

\[ \omega = \Omega_0 \left[1 + (\Omega_0 r/c)^2\right]^{-1/2}, \tag{23} \]

where \( r = (x^2 + y^2)^{1/2} \). The linear velocity of the BR points tends to \( c \) when \( r \to \infty \).
4 GRAVITY IN INERTIAL AND PROPER REFERENCE FRAME.

For the given CRF

\[ ds = d\sigma + \left(\frac{\Omega_0}{2c}\right) (ydx - xdy). \tag{24} \]

In the above CRFs \( ds \) is of the form

\[ ds = \mathcal{F}(x, dx), \tag{25} \]

where \( \mathcal{F}(x, dx) = d\sigma + f_\alpha(x)dx^\alpha \), \( f_\alpha \) is a vector field. Therefore, \( \mathcal{F}(x, dx) \) is a homogeneous function of the first degree in \( dx^\alpha \), that is, \( \mathcal{F}(x, ky) = k\mathcal{F}(x, y) \). Thus the space-time in the above NRFs are Finslerian [?]

By using the identities

\[ \mathcal{F}(x,y) = \frac{\partial \mathcal{F}(x,y)}{\partial y^\alpha} \xi^\alpha; \quad \frac{\partial^2 \mathcal{F}(x,y)}{\partial y^\alpha \partial y^\beta} = 0, \tag{26} \]

the function \( \mathcal{F}^2(x,y) \) can be written as

\[ 6\mathcal{F}(x,y)^2 = G_{\alpha\beta}(x,y) y^\alpha y^\beta, \tag{27} \]

where

\[ G_{\alpha\beta}(x,y) = \frac{1}{2} \frac{\partial^2 \mathcal{F}(x,y)^2}{\partial y^\alpha \partial y^\beta} \tag{28} \]

is an analog of the fundamental metric tensor of the Riemannian space-time. Consequently, the modulus of the vector \( y^\alpha \) in point \( x^\alpha \) is \( |y| = \mathcal{F}(x,y) = [G_{\alpha\beta}(x,y)y^\alpha y^\beta]^{1/2} \).

For \( ds \) of the form (25) we have

\[ G_{\alpha\beta}(x,y) = \eta_{\alpha\beta} + f_\alpha f_\beta + (\eta_{\alpha\beta} - \tau_\alpha \tau_\beta)f_\sigma \tau^\sigma + f_\alpha \tau_\beta + f_\beta \tau_\alpha, \tag{29} \]

where \( \tau^\alpha = dx^\alpha/d\sigma \).

A covariant vector in the Finslerian space-time, therefore, can be defined as

\[ y^*_\alpha = \mathcal{F}(x,y) \frac{\partial \mathcal{F}(x,y)}{\partial y^\alpha} = G_{\alpha\beta}(x,y) y^\beta. \tag{30} \]

The orthogonality of vectors \( y^\alpha \) and \( y_1^\alpha \) can be defined by the equality \( y^*_\alpha y_1^\alpha = 0 \). It must be noted that the orthogonality of two vectors is not symmetric.

Let us find the connection between an element of time in the CRF and the IRF.

It follows from the Stokes theorem that in this case the modulus \( A \) of the potential \( A_1 \) of a particle at the distance \( r \) from the center of the orbit of the reference body is equal \( A = Hr/2 \), which shows that \( A \) is a modulus of the 3-vector \( \vec{A} = \frac{1}{2} \vec{B} \times \vec{r} \), and the \( \vec{A} \) is directed tangentially to the orbit circle. For this reason, according to (19),

\[ dT/dt = 1 - \frac{e}{mc^2} \frac{Hr^2\omega}{2} = 1 - \frac{F_1r}{2mc^2}, \tag{31} \]

where \( \omega = d\varphi/dt \) is the angular velocity of the body reference, and \( F_1 = eHr\omega/c \) is Lorentz force. Since the centrifugal force \( F_c = F_1 = mw_c \), where \( w_c \) is the centrifugal
acceleration, we arrive at the conclusion that this result is equivalent to the well known one:

$$\frac{dT}{dt} = 1 - \frac{\omega^2 r^2}{2c^2}. \quad (32)$$

This result coincides with the result for the rotating disk due to the fact that the motion of points on the disk can be described by a similar Lagrangian.

Example 2. Isentropic fluid

The reference body is formed by macroscopically small elements of an isentropic flow of the perfect fluid.

