 doi:10.20944/preprints202101.0044.v1

Article

Effect of savings on a gas-like model economy with credit and debt

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Abstract: In this work, we apply ensemble formalism to a geometric agents model to study the effect of saving propensity in a system with money, credit, and debt. We calculate the partition function to obtain the total money of the system, with which we give an interpretation of the economic temperature in terms of the different payment methods available to the agents. We observe an interplay between the fraction of money that agents can save and the debt that can be financed. We also observe that the system's entropy increases as the saved proportion increases and increases, even more, when debt is present.

Keywords: Econophysics, Savings propensity, Geometric models.

1. Introduction

Econophysics brings together a set of theoretical and empirical achievements that came from using well-known tools and physics results, particularly from thermodynamics and statistical mechanics, applied to economics and financial markets. Although its scope and applicability are still discussed [1], this area has influenced financial economics studies over the past twenty years and can be considered a well-established current research branch [2]. One of the directions where it has advanced is in the study of the distribution of income and wealth from the perspective of kinetic exchange models [3]. Over the last few years, it has been shown that gas-like models can reproduce some patterns observed in capitalist economic systems, such as the Pareto rule of wealth distribution [4]. These models' main advantage, and the reason they have become attractive in various disciplines, is that their mathematical formulation and numerical implementation is very simple and straightforward [5]. Exchange agent models have been introduced to model a simple economy in which agents exchange money, and it is possible to average their behavior in analogy with the molecular models of gases formed by colliding particles. As particles of a gas exchange energy during the collisions, agents exchange a fraction of their capital in these models. This analogy is possible given the conservation of total capital [6]. The capital of the agents comes mainly from their income, however, such agents can increase their capital through a credit, which in the long run will generate a debt. Credit and debt are introduced through a new variable different from the money coming from income, in such a way that it could take negative values to a minimum, indicating the acquired debt. The economic model, including credit and debt, was first studied by Viaggiu et al. using the tools of the statistical ensembles [7]. There they adopt the Boltzmann-Gibbs distribution where energy is replaced by total money, including income, credit, and debt. This method was extended to studying markets and exchange economies through complex

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networks [8] and stock price formation processes from the order book [9]. This last system has also been addressed with the Boltzmann equation in a non-equilibrium situation [10].

The resulting aggregated economic variables could be related to macroeconomics in the same way that the gas particles' microscopic energy gives rise to thermodynamics. For example, the economic temperature can be used as an index that indicates, on average, how total available money is distributed among agents, it changes with debt, and when considering various types of microscopic economic transactions, [7]. This idea of establishing a link between classical thermodynamics and economics is due to Samuelson [11]. In kinetic agent models, it is standard to introduce savings propensity λ as the fraction of an agent's money that will not spend during the transaction, i.e., it is introduced in the exchange rule [12]. Also, through numerical simulation, saving agents' income distribution follows the so-called Gamma distribution when the system reaches equilibrium. If the agents do not save, the Boltzmann-Gibbs distribution, which usually models the lower-middle class, is recovered. Indeed, Pareto's law can be recovered for random savings within these models [13]. Furthermore, by considering that some agents have fixed savings and another sector has a random saving propensity, this leads to a distribution that has an exponential zone and another zone modeled by a power law, as observed in real economies [14].

In this work, we study the thermostatistical properties of kinetic models of exchange agents that describe a closed economy with income, savings, credit, and debt. Here we use a different approach first studied by Lopez-Ruiz et al. [15]. The analytical geometric model considers that the interacting agents obey an additive constraint that defines an N-dimensional equiprobability surface. The corresponding Hamiltonian contains the saving propensity as an exponent of the monetary variable. In this work, we calculate the canonical partition function through the statistical ensemble formalism. From the partition function, we obtain the economic temperature and entropy, where we observe an interplay between the fraction of money agents can save and the debt they can acquire.

This work is structured as follows. In section 2 we review the fundamental concepts of the geometric model for the distribution of the income for saving agents, and in section 3 we review the ensemble theory for money, credit, and debt. In section 4, we present the statistical ensemble for saving agents. We study the cases with and without debt and the corresponding limit for when the agents do not save money in their transactions, recovering the case studied by Viaggiu et al. [7], and Patriarca et al. [12], respectively. Finally, in the last section, we present a summary and discussion of the obtained results and possible future work routes.

