

Entangled particles spinning on the black hole horizon

M. Bousder*

Lab of High Energy Physics, Modeling and Simulations,
Faculty of Science, University Mohammed V-Agdal, Rabat, Morocco

Abstract

In this paper, we present a technique to unify the Reissner–Nordström metric and the Kerr–Newman metric. We construct a specific model and calculate the entanglement entropy of black horizon. We are interested in the entangled particle and antiparticle spinning on the black hole horizon. The two Reissner–Nordström horizons r_{\pm} , are the results of the rotation of several entangled particle-antiparticle on the real horizon. The energy absorbed by a black hole is transformed into a kinetic energy of the entangled particle-antiparticles. This study provides a new type of black hole metric. We show that the rotation of an entangled system of a particle and an antiparticle can create a extremal black hole. We also explore some of the implications of this point of view for the black hole entanglement.

Keywords: black hole, entanglement, entropy.

1 Introduction

Juan Maldacena and Leonard Susskind, devised a theory linking two phenomena both discovered by Einstein: “Einstein-Rosen bridges” (or wormholes) and quantum entanglement. According to them, if we move the two entangled particles apart would in fact amount to digging an ER bridge around a single particle which would manifest its properties in several places in space-time. This theory sheds light on a problem called the EPR paradox which highlights the non-locality of quantum mechanics, Which he opposes to the principle of locality which is the basis of the theory of relativity. However,

*mostafa.bousder@gmail.com & mostafa.bousder@um5.ac.ma

this ER=EPR correspondence is only demonstrated in a very simplified universe model, where gravity is generated in the absence of mass [1]. The Hawking radiation of a black hole is a scrambled cloud of radiation entangled with the black hole [1]. In this paper, we are interested in studying a two entangled particles on the black hole horizon. It is well known that the "Kerr metric"[2] is a metric that describes a rotating black hole, which is static and axisymmetric. When the black hole has an electric charge, the Schwarzschild solution is no longer valid. A non-rotating black hole corresponds to an isotropic black hole of mass M and charge Q , which described by the "Reissner–Nordström metric" [3, 4]. For a charged black holes with $|Q| \ll M$, are similar to Schwarzschild black holes. The Reissner-Nordström black holes have two horizon, the innermost is a Cauchy horizon. It is believed that black holes with $|Q| \succ M$ don't exist in nature, since they would contain a naked singularity. Their existence would be in contradiction with the principle of cosmic censorship of the Roger Penrose [5].

The present paper is organized as follows: The second section introduces a concept Entanglement on the horizon. Section 3 involves the ntangled particle and antiparticles system on the horizon. In section 4 we introduce a unique description of the Reissner–Nordström metric and the Kerr–Newman metric. Section 5, is devoted to calculate the geodesic of a new metric which describes the entangled system on the horizon. We will conclude in the last section.

2 Entanglement on the horizon

We consider a C,auchy slice Σ of the black hole spacetime with Minkowski coordinates (t, \vec{x}) is divided into two parts Σ_+ and Σ_- . We assume that the "horizon" is at $x = -a$, the thin region (of the order of the Planck length) near the horizon becomes is at $x = -a + \epsilon$, where $\epsilon \sim 0$. The absolute value function $\chi \rightarrow |\chi|$ is continuous but is not differentiable in 0. We will use this property of the absolute function to describe the entangled states in the black hole horizon. We define a general field $|\chi|$ as a component of two fields:

$$|\chi| = (-\chi \text{ or } \chi) \quad (2.1)$$

we consider that $|\chi|$ is free scalar field in a background spacetime. Next, we considere the entanglement entropy between the outside and the thin region of the inside the horizon, based on the new field $|\chi|$

$$\partial_\chi |\chi| = (-1 \text{ or } 1) \quad (2.2)$$

this the derivative of $|\chi|$, seems to describe a two entangled states. It is possible to find both cases -1 and 1 at the same time at point O , i.e. we can find two states at a single

point on the surface Σ .

