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QUINTESSENCE MODEL CALCULATIONS FOR BULKVISCOUS FLUID AND LOW VALUE PREDICTIONS OF THECOEFFICIENT OF BULK VISCOSITY IN GENERAL AS WELL AS MODIFIED GRAVITY WITH THE FORM f(R,T)=R+f(T) AND f(T)=λT.

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Abstract

In this paper we have defined the effect of bulk viscosity on Quintessence model and scaler field potential as well as on classical field. We have shown the same effect for modified gravity with f(R,T)= R+f(T). In the derivation we have predicted the possibility of time dependent evolution of gravitational constant G and anisotropy.

Keywords: Modified Gravity ; Fluid mechanics ; Quintessence model ; cosmological inflation ;Viscosity ; Gravitational physics.

1. Introduction

Quintessence is a hypothetical form of Dark Energy postulated as an explanation of observation of an accelerating universe. Here in this paper We have discussed only the quintessence model i.e the model with canonical Lagrangian and kinetic energies. The paper has been proceeded as follows.

In Section 2 we have discussed the quintessence model shortly for FRW model in perfect fluid in reference to the publication by Edmund J Copeland ; M. sami, and shinji Tsujikawa.

In Section 3 we have given the quintessence model and inflationary calculations w.r.t the bulk viscous fluid and applied the inflation condition. We have also shown that the coefficient of bulk viscosity should be low for the stabilization of universe. We have used the divergenceless condition of energy-momentum tensor of bulk viscous fluid and proved that the gravitational constant should vary with time to stabilize the universe having high value of coefficient of viscosity.

In Section 4 we have repeated the same in case of modified gravity and analyzed the result, and in Section 5 we have repeated the procedure for perfect fluid with anisotropic and homogeneous cosmology(Bianchi type-1).

2. Quintessence model for FRW model for perfect fluid

Quintessence is a scalar field which has a lagrangian of the form

$$L = 1/\,2(\nabla\varphi)^2 - V(\phi)$$

We will explore the type of potential that is necessary for inflation. The action for Quintessence is given by

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$$S = \int d^4x \sqrt{-g}[1/\,2(\nabla\varphi)^2 - V(\phi)] \quad (1)$$

where $(\nabla\varphi)^2 = g^{\mu\nu}\,\partial_\mu\varphi\,\partial_\nu\phi$ and V (ϕ) is the potential of the field. In a flat FRW spacetime the variation of the action with respect to ϕ gives

$$\ddot{\varphi} + 3H\dot{\varphi} + dV/d\phi = 0 \quad (2)$$

The energy momentum tensor of the field is derived by the action in terms of$g^{\mu\nu}$:

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} \quad (3)$$

We can write that $\delta\sqrt{-g} = -(1/2)\sqrt{-g}g_{\mu\nu}\delta g^{\mu\nu}$,then

$$T_{\mu\nu} = \partial_\mu\varphi\,\partial_\nu\phi - g_{\mu\nu}\,[\,½\, g^{\alpha\beta}\,\partial_\alpha\varphi\,\partial_\beta\phi + V(\phi)\,] \quad (4)$$

In the flat Friedmann background we obtain the energy density and pressure density of the scalar field:

$$\rho = -\,T_0^0 = ½\,\dot{\varphi}^2 + V(\phi) \quad ; \qquad p = T_1^1 = ½\,\dot{\varphi}^2 - V(\phi) \quad (5a)$$

Then we get ;

$$H^2 = 8\pi G/3\,[\,½\,\dot{\varphi}^2 + V(\phi)\,] \quad (6a)$$

$$\frac{\ddot{a}}{a} = -\,8\pi G/3\,[\,\dot{\varphi}^2 - V(\phi)\,] \quad (6\,b)$$

From (5a) we get

$$W = p/\rho = \frac{½\,\dot{\varphi}^2 - V(\varphi)}{½\,\dot{\varphi}^2 + V(\varphi)} \quad (5b)$$

From equation (5b) if we get $V(\phi) \gg \dot{\varphi}^2$ then we can write

$$W = -1 \quad ; \qquad \frac{w+1}{w-1} = 0 \quad (7)$$

This is the condition for cosmological inflation.