2. Microscopic and geometric models for a system of saving agents

Let us consider a simple discrete closed economy model in which N agents can exchange money by pairs. In the beginning, each agent has the same amount of money, say M/N, where M is the total money in the system, so the initial distribution of capital is uniform. Then agents begin to interact, at each time step a pair of agents (i, j) randomly chosen will interact each other, where $i \neq j$, and $i, j = 1, 2, ..., N$.

During the exchange, the capital of each agent changes following the next exchange rule constrained to the fact that the total money is conserved

$$u_i' = u_i + \Delta u, \quad u_j' = u_j - \Delta u, \tag{1}$$

where u_i and u_i' is the money of agent i before and after transaction, respectively. The amount exchanged Δu is taken randomly and depends on the details of the transaction, for instance for constant saving propensity λ is

$$\Delta u = (1 - \lambda) \left[\epsilon u_j - (1 - \epsilon) u_i \right], \tag{2}$$

where ϵ is a uniformly distributed random variable. In this case, during the transaction between agents i and j, the money that can be reallocated is reduced by $1 - \lambda$ corresponding to the fraction of the initial capital each agent has decided to use in the exchange. For this model, the agents' equilibrium distribution has been studied both analytically and numerically, and it depends on the value of the saving propensity parameter. Specifically, simulations points to a Gamma distribution for the money [12]

$$f(z)dz = \frac{z^{n-1}}{\Gamma(n)} e^{-z} dz, \tag{3}$$

where $z = \frac{nu}{\langle u \rangle}$, $\langle u \rangle$ is the average money and

$$n(\lambda) = \frac{1+2\lambda}{1-\lambda}, \tag{4}$$

is the shape factor of the Gamma distribution, that in this context is a function of the saving propensity. For $\lambda = 0$, where there is no saving criterion Eq. (3) reduces to theBoltzmann-Gibbs distribution[6].

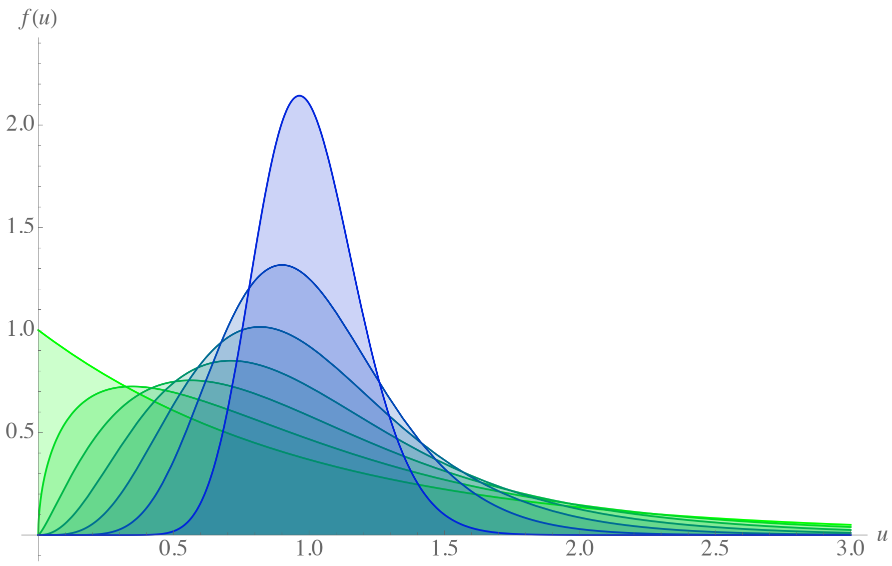

Figure 1. Gamma distribution function for distribution of money Eq. (3) for different values of the saving propensity λ. Green curve corresponds to $\lambda = 0$, saving factor increases as it tends to the blue, so the darker blue curve corresponds to $\lambda = 0.9$.

The same distribution can be obtained from a geometric perspective [15]. Let be a set of positive variables $\{x_i\}_{i=1,..,N}$ satisfying the constraint

$$x_1^b + x_2^b + \ldots + x_N^b = \sum_{i=1}^{N} x_i^b \leq M, \tag{5}$$

with b a positive real constant and M the total money. The equality in (5) defines a symmetrical surface, it also defines a transaction between agents. Here x is an internal geometrical variable, which can be related with the capital per agent through the probability density [15]. The probability $f(x)dx$ of finding an agent with generic coordinate x is proportional to the volume $V_{N-1}\left((M - x^b)^{1/b}\right)$ of all the points contained into the $(N-1)-$dimensional symmetrical region limited by the constraint $(M - x^b)$. Thus, by considering the normalization condition the distribution function is $f(x) = \frac{V_{N-1}\left((M-x^b)^{1/b}\right)}{V_N\left(M^{1/b}\right)}$. By assuming that the volume is proportional to the radius of the region, and for large N, is possible to find the same Gamma distribution (3) for the desired probability, with $z = \frac{x^b}{b\,\langle x^b \rangle}$. Thus, by comparing both expressions we find