It is well known that besides of traditional continual description, a perfect fluid can be considered as a collection of a finite number of identical macroscopic small particles which are under influence of interparticles forces which mimic the effect of pressure, viscosity etc [16], [17]. In particular, the fluid velocity in a given point is simply velocity of the particle being in this point. At such description the motion of the fluid particles is governed by solutions of ordinary differential equations of Newtonian or relativistic dynamics.

The motion of the fluid particles in an IRF is described by Lagrangian [15].

$$L = -mc \left( G_{\alpha\beta} \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} \right)^{1/2} d\lambda, \quad (33)$$

where $\lambda$ is a parameter along 4-path of particles, $w$ is enthalpy per unit volume, $G_{\alpha\beta} = \chi^2 \eta_{\alpha\beta}$,

$$\chi = \frac{w}{\rho c^2} = 1 + \frac{\varepsilon}{\rho c^2} + \frac{P}{\rho c^2},$$

$mc^2 + \varepsilon$ is the rest internal energy, $\rho = mn$, $m$ is the mass of the particles, $n$ is the particles number density, $P$ is the pressure, $\varepsilon$ is the internal energy without and $\eta_{\alpha\beta}$ is the metric tensor in Minkowski space. According to (36) the line element of space-time in the comoving reference frame is given by

$$ds^2 = G_{\alpha\beta} dx^\alpha dx^\beta. \quad (34)$$

The covariant derivative of tensor $G_{\alpha\beta}$ is equal to zero. Therefore, space-time in such CRF is Riemannian with the curvature is other than zero. The last description is equivalent to the one by the Lagrangian (33) [15].

Example 3. Gravitational field

Now we are ready to consider the case of gravitation.

An attempt to combine the theory of gravity in the Minkovski space with Einstein’s theory was made in 1961 by Thirring in [12]. He suggested that gravity in Minkowsky...
space is described by a tensor function $\psi_{\alpha\beta}(x)$, and the motion of test particles is described by Lagrangian

$$L = -mc\sqrt{g_{\alpha\beta}(\psi) \dot{x}^{\alpha} \dot{x}^{\beta}}^{1/2}.$$  \hspace{1cm} (35)

This means that the metric tensor in Einstein’s theory is a function of the field $\psi_{\alpha\beta}(x)$. Taking this as reasonable assumption, we will suppose that the motion of test particles in a gravitational field in inertial reference frames is described by the above Lagrangian.

Then it is obvious from (35) that space-time in the PRF is Riemannian with a linear element of the form

$$ds = \left(g_{\alpha\beta}(x)dx^\alpha dx^\beta\right)^{1/2}.$$  \hspace{1cm} (36)

This conclusion leads to the possibility of a double interpretation of gravity. Gravity can be considered as physical field $\psi$ in inertial reference frames and manifests itself as curvature in its own reference frames (PRFs) of this field.\(^7\)

Thus, this result allows us to interpret the classical Einstein's gravitation equations as a description of gravity in the reference frames formed by particles moving in a certain classical gravity field $\psi_{\alpha\beta}(x)$ which exists in Minkowski space\([9, 10]\). Of course, the problem of the existence of such correct equations of gravity is a separate task.

Hence it follows that the correct equations of gravitation should be bimetric, as Rosen had long guessed \([11]\). However, now flat space-time has a physical meaning, and the relation of two types of gravity description is achieved in quite definite way, namely, by a transition from one reference system to another. This is not a coordinate transformation. For this, e.g., the formalism 3+1 developed in General Relativity can be used.

## 5 The energy of the gravitational field

### 5.1 Motivation

In the Einstein theory and its modifications, which are based on the identification of gravity and curvature of space-time, the strict meaning of gravitational energy is not clear. However, a correct definition of the gravity energy is necessary to find gravitational binding energy of the Universe. Such a definition is possible in the Minkowski space.

Einstein’s vacuum equations coincide with eq. (5) at the gauge condition $Q_\alpha = 0$. Therefore, following Weinberg \([18]\), we can suppose that the nonlinear term of the Einstein equations is associated with gravitational energy. To be more precise, we assume that equations (5) can be written as:

\(^7\)1) The question: “Where is this inertial reference frame?” does not make sense, since, as noted above the properties of the reference system do not have a physical meaning in themselves. In the concrete, we can imagine that we are in an inertial reference frame and consider the gravity of the Universe as a physical field in Minkowski space, but we can also assume that we are in the reference frame comoving to the radial flow of galaxies and consider space-time as Riemannian.

