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Article

Conceptual framework for quantum affective computing and its use in fusion of multi-robot emotion

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Abstract: This study presents a modest attempt to interpret, formulate, and manipulate emotion of robots within the precepts of quantum mechanics. Our proposed framework encodes the emotion information as a superposition state whilst unitary operators are used to manipulate the transition of the emotion states which are recovered via appropriate quantum measurement operations. The framework described provides essential steps towards exploiting the potency of quantum mechanics in a quantum affective computing paradigm. Further, the emotions of multi-robots in a specified communication scenario are fused using quantum entanglement thereby reducing the number of qubits required to capture the emotion states of all the robots in the environment, and fewer quantum gates are needed to transform the emotion of all or part of the robots from one state to another. In addition to the mathematical rigours expected of the proposed framework, we present a few simulation-based demonstrations to illustrate its feasibility and effectiveness. This exposition is an important step in the transition of formulations of emotional intelligence to the quantum era.

Keywords: Affective computing; Quantum emotion; Emotion fusion; Social robots

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1. Introduction

Affective computing is the study and development of systems and devices that can recognise, interpret, process, and simulate affective phenomena [1]. Although some kernels in the field may be traced back to early philosophical inquiries into emotion [2], the more modern branch of computer science originated with Picard's extensive works in [3] and flourished into an interdisciplinary field spanning computer science, psychology, and cognitive science [4]. One of the motivations for the field is the ability to give machines emotional intelligence so that the machine could interpret emotional states of humans and adapt its behaviour to them as well as providing an appropriate response to those emotions [6].

In artificial intelligence, interaction between artificial systems and their users is improved by making such systems not just intelligent but also emotionally sensitive [11]. Research in affective computing is encumbered by fundamental issues of seriality in the computational hardware whereas the human brain fundamentally works differently [7]. The physical make-up of the human brain (often divided into rational and emotional brains) is such that information is simultaneously processed rationally and emotionally, while in the area of social robotics, this conundrum clumsiness in a robot's behaviour, i.e., conventional computers cannot effectively process information in parallel. This would suggest that incongruity of using such frameworks to model the human affective system, which is known to be capable of manifesting concurrent emotional experiences (for example, happy and sad) [9].

Exploiting the overwhelming superiority that quantum computing offers [24], numerous studies exploring the use of quantum mechanisms in affective computing have been proposed. For example, in [28], Schwartz et al. enthused the need for current neuropsychology to incorporate the mathematics of quantum physics in accounting for human observational bias in the measurement of physical properties of the human brain. Building on his previous efforts, Aerts asserted that a number of quantum mechanical principles are at the origin of specific effects in cognition, and based on which a general hypothesis about the quantum structure of human thought was investigated [8]. Similarly, Narens et al. argued that psychology may not be effective in transferring contextual dialogues into probabilistic models but found solace in the belief that quantum probability theory could better handle the dynamics of contextual impact on behaviours [29].

Following these insights, in [34], Lukac et al. provided a quantum mechanical model of robot-specific emotions based on quantum cellular automata, where a robot is described as a set of functional agents each acting on behalf of its own emotional function. Years later, Raghuvanshi et al. proposed concepts that apply quantum circuits to model fuzzy sets, which birthed a new method to model emotional behaviour for a humanoid robot [35]. More recently, in [36], Yan et al. proposed a framework portraying emotion transition and fusion, which is encoded as quantum emotion space with three entangled qubit sequences to locate an emotion point within a three-dimensional emotion space. They surmised that such a framework facilitates easier tracking of emotion transitions over different intervals in the emotion space. However, as a drawback, their definition of emotion relies on the two qubit sequences, which is insufficient when more sophisticated emotions are desired.

To summarise, to date, efforts to integrate quantum mechanics into different aspects of affective computing can be divided into two groups: (1) applications that exploit some of the properties responsible for the potency of quantum computing algorithms as tools to improve some available affective computing tasks (mainly psychologists and physiologists' interest); and (2) applications that derive their inspiration from the expectation that quantum computing hardware will soon be physically realised and, hence, such studies focus on extending affective computing tasks to the quantum computing framework (typically physicists and engineers are interested in). These directions motivate two fundamental questions regarding the notion of quantum emotion: First, what advantages do the resulting quantum emotion offer us? And, second, how does this "quantumness" relate to possible descriptions of future quantum robots [13]?

While there may be other questions, answering these two questions as well as exploring the integration of some of the remarkable advances recorded in quantum machine learning [32], quantum artificial intelligence [37], and quantum image processing [25] provide few of the other motivations for this study. For the first question, the use of quantum mechanics in describing emotion presupposes the potential use of parallel processing power and demand lower computing resources since quantum computing offers exponential storage and speed up relative to the required number of qubits as well as the sufficient (polynomial) number of CNOT gates that are needed to manipulate quantum emotion. Regarding the second question, by conceptualising a quantum robot as a mobile quantum system that includes an on-board quantum computer and its required ancilliary systems [27], it is reasonable to imagine that quantum robots will carry out tasks whose goals include specified changes and adjustments to emotions in conformity with the state of the environment or even carrying out measurements on the environment itself. Similarly, integrating quantum mechanics into descriptions of robot emotion supports potential development of quantum robots as well as its use to manipulate classical robot devices to perform advanced emotion-related actions [23].

Consequent upon the foregoing survey, in this study, we propose a conceptual framework for quantum affective computing (QAC) as a quantum computing unit within a quantum robot system (defined here as a quantum robot that includes many quantum computing units and each unit is supposed to accomplish specific tasks and exchange of information with each other). While affective

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computing is defined as presented at the beginning of this section, the primary purpose of QAC is to enhance the capacity for storing, processing, and retrieving the affective information in the human-robot interaction either by transitioning from traditional to quantum paradigms (e.g., used for quantum robots) or by complementing traditional procedures with quantum techniques (e.g., the quantum machine learning algorithm mentioned earlier). A QAC unit is one ingrained with the capability to receive external stimulus, its manipulation as quantum signals which can motivate (or compel) the quantum robot to carry out some actions on its environment, such as emotion expression and path planning. The first step in accomplishing this is formulating a representation for emotion within the precepts of quantum mechanics and then realising its manipulation and fusion for different applications in quantum-based affective computing.

The remainder of the study is organised as follows. In Section 2, we present some of quantum properties and propose our framework for QAC. In Section 3, we define the quantum representation of robot emotion and discuss their basic transitions. In Section 4, we discuss the intrigues involved infusing multiple robot emotions and illustrate both the fusion and retrieving procedures.

2. Conceptual framework of quantum affective computing

As discussed earlier, affective computing aims to leverage the capability of computing technologies to recognise, understand, represent, and adapt human's emotion towards building harmonious human-robot environment. As a prelude to our discussion on QAC, in this section, we highlight a brief background on quantum computing with focus on its bewildering properties of superposition and entanglement, and discuss how such properties to imitate human behaviour as well as working mechanisms the brain uses to process emotions. Based on this background, later in the section, we propose our conceptual framework of QAC.