$$\partial_\chi |\chi| \equiv (-1)^\delta \quad (2.3)$$

there is a different tangent to the left and a tangent to the right at the point O , when $\delta = \{2k, 2k+1\}$ of $k \in \mathbb{Z}$. We can represent the creation of a right particle and a left particle in the horizon by the lines $|\chi|$. In this case, Eq.(2.3) also has a T -symmetry, For spin systems, the Eq.(2.3) obeys $\partial_\chi |\chi| \subset \exp(i4\pi s)$, where s is the spin of the state in the defect Hilbert space [7]. We suppose that a field χ describes a particle which creates outside the horizon, and that $-\chi$ describes the particle created inside. While the field $|\chi|$ can describe both particles at the same time. The field $|\chi|$ seems classical to on the parts Σ_L and Σ_R . But in the point O , $|\chi|$ becomes a quantum field. we associate to the field $-\chi$ and χ a state $|\Psi_+\rangle$ and $|\Psi_-\rangle$ respectively, in the basis $\{|-\rangle, |+\rangle\}$. The entanglement between the two states does not describe the complete field. Firstly, we associate with the field $|\chi|_O$ in the point O by entangled states:

$$|\Psi_\pm\rangle \sim |+\rangle |-\rangle \pm |-\rangle |+\rangle \quad (2.4)$$

this state is invariant under the \mathbb{Z}_2 symmetry: $|+\rangle \rightarrow (-1)^\delta |+\rangle$ and $|-\rangle \rightarrow (-1)^{-\delta} |-\rangle$. The two states (2.4) exist at the same time on O , which means that we can replace the term \pm by $(-1)^\delta$.

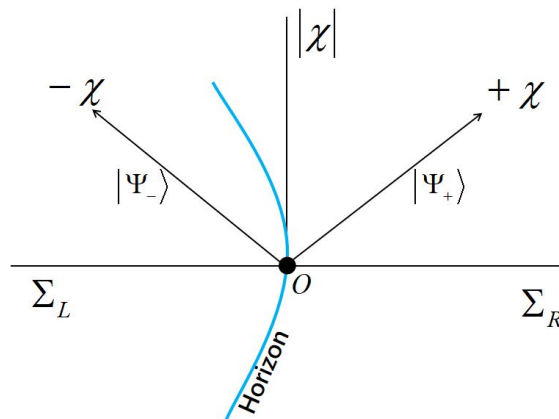


Figure 1: We can't distinguish between the two entangled states $|\Psi_\pm\rangle$ at a point O of horizon. When separating two states, the state $|\Psi_-\rangle$ enters to the black hole singularity, and the state $|\Psi_+\rangle$ exits black hole. The two states are still entangled.

Next, we assume that there is an infinity of tangents in the point O , i.e. the creation of particles in horizon propagates with a spherical forms in all directions. So we replace $\pi\delta$ by a temporal or spatial parameter $\theta \in \mathbb{R}$.

$$\partial_\chi |\chi| \subset e^{i\theta} \in U(1) \quad (2.5)$$

In this case we generalize Eq.(2.4) by the created hybrid-entangled bi-particles state shared between Alice and Bob reads as follows

$$|\Psi_\theta\rangle \sim |+\rangle |-\rangle + e^{i\theta} |-\rangle |+\rangle \quad (2.6)$$

the states $|\Psi_\theta\rangle$ and $e^{-i\theta} |\Psi_\theta\rangle$ describe the same physical quantum state. $|\Psi_\theta\rangle$ and $|\Psi_+\rangle$ will not correspond to the same physical state. Let H be a fixed observable (Hermitian operator) on some Hilbert space of quantum states representing a certain conserved quantity. Let $\rho_\theta = e^{-i\theta H} \rho e^{i\theta H}$ be the evolution of density operator ρ generated by H . Clearly, ρ_θ satisfies the von Neumann-Landau equation

$$i\partial_\theta \rho_\theta = H\rho_\theta - \rho_\theta H = [H, \rho_\theta] \quad (2.7)$$

in terms of the eigenvectors $|\Psi_\theta\rangle$ and the associated eigen values λ_θ depend on θ , therefore, the quantum Fisher information is zero. We propose that the particles created with $\theta = 0$, don't escape the horizon but are trapped. We propose that these particles which corresponds to $\theta = 0$, spins on the black hole horizon [10].

$$|\Psi_0\rangle \sim |+\rangle |-\rangle + |-\rangle |+\rangle \quad (2.8)$$

the entangled states $\{|\Psi_0\rangle\}$ are trapped on the horizon. Our goal is to compute the angular momentum in the classical frame and then in the quantum frame, to describe both the classical and quantum nature of a black hole. The dynamics of the particles rotating on the horizon, requires to calculate the orbital angular momentum vector of the entangled particles in the horizon.