2) Are both descriptions of gravity fully equivalent or just locally? This question remains open, but this plays no role for problem under consideration.
where the energy-momentum tensor of the field is $t_{\alpha \gamma} = k^{-1} B^c_{\alpha \delta} B^d_{\beta \epsilon}$ and $k$ is some constant.

It is not hard to show that this assumption makes sense.

We use the previously mentioned expression for the gravitational force in a spherically symmetric gravitational field, which follows from the gravity equations used here

$$F = -\frac{mMG}{r^2} \left( 1 - \frac{r_g}{f} \right)$$

for a spherically symmetric field.

We define the potential of the force $F$ as $U(r) = \int_0^r \frac{F(r')}{m} dr'$.

Double differentiation of function $U(r)$ gives

$$U'' = -\frac{2U'}{r} - \frac{2G^2 M^2}{c^2 f^4}.$$  

Now denote

$$t_{00} = \frac{GM^2}{2\pi f^4}.$$  

As a result, we obtain that function $U(r)$ satisfies the differential equation

$$\nabla^2 U = -4\pi \bar{\rho}$$

where $\bar{\rho} = t_{00}/c^2$.

This equation has a clear physical meaning. This is the Poisson equation for spherically symmetric field, in which $\bar{\rho}$ is the mass density of the gravitational field created by the mass $M$. In the absence of another matter, only the density of the gravitational field can lead to the fact that the potential will differ from zero.

This result shows clearly that $t_{00}$ is the density of the gravitational field in the considered theory where the gravitational force is given by eq. (37).

5.2 Energy-momentum tensor in Minkowski space

More information about the energy of the gravitational field can be obtained by the following method.

From the field equations (7) can be obtained equations for a function $\psi_{\alpha \beta}(x)$ which describes gravitation as a physical field in the Minkowski space [9], existence of which was supposed by Tiring [12].

For this, we set $B^\alpha_{\beta \gamma} = \nabla^\alpha \psi_{\beta \gamma}$, where the operator $\nabla^\alpha$ is a derivative in Minkowski space.

The object $\nabla^\alpha \psi_{\beta \gamma}$ is transformed the same as the tensor $B^\alpha_{\beta \gamma}$ at projective mappings (1). Consequently, the mapping $\nabla^\beta \psi_{\alpha \beta} \rightarrow \nabla^\beta \psi_{\alpha \beta} + (n + 1) \varphi_\beta$ preserve the projectively invariant object $B^\alpha_{\beta \gamma}$, and consequently, are some gauge transformations of function $\psi_{\alpha \beta}$. 
Evidently, conditions $\nabla^\sigma \psi_{\sigma \gamma} = 0$ can be select as an additional condition to fixate the gauge. At this condition, eqs. (5) take the form

$$\nabla^2 \psi_{\alpha \beta} = \chi (t_{\alpha \beta} + T_{\alpha \beta}),$$

$$\nabla^\sigma \psi_{\sigma \gamma} = 0,$$

$$\nabla^\beta (t_{\alpha \beta} + T_{\alpha \beta}) = 0$$

where

$$t_{\alpha \beta} = \chi^{-1} B^\nu_{\alpha \mu} B^\mu_{\beta \nu} = \chi^{-1} \nabla^\sigma \psi_{\alpha \gamma} \nabla^\gamma \psi_{\sigma \beta}.$$  

$\chi$ is a constant, operator $\nabla$ defined in the Minkowski space, and a possible material tensor $T_{\alpha \beta}$ has been added for completeness.

Obviously, equality

$$\nabla^\beta (t_{\alpha \beta} + T_{\alpha \beta}) = 0$$

holds [9].

Equation (40) can be considered as an equation to determine the function $\psi_{\alpha \beta}(x)$, which determines the properties of gravity in Minkowski space in the spirit of Tirring [12]. (See section 6).

In the Newtonian limit, equation (39) for component $t_{00}$ should coincide with Poisson’s equation for a self-interacting field. Consequently, $\chi = 4\pi G/c^4$.

Thus, the tensor $t_{\alpha \beta}$ has a property of tensor-energy of the field and is consistent with non-relativistic Poisson’s equations (38).

### 5.3 Test

The resulting expression for the energy-momentum tensor withstands the most difficult test: the calculation of the field energy of a point mass.