2.1. Quantum effects in human behaviour

Analogous to the fundamental concept of classical computation, the bit, a quantum bit (or qubit) is the smallest unit of information in the quantum system [24]. The difference between bits and qubits lies in the latter's ability to propagate in a superposition state, which can be described as a unit vector in two-dimensional Hilbert space. As shown in Fig. 1(a), the vector can always be written as $|\psi\rangle = \alpha|\uparrow\rangle + \beta|\rightarrow\rangle$, where $|\uparrow\rangle$ and $|\rightarrow\rangle$ are orthogonal basis states and α and β are complex numbers for probability amplitudes. The probabilities for $|\psi\rangle$ to be in the $|\uparrow\rangle$ and $|\rightarrow\rangle$ states are, respectively, $|\alpha|^2$ and $|\beta|^2$, where $|\alpha|^2 + |\beta|^2 = 1$. Geometrically, this can be interpreted as the condition that the qubit's state is normalized to length 1. If we let $|\uparrow\rangle = |0\rangle$ and $|\rightarrow\rangle = |1\rangle$, then $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, so the states $|0\rangle$ and $|1\rangle$ are known as computational basis states, and they form an orthonormal basis for this vector space.

As espoused via the mechanism of human emotional response (depicted in Fig. 1(a)), when a stimulus is received from the outside world, there are usually two routes to transmit information to the brain and make corresponding responses. They are the short route from the thalamus to amygdala directly in emotional brain, and the long route from the thalamus to amygdala through the cortex in rational brain. Absence of information processing by the cortex in the short route makes it faster to execute, which ensures the rapid response to external stimulation. In contrast, stimulus information via the long route is processed by the sensory cortex, which is conducive to the control of emotion and the adoption of appropriate coping style. The main function of the amygdala in the brain is to produce emotions (which are adapted to various external information introduced into the neocortex of the brain). Similar to this, in robotic aspect of affective computing, i.e., receiving the stimulus and after the processing to make the corresponding response, received stimuli are processed to determine the best response. Consequently, interpretations of affective computing from the quantum viewpoint, i.e., QAC, entails assimilating the mechanism of the amygdala and its role in generating and responding to emotions within the precepts of quantum mechanics [20][21], for example, the state the amygdala is in can be represented as a vector (red arrow in Fig. 1(a)) of the two eigenstates in a Bloch sphere [9].

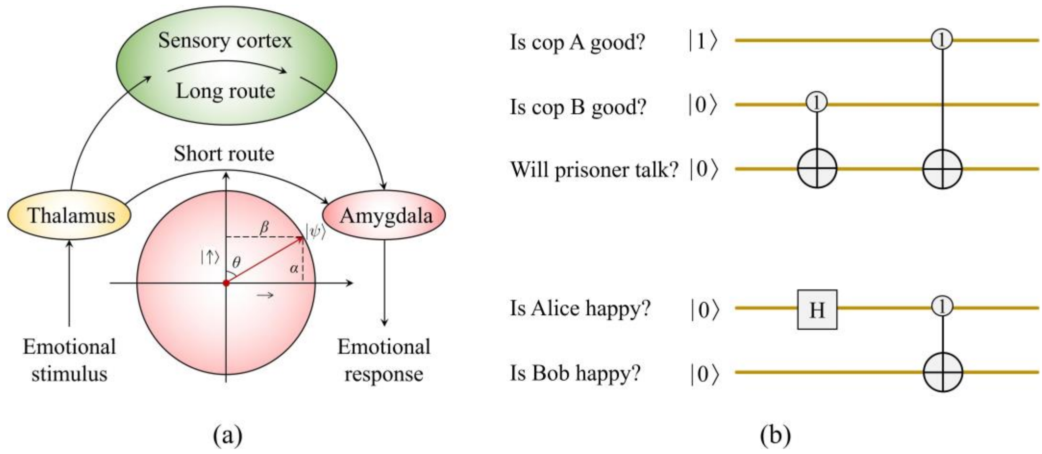

Fig. 1. Diagram of a qubit's superposition and quantum logic circuits.

In quantum computing, a quantum circuit is a computation model in which a computation is a sequence of quantum logic gates (or simply quantum gates) [18]. Quantum gates are represented by unitary matrices, suppose an operator U_f acting on a quantum state is a unitary matrix, i.e., $U_f U_f^\dagger = I$, where $U_f^\dagger$ is the conjugate-transpose matrix of U_f and I is an identity matrix as shown in Fig. 2. Since the unitary transform is reversible, it is logical that using quantum gates composed of the U_f and $U_f^\dagger$ transforms each of the input and output states can be reversed. Mathematically, this can be expressed via $U_f\,|x\rangle = |\,f(x)\rangle$ and $U_f^\dagger|\,f(x)\rangle = |x\rangle$. It is noteworthy that quantum parallelism allows quantum computers to simultaneously evaluate a function $f(x)$ for many values of x [24]. Under such a property, the fusion of quantum emotions (i.e. mixed emotions) for multi-robots and their emotion processes can be manipulated in parallel.

The notion of quantum circuits that correspond to affective behaviour is introduced by Perkowski et al. in [35], i.e., to employ EPR (Einstein-Podolsky-Rosen) circuits to set qubits into an entangled state to influence the human behaviour. For instance, the first circuit in Fig. 1(b) presents the behaviour of the prisoner. The prisoner is willing to talk when one cop is good and one cop is bad (i.e. utilises a strategy combing both "hard " and "soft" means), otherwise, his willingness level does not chance, i.e. the prisoner will never talk. Further, let's see another example to show the behaviour between Alice and Bob using the second circuit in Fig. 1(b). If we define 1 as indicating happy and 0 as unhappy, after the quantum gate operation, the behaviour between Alice and Bob is entangled. So if both Alice and Bob are unhappy, so the input is $|00\rangle$ and output becomes $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$, there is one half probability that they will talk to each other and still be unhappy, and there is one half probability that they will talk to each other (and perhaps they comfort and encourage mutually), so they will both become happy.

The final step in any quantum computation task is the measurement operation, which converts the transformed quantum information into the classical information in form of probability distributions [31]. Even though a qubit can represent many states, when it is observed, the measurement results can only be either 0 or 1, with a certain probability. This operation converts a single qubit state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ into a probabilistic classical bit, which is 0 with probability $|\alpha|^2$ or 1 with probability $|\beta|^2$. In affective computing that considers quantum mechanics, quantum properties allow for existence of multiple emotions in the Hilbert space represented by the phases. After measurements, however, only one emotion will be observed. This is analogous to the human behaviour where in the human mind, there are numbers of emotions, but the actions typically represent one emotion [35].