3. Quintessence model for FRW model with bulk viscosity

- **Prediction Of variable G:**

Now if we consider the universe as bulk viscous fluid, then we have to consider the viscous dissipation for energy momentum tensor which will eventually reduce the pressure.

Now consider due to bulk viscosity we have the coefficient of viscosity Ɛ. Now the apparent pressure will be P=p- ƐΘ where $\Theta = 3\frac{\dot{a}}{a}$= expansion.

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So from the equation (5a) we can get

$$\rho = -T_0^0 = ½\dot{\varphi}^2 + V(\phi) \quad ; p = T_1^1 = ½\dot{\varphi}^2 - V(\phi) + \mathcal{E}\Theta \quad (8)$$

Now as we know

$$\Theta = 3\frac{\dot{a}}{a}. \ ; \text{and } H = \frac{\dot{a}}{a}. \quad \text{And so } \Theta = 3H \quad (9)$$

so we get ;

$$\rho = -T_0^0 = ½\dot{\varphi}^2 + V(\phi) \quad ; p = T_1^1 = ½\dot{\varphi}^2 - V(\phi) + 3\mathcal{E}H \quad (10)$$

Now from (6a) we get

$$H = \sqrt{8\pi G/3[½\dot{\varphi}^2 + V(\varphi)]} \quad (11)$$

So from (5b) ; (10) and (11) we can get

$$W = \frac{½\dot{\varphi}^2 - V(\varphi) + 3\mathcal{E}\sqrt{8\pi G/3[½\dot{\varphi}^2 + V(\varphi)]}}{½\dot{\varphi}^2 + V(\varphi)} \quad (12a)$$

Now if we get $V(\varphi) \gg \dot{\varphi}^2$ then from the equation (12a) we can get ;

$$W = -1 + 3\mathcal{E}K/V > -1 \text{ where } K = \sqrt{8\pi G/3} \quad (12b)$$

so if $\mathcal{E} > 0$ then we can get w=-1 for low value of K otherwise it will be greater than -1 and thus it will not produce the inflation condition and that's why we can say that the G will change with time during cosmological inflation condition.

From the FRW model using the divergence-less condition of energy momentum tensor

$$T_{\mu\nu} = \rho U_\mu U_\nu + p\ h_{\mu\nu}$$

We can write

$$\dot{\rho} + 3(\rho + p)H = 0 \quad (13)$$

So using the definition (8) we can get

$$\dot{\varphi}\,\ddot{\varphi} + 3(½\dot{\varphi}^2 + V(\varphi) + \dot{\varphi}^2 - V(\varphi) + 3\mathcal{E}H)H + (\frac{\partial V}{\partial \varphi})\dot{\varphi} = 0$$

Or ; using the definition of H from Via we get

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{\partial V}{\partial \varphi} + (9\mathcal{E}K^2/2)\dot{\varphi}^2 + 9\mathcal{E}K^2 V(\phi) = 0 \quad (14)$$

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Now during inflation as $V(\phi)$ is very high and for slow roll condition this potential is almost constant ; so we can get

$$3H\dot{\varphi} + \frac{\partial V}{\partial \varphi} + (9\mathcal{E}K^2/2)\dot{\varphi}^2 + 9\mathcal{E}K^2 V(\phi) = 0$$

Again ;

$$3H\dot{\varphi} + (9\mathcal{E}K^2/2)\dot{\varphi}^2 + 9\mathcal{E}K^2 V(\varphi) = -\frac{\partial V}{\partial \varphi} = 0$$

$$\text{Or ; } \dot{\varphi} = \frac{-3H \pm \sqrt{[9H^2 - 2K_2(K_2 V + \frac{\partial V}{\partial \varphi})]}}{K_2} \tag{15}$$