According to eq. (40) that zero-component $t_{00}$ of the energy-momentum tensor of a point mass $M$ is given by

$$t_{00} = \chi^{-1} \frac{r_g^2}{2 f^4}$$

which gives, when integrating over entire space,

$$\int t_{00} dV = Mc^2$$

and $\int t_0 dV = 0$ for $i = 1, 2, 3$.

Thus, the energy of a point mass is finite and equal to the expected value.

In addition, it should be noted also that the integration of density $t_{00}$ over the volume of the visible Universe gives the correct value of its mass $\sim 10^{56}g$.

For this reason, we can consider (41) as the density energy of the gravitational field of the mass $M$.

It follows from (41) that the energy $\int_0^R t_{00} dV$ in the volume inside radius $R$ around the central mass $M$ is
6 GRAVITATIONAL MASS-ENERGY DEFECT

\[ E_{\text{sph}} = Mc^2 \left[1 - \frac{r_g}{f}\right] \]  (42)

where \( f = \left(\frac{r^3}{r_g^3} + r^3\right)^{1/3} \) and \( f = \left(\frac{r^3}{r_g^3} + r^3\right)^{1/3} \). The total energy is \( E_{\text{sph}} + E_{\text{out}} \) where

\[ E_{\text{out}} = \int_{R}^{\infty} t_{00} dV = MC^2 \frac{r_g}{f} \], so that \( E_{\text{out}} + E_{\text{sph}} = MC^2 \).

Since the solution of the field equations under consideration is refer to any spherically symmetric mass, it implies that the above formulas give the gravitational energy of any spherically symmetric mass \( M \) with radius \( R \).

Expression (42) is the difference of the two terms. The second term is the gravitational binding energy \( E_b = MC^2 r_g/f \).

6 Gravitational mass-energy defect

The internal energy of the gravitational field of the visible region of the Universe is \( E = E_0 - E_b \) where \( E_0 = M_0 c^2 \) is the energy of a newborn mass \( M_0 \) and \( E_b \) is the gravitational binding energy at radius \( R \).

According to (42),

\[ E = E_0 \left(1 - \frac{R_g}{(R_g^3 + R^3)^{1/3}}\right) \]  (43)

where \( R_g = 2GM_0/c^2 \), \( M_0 = E_0/c^2 \). The quantity \( E_b = E_0R_g/f \), where \( f = (R_g^3 + R^3)^{1/3} \), is the gravitational binding energy of the mass inside radius of \( R \).

It can be seen from (43) that the internal energy of the region inside \( R_g \) can be zero only in the case when the radius of matter tends to zero. Only in this case, a point mass of the newborn Universe satisfies the law of energy conservation.

Fig. 6 shows the graph the internal energy \( E/c^2 \) as the function of the radius \( R \) of the matter for the two magnitudes of \( E_0 \) which are close to the energy \( M_0 c^2 \) of the observed Universe. This graph can be considered as the dependence of the internal energy of the expanding Universe on its radius in the past in the most interesting interval of \( R \).

![Figure 6: Mass-energy vs. radius R of the Universe.](attachment:figure6.png)
The graph shows that the increase in mass ends at a certain radius of matter, which indicates its size since the density here must rapidly decrease. This implies that the gravitational binding energy determines the size of a self-gravitating matter.

A spontaneous appearance of mass \( M_0 = E_0/c^2 \) causes the immediate appearance of the gravitational binding energy (mass-energy defect), whose release is the necessary condition of the appearance \( M_0 \) as a real object.

It is very important that the model under consideration can be compared with observations since we know the density \( \rho \gamma \) of cosmic microwave radiation and the corresponding redshift. The density is \( \rho \gamma = 2 \cdot 10^{-13} \text{erg/cm}^3 \), and redshift is \( z \approx 10^3 \).

Figure 7 shows the dependence of the radiation density \( E_b/V \) in the volume \( V \) due to the gravitational mass-energy defect on the radius \( R \) of the expanding Universe.

![Figure 7: The energy density of the radiation as the function of the Universe radius.](image)

We see that the radiation density due to the gravitational mass-energy defect is \( 2 \cdot 10^{-13} \text{erg/cm}^3 \) at the radius \( \sim 5 \cdot 10^{29} \text{cm} \).

This distance is consistent with the graph in Fig.4 where the acceleration of the particles is shown.

