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Type of the Paper (Short Communications)

Introducing uncertainty in risk calculation along road using simple stochastic approach

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Abstract: Based on a previous risk calculation study along a road corridor, risk is recalculated using stochastic simulation by introducing variability for most of the parameters in the risk equation. This leads to an exceedance curve comparable to that of catastrophe models. This approach introduces uncertainty into the risk calculation in a simple way, which can be used for poorly documented cases to fulfil lack of data. This approach seems to tend to minimize risk or to question risk calculations.

Keywords: landslide; rockfall, risk, stochastic, uncertainty, transportation corridors

1. Introduction

Several authors have used power-laws to assess hazards as a function of the volume or area of instability [1, 2, 3, 4] or risk [5]. Volumes are often used as a quantification of magnitude of landslides. The frequency of failure of a volume greater than a given volume *Vol* [3] for a given region and several observations N_0 during a period Δt is given by:

$$\lambda(v \geq Vol) = \frac{N_0}{\Delta t}\left(\frac{\mathrm{Vol}}{V_0}\right)^{-b} = a\,Vol^{-b} \qquad (1)$$

In general, the analysis is based on the following conceptual formula (modified from [6]):

$$R = \lambda_r \times f_r \times PS \times Pp\,'\,Exp\,'\,E'\,V \qquad (2)$$

Where λ_r, is the temporal frequency of rupture for a given period in a given perimeter, f_r the probability of rupture associated with a given magnitude (here $\lambda = \lambda_r \times f_r$). *PS* is a spatial weight if the exact location is not known, *Pp* the frequency of propagation for a given location, *Exp* is the exposure, *E* corresponds to the value or unit of the object at risk and *V* its vulnerability.

One of the problems is that this formulation does not explicitly incorporate uncertainty. Uncertainty has mainly been applied by introducing random variables into the calculation of the factor of safety [7, 8]. Uncertainty can also be inserted by using first-order second-moment (FOSM) methods for which an objective function is chosen which is supposed to respect a Gaussian distribution, for example the safety factor, whose analytical expression is known, as well as the variances of the variables [9, 10, 6]. [11] applied the FOSM technique for inserting uncertainty in the risk analysis of block falls potentially

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affecting a tourist area shows that the 1-sigma confidence interval varies from 48 to 132% of the mean value. Simulations of block trajectories can provide probabilities of excess as a function of impact energy on objects [12]. [13] showed that by inserting uncertainty by Monte Carlo simulations, the risk of rockfall on a section of railway track is reduced.

Here the analysis carried out by [5] along a road section is taken up again, and simplified, by replacing some parameters by random variables and by using Monte Carlo simulations using MATLAB 2016a. The approach is comparable to that of [13], but the intention is to show that such an approach can be applied, particularly when data are lacking, in a similar way to the disaster model [14], which presents the results according to a surplus curve with no particular constraints.

2. Model data

[5] use equation (1) and provide a simple synthetic example of risk calculation along a stretch of road in British Columbia that is adapted to follow the ratings used in this chapter. On average, $N_0 = 100$ events reach the road per year for volumes greater than $V_0 = 0.001$ m^3, they are distributed according to a cumulative power with the observed b equal to 0.434 and $a = N_0 \times V_0^b = 4.99$ (Figure 1):

$$\lambda(v \geq Vol) = \frac{100}{1\ year}\left(\frac{Vol}{0.001}\right)^{-0.434} = 4.99\ Vol^{-0.434} \quad (3)$$

By integrating by classes, we obtain the frequencies of each class, i.e. by making the difference between the values obtained for the two limits of a volume class by the equation (3). PS is equal to 1 since it is known that it reaches the road section under consideration. The probability of propagation is relative to the location of the object, according to [5] as it is a two-way road, small volumes (< 5 m^3) affect only one of the lanes, and for smaller volumes they do not necessarily affect the car passing over them, but for volumes above 100 m^3 the affected road section width D is completely covered and $Pp = 1$. Exposure is calculated according to D, which increases roughly like the cubic root of the volume. The average vehicle length L_v is 5.4 m and 5,000 vehicles travel per day. Here only fatal accidents of at least one occupant are counted and therefore vulnerability is equal to lethality, injuries are not considered and therefore E is implicitly set to 1. The values of vulnerability or probability of death and probability of impact are modified according to functions instead of discrete sets of values (Figure 2). As an example, the class of blocks from 0.1 to 1 m^3 we obtain (Table 1):