Now if $V(\phi) >> \dot{\varphi}^2$ then slow roll mechanism for cosmological inflation should follow and $V(\phi)$ remains almost constant. So we can assume that

$$\frac{\partial V}{\partial \varphi} = 0 \quad \text{and} \quad H^2 = V/3$$

So using this in the equation (15) we get

$$\dot{\varphi} = \frac{-3H \pm \sqrt{[9V - 2K_2^2 V]}}{K_2} = -3HK_2^{-1} \pm \text{constant} \tag{16}$$

(as the term under the square root is very low and denominator is also very low so K_2^{-1} is very high) Now as K_2 is directly proportional to G and the value of G is of the order of 10^{-11} so K_2 is very low in it's value.On the other hand K_2^{-1} is too high.

So we see that from equation (16) the value of damping term $\dot{\varphi}$ is high but this we initially used the condition of $\dot{\varphi}$ being small. This shows that viscosity creates ambiguity. Now if we consider the value of G was very high at the time of inflation then only we can resolve the ambiguity of that problem. Otherwise the slow-roll model breaks down. On the other hand the measurement shows the low value of G. So We consider the idea of time varying G (inversely proportional with time).

- **Proof of the coupling between the Viscosity and Scalar field and its potential**:

From equation 10 we can write as

$$H^2 = \frac{8\pi G}{3}[\tfrac{1}{2}\dot{\varphi}^2 + V(\varphi)] \qquad \text{and ;}$$

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{8\pi G}{3}[\tfrac{1}{2}\dot{\varphi}^2 - V(\varphi) + 3\mathcal{E}\theta] \tag{17}$$

Now from this above two equation we can get the potential as

$$V(\varphi) = \frac{3H^2}{8\pi G}\left[1 + \frac{\dot{H}}{3H^2}\right] + \frac{\mathcal{E}\theta}{3} \tag{17a}$$

And

$$\varphi = \int dt[-\frac{\dot{H}}{4\pi G} - \frac{2\mathcal{E}\Theta}{3}]^{1/2} \tag{17b}$$

Now from slow roll parameter we can deduce a relation between scalar field and its potential as

$$V(\varphi) = V_0 \exp[-\sqrt{\frac{16\pi}{p}}\frac{\varphi}{m_{pl}}] \tag{17c}$$

Here p comes from power law expansion concept as($a=a_0 t^p$)

From the set of equations we can easily observe that the scalar field potential and coefficient of viscosity is coupled and the increase in viscosity causes the increase of scaler field potential. So we can predict that at the time of inflation, the high value of potential is basically due to the high viscosity. For single inflationary expansion in universe evolution, the viscosity should vary with time and should decrease with time after inflation.

4. Quintessence model for FRW model with bulk viscosity in modified gravity

We have used f(R,T) = R + f(T) relation in the modified Einstein Hilbert action and considered f(T)= ƛT.

If we include bulk viscosity for modified gravity, then we can also consider the viscous dissipation for energy momentum tensor as earlier.

Now consider the coefficient of viscosity Ɛ. The apparent pressure will be P=p- ƐΘ where Θ = 3 $\frac{\dot{a}}{a}$= expansion.

In the definition of Einstein tensor the definition of energy momentum tensor should be modified. We know that the energy momentum tensor may be written as $T_{\mu\nu} = \rho U_\mu U_\nu + P\ h_{\mu\nu}$

Now from the modified gravity using the assumptions f(R,T) = R + f(T) and f(T)= ƛT ; we can get that;

$$R_{\mu\nu} - ½ R\, g_{\mu\nu} = 8\pi T_{\mu\nu} + 2f_T(T)T_{\mu\nu} + [2pf_T(T) + f(T)]g_{\mu\nu} \tag{18a}$$