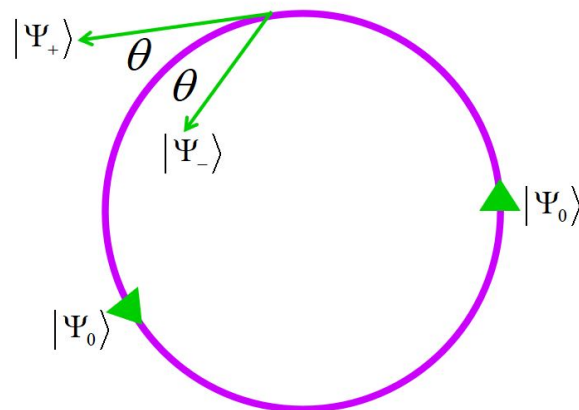


Figure 2: particles in state $|\Psi_+\rangle$ go out of the horizon on the other hand particles in state $|\Psi_-\rangle$ directs to the singular. And particles in the states $|\Psi_0\rangle$ are trapped on the horizon.

The orbital angular momentum operator of a particle in the horizon can then be defined as the vector operator $L = -ix^\mu \times \partial_\mu$. Moreover, we assume that we can't

distinguish between the two particles χ and $-\chi$, during the rotation in the horizon. We propose that the orbital angular momentum of the particles which turn in the horizon, has a quantum and classical aspect at the same time we start with the classical orbital angular momentum

$$L_{zC} = p_y x \frac{d}{dy} y - p_x y \frac{d}{dx} x \quad (2.9a)$$

in the quantum frame the eigenvalue of the orbital angular momentum operator L_{zQ} in the state $|\tilde{\Psi}_0\rangle$ is expressed as below

$$\langle \tilde{\Psi}_0 | L_{zQ} | \tilde{\Psi}_0 \rangle = -i \langle \tilde{\Psi}_0 | x \frac{d}{dy} y | \tilde{\Psi}_0 \rangle + i \langle \tilde{\Psi}_0 | y \frac{d}{dx} x | \tilde{\Psi}_0 \rangle \quad (2.10)$$

we compare the two orbital angular momentums (2.9a) and (2.10), we obtain a new state $|\tilde{\Psi}_0\rangle$, is a conformal transformation of vacuum state $|\Psi_0\rangle$ as

$$|\tilde{\Psi}_0(t)\rangle \equiv e^{ix^\mu p_\mu(t)} |\Psi_0\rangle \quad (2.11)$$

to obtain a classical and quantum description of horizon particles ($\theta = 0$), the state $|\Psi_0\rangle$ changes to new vacuum state $|\tilde{\Psi}_0(t)\rangle$ by the phase $\exp ix^\mu p_\mu(t)$. We propose a new notation of the conformal state of vacuum $|\varphi_+\rangle := |\tilde{\Psi}_0(t)\rangle$ and $|\varphi_-\rangle := |\tilde{\Psi}_0^*(t)\rangle$, we will use this notation later. To study the behavior of particles spinning on the horizon, we propose to start studying first the rotation of the Kerr Newman black hole.

3 Entangled particle and antiparticle on the horizon

In spherical coordinates (t, r, θ, φ) , the Kerr–Newman metric is

$$ds^2 = -\frac{\Delta}{\rho^2} (dt - J/M \sin^2 \theta d\phi)^2 + \frac{\sin^2 \theta}{\rho^2} [(r^2 + J^2/M^2) d\phi - J/M dt]^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 \quad (3.1)$$

where

$$\Delta \equiv r^2 - r_s r + J^2/M^2 + Q^2/4 \quad (3.2)$$

$$\rho^2 = r^2 + J^2/M^2 \cos^2 \theta \quad (3.3)$$

with $r_s = 2M$ is the Schwarzschild radius. We take the speed of light $c = 1$ the gravitational constant $G_N = 1$ the vacuum permittivity $4\pi\epsilon_0 = 1$.