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Gate	Equation	Matrix	Transform	Notation
Identity ($\boldsymbol{I}$)	$\boldsymbol{I} = \|0\rangle\langle 0\| + \|1\rangle\langle 1\|$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\boldsymbol{I}\|0\rangle = \|0\rangle$ $\boldsymbol{I}\|1\rangle = \|1\rangle$	I
Pauli-X (X or **NOT**)	$\boldsymbol{X} = \|0\rangle\langle 1\| + \|1\rangle\langle 0\|$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\boldsymbol{X}\|0\rangle = \|1\rangle$ $\boldsymbol{X}\|1\rangle = \|0\rangle$	X
Hadamard ($\boldsymbol{H}$)	$\boldsymbol{H} = \frac{\|0\rangle + \|1\rangle}{\sqrt{2}}\langle 0\| + \frac{\|0\rangle - \|1\rangle}{\sqrt{2}}\langle 1\|$	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$\boldsymbol{H}\|0\rangle = \frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)$ $\boldsymbol{H}\|1\rangle = \frac{1}{\sqrt{2}}(\|0\rangle - \|1\rangle)$	H
Controlled-NOT (**CNOT**)	$\mathbf{CNOT} = \|0\rangle\langle 0\| \otimes \boldsymbol{I} + \|1\rangle\langle 1\| \otimes \boldsymbol{X}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$\mathbf{CNOT}\|00\rangle = \|00\rangle$ $\mathbf{CNOT}\|01\rangle = \|01\rangle$ $\mathbf{CNOT}\|10\rangle = \|11\rangle$ $\mathbf{CNOT}\|11\rangle = \|10\rangle$	
Toffoli ($\boldsymbol{T}$ or **CCNOT**)	$\mathbf{T} = \|0\rangle\langle 0\| \otimes \boldsymbol{I} \otimes \boldsymbol{I} + \|1\rangle\langle 1\| \otimes \mathbf{CNOT}$	$\begin{pmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&1&0 \end{pmatrix}$	$\mathbf{T}\|000\rangle = \|000\rangle$, $\mathbf{T}\|001\rangle = \|001\rangle$ $\mathbf{T}\|010\rangle = \|010\rangle$, $\mathbf{T}\|011\rangle = \|011\rangle$ $\mathbf{T}\|100\rangle = \|100\rangle$, $\mathbf{T}\|101\rangle = \|101\rangle$ $\mathbf{T}\|110\rangle = \|111\rangle$, $\mathbf{T}\|111\rangle = \|110\rangle$	

Fig. 2. Commonly used quantum gates.

2.2. Quantum framework of affective computing

In [12], Dong et al. presented the structural background of a quantum robot including description of its fundamental components and frameworks for multi-quantum computing units (MQCU), quantum actuator, and information acquisition units. Based on this structure, a quantum robot uses the information acquisition units to perceive its environment and acquire information, and then send sensing information to the MCQU. An MQCU is made up of many quantum computing units (QCUs) and, as well as accomplishing specified tasks, QCU exchanges information with each other through a quantum bus (which may be a refreshable entanglement resource [16]). The MQCU processes the information and generates new signals or learning control algorithms that are sent to the quantum actuator until the task is accomplished. Finally, the quantum actuator receives information from the MQCU and carries out specified actions on its environments, which is regarded as an interaction channel between a quantum robot and its environments (i.e., in addition to the information acquisition units).

Moreover, quantum robots have communication interfaces to exchange information with distant mainframes or other quantum robots, which can constitute a multi quantum robot system as shown in Fig. 4. With external communication, quantum information can be exchanged and the advantages of quantum communication such as high channel capability, perfect security and quantum teleportation can be fully utilized [12]. These features could be also used to facilitate the fusion of emotion for multi-robots in a communication scenario, which will be discussed in Section 4.

Guided by the intuition highlighted above, we present the framework of QAC which is regarded as a unit within the MQCU. This framework is envisioned as a QCU that facilitates exchange of information with other components of the MQCI. To encapsulate this intuition, the proposed framework is built on the use of several hypothetical quantum devices, such as emotion generator (EG), emotion controller (EC), emotion sensor (ES), and emotion reader (ER), as required to achieve the initialisation, manipulation, and retrieval of emotions. As presented in Fig. 3, the EG initialises the quantum emotion by transforming the quantum system from its initial state (i.e., an array of $|0\rangle$ states) to a specified emotion state. Usually, this information consists of both quantum and classical information, which precludes frequent interactions with the outside world since quantum

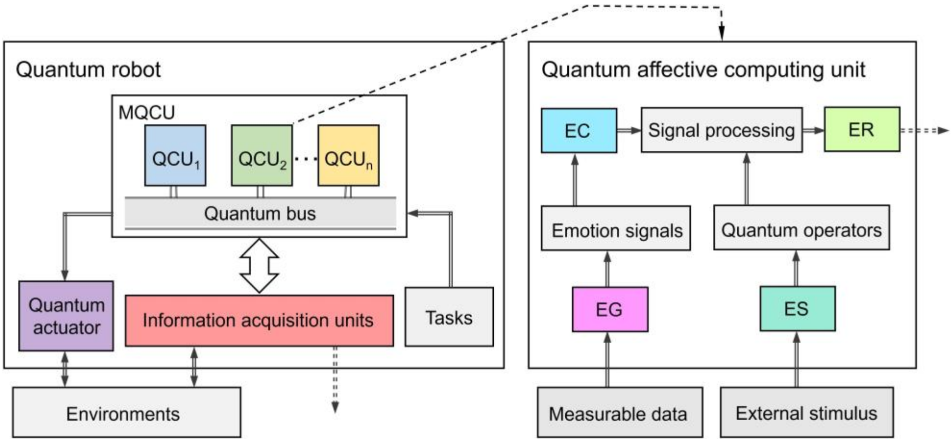

Fig. 3. Quantum robot system showing three interacting components and framework depicting QAC unit.

measurement destroys the quantum system. This pitfall can be avoided by adopting either of two viewpoints to affective computing. First, from the point of view of physiology, we may record the human body's various physiological parameters by means of various measurements, e.g., data on human motions, facial expressions, pulses, and brain waves. Subsequently, all such information can be used to compute the human's emotional status. Second, based on the psychological viewpoint, where different sensors can be used to receive and process the data. According to this, information to compute the robot's instantaneous emotional state can be computed. The ES receives the stimulus (such as an environment with music playing [22]) received from the external world into a set of unitary operations in a format that is amenable to manipulation using quantum resources. In this regard, a quantum sensor is a kind of microstructural sensor, that is designed by applying quantum effect. To detect faint classical signals (external stimulus), as presented in [12], two types of quantum sensors, superconduction quantum interference device sensor [14] and quantum well Hall sensor [15] can be considered. The EC transforms and controls the quantum emotion signal using quantum operators, i.e., quantum gates. The quantum controller acts as the connection between the current emotion signals and the external stimulus (quantum operators) to produce the desired/undesired signal processing outcomes. Usually, the quantum controllers are a combination of quantum gates that can the rapidly developing quantum control theory [17] in its design. Finally, the ER retrieves the quantum emotional state of the robot by applying quantum measurement operations. This could be a pure quantum apparatus or a hybrid semiclassical apparatus capable of reading quantum information [30]. Final choice of either would depend on how we intend to use the produced quantum signals. For example, it may be used in the quantum system to direct an actuator to carry out corresponding actions on external environments or a readout to control some classical device.