From the RHS of equation 17a we can derive

$$R_{\mu\nu} - ½ R\, g_{\mu\nu} = (8\pi+3ƛ)(\rho + p)U_\mu U_\nu + [ƛ\rho - (8\pi+3ƛ)p]g_{\mu\nu} \tag{18b}$$

$$\text{Or ; } R_{\mu\nu} - ½ R\, g_{\mu\nu} = (\rho + p)_{modified} U_\mu U_\nu - p_{modified}\, g_{\mu\nu} \tag{18c}$$

Now comparing this RHS in eqn (17c) with $T_{\mu\nu} = (\rho+p)U_\mu U_\nu - P\ g_{\mu\nu}$ we can get

$p_{modified}$ =(8π+3ƛ)p – ƛρ ;

$\rho_{modified}$ =(8π+3ƛ)ρ – ƛp ;

Now for viscous fluid the modified pressure will become

$$P_{modified} = (8\pi + 3ƛ)p - ƛ\rho - \mathcal{E}\Theta \quad (18d)$$

So from the equation (5a) we can get

$$\rho_{modified} = -T_0^0 = ½\,\dot{\varphi}^2 + V(\phi) \;\; ; \; p_{modified} = T_1^1 = ½\,\dot{\varphi}^2 - V(\phi) + \mathcal{E}\Theta \quad (19)$$

Now as we know

$$\Theta = \frac{\dot{a}}{a} \text{and} \qquad H = \frac{\dot{a}}{a} \text{ So we can write } \Theta = 3H \quad (20)$$

So from equations (19) and (18a),(18b),18c and 18d we get

$$\rho_{modified} = -T_0^0 = ½\,\dot{\varphi}^2 + V(\phi) \;\; ; \; p_{modified} = T_1^1 = ½\,\dot{\varphi}^2 - V(\phi) + \mathcal{E}\Theta \quad (21)$$

Deriving the original pressure and density from the modified part, we get;

$$p = \frac{½(8\pi+4ƛ)\dot{\varphi}^2 - (8\pi+2ƛ)V(\varphi)}{(8\pi+4ƛ)(8\pi+2ƛ)} + \mathcal{E}\Theta \text{ ; and } \rho = \frac{½(8\pi+4ƛ)\dot{\varphi}^2 + (8\pi+2ƛ)V(\varphi)}{(8\pi+4ƛ)(8\pi+2ƛ)} \quad (21a)$$

So from (6a),(5b) ,(20a) and (21) I can get

$$w = \frac{½(8\pi+4ƛ)\dot{\varphi}^2 - (8\pi+2ƛ)V(\varphi) + 3\mathcal{E}\sqrt{8\pi G/3\,[\,½\,\dot{\varphi}^2 + V(\varphi)\,]}\,(8\pi+4ƛ)(8\pi+2ƛ)}{½(8\pi+4ƛ)\dot{\varphi}^2 + (8\pi+2ƛ)V(\varphi)} \quad (22a)$$

Now if we get $V(\phi) >> \dot{\varphi}^2$ then from the equation (21a)we can get

$$w = -1 + 3\mathcal{E}(8\pi + 4ƛ)KV^{\frac{-1}{2}} > -1 \text{ where } K = \sqrt{8\pi G/3} \quad (22b)$$

so if $\mathcal{E} > 0$ then we can get w=-1 for low value of K otherwise it will be greater than -1. Thus it will not produce the inflation condition and that's why we can say that the G will change with time during cosmological inflation condition, in modified gravity also. As a modification there is another term and i.e ƛ which should be small, otherwise same problem will arise and inflation theory will break down. So in modified gravity we may conclude that both Ɛ and ƛ both should have small value.