$$\begin{aligned} R(0.1 - 1\ m^3) &= (\lambda_r \times f_r) \times PS \times Pp\ ´\ Exp\ ´\ E´\ V \\ &= 8.56 \times 1 \times 0.4\ ´\ 0.0167\ ´\ 1´\ 0.2 \\ &= 0.011\ fatal\ accidents\ per\ year \end{aligned} \quad (4)$$

The exposition is recalculated according to [15]:

$$RExp = N_v \frac{(L_v + D)}{v_v} = \frac{5000}{24}\frac{(5.4+1)}{80 \times 1000} = 0.0167 \quad (6)$$

where v_v is the speed of the vehicle and N_v is the number of vehicles per year. The sum of all classes up to 10^5 m^3 indicates an average annual frequency of fatal accidents of 0.106, i.e. approximately one accident every 10 years. This way of calculating is conservative, the risk is increased by using the upper bounds of the classes. The following paragraph attempts to overcome this problem by introducing simulations, which allow the uncertainty to be incorporated.

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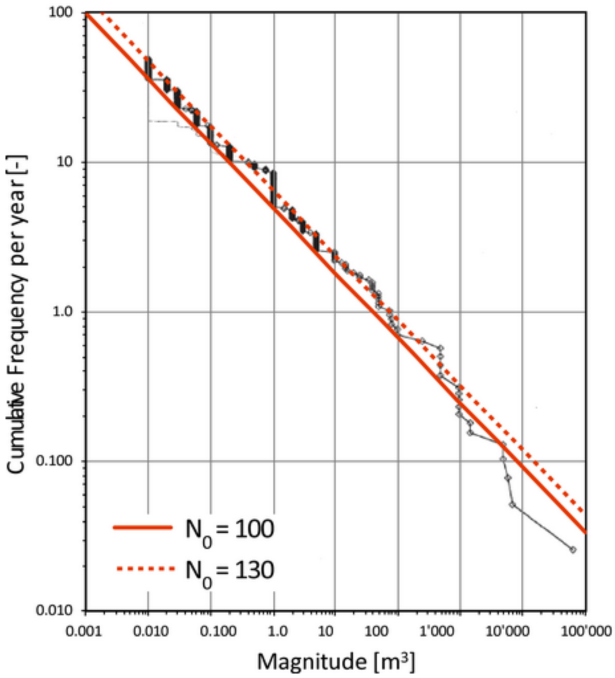

Figure 1. Cumulative frequency distribution as a function of magnitude (volume) of 390 events along 75 km of Highway 99 in British Columbia and adjustment proposed by [5] for 100 event per year and modified to 130 event per year (modified from [5]).

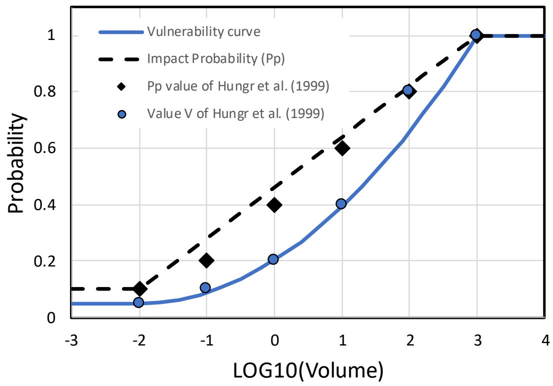

Figure 2. Model for the probability of impact or spread and vulnerability created from data from [5] to make the functions continuous.

Table 1. Details of risk calculations for different classes (modified from [5]).