3. Quantum representation of robot emotion

A traditional two-dimensional pleasure-arousal (PA) plane is characterised by a pleasure-displeasure and an arousal-sleep plane that indicate both the specific emotions and general features common to many different emotions [26]. In addition, unlike the expression of sensitive feelings in human communication, discussions about robot emotion are relatively straightforward. Therefore, we modify the original PA plane in [26] and represented in the form depicted in Fig. 5(a). As seen in that figure, there are four quadrants that are labelled as "Excitement", "Distress", "Depression", and "Contentment". Since our study is focused on the robot emotion, each quadrant of the modified PA plane can be temporarily roughly divided into two key emotions illustrated in the figure.

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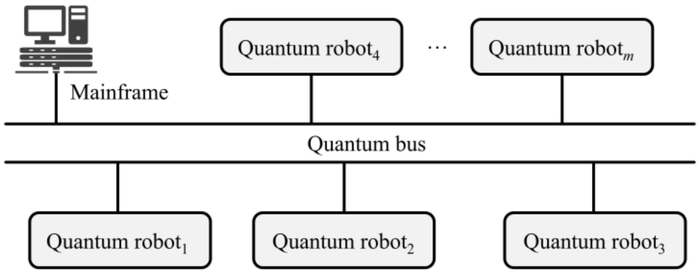

Fig. 4. Quantum system for multi-robots in a communication scenario (figure adapted from [12]).

We adopt the PA plane with 8 emotions (using angles in different intervals) in the four quadrants as shown in Fig. 5(b). The colours are selected according to established psychological interpretations, and the colour information corresponds to the selected emotion for visualisation [5]. It is noteworthy that the researchers in different fields have different opinions about the color coding of emotion. The correspondence used in this study is only for the visualization of these emotions to make the proposed module more understandable. The truly-used parameters are the angles presented in Fig. 5(b), which are involved into the quantum superposition states. The type of emotions is chosen according to the coordinate values in the original PA plane. Since angle values in the range 0 to 2π are continuous, by this spectrum, we can add more intervals (representing more emotions) according to the situation.

Recalling the equation $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ and its the discussion in Section 2, we let $\alpha = \cos\theta$ and $\beta = \sin\theta$, $\theta \in [0, 2\pi)$, then $|\psi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$. Further, if we let $\xi = \theta/4$, then $\xi \in [0, \frac{\pi}{2}]$. We do this because the trigonometric function is monotonically changing in the period. In this case, we need to proportionally map the emotions in the range 0 to 2π to a new interval of $[0, \pi/2]$. Since the angles in these two intervals are continuous, bijection is a useful tool to accomplish the desired transformation. In other words, we can map all the eight types of emotions via the angles in the interval of $[0, \pi/2]$. Consequently, mathematically, the quantum emotion of a robot can be expressed as

$$|E(\xi)\rangle = \cos\xi|0\rangle + \sin\xi|1\rangle, \tag{1}$$

where $\xi \in [0, \frac{\pi}{2}]$ is regarded as the emotional parameter (or variable); $|0\rangle$ and $|1\rangle$ are 2-D computational basis quantum states. Following this formalism, it is important to discuss how a desired quantum emotion state of a robot can be generated. Given an angle $\xi \in [0, \frac{\pi}{2}]$, a rotation matrix (rotation around the y-axis of a Bloch sphere by angle 2ξ) is defined as

$$R_y(2\xi) = \begin{pmatrix} \cos\xi & -\sin\xi \\ \sin\xi & \cos\xi \end{pmatrix}, \tag{2}$$

which transforms the computational basis state into the desired quantum emotion state, expressed as

$$R_y(2\xi)|0\rangle = \begin{pmatrix} \cos\xi & -\sin\xi \\ \sin\xi & \cos\xi \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \cos\xi|0\rangle + \sin\xi|1\rangle. \tag{3}$$

Now, to transform the emotional state, we consider several single quantum gates and apply them on the quantum emotion state in Eq. (1). X gate exhibits the property that $X|0\rangle = |1\rangle$ and $X|1\rangle = |0\rangle$; so, when it is applied on an emotion state, its impact can be expressed as

$$X(|E(\xi)\rangle) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \cos\xi \\ \sin\xi \end{pmatrix} = |E\left(\frac{\pi}{2} - \xi\right)\rangle, \tag{4}$$

where $|E(\xi)\rangle$ is the emotion state as defined in Eq. (1). The function of the X gate is like the emotion inversion operation that flips every given emotion.

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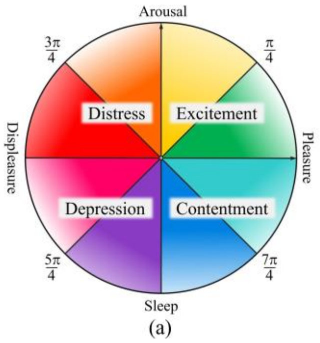
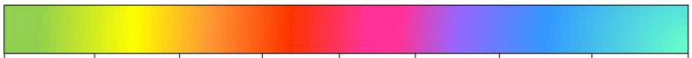


(a)



Green	Yellow	Orange	Red	Pink	Purple	Blue	Indigo
$[0,\frac{\pi}{2})$		$[\frac{\pi}{2},\pi)$		$[\pi,\frac{3\pi}{2})$		$[\frac{3\pi}{2},2\pi)$	
Excitement		Distress		Depression		Contentment	
$[0,\frac{\pi}{4})$	$[\frac{\pi}{4},\frac{\pi}{2})$	$[\frac{\pi}{2},\frac{3\pi}{4})$	$[\frac{3\pi}{4},\pi)$	$[\pi,\frac{5\pi}{4})$	$[\frac{5\pi}{4},\frac{3\pi}{2})$	$[\frac{3\pi}{2},\frac{7\pi}{4})$	$[\frac{7\pi}{4},2\pi)$
Happiness	Surprise	Fear	Anger	Sadness	Boredom	Relaxation	Expectation

(b)

Fig. 5. Description of the modified PA plane showing (a) four quadrants and (b) 16 emotions as well as colour and angle interval.

Mathematically, the Z gate has the properties that $Z|0\rangle = |0\rangle$ and $Z|1\rangle = -|1\rangle$; so, when it is applied on the emotion state, we have the transformation:

$$Z(|E(\xi)\rangle) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \cos\xi \\ \sin\xi \end{bmatrix} = |E(-\xi)\rangle, \tag{5}$$

whose function is the negation operation, i.e., to change the sign of the angle. On the QAC framework, when combined with other transformations, this operation produces many useful outcomes. This will be discussed later.