From the FRW model using the divergenceless condition of energy momentum tensor

$$T_{\mu\nu} = \rho U_\mu U_\nu + P\; h_{\mu\nu}$$

We can write

$$\dot{\rho} + 3(\rho + p)H = 0 \quad (23)$$

So using the definition (19) we can get

For $k_1 = \frac{½(8\pi+4ƛ)\dot{\varphi}^2}{(8\pi+4ƛ)(8\pi+2ƛ)}$; $k_2 = \frac{1}{(8\pi+4ƛ)}$ we get for bulk viscosity;

$$k_1\dot{\varphi}\,\ddot{\varphi} + 3(\,½\,k_1\dot{\varphi}^2 + k_2V(\varphi) + k_1\dot{½}\varphi^2 - k_2V(\varphi) + 3H\mathcal{E})H + k_2\left(\frac{\partial V}{\partial\varphi}\right)\dot{\varphi} = 0$$

Or , using the definition of H from (6a) we get

$$k_1 \ddot{\varphi} + 3k_1 H\dot{\varphi} + k_2 \frac{\partial V}{\partial \varphi} + (9\mathcal{E}K^2/2)\, \varphi^{\cdot 2} + 9\mathcal{E}K^2 V(\varphi) = 0 \quad (24)$$

Now during inflation as $\ddot{\varphi}$is very high and for slow roll approximation we can get

$$3k_1 H\dot{\varphi} + k_2 \frac{\partial V}{\partial \varphi} + (9\mathcal{E}K^2/2)\, \varphi^{\cdot 2} + 9\mathcal{E}K^2 V(\varphi) = 0$$

$$\text{Or, } \dot{\varphi} = \frac{-3k_1 H \pm \sqrt{(3k_1 H)^2 - 2K_2(K_2 V + k_2 \frac{\partial V}{\partial \varphi})}}{K_2} \quad (25)$$

Now if V(ϕ) >>$\dot{\varphi}^2$ then slow roll mechanism for cosmological inflation should follow and V(ϕ) remains almost constant. So we can assume that

$$\partial_\varphi V = 0 \qquad \text{and} \qquad H^2 = V/3$$

So using this in the equation (24) we get

$$\dot{\varphi} = \frac{-3k_1 H \pm \sqrt{[3k_1^2 V - 2K_2{}^2 V]}}{K_2} = -3k_1 H\, K_2^{-1} \pm \text{constant.} \quad (26)$$

(as the term under the square root is very low and denominator is also very low so $K_2^{-1}$is very high so considered as constant)

Now as K_2 is directly proportional to G and the value of G is of the order of 10^{-11} so $K_2$is very low. K_2^{-1} is too high. if $\bar{\lambda}$ is very high then $k_1$will become very low. As it is considered that the $\bar{\lambda}$ should have low value to get cosmic inflation condition, we can say that the value of k_1 should be high. So for both high value of k_1 and K_2^{-1} the friction term $\dot{\varphi}$ will become high.

We see that from equation 26 the value of damping term $\dot{\varphi}$ is high, initially we considered the condition $\dot{\varphi}$ is small. This shows that viscosity creates ambiguity. Now if we consider the value of G was very high at the time of inflation then only we can resolve the ambiguity of that problem. Otherwise the slow roll model breaks down. On the other hand the measurement shows the low value of G. So We have predicted time varying G (inversely with time). So the conclusion regarding variable G comes also in the case of modified gravity.

- **Proof of the coupling between the Viscosity and Scalar field and it's potential**:

From equation 10 we can write as

$$H^2 = \frac{8\pi G}{3}[\rho_{\text{modified}}] \quad \text{and ;}$$

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{8\pi G}{3}[\rho_{\text{modified}} + 3p_{\text{modified}}] \quad (27)$$

Now from this above two equation we can get the potential as

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$$V(\varphi) = \frac{3H^2(8\pi+4\lambda)}{8\pi G}\left[1+\frac{\dot{H}}{3H^2}\right]+\frac{\varepsilon\theta}{3} \tag{27a}$$

And

$$\varphi = \sqrt{(8\pi+2\lambda)}\int dt[-\frac{\dot{H}}{4\pi G}-\frac{2\varepsilon\theta}{3}]^{1/2} \tag{27b}$$

Now from slow roll parameter we can deduce a relation between scaler field and it's potential as

$$V(\varphi) = V_0 \exp[-\sqrt{\frac{16\pi}{p}}\frac{\varphi}{m_{pl}}] \tag{27c}$$

Here p comes from power law expansion concept as($a=a_0 t^p$)

From the set of equations we can easily observe that the scalar field potential and coefficient of viscosity is coupled and the increase in viscosity causes the increase of scaler field potential. So we can predict that at the time of inflation the high value of potential is basically due to the high viscosity. For single inflationary expansion in universe evolution the viscosity should vary with time and should decrease with time after inflation. The results for modified gravity is similar to the general gravity except some modification.