Volume	**4.99×*Vol*$^{-0.434}$**	$l_r \times f_r$	*D* ~*Vol*$^{(1/3)}$	*Exp*	*Pp*	*V*	*H* × *Pp* × *Exp* × *V*	*1/R*
[m³]	[#/yr]	[#/yr]	[m]	[-]	[-]	[-]	[-]	[yr]
0.001	100.000							
0.010	36.813	63.187	0.2	0.0146	0.1	0.05	0.005	217.0
0.100	13.552	23.261	0.5	0.0154	0.2	0.1	0.007	139.9
1.0	4.989	8.563	1	0.0167	0.4	0.2	0.011	87.6
10	1.837	3.152	2	0.0193	0.6	0.5	0.018	54.9
100	0.676	1.160	5	0.0271	0.8	0.8	0.020	49.7
1'000	0.249	0.427	10	0.0401	1.0	1.0	0.017	58.4

10'000	0.092	0.157	30	0.0922	1.0	1.0	0.014	69.0
>10'000		0.092	50	0.1443	1.0	1.0	0.013	75.7
						Total	0.106	9.4

3. Introduce uncertainty into risk calculation

Nowadays, the related uncertainty for risk management is more and more required, one of the means to obtain it, is to use risk calculation simulations. This is presented through a previous example of risk calculation by modifying the procedure of [5]. The first step of the simulation consists in simulating according to the distribution the volumes of blocks that will fall, it is necessary to define the minimum and maximum frequencies corresponding to the maximum (10^5 m^3) and minimum (10^{-3} m^3) volumes of the distribution function. Let F_{max}= 4.99 × $0.001^{0.434}$ = 100 and F_{min}= 4.99 × $100'000^{0.434}$ = 0.0337. Starting from the power law cumulative distribution, it is quite easy to invert it and thus by drawing at random in an equiprobable way values between F_{min} and F_{max} such that the simulated frequency is given by:

$$F_{sim} = F_{min} + rnd \times (F_{max} - F_{min}) \quad (6)$$

Knowing that *rnd* is a random variable varying from 0 to 1 according to a uniform distribution. Thus, the corresponding volume is:

$$V_{sim} = \left(\frac{F_{sim}}{a}\right)^{-\frac{1}{b}} \quad (7)$$

This makes it possible to simulate a distribution of rockfall events per year. Instead of calculating by class, the calculation is performed for each of the 100 simulated volumes. Based on these simulations, it is possible to add distributions for several variables in the risk calculation. First the number of events is on average 100 events per year, which can become a random variable by using an inverse Poisson distribution, which allows to simulate random values from a mean for discrete values. 10'000 years are simulated (Figure 3).

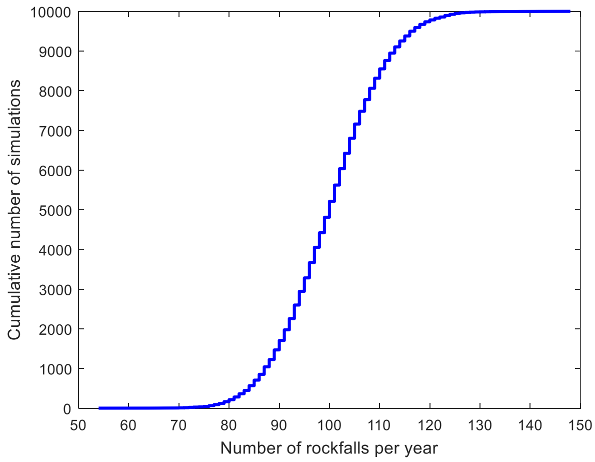

Figure 3. Cumulative distributions of the number of events per year for the 10,000 simulations, based on Poisson distribution using an average of 100.

In the example of [5] there are two estimated variables that are discrete *Pp* and *V*. The idea is to make them continuous, a linear fit for *Pp* and by a second-degree polynomial for *V* from the log base 10 values of the volumes (Figure 2):

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$$Pp = 0.180\ Log_{10}(Flight) + 0.460 \tag{8}$$

$$V = 0.038\ (log_{10}(Flight))^2 + 0.152\ log_{10}(Flight) + 0.202 \tag{9}$$

The value of D is given by the cubic root of the volume. The last step is to add distribution functions to the other variables. For simplification uniform distribution functions are used here, i.e. values are equiprobable between two limits (Table 2). This applies to the variables related to the exposure D, v_v, N_v. We did not randomized L_v because the length of the zone affecting the passengers are not easy to estimate, and does not change much, the goal is also to be coherent with [5].