$$\frac{u}{\langle u \rangle} = \frac{x^b}{\langle x^b \rangle}. \tag{6}$$

where $b = b(\lambda) = n(\lambda)^{-1}$. Note that in the particular case when $b = 1$, corresponding to no savings, x is exactly the money of the agents. In this way, saving propensity appears in the constriction through the power of the geometric variables. This model will allow later to propose a Hamiltonian formulation that we will use when defining the saving agents ensemble.

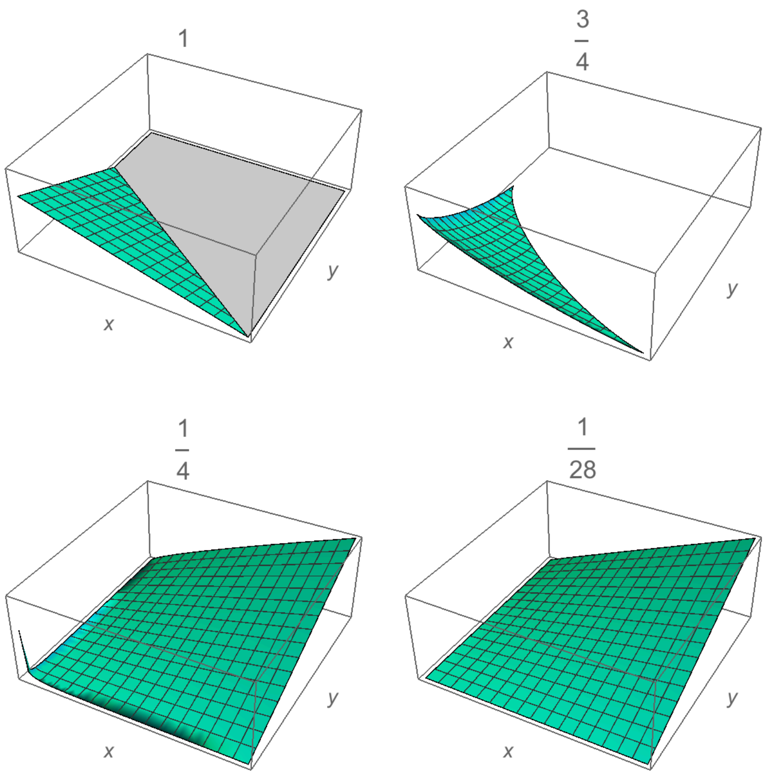

Figure 2. Constriction surfaces Eq. (5) for three agents with $M = 1$, for different values of $b = 1, 3/4, 1/4, 1/28$, which correspond to a saving propensity of $\lambda = 0, 0.1, 0.5, 0.9$, respectively.

3. Statistical ensembles for agents with credit and debt

As noted by Viaggiu et al. [7], to build an ensemble, it is only necessary to have a conservation law. In their case, they introduce the total money in the system as a conserved quantity, which depends on two possible variables, u the money of each agent coming form income and v, which is a monetary variable whose positive values correspond to the credit obtained by the agent, and its negative values to the acquired debt. Therefore, in general

$$m(u, v) = \sum_{i=1}^{N} (u_i + v_i). \tag{7}$$

The canonical partition function is introduced as follows[7]

$$Z = \int\int d^N u\, d^N v\, e^{-\frac{m(u,v)}{T}}, \tag{8}$$

where T is the economic temperature related to the average money per agent but depends on agents' interactions. With this definition of the partition function, it is possible to find the thermodynamic

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variables in the same way as in equilibrium statistical mechanics. In particular, entropy has the same interpretation being proportional to the number of microscopic configurations of the system, so the equilibrium state corresponds to the configuration that maximizes the entropy.

In order to interpret the meaning of economic temperature, let us briefly review three cases: Firstly, let us consider the simple case $M(u) = \sum_i u_i$, so the integral of the partition function (8) is simply

$$Z = V_v^N T^N, \tag{9}$$

where V_v is the integral over the v-variable, which is constant and depends only on the domain of v, it can be interpreted as the maximum credit accessible in the system. From the usual thermodynamic relationship for the internal energy, namely

$$M = T^2 \frac{\partial}{\partial T} \ln Z = NT, \tag{10}$$

we can obtain the system's total capital and the economic temperature as the mean capital per agent $T = M/N$.