The Kerr-Newman metric [8] describes a black hole if and only if

$$J^2 \leq (M^2 - Q^2) M^2 \quad (3.4)$$

where M is the mass of the black hole, Q is the electric charge and J is the angular momentum. The case $J^2 \leq M^4 - Q^2 M^2$ describes an extremal black hole. In the case where $(Q = 0, J \neq 0)$, we will have a Kerr black metric. For $(Q \neq 0, J \neq 0)$, we get a Kerr–Newman black hole. For $(Q \neq 0, J = 0)$, we get a Reissner–Nordström black hole. Finally for $(Q = 0, J = 0)$, we obtain a Schwarzschild metric. We know that the Schwarzschild radius is written as $r_s = 2M$. We can rewrite Eq.(3.4) by a more general form:

$$J^2 \lesssim -\frac{r_s^2}{4} Q^2 e^{-\frac{r_s^2}{4Q^2}} \quad (3.5)$$

the objective behind this general form, is to make appear the Kruskal–Szekeres coordinates to cover the entire spacetime manifold around the horizon. We take $u = -\frac{r_s^2}{4Q^2}$, one can obtain

$$J^2/Q^4 \lesssim u e^u \quad (3.6)$$

the term $u e^u$ is the lightlike Kruskal coordinate, where

$$u = -M^2/Q^2 \equiv -r/2M \quad (3.7)$$

this last equation makes it possible to find the charge of the particles rotating in the horizon

$$Q_{\pm} = \pm M \quad (3.8)$$

every particle on the horizon, has a charge which depends directly on the black hole mass; if the black hole mass larger, the charge of a horizon particle will be more important. This equation shows that there are two types of spinning particles in the horizon. And each particle of the charge $Q_+ = +M$, is entangled with another antiparticle of the charge $Q_- = -M$. The particle and antiparticle have opposite electric charges Q_+ and Q_- , i.e. CPT anticommutes with the charges. The sum of all the charges of the horizon particles is zero. If the number of particles N on the horizon is limited, the horizon charge will be $Q_H = 0$. If we take that the number of horizon particles $N \rightarrow \infty$, we'll have $\sum Q_{r_s} = M \sum_{n=0}^{\infty} (-1)^n = M/2$. Then the charge of the black hole horizon is

$$Q_H = M/2 \quad (3.9)$$

this charge corresponds exactly with a physical event horizon, because (3.9) check the condition of existence of the event horizon: $2Q_H \prec r_s$. The notion of an electrically charged black hole horizon, is already found by [9]. Therefore the Eq.(3.4) becomes

$$J^2 \preceq \frac{3}{4} M^4 \quad (3.10)$$

the term J in the last equation represents a classical state. In the quatum framework, we take \hat{J} as an operator on the other hand J are its eigenvalues in the states $\{|\varphi_{\pm}\rangle\}$

$$|J| \leq \frac{\sqrt{3}}{2} M^2 \quad (3.11)$$

according to Eq.(2.1), the field $|\chi|$ describe the particles which escape from black hole horizon, and $|J|$ describe the spinning particles that trapped on the horizon, in this case we take $|\chi|(\theta = 0) = |J|$. For a extremal 4d Kerr black hole, the Eq.(3.11) become a two equations, are given by

$$J_- = -\frac{\sqrt{3}}{2} M^2 \text{ or } J_+ = +\frac{\sqrt{3}}{2} M^2 \quad (3.12)$$

which induces that the extremal black hole is a set of two entangled particles with the low mass M . This equation indicate that the two entangled particles on the horizon, are created from the point O , and rotate in opposite directions then it is particle-antiparticle annihilation on another point O' on the horizon. Therefore, the extremal black hole is considered like a particle accelerator. The rotation of the two entangled particles (particle and antiparticle) creates a black hole is equivalent to the pair production. This requires there is a enough energy available in the center of mass to create the pair. In this case we suppose that all the energy absorbed by a black hole is transformed to kinetic energy of the particles-antiparticles in the horizon.

the ADM mass and angular momentum are function of the horizon size a

$$M \equiv a \quad (3.13)$$

The black hole has a Bekenstein-Hawking entropy checked

$$S_{BH} \geq \frac{4\pi}{\sqrt{3}} |J| \quad (3.14)$$

this relation presents a minimum value for the entropy of a black hole. We will use this entropy in the next section.