5. Quintessence model for anisotropic cosmologic model (Bianchi I) for perfect fluid

The canonical Lagrangian for the anisotropic cosmology can be written as ;-

$$L = 1/2(\nabla\varphi)^2 - V(\phi) + L_g \tag{27}$$

So the action will become ;-

$$S = \int d^4x\sqrt{-g}[1/2(\nabla\varphi)^2 - V(\phi) + L_g] \tag{28}$$

So using this action we get the energy momentum tensor as ;-

$$T_{\mu\nu} = \partial_\mu\varphi\,\partial_\nu\phi - g_{\mu\nu}\,[\tfrac{1}{2}\,g^{\alpha\beta}\,\partial_\alpha\varphi\,\partial_\beta\phi + V(\phi) - L_g] + 2\frac{\partial L_g}{\partial g^{\mu\nu}} \tag{29}$$

So ; using this equation (28) and from the concept of density and pressure in cosmology we get ;

$$\rho = -T_0^0 = \tfrac{1}{2}\dot{\varphi}^2 + V(\phi) - L_g + 2\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} \;;\quad p = T_1^1 = \tfrac{1}{2}\dot{\varphi}^2 - V(\phi) + L_g + 2\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=1;\,\nu=1} \tag{30}$$

so we get ;

$$H^2 = 8\pi G/3\,[\tfrac{1}{2}\dot{\varphi}^2 + V(\phi) - L_g + 2\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0}] \tag{31}$$

$$\frac{\ddot{a}}{a} = -8\pi G/3\,[\dot{\varphi}^2 - V(\phi) + L_g + \left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} + 3\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=1;\,\nu=1}] \tag{31a}$$

From (5b) we get

$$W = p/\rho = \frac{½\,\dot{\varphi}^2 + V(\varphi) - L_g + 2\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0}}{½\,\dot{\varphi}^2 - V(\varphi) + L_g + 2\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=1;\,\nu=1}} \qquad (31b)$$

From equation(31b)if we get V(ϕ) >>$\dot{\varphi}^2$ then we can write

$$W = \frac{V(\varphi) - L_g + 2\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0}}{-V(\varphi) + L_g + 2\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=1;\,\nu=1}} \quad ; \quad \frac{w+1}{w-1} = \frac{1}{(V(\varphi) - L_g)}\left[\frac{\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} + \left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=1;\,\nu=1}}{\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} - \left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=1;\,\nu=1}}\right] \qquad (31c)$$

So from 31c we get ;

$$W \neq -1 \quad \text{as well as} \quad \frac{w+1}{w-1} \neq 0 \qquad (31d)$$

So we see that the inflation condition breaks if the values of L_g ; $\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0}$; $\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=1;\,\nu=1}$ are considerably large in the system. So to get inflation condition those values should be too low i.e the anisotropy should not remain on the Geometry. We may say that during the inflation the anisotropy of the universe was too low and presently it is almost zero. If we want to consider anisotropy then G must vary with time. We have given this proof in this following section.