Let us now consider the money function for credit and debt $m(u,v) = \sum_i (u_i + v_i)$, where $y \in [-d, \infty)$, $d \geqslant 0$. In such a case the integral for the partition function and the thermodynamic quantities are as follows

$$Z = T^{2N} e^{\frac{Nd}{T}}, \qquad T = \frac{1}{2}\left(\frac{M}{N} + d\right). \tag{11}$$

Let us explore a couple of features. In this case the temperature is not simply the average money per agent, but it is distributed among the different payment alternatives. In this case, T is the arithmetic average between two alternatives, the agent's money and his approved credit.

Economic temperature T can be thought of as an index that relates the average money to an agent's debt capacity. For instance, if $M/N < d$ then $T/d < 1$, which indicates that in such an economy, the agents could not cover a debt d on average. Indeed, this ratio is bounded when the average money is much lesser than the amount of credit, then T/d tends to $1/2$. In the case that $M/N \propto d$, then $T/d \approx 1$, which is the limit of the debt that could be covered. Then, for an economy to have no problem with a credit amount d, we should look for at least $T/d > 1$, which implies that on average $M/N > d$. Indeed, when we ask for $M/N \gg 2d$, then $T/d \gg 1$, so for these values of this ratio, there will be enough liquidity to cover the debt.

On the other hand, recalling the standard interpretation of the partition function Z as the number of accessible microstates of the system for a given temperature, according to the expression (11), the number of accessible states grows exponentially with the ratio d/T. In fact, given those mentioned above, the more microscopic states are available for the system, the agents will have more difficulties to pay the debt.

We can go further, removing the credit from the agents and leaving them with the debt d, which can be done if $y \in [-d, 0)$, $d \geqslant 0$. In this case the partition function and the thermodynamic quantities result as follows

$$Z = T^{2N}(e^{d/T} - 1)^N, \qquad M = 2NT - \frac{Nd\, e^{d/T}}{e^{d/T} - 1}. \tag{12}$$

Although apparently in the limit $d/T \gg 1$, the expressions of the previous case are recovered, this case is not valid in the corresponding regime, besides having the aforementioned problems. However, if the debt is small an expansion of the exponential can be performed for $d/T \ll 1$, which implies that

$$Z \approx T^N d^N \left(1 + \mathcal{O}\left(\frac{d}{T}\right)\right), \qquad M \approx N\left(T - \frac{d}{2} + \mathcal{O}\left(\frac{d^2}{T^2}\right)\right). \tag{13}$$

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In this limit economic temperature goes as

$$T \simeq \frac{M}{N} + \frac{d}{2} + \ldots, \tag{14}$$

which is consistent with $M/N \gg d/2$. Temperature of the system increases a little as long as the debt is much less than twice the average capital.

4. Statistical ensambles for money, credit and debt with saving propensity

In this section, we study the effect of the saving propensity on thermodynamic quantities, especially in economic temperature and in the partition function. Given the Hamiltonian formulation given in section 2, we can construct different money functions to calculate the corresponding thermodynamics and partition functions.

4.1. Case 1: Money and savings

Let us consider directly the money function (5) and calculate straightforwardly the partition function

$$Z = \int\int d^N x\, d^N y\, e^{-\frac{1}{T}\sum_i x_i^b} = V_y^N \left(\int e^{-\frac{x^b}{T}} dx\right)^N, \tag{15}$$

where b is given by the inverse of Eq. (4). This integral can be easily related to the Gamma function through a variable change, such that

$$Z = V_y^N T^{\frac{N}{b}} \Gamma\left(\frac{1}{b} + 1\right)^N. \tag{16}$$

With use of the definition (10), we can determine the economic temperature

$$M = \frac{NT}{b}, \quad \Rightarrow \quad T = b\frac{M}{N}. \tag{17}$$

The expression (17) looks like the equipartition theorem of energy, which states that each microscopic degree of freedom contributes by a term proportional to T to the total energy. In this case, we can say that each agent contributes to the total money an amount proportional to the economic temperature by a coefficient that function of the saving propensity Eq. (4) such that $0 \leq b \leq 1$. The ratio between temperature and the money per agent decreases as the fraction of money saved increases, Fig. 3.