4 Mirage horizon of the entangled particle-antiparticle

In this section, we want to find a unique description for the Reissner–Nordström metric and the Kerr–Newman metric. The Reissner–Nordström metric [12] reads

$$ds^2 = -\left(1 - \frac{r_s}{r} + \frac{Q^2}{r^2}\right) dt^2 + \left(1 - \frac{r_s}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2 d\omega^2 \quad (4.1)$$

where $d\omega^2 = d\theta^2 + \sin^2 \theta d\varphi^2$.

The event horizons for the spacetime are located where $\frac{1}{g^{rr}} = 0$, which gives two solutions, i.e. we will have two event horizons are located in

$$r_{\pm} = M \pm \sqrt{M^2 - Q^2} \quad (4.2)$$

these concentric event horizons become degenerate for (3.8), which corresponds to an extremal black hole, which explains the result found by (3.12), i.e. the black hole is a set of two entangled particles, one rotates with respect to the other. This comparison opens a description of a black hole with entangled particles (particle and antiparticle) (3.8) on the horizon (3.9) by two the metrics Eq.(3.1) and Eq.(4.1). But we have a problem, the Kerr- Newman metric Eq.(3.1) describes a rotating black hole, on the other hand, the Reissner- Nordström metric Eq.(4.1) describes a black hole without rotation. To unify the two metrics without a generalization, we propose that the concentric event horizons described by the horizon angular momentum $J_H = 0$ and angular momentum $J_{\pm} \neq 0$ of the entangled particle-antiparticle system on the horizon. The set $\{J_H, J_{\pm}\}$ is described by Reissner-Nordström metric and the Kerr-Newman metric at the same time. When we have an annihilation between particle and antiparticle in the horizon, therefore, $J_H = 0$. On the other hand J_{\pm} describes the horizon with the creation of these entangled particles. Which means that the geometry of the space-time close to the horizon changes according to the dynamics of the entangled particles. which means that the two horizons r_{\pm} are a mirage created by the particle-antiparticle system. In this case we propose that the singularity does not rotate. The rotation and the load present only on the horizon. To find the intersection between these two metrics we will use a technique. For $Q = Q_H$ (3.9), we can use the two angular momentum (3.12) in (4.2), and we obtain

$$r_{\pm} = M + \frac{J_{\pm}}{M} \quad (4.3)$$

r_{\pm} are two Reissner-Nordström horizons, it depends on J_{\pm} . According to our model, r_{\pm} are a virtual horizons or mirage horizons, because there are a results of rotation of a several entangled particles-antiparticles on the real horizon ($Q_H = M/2$, $J_H = 0$, $r = r_s$). Since r_{\pm} depends on J_{\pm} , then, r_{\pm} describe the two particles. Therefore, we propose to replace r_s in Eq.(4.1) by the last values of $|r_{\pm}|$. To find a metric which describe the space-time around the horizon with the presence of the entangled particles, we propose first this new metric

$$ds^2 = - \left(1 - \frac{M}{r} + \frac{M^2}{r^2} - \frac{|J_{\pm}|}{rM} \right) dt^2 + \left(1 - \frac{M}{r} + \frac{M^2}{r^2} - \frac{|J_{\pm}|}{rM} \right)^{-1} dr^2 + r^2 d\omega^2 \quad (4.4)$$

we start with a region near to singularity $0 \lesssim r \preceq r_s$, in this case we propose to use the Taylor series for this metric:

$$|J_{\pm}| \equiv \frac{M^4}{r^2} - \frac{M^5}{r^3} + \dots + O \left(\pm \frac{M^{n+2}}{r^n} \right) \quad (4.5)$$