In addition, we can also find the relationship between the geometric distance from the observer and the redshift \( z \). It is obtained with a formula obtained in [9]:

\[
z = \left( C(r)^{-1} \left( 1 + \frac{v(r)}{c} \right)^{1/2} \right) - 1
\]

where \( C(r) = 1 - r_g/(r^3 + r^3)^{1/3} \) and \( v(r) \) is the recession velocity in Minkowski space at zero-density and pressure.

It can be seen from figure 8 that the distance \( r \approx 5 \cdot 10^{29} \text{cm} \) corresponds to the \( z = 10^3 \) which is CMBR density. This is consistence with observations.

### 7 Direct evidence for inflation

Although the model of inflationary cosmology is almost generally accepted, the nature of the inflaton remains unclear. Therefore, it is not surprising that attempts have been made to explain inflation without the need to use hypothetical fields, and to obtain inflation as a consequence of the properties of space-time in modified versions of the
Einstein equations \[23, 21, 22, 25, 24\]. However, despite this, the problem cannot be considered solved.

However, the triggering of inflation could also be the result of a gravitational mass-energy defect in the newborn Universe.

Let us find the dependence of the radius of the spherically symmetric mass \(E/c^2\) on time during its expansion, due to the gravitational mass-energy defect.

Consider a test particle located on the edge of the expanding spherical mass \(M\). The desired dependence is determined by the change in the force acting on this particle by the substance of the mass \(M\) in the process of the sphere expansion.

External matter does not affect this due to the supposed homogeneity and isotropy of the Universe, as well as due to the existence of the gravitational event horizon.

To avoid doubts about the correctness of the result obtained by numerical methods, we consider the solution of this problem in two different ways.

1. The Lagrangian for the motion of a particle during its radial expansion of mass \(M\) in Minkowski space has the form (9)

\[
L = -mc(Cc^2 - Ar^2)
\]

where \(C\) and \(A\) (11) are the functions of the distance \(r\) from the mass center. The corresponding particles motion equation is

\[
\ddot{r} = -c^2 \frac{C'}{A} + \left( \frac{C'}{C} - \frac{A'}{2A} \right) \dot{r}^2.
\]

Sometimes, the solution of this equation it is easy all obtained by the expression for the velocity (14) which follows from an integral of motion considered in section 2:

\[
\dot{r}^2 = c^2 \frac{C^4 f^4}{r^4} \left( \frac{r_g}{f} \right).
\]
The speed and acceleration do not have a singularity in the mass center, they tend
to zero together with distance $r$. The dependence of $r$ on $t$ can be found both for the
expansion of mass from zero radius, and for history back in time, starting from the radius
of the visible Universe was approximately $1 \cdot 10^{-100}$ cm. Figure 9 shows this solution.

2. Of course, the above method does not allow us to take into account the influence
of pressure, since we are dealing with a dust-like model of the Universe.

The expansion of the Universe as a function of time, taking into account pressure
inside the mass $M$, can also be obtained by another method.

It was shown in [15] that the motion of macroscopically small elements of an isentropic
fluid ("particles") of a mass $m$ is described by the following Lagrangian

$$L_{FL} = -mc\chi \left( \eta_{\alpha\beta} d\dot{x}^\alpha d\dot{x}^\beta \right)^{1/2}$$

(46)

where $\eta_{\alpha\beta}$ is metric tensor of Minkowski space in coordinate system used,

$$\chi = \frac{w}{\rho c^2},$$

$w$ is the enthalpy per unit volume, $\rho = mn$, $n$ is the particle number density.

This means that macroscopically small elements of the isentropic fluid move along
the geodesic lines of the Riemannian space with a line element $ds = (mc)^{-1} dS$ where

$$S = \int L_{FL}(x, \dot{x}) dt,$$

\footnote{This approach is inspired by the existence of an effective numerical solution of problems of hydrodynamics [16, 17], known as Smoothed Particle Hydrodynamics (SPH). In this method, a fluid is considered as composed by finite number of particle. These particles move under the action of inter-particle forces which mimic effects of pressure, viscosity, and so on. Due to the replacement of integration by summation over number of particles, continual derivatives become the time derivative along the particle trajectory, and as a result, the motion of particles governed by ordinary differential equations of classical mechanics.}
$ds = \left(G_{\alpha\beta}(x)dx^\alpha dx^\beta\right)^{1/2}$

where $G_{\alpha\beta} = \chi^2\eta_{\alpha\beta}$.