The H gate executes the transformation $H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. Therefore, when it is applied on the emotion sate the outcome can be expressed as

$$H(|E(\xi)\rangle) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \cos\xi \\ \sin\xi \end{bmatrix} = |E(\frac{\pi}{4} - \xi)\rangle. \tag{6}$$

As we shall see later, this operation can be used to neutralise an emotional state.

The general form resulting from the combination of the three transformations (i.e. X, Z, and H) can be expressed as a unitary matrix in the form:

$$C(2\eta) = \begin{bmatrix} \cos\eta & \sin\eta \\ \sin\eta & -\cos\eta \end{bmatrix}, \tag{7}$$

where $\eta \in [0, \pi/2]$. When applied on the emotion state of a robot, the $C(2\eta)$ operator transforms the emotion to a state:

$$C(2\eta)(|E(\xi)\rangle) = \begin{bmatrix} \cos\eta & \sin\eta \\ \sin\eta & -\cos\eta \end{bmatrix} \begin{bmatrix} \cos\xi \\ \sin\xi \end{bmatrix} = |E(\eta - \xi)\rangle. \tag{8}$$

This operation transforms a given emotion to a new one, within the bound encoded ξ and $\eta - \xi$, respectively. As inferred in Eq. (5), combining this operation with the Z gate changes the emotion to a new state whose angles is $\eta + \xi$. Moreover, transformations, X, Z, and H, are the special cases of $C(2\eta)$ in which η is equal to $\pi/2$, 0, and $\pi/4$, respectively.

Furthermore, by using the definition of $C(2\eta)$, that of the rotation matrix in Eq. (2) and the property of the Pauli Z matrix, we obtain the useful operation exhibited as

$$C(2\eta) = \begin{bmatrix} \cos\eta & -\sin\eta \\ \sin\eta & \cos\eta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = R_y(2\eta) \cdot Z. \tag{9}$$

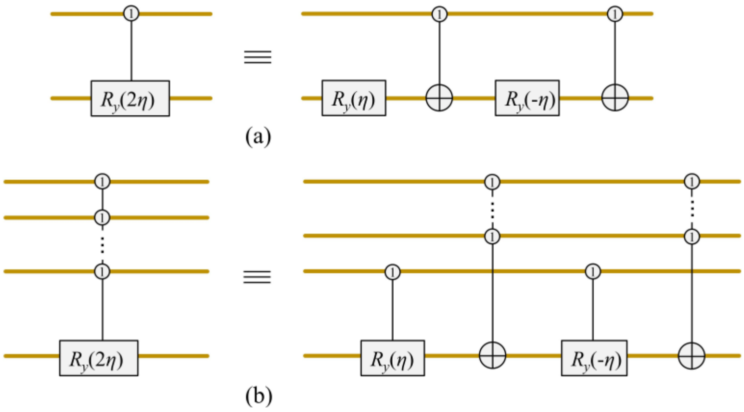


Fig. 6. Circuit implementation of controlled-$R_y\,(2\eta)$ operation.

An implication arising from Eq. (9) is that trivial tasks to show increase or decrease of emotion from ξ to $\xi + \eta$ or from ξ to $\xi - \eta$ can be achieved by using $R_y(2\eta)$ or $R_y(-2\eta)$, respectively. These two operations are expressed by

$$R_y(2\eta)|E(\xi)\rangle = |E(\xi + \eta)\rangle, \tag{10}$$

and

$$R_y(-2\eta)|E(\xi)\rangle = |E(\xi - \eta)\rangle. \tag{11}$$

The matrix $R_y(2\eta)$ has unit determinant and exhibits the following properties:

$$\begin{aligned} R_y(2\eta) \cdot R_y(-2\eta) &= I, \\ R_y(-2\eta) &= X \cdot R_y(2\eta) \cdot X, \\ R_y(2\eta_1 + 2\eta_2) &= R_y(2\eta_1) \cdot R_y(2\eta_2). \end{aligned} \tag{12}$$

Following the above mathematical formalism, in the quantum circuit model of computation, we can use circuit elements can be used to illustrate the execution of the same operations highlighted earlier. The $R(2\eta)$ operation can be realised using a combination of two rotation gates and two controlled-NOT (CNOT) gates as presented in Fig. 6(a). Here, when the control wire has an input $|0\rangle$ state, the operation $R(\eta) \cdot R(-\eta)$, which is equal to the identity operation is executed. This translates to the first property in Eq. (12). Therein, when the control wire is in the $|1\rangle$ state, the operation $R(\eta) \cdot X \cdot R(-\eta) \cdot X$ is equal to the $R(2\eta)$ that is executed. An extended version of this operation with multiple control conditions is presented in Fig. 6(b).

In quantum computing, for the discussion of the circuit implementation and complexity evaluation of the circuit-based operations, the decomposition of seemingly complicated circuits (on the left) into simpler circuit networks composed entirely of basic or elementary quantum gates (on the right), i.e. NOT, CNOT, and Toffoli gates (as shown in Fig. 2), is illustrated in Fig. 7. Furthermore, as indicated in Fig. 7(c), an n-controlled NOT gate can be decomposed into $2(n\text{-}1)$ Toffoli gates as well as 1 CNOT gate, and 1 Toffoli gate can be further approximately simulated by 6 CNOT gates [25].

4. Fusion of quantum emotions for multi-robots

Emotion fusion entails the combination (mixture) of the emotions of multi-robots in a specified communication scenario or environment. The emotion fusion state of all robots (or part of them) can be controlled or changed using external stimulus.

We assume an enclosure (such as a room) containing N robots, and each robot is identified by a number ($k = 0, 1, 2 \cdots N - 1$). To simplify things, we divide the enclosure into $2^n \times 2^n$ lattices

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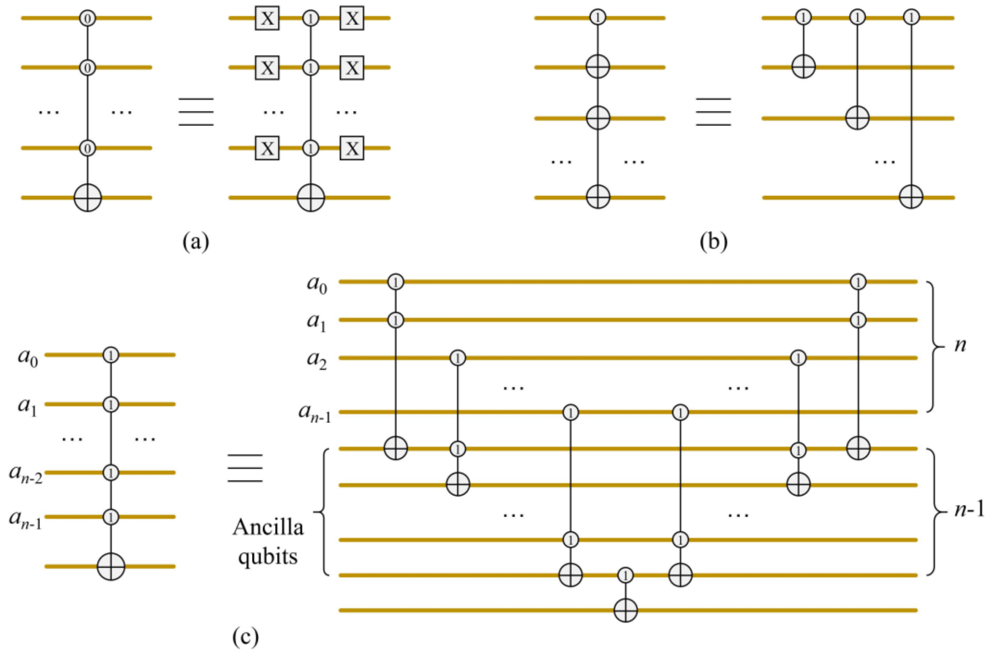

Fig. 7. Circuit implementation of controlled-R_y (2η) operation.