this last expression explains the presence of the entanglement in the horizon. When $n \rightarrow \infty$, we can't separate precisely between two cases; when n even or odd. Clearer,

for $n \rightarrow \infty$, n will be even and odd at the same time. In the case where $r = r_s$, one can obtain $|J_{\pm}| = \frac{M^2}{6}$, this value verifies the condition (3.11), and they are different from the values (3.12), which shows that the unification between Reissner–Nordström metric and the Kerr–Newman metric is done for a non-external black hole. The term $O\left(\pm \frac{M^\infty}{r^\infty}\right) \subset J_{\pm}$, where $\infty + (2k) = \infty + (2k + 1)$, $k \in \mathbb{N}$, is a little negligible, but his effect on particle and antiparticle is very strong, he represents the entanglement of this two particles

$$O\left(\pm \frac{M^\infty}{r^\infty}\right) \equiv (-1)^\infty O\left(\frac{M^\infty}{r^\infty}\right) \quad (4.6)$$

indeed, this equation shows that we can have $(-1)^\infty = \{-1, 1\}$, which is equivalent with Eq.(2.2), which really describes the entangled particle-antiparticle by the two angular momentums J_{\pm} . We remark that $(-1)^\infty$ becomes an operator which has eigenvalues in the base $\{|\varphi_{\pm}\rangle\}$, which describes the state of an entangled particle and antiparticle

$$(-1)^\infty = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad (4.7)$$

we compare Eq.(2.3) with Eq.(4.7) we find

$$\partial_{J_+} |J_{\pm}| = (-1)^\infty \quad (4.8)$$

The term Eq.(4.6) is essential, for these the entangled particle-antiparticle create a extremal black hole, because this term is a responsible in (4.5), which creates a rotation between the entangled particles on the horizon.

$$S_{BH} \succeq \frac{4\pi}{\sqrt{3}} \left| (-1)^2 \frac{M^4}{r^2} + (-1)^3 \frac{M^5}{r^3} + \dots + (-1)^\infty O\left(\frac{M^{n+2}}{r^n}\right) \right| \quad (4.9)$$

this relation describes a minimal entropy $S_{BH} \succeq S_{\min}$ for a black hole:

$$S_{\min} = \frac{4\pi}{\sqrt{3}} \frac{M^4}{r^2} \frac{1}{1 + \frac{M}{r}} \quad r \neq 0 \quad (4.10)$$

for $r = r_s$, we get $S_{\min,H} = \frac{2\pi}{3\sqrt{3}} M^2$.

5 Geodesic of the two entangled particle-antiparticle

In what follows, we want to determine a metric which describes the entangled system of a particle and antiparticle. Taking into account Eq.(4.5), we obtain a new metric which is written

$$ds^2 = \frac{-1}{1 + \frac{M}{r}} dt^2 + \left(1 + \frac{M}{r}\right) dr^2 + r^2 d\omega^2 \quad (5.1)$$

Substituting Eqs.(3.8,4.5) into Eq.(5.1) one can obtain

$$ds^2 = -\frac{|J_{\pm}|}{Q_{\pm}^4} r^2 dt^2 + \frac{1}{r^2} \frac{Q_{\pm}^4}{|J_{\pm}|} dr^2 + r^2 d\omega^2 \quad (5.2)$$

where $d\omega^2 = d\theta^2 + \sin^2 \theta d\varphi^2$.

we first provided the equations of motion of the particle-antiparticle system. Now, we consider the spherical coordinates $x^\mu = (x^0 = t, x^1 = r, x^2 = \theta, x^3 = \varphi)$: the Christoffel symbols are

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2} g^{\alpha\sigma} \left(\frac{\partial g_{\sigma\nu}}{\partial x^\mu} + \frac{\partial g_{\mu\sigma}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^\sigma} \right) \quad (5.3)$$

one can compute the geodesics of a two-entangled particles system $|J_{\pm}|$. The Euler-Lagrange equations of motion for $|J_{\pm}|$ are then given in local coordinates by

$$\frac{d^2 x^\alpha}{dt^2} + \Gamma^\alpha_{\mu\nu} \dot{x}^\mu \dot{x}^\nu = 0 \quad (5.4)$$

we find differential equations, (see appendix), then we calculate their solution

$$\theta(t) = \pm \cos^{-1} \omega (t - t_0)$$

where ω is the angular frequency for both particles, the rotation of the two particles begins at time t_0 . For $t = t_0$, we obtain $\theta(t_0) = \pm 1$; this result is equivalent to the rest of particle-antiparticle.