This space-time is conformal to the Minkowski one.

This result allows us to study the properties of a fluid by studying the motion of its elements instead of studying the streamlines obtained by solving the complex relativistic Euler equations.

For a radial motion in the expanding Universe in Minkowski space we have:

$L_{lf} = -mc\chi(r)\left(c^2 - \dot{r}^2\right)^{1/2}$

The enthalpy of a fluid is $W = E + pV$, and $w = E/\rho c^2 + p$ where $p$ is the pressure.

Using equality $M_0 = \rho V = Const$ we obtain from (43) that

$$\chi = 1 - \frac{Rg}{f} + \frac{P}{\rho c^2}$$

and the above Lagrangian yields the following equation of the radial motion of “particles” at the Universe expansion:

$$\ddot{r} = -c^2\frac{\chi'(r)}{\chi(r)} \cdot \left(1 - \frac{\dot{r}^2}{c^2}\right). \quad (47)$$

In order to solve this equation, we can sometimes use an integral of the motion

$$\frac{mc^2\chi}{[1 - (\dot{r}/c)]^{1/2}} = Const. \quad (48)$$

that yields the following equation of the motion of macroscopic small element (“particle”) of the isentropic fluid

$$\left(\frac{\dot{r}}{c}\right)^2 = 1 - \frac{\chi^2}{E^2} \quad (49)$$

where $E = E/mc^2$ and $E$ is the energy of the “particle”.

Since Hubble’s law in the absence of pressure must be satisfied, the constant $E$ must be equal to 1 in vacuum.

Below graphs of the solutions $r = r(t)$ of equation (47) are given at $p = 0$.

It should not be thought that there was a singularity before the exponential expansion, since all the equations of motion of the test particles give zero velocity when radius of the Universe tends to zero.

It should be noted that the above-obtained graphs of evolution of the radius of the expanding universe can be obtained in another way. The solution (11) of equations (7), in Minkowski space, which is similar to the solution of Schwarzschild in GR, allows you to get the distribution of velocities $v(r)$ and accelerations $w = dv/dr$ at an arbitrary radius $r$. The solution of the differential equation $dr/dt = v(r)$ with the initial condition
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Figure 10: The Universe radius $r(t)$ starting for the initial conditions: $r(0) = 10^{-100}\text{cm.}$ and $\dot{r}(0) = 0$.

Figure 11: The Universe velocity $\dot{r}(t)$ for the initial condition: $r(0) = 10^{-100}\text{cm.}$ and $\dot{r}(0) = 0$.

$r(0) = r0$ close to $r0 = 0$ gives the evolution of the geometric radius of the visible universe. It is similar to that obtained on the graph above.

7.1 The time and energy before inflation

The purpose of this paper is to show that the gravitational mass-energy defect is a likely cause of the Big Bang energy.

There are at least two consequences of the considered model, which can help further study the structure of the substance before inflation and study its results.

1) Time $t$ on the graphs is not of course physical time. Physical time is measured by the length of its own world line of particle motion. When the particles of the newborn Universe move radially, the proper time is $\tau = ds/c$ where $ds$ is the line element

$$ds = \left( C - A \left( \frac{v}{c} \right)^2 \right)$$

where $A$, $C$ and the velocity $v$ are given above in this Section. The dependency $\tau$ on $t$ is given by fig. 12.

It is very interesting that these results are consistent with well-known results obtained on the basis of General Relativity, except for the absence of singularity at the time of the birth of the Universe.

2) In the model of the dynamics of the universe considered here, the energy of the newborn Universe is quantity determined by the Schwarzschild radius $R_g$, which in turn is given by the average density $\rho$ of the visible Universe.

The energy released is the gravitational binding energy, which, unlike GR, can be found for any radius $r$ of the
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Figure 12: The proper time \( \tau \) vs. radius \( r \)

Figure 13 shows the dependence of the density of released energy on the radius of the Universe on the radius \( r \) before inflation. This is important information for assumptions about the structure of matter, from the Planck era to the time of inflation.

Figure 13: Energy released vs. radius \( r \)
8 Conclusion

Thus, we have sufficient reason to think that the main cause of the Big Bang, the hot Universe, and the triggering inflation was the gravitational mass-energy defect of the newborn point mass, which is contained within radius $R_g$.

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References


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