From the Bianchi Type I model using the divergenceless condition of energy momentum tensor

$$T_{\mu\nu} = \rho U_\mu U_\nu + p\, h_{\mu\nu}$$

We can write

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\right) = 0 \qquad (32)$$

$$\text{Or; } \dot{\rho} + 3(\rho + p)H = 0 \qquad (32a)$$

Now from (29) we get ;

$$\dot{\varphi}\,\ddot{\varphi} + \left(\frac{\partial V}{\partial \varphi}\right)\dot{\varphi} - \dot{\varphi}\frac{\partial L_g}{\partial \varphi} + 2\dot{\varphi}\frac{\partial}{\partial \varphi}\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} + 3H\left[\dot{\varphi}^2 + 2\left(\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} + \left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=1;\,\nu=1}\right)\right] = 0$$

$$\text{Or; } \ddot{\varphi} + 3H\dot{\varphi} + \frac{\partial V}{\partial \varphi} + 2\frac{\partial}{\partial \varphi}\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} - \frac{\partial L_g}{\partial \varphi} + \frac{6H}{\dot{\varphi}}\left(\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} + \left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=1;\,\nu=1}\right) = 0 \qquad (33)$$

Here $\dot{\varphi}$ act as friction in the second order differential equation. For inflation the potential should be flat and we can neglect the acceleration $\ddot{\varphi}$. For if the field ϕ starts off with a huge acceleration $\ddot{\varphi} \gg 1$, the friction term will take care of it.

So now if we apply the idea of slow roll mechanism we say that acceleration is huge and so ;

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$$3H\dot{\varphi} + \frac{\partial V}{\partial \varphi} + 2\frac{\partial}{\partial \varphi}\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} - \frac{\partial L_g}{\partial \varphi} + \frac{6H}{\dot{\varphi}}\left(\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} + \left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=1;\,\nu=1}\right) = 0\ ;$$

$$\text{Or; } \dot{\varphi}^2 + \frac{1}{3H}\left[\frac{\partial V}{\partial \varphi} + 2\frac{\partial}{\partial \varphi}\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} - \frac{\partial L_g}{\partial \varphi}\right]\dot{\varphi} + 6\left(\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} + \left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=1;\,\nu=1}\right) = 0 \quad (33a)$$

Now from the solution of this quadratic equation we can say that ;

If

$$A = \frac{1}{3H}\left[\frac{\partial V}{\partial \varphi} + 2\frac{\partial}{\partial \varphi}\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} - \frac{\partial L_g}{\partial \varphi}\right];$$

$$B = 6\left(\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} + \left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=1;\,\nu=1}\right);$$

Then we can write ;-

$$\dot{\varphi} = \frac{-A \pm \sqrt{A^2 - 4B}}{2}\ ;$$

$$\text{Or; } \dot{\varphi} = -\frac{1}{6H}\left[\frac{\partial V}{\partial \varphi} + 2\frac{\partial}{\partial \varphi}\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} - \frac{\partial L_g}{\partial \varphi}\right] \pm \sqrt{\frac{1}{4}\left(\frac{1}{3H}\left[\frac{\partial V}{\partial \varphi} + 2\frac{\partial}{\partial \varphi}\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} - \frac{\partial L_g}{\partial \varphi}\right]\right)^2 - 6\left(\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} + \left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=1;\,\nu=1}\right)} \quad (34)$$

Now if $\partial_\varphi V = 0$ and $H^2 = V/3$

Then we get ;

$$\dot{\varphi} = -\frac{1}{6H}\left[2\frac{\partial}{\partial \varphi}\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} - \frac{\partial L_g}{\partial \varphi}\right] \pm \sqrt{\frac{1}{4}\left(\frac{1}{3H}\left[2\frac{\partial}{\partial \varphi}\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} - \frac{\partial L_g}{\partial \varphi}\right]\right)^2 - 6\left(\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} + \left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=1;\,\nu=1}\right)}$$