($N = 2^n \times 2^n$), as shown in Fig. 8. Meanwhile, Fig. 8(a) shows the multi-robots in a room, while the X, Y, and Z axes constitute H, W, and V planes. As an example, Fig. 8(b) shows, the details of the H plane where the rows and columns along the plane indicate the levels of the qubits encoding the emotions in the affected area. For instance, spatially the robots in the square "ABCD" can be presented by $|y_1y_0x_1x_0\rangle = |1100\rangle$, while those in the square "AEFG" comprising of four robots can be simultaneously controlled by the qubit sequence $|y_1x_1\rangle = |10\rangle$. This illustrates that the more robots we want to control, the less qubits we need.

Therefore, the emotion fusion of these N robots can be represented as

$$|\widetilde{E}(\xi)\rangle = \frac{1}{2^n} \sum_{k=0}^{2^{2n}-1} (\cos \xi_k |0\rangle + \sin \xi_k |1\rangle) \otimes |k\rangle, \tag{13}$$

$$\xi_k \in \left[0, \frac{\pi}{2}\right], k = 0, 1, \cdots, 2^{2n} - 1, \tag{14}$$

where $\otimes$ is the tensor product notation, $|k\rangle$, $k = 0, 1, \cdots, 2^{2n} - 1$, are 2^{2n}-D computational basis quantum states and $\xi = (\xi_0, \xi_1, \cdots, \xi_{2^{2n}-1})$ is the vector of angles encoding emotions. There are two parts in a typical scenario for multi-robot emotion representation: $\cos \xi_k |0\rangle + \sin \xi_k |1\rangle$ encodes the emotional information and $|k\rangle$ is about the corresponding locations (i.e., the identity) of each robot in the scenario, respectively. Therefore, to facilitate the emotion fusion of these N robots requires $2n$+1 qubits, where $2n$ qubits specify the robot's spatial location in the enclosure and the remaining one qubit is used to identify the emotional states of all robots, i.e., the controlled condition operations applied on the locations (Y and X axes) are used to confine the present emotion to a targeted robot. In this manner, transformations on a multi-robot quantum atmosphere facilitate parallel manipulation (communication atmosphere is described as the psychological factor and feeling that can affect the behavior process and result in the space, which is usually obtained by fusing individual emotional states). Finally, the fused emotional state of many or multi-robots is a normalised state (i.e., similar to

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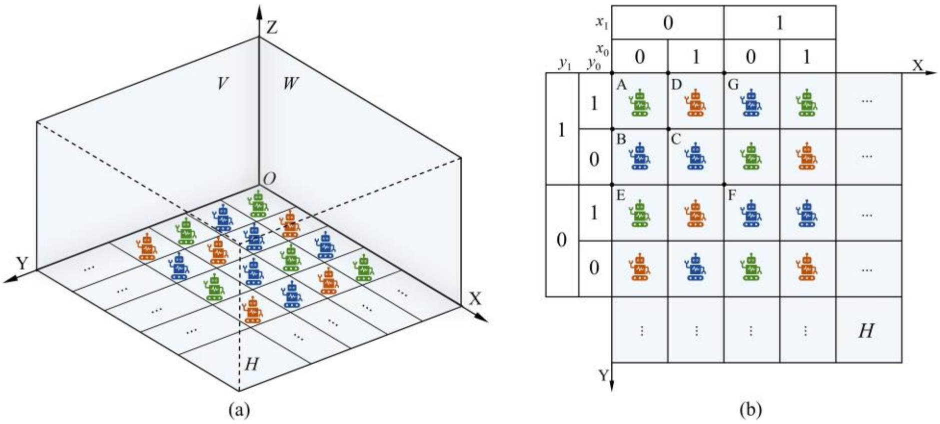

Fig. 8. Multi-robots in the same enclosure or environment (e.g., a room), and the quantum binary representation of the location of each robot in the room.

the one composed of intricacies of many humans in some environment scenario), i.e. $\||\widetilde{E}(\xi)\rangle\| = 1$ as given by

$$\||\widetilde{E}(\xi)\rangle\| = \frac{1}{2^n}\sqrt{\sum_{i=0}^{2^{2n}-1}(\cos^2\xi_k + \sin^2\xi_k)} = 1. \quad (15)$$

Next, we discuss how to realise the emotion fusion. Given a vector $\xi = (\xi_0, \xi_1, \cdots, \xi_{2^{2n}-1})$ of angles ($\xi_k \in [0, \frac{\pi}{2}]$, $k = 0, 1, \cdots, 2^{2n}-1$), we turn quantum computers from the initialized state, $|0\rangle^{\otimes 2n+1}$, to the quantum emotion state in Eq. (13), composed of a polynomial number of simple gates.

First, using the two-dimensional identity matrix I and the $2n$ Hadamard matrices $H^{\otimes 2n}$, the transform $\mathsf{H} = I \otimes H^{\otimes 2n}$ applied on $|0\rangle^{\otimes 2n+1}$ produces the state $|H\rangle$, presented as

$$\mathsf{H}(|0\rangle^{\otimes 2n+1}) = \frac{1}{2^n}|0\rangle \otimes \sum_{i=0}^{2^{2n}-1}|i\rangle = |H\rangle. \quad (16)$$

So far, we have formally separated the room into a $2^n \times 2^n$ lattices, i.e. each robot in the enclosure has a unique identification. Subsequently, we relate each robot a specified emotion.