Table 2. Limitations of uniform distributions of random variables.

Variables	Units (remarks)	Minimum	Maximum
Debris width D	m	D/2	3D/2
Vehicle speed v_v	km/h	57.5	102.5
Number of vehicles N_v	Vehicles/day	4’500	5’500
Probability of impact or propagation at the vehicle location Pp	[-] ; Integrated in the calculation from the integration of an order of magnitude of the volume	$\log_{10}(V(d))$-0.5	$\log_{10}(V(d))$+0.5
Vulnerability V(lethality)	idem	idem	idem

4. Results

The simulation programme with a realization for 10000 blocks with the same data as [5], except for the continuous functions for V and Pp, the frequency or probability of accident is 0. 0992, i.e. one fatal accident every 10 years. By simply adding the variabilities shown in the **Error! Reference source not found.**, for 10,000 simulations we obtain 0.103 (1 accident every 9.7 years), which shows the validity of the simulation compared to [5] data.

Table 3. Characteristics of the excess supply curves in the Figure 4 for the two first columns and for two other scenarios by changing the number of occupants in the car and the total number of rockfall per year.

Thresholds	Frequency	Return period T [year]			
Case	**A**	**A**	**B**	**C**	**D**
	[ev./year]	**1 occ. N_0 = 100**	**1 occ. N_0 = 130**	**1-2 occ. N_0 = 100**	**1-2 occ. N_0 = 130**
Average	0.060	16.8	12.9	11.3	8.6
Minimum (max. T)	0.011	89.8	69.3	58.8	44.0
2.50%	0.019	51.6	34.9	35.0	24.0
5%	0.022	45.3	31.3	31.4	21.6
Median	0.048	21.0	15.3	14.1	10.4
95%	0.138	7.20	5.9	4.8	3.9
97.5	0.167	6.00	5.0	3.9	3.3
Maximum (Min. T)	0.4080	2.5	2.6	1.8	1.6

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By carrying out 10,000 simulations of one year with a number of annual rockfalls distributed according to the Figure 3, we obtain an average frequency of 0.059 events per year, i.e. one event every 17 years (Table 2). The median is 0.048, i.e. a longer time than that obtained by [5] separates the potential accidents. The fact that no longer working in classes reduces the average frequency is divided almost by a half. The so-called excess curves indicate that there is a 95% chance that there is less than 46 years between two events (Figure 4). The probability of having an event every seven and a half years is 5%, which is not negligible.