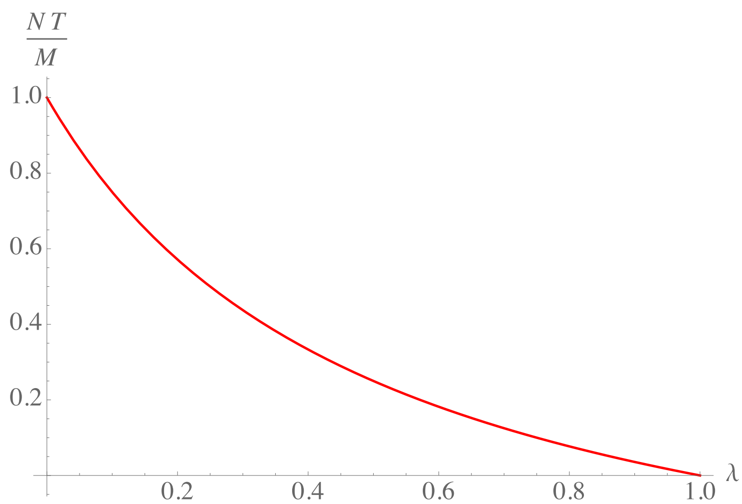

Figure 3. Plot of the ratio of economic temperature and the average money per agent as function of the saving propensity.

For the ideal gas, the equivalent ratio of T by the energy per particle is a constant equal to $2/3$, which depends on the functional form of the microscopic kinetic energy and the dimension. Expression (17) was previously obtained by Patriarca et al. [12] where b^{-1} is interpreted as an effective dimension.

It is also possible to obtain the entropy of the system as usual in thermodynamic systems $TS = M + T\ln Z$, with which the following expression is obtained

$$S = \frac{N}{b}\left[\ln T + 1 + \ln\left(1 + \frac{1}{b}\right)\right] + N\ln V_y, \tag{18}$$

where there is no emphasis on the indistinguishability of the agents. This expression is plotted in Fig. 4, where one can see an increasing behavior with T, as usual. However, in this case, we see that the entropy begins to grow from a certain minimum temperature that changes according to the agents' saving capacity; the more they save, the lower is the minimum temperature. Indeed, we can also observe that the more the agents save, the more rapidly the entropy grows.

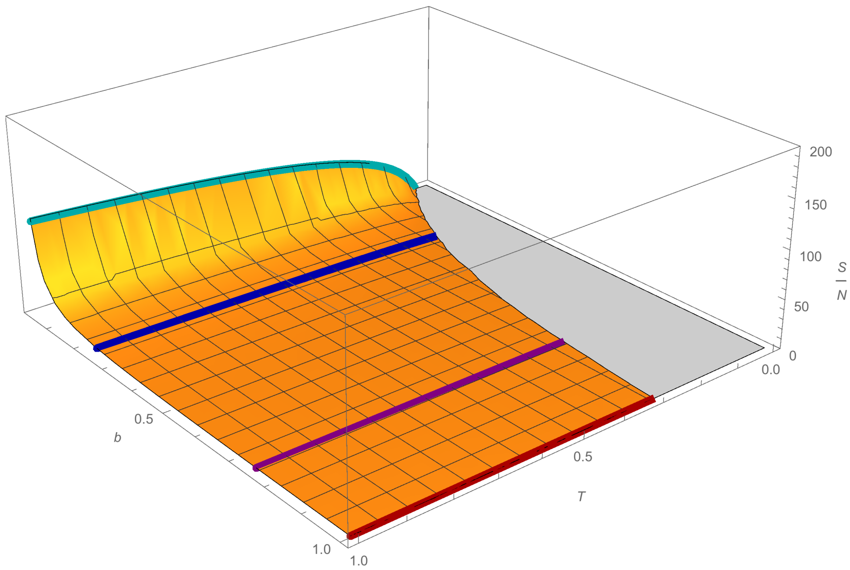

Figure 4. Plot of the ratio between the entropy and the number of agents (18) as a function of the temperature T and the parameter b, with $V_y = 1$. The curves are presented for the values of $b = 1$ (red), $3/4$ (purple), $1/4$ (blue), $1/28$ (cyan), which correspond to a saving propensity of $\lambda = 0$, 0.1, 0.5, and 0.9, respectively.