Now again from equation 30 we can write it as ;

$$\dot{\varphi} = -\frac{1}{6\sqrt{8\pi G/3\,[\,½\,\dot{\varphi}^2 + V(\varphi) - L_g + 2\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0}\,]}}\left[2\frac{\partial}{\partial \varphi}\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} - \frac{\partial L_g}{\partial \varphi}\right] \pm \sqrt{\frac{1}{4}\left(\frac{1}{3\sqrt{8\pi G/3\,[\,½\,\dot{\varphi}^2 + V(\varphi) - L_g + 2\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0}\,]}}\left[2\frac{\partial}{\partial \varphi}\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} - \frac{\partial L_g}{\partial \varphi}\right]\right)^2 - 6\left(\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} + \left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=1;\,\nu=1}\right.}$$

Using the condition for slow roll mechanism we get ;

$$\dot{\varphi} = -\frac{1}{6\sqrt{8\pi G/3\,[\,V(\varphi) - L_g + 2\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0}]}}\left[2\frac{\partial}{\partial\varphi}\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} - \frac{\partial L_g}{\partial\varphi}\right] \pm$$

$$\sqrt{\frac{1}{4}\left(\frac{1}{3\sqrt{8\pi G/3\,[V(\varphi) - L_g + 2\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0}]}}\left[2\frac{\partial}{\partial\varphi}\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} - \frac{\partial L_g}{\partial\varphi}\right]\right)^2 - 6\left(\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} + \left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=1;\,\nu=1}\right)}$$

(35)

So from this equation we see whether the degree of anisotropy is considerable or not and due to very low value of G ; $\dot{\varphi}$ is becoming too high but at the starting of this calculation for slow roll mechanism it was already considered that this friction term is small and is not capable of changing $V(\varphi)$ significantly. So it seems this friction parameter is considered small initially and showing very large value at the end. This is an ambiguity. It is better to consider the G as a variable of time which has high value during inflation and decreases with increase of time. Thus this ambiguity is resolved with small value of anisotropy in cosmological model.

- **Proof of the coupling between the anisotropy and Scaler field and it's potential**:

From equation 10 we can write as

$$H^2 = 8\pi G/3\,[\,\tfrac{1}{2}\dot{\varphi}^2 + V(\phi) - L_g + 2\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0}] \qquad \text{and ;}$$

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -8\pi G/3\,[\,\dot{\varphi}^2 - V(\varphi) + L_g + \left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} + 3\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=1;\,\nu=1}]$$

(36)

Now from this above two equation we can get the potential as

$$V(\varphi) = \frac{3H^2}{8\pi G}\left[1 + \frac{\dot{H}}{3H^2}\right] - [L_g - \left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=0;\,\nu=0} + \left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=1;\,\nu=1}] \qquad (36a)$$

And

$$\varphi = \int dt[-\frac{\dot{H}}{4\pi G} - 2[\left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)^{\cdot}_{\mu=0;\,\nu=0} + \left(\frac{\partial L_g}{\partial g^{\mu\nu}}\right)_{\mu=1;\,\nu=1}]]^{1/2} \qquad (36b)$$

Now from slow roll parameter we can deduce a relation between scaler field and it's potential as

$$V(\varphi) = V_0 \exp[-\sqrt{\frac{16\pi}{p}}\frac{\varphi}{m_{pl}}] \qquad (36c)$$

Here p comes from power law expansion concept as($a = a_0 t^p$)

From the set of equations we can easily observe that the scaler field potential and anisotropy are coupled and the decrease in anisotropy causes the change of scaler field potential. Now to get the exact evolution profile we need to get any coupling relation between anisotropy and viscosity.

6. Discussion and Conclusion

Here we have reached the following conclusions and predictions.

- The coefficient of viscosity should increase before inflation and should decrease after inflation as the scalar field potential is coupled with viscosity.
- The gravitational constant G should vary with time.
- The variation of scalar field potential with respect to scalar field will always remain unchanged.
- The evolution profile of scalar field potential w.r.t time will not be effected too much due to modify the gravity.
- Scaler field changes with the change of viscosity.

From the sets of equations 17(a), 17(c) ,36(a) and 36(c) we get the scalar field potential necessary for accelerated expansion. We have also established the relationship between viscosity , scalar field potential and scalar field. Here we have followed mainly power law expansion idea.

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