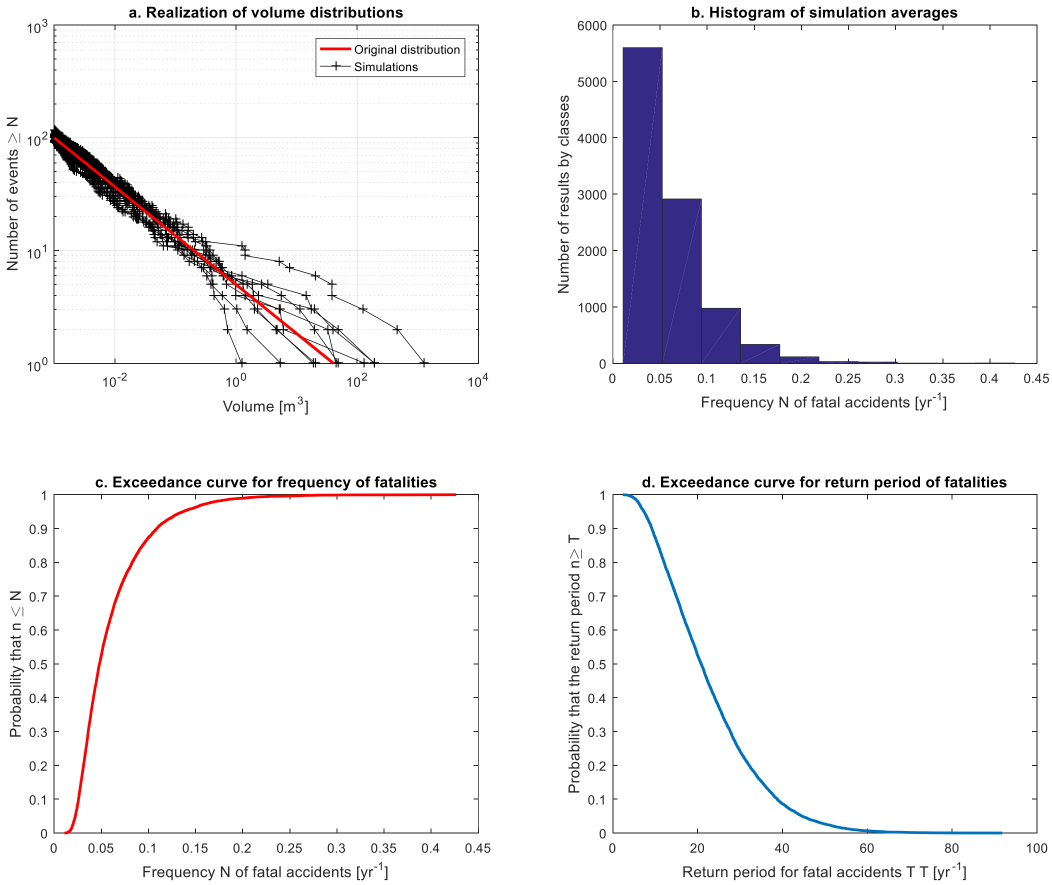

Figure 4. Simulation results. a. 10 realisation of the volume distribution; b. histogram of simulated fatal accident frequencies; c. excess curve or probability that the frequency is greater than a given value; d. probability that the accident return period is greater than a given value.

5. Discussion and conclusion

The orders of magnitude are respected since [5] indicate that the return period of fatal accidents observed the same Highway 99 between 1960 and 1996 is 12 years and 8 years from 1980 to 1996 as traffic increased. Here the mean and median values are T = 17 and 21 years and 95% of the simulated return periods are greater than 7.5 years, which is close to the observation. This result can be interpreted in different ways, either by using high probability thresholds or by modifying the distributions of the random variables introduced, which are nevertheless symmetrical. Or the recent accidents statistics and an analysis of accidents by collisions must be questioned, which could be added and halve the simulated return period.

By increasing the number of events per year to N_0 = 130, it also fit the data (Figure 1) by maximizing the frequency, the average return period is 12.9 years (median 15.3) (Table case B), by adding a randomized number of occupants being 1 or 2 randomly it provides T = 11.2 years (median 14.1; case C) and if both are used, the result is 8.6 years (median

10.4; case D). It shows that reasonable hypothesis can lead to an agreement with the observed data. It also shows that there is still 5% chance the return period ranged between 7.2 and 3.9 years. It is also noteworthy that the centred 95% confidence levels ranges for return period decrease with hazard increase and the occupants increase case A to D, 51.6 to 6.0 years (range 45.6), 34.9 to 5.0 years (29.9), 35.0 to 3.9 years (31.1) and 24.0 to 3.3 years (20.7) respectively.

This approach makes it possible to add probabilities of realization to frequencies or return period, which is useful for decision-making, the above example permits to analyse the risk calculation sensitivity. Randomizing the original data of [5] it minimizes the average risk because it calculates values for all realizations and not just for classes, but at the same time it provides elements for the quantification of uncertainties. [16] have also shown that the risk calculation using probabilistic approach reduced the risk compared to average value. This type of approach is likely to be developed in the landslide risks assessment, by also introducing variability such as those of propagation models. It is a way to introduce the catastrophe model in the landslide risk assessments.

The main objective of this note is to show that this kind of method can be applied easily, by adding other random variables, while using other distribution functions, such as the normal distribution, the log-normal distribution, the triangular distribution, etc. In any case the use of Poisson's distributions is a valid approach when nothing is known. This method becomes especially useful when the knowledge of the data is partial, meaning that it is possible to obtain an excess curve using expert input, as proposed by [13] and [11]. Such sensitivity studies should be used more often in a near future, but at the same time recommendations should be issued so that the results can be compared for risk management purposes.

Author Contributions: the first authors proposed the method and wrote the computer code, and the design of the study was setup during a workshop were all the authors contributed.

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