4.2. Case 2: Savings, money, credit and debt

In order to introduce credit and debt we must add to (5) a term with monetary units, similarly as in Eq. (7). Let us consider the geometric variable $y \in [-d^{1/b}, \infty)$, where d is the maximum debt. Thus, for $m(x,y) = \sum_i (x_i^b + y_i^b)$ we can calculate the partition function as follows

$$Z = \int\int d^N x\, d^N y\, e^{-\frac{1}{T}\sum_i (x_i^b + y_i^b)} = \left(\int_0^\infty e^{-\frac{x^b}{T}} dx\right)^N \left(\int_{-d^{1/b}}^\infty e^{-\frac{y^b}{T}} dy\right)^N. \tag{19}$$

The first integral is again a gamma function, while the second one can be rewritten as the upper incomplete gamma function $\Gamma(b^{-1}, zdT^{-1})$, where $z = e^{i\pi b}$. This last term appears due to the negative lower integration limit introduced by the debt. However, this function can be analytically continued to complex numbers, maintaining many of its real-valued counterparts' properties. In particular it satisfies [16]:

$$\Gamma(a, rz) = z^a\Gamma(a, r) + (1 - z^a)\Gamma(a).$$

With this property together with the relation $\gamma(a,x)+\Gamma(a,x)=\Gamma(a)$, with $\gamma(a,x)$ the lower incomplete gamma function [16], the partition function can be written as

$$Z=\left\{\frac{T^{2/b}}{b^2}\Gamma\left(\frac{1}{b}\right)\left[\Gamma\left(\frac{1}{b}\right)+\gamma\left(\frac{1}{b},\frac{d}{T}\right)\right]\right\}^N. \tag{20}$$

By taking the derivative of the partition function we find the total money of the system

$$M=\frac{2NT}{b}\left[1-\frac{b}{2}\left(\frac{d}{T}\right)^{1/b}\frac{e^{-d/T}}{\Gamma\left(\frac{1}{b}\right)+\gamma\left(\frac{1}{b},\frac{d}{T}\right)}\right]. \tag{21}$$

This expression is much more involved than in the previous cases, particularly let us realize that it is no longer possible to directly obtain the temperature, instead it is necessary to solve a transcendent equation. This implies that economic temperature will no longer be directly proportional to the mean capital per agent. Nevertheless, this also occurs in other physical systems, consider for example the relativistic gas [17], where the proportionality between temperature and energy can only occur in the non-relativistic and ultra-relativistic limits, for low and high temperatures respectively. For intermediate values of T, from a certain critical temperature [18], it is not possible to have a simple relationship between these variables.

In order to have a more manageable expression of Eq. (21), let us consider the same expansion as in section 3 for $d \ll T$. In such a case, the first terms of the expansion of the exponential can be considered, and for the incomplete gamma function, the first terms of the following series [16]

$$\gamma(a,x)=x^a\sum_{k=0}^{\infty}(-1)^k\frac{x^k}{k!(k+a)}, \quad \text{for } x\ll 1. \tag{22}$$

Hence, we obtain the following:

$$M\simeq\frac{2NT}{b}\left\{1-\frac{b}{2\Gamma\left(\frac{1}{b}\right)}\left(\frac{d}{T}\right)^{1/b}\left[1-\frac{d}{T}+\frac{1}{2}\left(\frac{d}{T}\right)^2-\frac{b}{\Gamma\left(\frac{1}{b}\right)}\left(\frac{d}{T}\right)^{1/b}\left(1-\frac{d}{T}\frac{b+2}{b+1}+\dots\right)+\dots\right]\right\},$$

$$M\simeq\frac{2NT}{b}\left\{1-\frac{b}{2\Gamma\left(\frac{1}{b}\right)}\left[\left(\frac{d}{T}\right)^{\frac{1}{b}}-\left(\frac{d}{T}\right)^{\frac{b+1}{b}}+\frac{1}{2}\left(\frac{d}{T}\right)^{\frac{2b+1}{b}}-\frac{b}{\Gamma\left(\frac{1}{b}\right)}\left(\frac{d}{T}\right)^{\frac{2}{b}}+\dots\right]\right\}. \tag{23}$$

First we see that in the case where the agents do not save, i.e. $b=1$, Eq. (23) reduces to $M\approx 2NT-dN$, which is precisely Eq. (11) that was obtained by Viaggiu et al. [7]. For $b\neq 1$ we can consider just the first term of the previous series

$$M\simeq\frac{2NT}{b}\left[1-\frac{b}{2\Gamma\left(\frac{1}{b}\right)}\left(\frac{d}{T}\right)^{\frac{1}{b}}\right]. \tag{24}$$