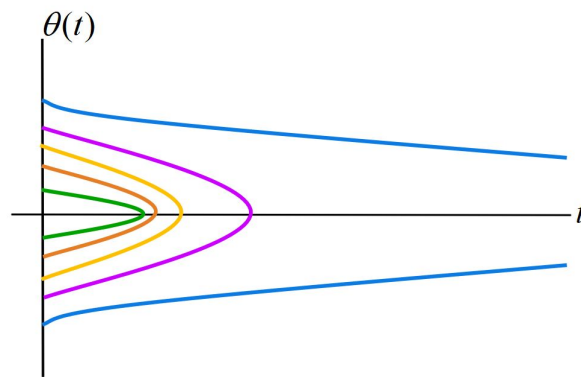


Figure 3: Some curves of the variation of θ as a function of t .

we also calculate the parameter φ and we find

$$\begin{aligned} \varphi(t) = & -C\varpi \ln(\csc(\omega t - \omega t_0) + \cot(\omega t - \omega t_0)) \\ & + C\varpi \cos(\omega t - \omega t_0) + \varphi_0 \end{aligned}$$

where $\varpi = \frac{\tan(\omega t - \omega t_0)}{\omega |\tan(\omega t - \omega t_0)|}$, C is a constant and φ_0 is a constant of integration.

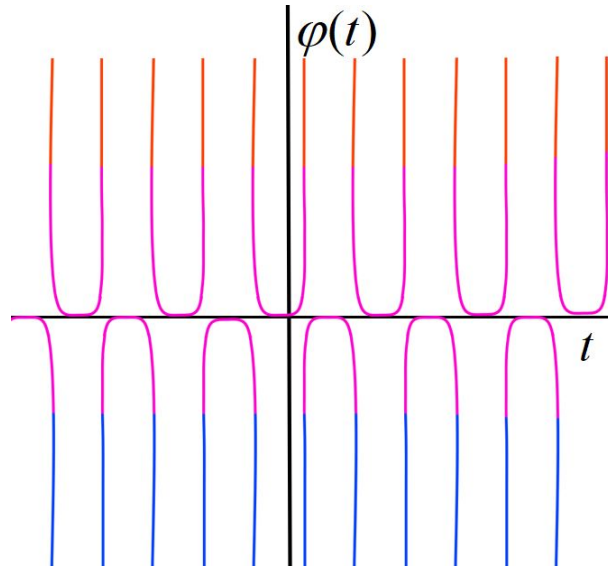


Figure 4: A possible curves of the variation of φ as a function of t .

The local velocity is therefore

$$v(r, t) = \pm \frac{\sqrt{\omega |J_{\pm}|} \tan \omega (t - t_0)}{Q_{\pm}^2 \cos \omega (t - t_0)} r^2$$

for $t = t_0$, the velocity of the entangled particles will be zero. If $|J_{\pm}| = 0$, $v(r, t)$ will be zero, which implies that the entanglement between the particle and the antiparticle is essentially to generate by the angular momentum.

6 Conclusion

In the present work, we used an absolute field to describe two entangled states in the horizon. Then we studied the entangled particle-antiparticle that revolve around on the horizon. We have studied these particles for Kerr Newman black hole. We have shown that the rotation of an entangled system of a particle and an antiparticle, can create a extremal black hole. The present study was designed to determine the black hole effect on entangled particles on the horizon. This study has shown that the system of two entangled particles has a single metric, which derives the motion of the electrically charged particle and antiparticle on the horizon. This study has raised important questions about the nature of the entanglement, since we have shown that the two angular momentums of the two entangled particles are perfectly connected with a Taylor by an operator $(-1)^{\infty}$ at infinite. This connection leaves the two entangled particles in rotation between them. The analysis of the entangled particles geodesic, has extended our knowledge of more about the rotations of the entangled particle-antiparticle.