The rotation matrices $R_y(2\xi_k)$ as defined in Eq. (2) and controlled-rotation matrices R_k with $k = 0, 1, \cdots, 2^{2n}-1$ are considered, i.e.,

$$R_k = \left(I \otimes \sum_{j=0, j\neq k}^{2^{2n}-1}|j\rangle\langle j|\right) + R_y(2\xi_k) \otimes |k\rangle\langle k|. \quad (17)$$

This controlled-rotation R_k is a unitary matrix, since $R_k R_k^\dagger = I^{\otimes 2n+1}$. Meanwhile, applying R_s and $R_t R_s$ on $|H\rangle$ produces:

$$\begin{aligned} R_s(|H\rangle) &= R_s\left(\frac{1}{2^n}|0\rangle \otimes \sum_{k=0}^{2^{2n}-1}|k\rangle\right) \\ &= \frac{1}{2^n}\left(I|0\rangle \otimes \sum_{k=0, k\neq s}^{2^{2n}-1}|k\rangle\langle k| \sum_{k=0}^{2^{2n}-1}|k\rangle \right. \\ &\quad \left. + R_y(2\xi_s)|0\rangle \otimes |s\rangle\langle s| \sum_{k=0}^{2^{2n}-1}|k\rangle\right) \\ &= \frac{1}{2^n}\left(|0\rangle \otimes \sum_{k=0, k\neq s}^{2^{2n}-1}|k\rangle + (\cos\xi_s|0\rangle + \sin\xi_s|1\rangle) \otimes |s\rangle\right), \end{aligned} \quad (18)$$

and

$$
\begin{aligned}
R_t R_s|H\rangle &= R_t(R_s|H\rangle) \\
&= \frac{1}{2^n}|0\rangle \otimes \Big(\sum_{k=0,k=s,t}^{2^{2n}-1} |k\rangle + (\cos\xi_s|0\rangle + \sin\xi_s|1\rangle) \otimes |s\rangle \\
&\quad + (\cos\xi_t|0\rangle + \sin\xi_t|1\rangle) \otimes |t\rangle \Big).
\end{aligned} \tag{19}
$$

In addition, it is clear from Eq. (19) that

$$
\mathsf{R}|H\rangle = \Big(\prod_{k=0}^{2^{2n}-1} R_k\Big)|H\rangle = |\widetilde{E}\rangle. \tag{20}
$$

Therefore, the unitary transform $\mathsf{P} = \mathsf{RH}$ turns a quantum computing hardware from the initialized state $|0\rangle^{\otimes 2n+1}$ to the emotion fusion state $|\widetilde{E}\rangle$. The total number of simple operations used to prepare the fusion emotion state is

$$
2n + 2^{2n} \times (2^{2n-1} - 1 + 2^{2n-1} - 2) = 2^{4n} - 3 \cdot 2^{2n} + 2n. \tag{21}
$$

Consequently, the computational complexity for fusing the emotional states of multi-robots can be calculated as $O(2^{4n})$.

While the storage of an emotional state is accomplished during the preparation process, the measurement of the quantum emotion state produces a probability distribution that is used to retrieve the emotion. As presented earlier, measurement of a quantum state produces only one result, which is one entry in a set of basis vectors. Therefore, with only one quantum state, it is impossible to obtain information from that state; hence, a measurement process requires many identical quantum states and multiple measurement operations on these identical states recovers information about the quantum state [33] which in our case is information about the emotion of each robot in the form of a probability distribution.

As discussed earlier, transformations on a multi-robot quantum emotion facilitate parallel manipulation in each enclosure or setting. In the sequel, we illustrate the use of control conditions to confine the rotation operation to transform the emotion of each robot or multi-robots in an environment. Suppose an 8×8 enclosure to accommodate 64 robots composed of a lattice that is divided into 4 squares, i.e., 16 robots in each square shown in the top figure in Fig. 9(a)). Further, we suppose that all the 16 robots in each square have the similar emotions that are encoded using the same colour. This is similar to a competition match, for example, there are four teams and usually each team has the analogous emotional states (e.g., expectation), and different team may have different emotions. After the match, the team's emotion will change according to the match result, e.g., the winner will be happy and the loser will be sad even anger. To encode this and further explain the example, we further divide the lattice into an upper and lower half, which can be used to constrict the operation by focusing on $R_y(-\pi/4)$ (using state "1" control condition) and $R_y(-\pi/2)$ (using state "0" control condition) to the upper and lower halves of the plane as shown in in Fig. 9(b), using which we can transform the emotion to its opposite state on the emotion spectrum. Together with Eq. (11), the bottom figure in Fig. 9(a) illustrates the original and resultant emotion states (we only use the central 2×2 lattice, i.e., 4 robots, to show their locations and change in emotional states).

Extracting and subsequently analysing the distributions of probability read outs resulting from measurement operations provides the information needed to retrieve a new emotion state. As mentioned in Section 2.1, the total probability of states $|0\rangle$ and $|1\rangle$ in an emotion state equals 1. Therefore, when the $|0\rangle$ is measured, the probability distribution of the kth robot can be recovered using

$$
Prob(k) = 2^{-2n}\cos^2\theta_k. \tag{22}
$$

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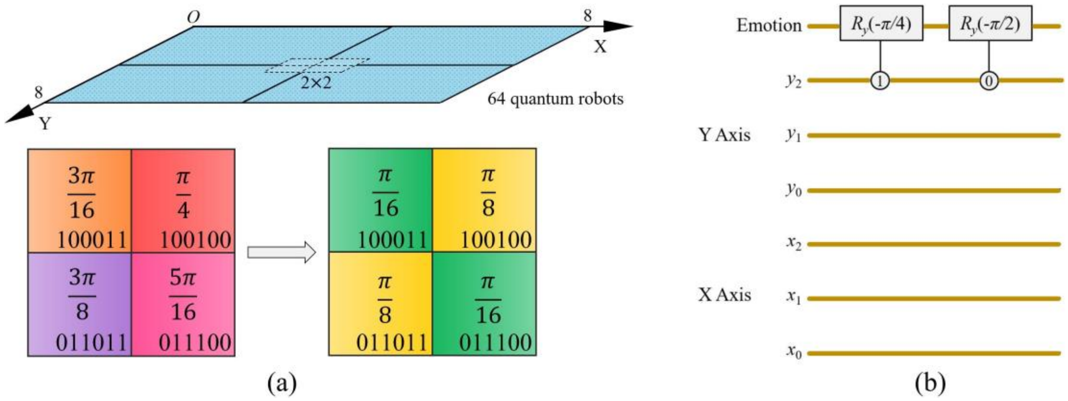

Fig. 9. Examples of 8×8 lattice divided into 4 quadrants that each includes 16 robots with similar emotion and circuit for transformation of its emotion.

Figure 10 presents the probability distribution for the fused emotion environment described earlier where (a) and (b) show the fused emotions prior to and after the transformation in Fig. 9 (i.e. via the circuit in Fig. 9(b)). These distributions provide a visual impact of the quantum rotation operation on the emotional states of each robot.