With this expression, it is possible to obtain an approximate expression for the entropy in the same way as in the case of the previous subsection

$$\frac{S}{N}\simeq\frac{2}{b}\left[1-\frac{b}{2\Gamma\left(\frac{1}{b}\right)}\left(\frac{d}{T}\right)^{\frac{1}{b}}\right]+\frac{2}{b}\ln T-2\ln b+2\ln\Gamma\left(\frac{1}{b}\right)+\frac{b}{\Gamma\left(\frac{1}{b}\right)}\left(\frac{d}{T}\right)^{1/b}, \tag{25}$$

where the last term corresponds to the approximation of the incomplete gamma function with the leading term of (22). In the case $b=1$, Eq. (25) reduces to $S\approx 2N\ln T+2N$, obtained in [7]. We show in Fig. 5 three graphs of S for different values of d. It can be seen that the behavior is qualitatively

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similar to that of Fig. 4. However, we note that when $d \neq 0$ entropy grows faster as b decreases, or when saving increases. Furthermore, as d increases, the behavior of the minimum T changes in each case.

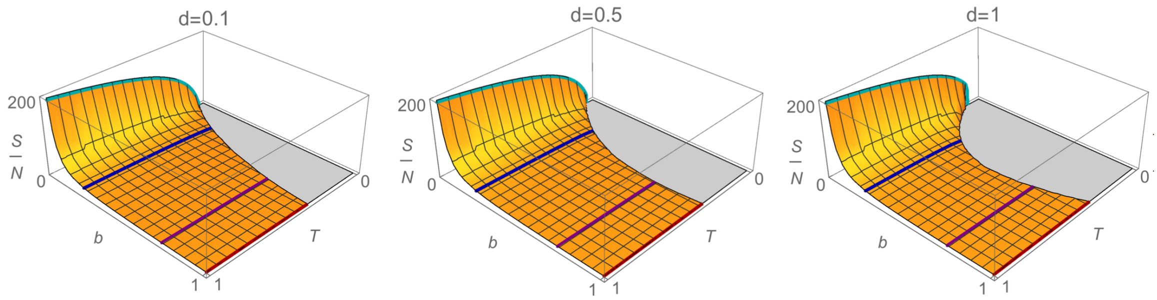

Figure 5. Plot of the ratio between the entropy and the number of agents (25) as a function of the temperature T and the parameter b for $d = 0.1, 0.5, 1$. We note a change in the behavior of S as d grows. Again the curves are for $b = 1$ (red), $3/4$ (purple), $1/4$ (blue), $1/28$ (cyan), and correspondingly $\lambda = 0, 0.1, 0.5$, and 0.9.

To interpret the economic temperature, in this case, consider Eq. (24) and solve for T as follows

$$T \simeq \frac{Mb}{2N}\left[1 + \frac{b}{2\Gamma\left(\frac{1}{b}\right)}\left(\frac{d}{T}\right)^{\frac{1}{b}}\right]. \tag{26}$$

Let us realize that T still appears on the right-hand side of the above expression. However, as a first approximation, we can consider $T_0 \sim Mb/2N$, and replace it into Eq. (26), to have an iterated solution, as shown below

$$T \approx \frac{Mb}{2N} + \frac{d^{1/b}}{2\Gamma\left(\frac{1}{b}\right)}\left(\frac{Mb}{2N}\right)^{1-\frac{1}{b}} = \frac{b}{2}\left[\frac{M}{N} + \frac{d^{1/b}}{\Gamma\left(1+\frac{1}{b}\right)}\left(\frac{Mb}{2N}\right)^{1-\frac{1}{b}}\right]. \tag{27}$$

We can observe that the second term corresponds to a non-trivial function of the debt that reduces to d in the case of no savings and plays a role of a generalized payment method that includes saving propensity. Another highlight is that in this case, the economic temperature is not just the arithmetical mean of payment methods, but it is a weighted average with $b/2$.

To analyze the behavior of the temperature given by Eq. (27), it is convenient to study the ratio T/d, since such approximation is valid when this ratio is much greater than one. In figure 6 the ratio T/d is plotted as a function of the saving propensity for the particular case in which the mean money per agent is equal to 1, in monetary units. For the case in which debt d is small compared to one, the savings decrease the relation T/d as λ increases. When $T = d$ the approximation is no longer valid, this defines a λ_{max} which is the maximum percentage that an agent can save. As the value of debt increases, λ_{max} decreases, i.e., agents can save less. When $d = 1$ the approximation is not valid for any λ. It is interesting to see that as d increases, the ratio T/d increases for high values of λ. This behavior has no meaning in terms of the model, although mathematically, it is consistent with the application regime. To summarize, as d increases, the agents' savings capacity decreases.

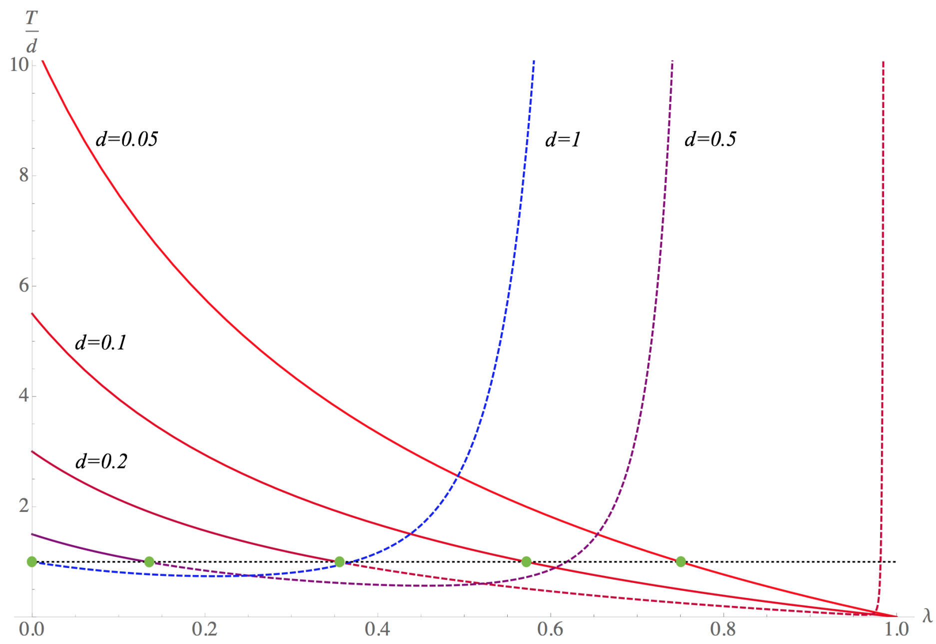

Figure 6. Graphic of the ratio $\frac{T}{d}$ from Eq. (27) as function of savings propensity λ for unitary average money per agent. The values of d =0.05, 0.1, 0.2, 0.5, 1. The approximation fails for $T = d$, so the dashed branches of the graph have no interpretation in the model.

5. Conclusion

In this work, we apply the statistical ensemble formalism to a geometric model of agents to study the effect of saving propensity on a system with money, credit, and debt. This formalism allows us to obtain analogous thermodynamic variables. Mostly, we study the so-called economic temperature as an index that relates the average money with the system's debt capacity. The economic temperature in the presence of saving propensity follows an analogous equipartition theorem, where the fraction of the temperature is determined by a function of savings. When money, credit, debt, and savings are considered in the system, we find that economic temperature is given by the solution of Eq. (21). In the case $T \gg d$, which corresponds to a system where agents can pay their debt, it was possible to find an approximation for the economic temperature expressed as Eq. (27). This expression for T has a term that considers the competition between debt and savings, and that can be considered a generalized alternative payment method. When studying their behavior, we realized that the relationship between T and d decreases as savings propensity grows to a maximum value that depends on d. The entropy of the saver agent system when debt is present was also calculated. In a general way, we can say that entropy increases as savings increase, always starting from a minimum temperature from which the entropy is greater than zero. In the case of debt, the entropy increases even more, and the behavior of the minimum temperature changes slightly. This formalism is interesting to study exchange agent models since it is possible to find macroscopic properties. It would be interesting to find an interpretation similar to that given here to the economic temperature, but for entropy or free energies. It would also be interesting to study some other economic processes with this formalism, such as economic inequality, which can be characterized by some indices such as the Gini or Kolkata [19,20] to be addressed elsewhere.

Author Contributions: Conceptualization, Guillermo Chacón-Acosta and Vanessa Ángeles-Sánchez; Formal analysis, Guillermo Chacón-Acosta and Vanessa Ángeles-Sánchez; Funding acquisition, Guillermo Chacón-Acosta; Investigation, Guillermo Chacón-Acosta and Vanessa Ángeles-Sánchez; Supervision, Guillermo Chacón-Acosta; Writing – original draft, Guillermo Chacón-Acosta and Vanessa Ángeles-Sánchez.

Funding: This research was partially funded by DSA-PRODEP under grant No. UAM-C-CA-38, IDCA 23886 (GCA), and under grant CONACYT 612794 (VA).

Acknowledgments: VA thanks R. López-Ruíz for the fruitful discussions and lessons on this subject during the stay at Universidad de Zaragoza. GCA thanks the organizers of the 2017 SigmaPhi International Conference held in Corfu, where an earlier version of this work was presented, and helpful comments were received.

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