At the end of these applications, even though we have discussed the idea of using quantum techniques in the affective computing through the whole manuscript, we still recall how the quantum properties plays roles during these operation, especially for those who are interested in the affective computing research but not familiar with quantum. In addition to the usage of the module could be used in future quantum robotic system, we mainly mention two advantages this paradigm offers us. First, in Eq. (13), when we discuss the emotion states of N robots, we can see how the quantum superposition works. We only use one superposition qubit to encode the emotional state of all the N robots and use the control functions to differentiate each robot and its personal emotion. This will definitely reduce a lot of computing resources, especially, when a lot of robots are involved. In addition, as shown in Fig. 9 and by referring to the discussion in Section 2.1, quantum computation has the ability to allow the simultaneous evaluation of a function $f(x)$ for many values of x, which is called quantum parallelism. So, in the example shown in Fig. 9, performing the rotation operation once is capable to change the emotional state of all the robots in the system, and with the control conditions, two rotation operations could separately divide the lattice into two parts and make their own transformation. This also give us an statement that, regarding the relationship between the number of control conditions and the size of the affected area in the lattice. The more controls a transformation has, the smaller the size of the affected area. Specifying the area in which the transformation will be applied increases the complexity of the new transformation in terms of the depth and the number of basic gates in the corresponding circuit.

5. Concluding remarks

With increasing importance in quantum computing on one hand and machine learning on the other, combined with fast advancements in design of more and more sophisticated robots, it is obvious that at least some of the advanced future robots will be controlled by quantum computers. With a massive amount of data that will be generated by future sensors and rising complexity of control, planning, interacting, and reasoning systems, as well as the fact that future robots will operate in all kinds of environments, especially interacting with humans, it is widely accepted that quantum algorithms and especially quantum machine learning will play a major role in robotics and automated integrated systems. It is foreseeable that quantum algorithms, quantum sensors, and quantum controls will be the main approach to next generation of robotics research.

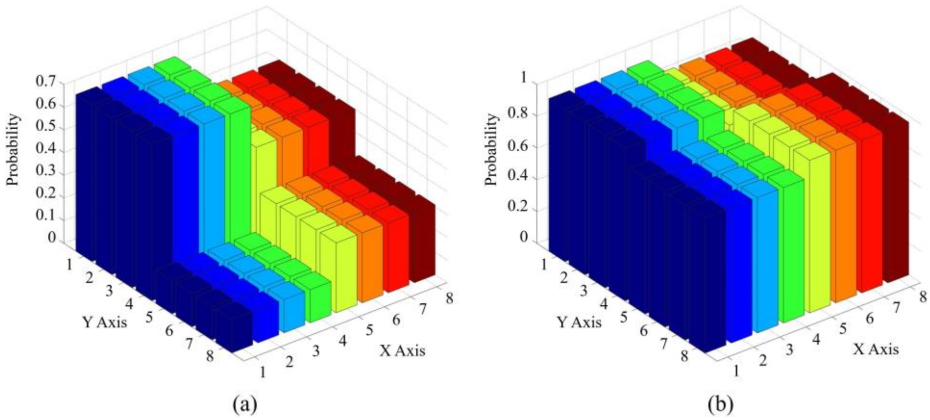

Fig. 10. Probability distribution obtained from the emotion before and after the application of the quantum rotation operation.

While the first ideas of quantum robots come from Benioff, he emphasized the importance of quantum computer in quantum robots, but the robot described there has no awareness of its environment and does not make decisions or measurements. In Dong's work which is accepted by most the engineers, they provide an alternative definition for quantum robots, which considers interaction with the external environment via sensing and information processing. A quantum robot is a mobile physical apparatus designed for using quantum effects of quantum systems, which can sense the environment and its own state, process quantum information and accomplish meaningful tasks. In such a engineering view, they formulated several fundamental components to compose a quantum robot for its information acquisition and communication.

This study explores the rudimentary processes for interpreting standard notations and operations in affective computing using properties of quantum computation. The main motivation are twofold: first is to arouse people to consider the affective computing both from the microcosmic and microcosmic aspects to discuss the ingenious and mysterious emotion and try to solve the usual engineering problems such as computing resources and computing speed, respectively, by using the quantum proptoses. Second, in Dong's quantum robot structure which is also accepted by most of the engineers, the multi-quantum computing units (MQCU) play significant roles, which is similar to the human brain. Even though the MQCU are coalesced in the system, but the concrete function and content therein are not designed and refined. We design one of them, a quantum affective computing (QAC) unit, to process the robot's emotion information and propose how this emotion can communicate with others (e.g., influenced by the environment or make emotional decisions) to make such a robot to become a cognitive or emotional robot. While facile in its presentation, the study presents a first step in quantum affective computing (QAC) where many of the bewildering properties of quantum computing could be coalesced into distributions and descriptions of quantum emotion as well as the fusion of emotions in different robot-robot and robot-human environments.

As advantages, among others, the parallel computing capability inherent to quantum computers ensures demand for fewer computing resources and considerable processing speed to execute the emotion representation and fusion of multi-robots in an enclosure on our proposed QAC framework. Meanwhile, as discussed in [38], signal processing by classical and quantum computers are completely different, e.g., on a classic computer, an $M \times L$ image can be represented as a matrix, encoded with at least 2^n bits $n = |log_2(ML)|$. Classical image transformation is conducted by matrix computation U. Alternatively, the same image can be represented as a quantum state and encoded in n qubits. Quantum image transformation is carried out by unitary evolution $\hat{U}$ under a suitable Hamiltonian. Therefore, the QAC will be useful for the development of the quantum robots, and of course, the other

QCUs (e.g., cognitive unit, since cognition and emotion have been fertilized each other leading to the robot's behaviours) will also make sense.

In the long road to harnessing the potency of quantum computation for affective computing, our ongoing and future work is motivated by the following considerations. First, the proposed QAC framework is supposed to be a dependent or interactive unit for exchange of information with other QCUs in the quantum robot system. Therefore, we may need to study how such a unit can communicate with others, including the necessary information acquisition, fusion, and distribution. IoW, we should define the interface of QAC unit with other QCUs to make the affective processing of quantum robots and its corresponding response seamless and practical. Second, the four hypothetical devices of the QAC framework need to be further studied to ascertain which technologies are suited for processing quantum sensory information. For example, typical multi-robot or robot-human scenarios as well as changes in emotion states are influenced by external factors and stimuli. Following that, how to design the ES to extract the useful information from the external stimulus and encode it for further computing in the quantum system. Third, in the fusion of multi-robot emotion, it is known that multiple measurement operations required to recover an emotion state would impose additional computing costs. This must be carefully studied for efficiency. Ancilla-driven and measurement-based quantum computation [30] are viable alternatives to the circuit-model of quantum computation that have been considered in similar quantum readers. Consequently, it is expedient to study required refinements for their integration to help provide required trade-off between accuracy and computational cost. Finally, considering the development of quantum computers, when we are expecting a full and true quantum computers to handle the intractable problems that traditional computers are unable to do, some peripheral devices to assist the traditional computers will be also important to make them more efficient. Similarly, as mentioned earlier, the signal encoding and processing between traditional and quantum computers are different, but if we could design a signal converter between them, the QAC module will also be possible to be applied into some available traditional affective computing systems to handle specific issues by making full use of its quantum properties.

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