Appendix

the metric (5.2) is written as

$$g_{\mu\nu} = \begin{pmatrix} -\frac{|J_{\pm}|}{Q_{\pm}^4}r^2 & 0 & 0 & 0 \\ 0 & \frac{1}{r^2}\frac{Q_{\pm}^4}{|J_{\pm}|} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{pmatrix}$$

the nonvanishing components of the Christoffel symbols $\Gamma_{\mu\nu}^{\alpha}$ are given by

$$\Gamma^0_{01} = \frac{1}{2}g^{00}\frac{\partial g_{00}}{\partial x^1} = \frac{1}{r} \quad (6.1)$$

$$\Gamma^1_{00} = -\frac{1}{2}g^{11}\frac{\partial g_{00}}{\partial x^1} = \frac{|J_{\pm}|^2}{Q_{\pm}^8}r^3 \quad (6.2)$$

$$\Gamma^1_{11} = \frac{1}{2}g^{11}\frac{\partial g_{11}}{\partial x^1} = -\frac{1}{r} \quad (6.3)$$

$$\Gamma^1_{22} = -\frac{1}{2}g^{11}\frac{\partial g_{22}}{\partial x^1} = -\frac{|J_{\pm}|}{Q_{\pm}^4}r^3 \quad (6.4)$$

$$\Gamma^1_{33} = -\frac{1}{2}g^{11}\frac{\partial g_{33}}{\partial x^1} = \frac{|J_{\pm}|}{Q_{\pm}^4}r^3 \sin^2 \theta \quad (6.5)$$

$$\Gamma^2_{21} = \frac{1}{2}g^{22}\frac{\partial g_{22}}{\partial x^1} = \frac{1}{r} \quad (6.6)$$

$$\Gamma^2_{33} = -\frac{1}{2}g^{22}\frac{\partial g_{33}}{\partial x^2} = -\sin \theta \cos \theta \quad (6.7)$$

$$\Gamma^3_{31} = \frac{1}{2}g^{33}\frac{\partial g_{33}}{\partial x^1} = \frac{1}{r} \quad (6.8)$$

$$\Gamma^3_{32} = \frac{1}{2}g^{33}\frac{\partial g_{33}}{\partial x^2} = \cot \theta \quad (6.9)$$

therefore, the Euler–Lagrange equations are given by

$$\ddot{r} + \frac{|J_{\pm}|^2}{Q_{\pm}^8}r^3 = 0 \quad (6.10)$$

$$\ddot{r} - \frac{1}{r}\dot{r}^2 = 0 \quad (6.11)$$

$$\ddot{r} - \frac{|J_{\pm}|}{Q_{\pm}^4}\dot{\theta}^2 r^3 = 0 \quad (6.12)$$

$$\ddot{r} + \frac{|J_{\pm}|}{Q_{\pm}^4}\sin^2 \theta \dot{\varphi}^2 r^3 = 0 \quad (6.13)$$

$$\ddot{\theta} + \frac{\dot{r}}{r}\dot{\theta} = 0 \quad (6.14)$$

$$\ddot{\theta} - \sin \theta \cos \theta \dot{\varphi}^2 = 0 \quad (6.15)$$

$$\ddot{\varphi} + \frac{\dot{r}}{r} \dot{\varphi} = 0 \quad (6.16)$$

$$\ddot{\varphi} + \cot \theta \dot{\theta} \dot{\varphi} = 0 \quad (6.17)$$

by the comparison between Eqs.(6.10,6.11,6.12,6.13), one eliminates Eqs.(6.10,6.13), it is to have real values for the physical quantities. According to Eqs.(6.14,6.15,6.16,6.17), we find this differential equation,

$$\ddot{\theta} + \dot{\theta}^2 \cot \theta = 0 \quad (6.18)$$

its solution is of the form

$$\theta(t) = \pm \cos^{-1} \omega(t - t_0) \quad (6.19)$$

from Eqs.(6.14,6.16,6.19), we find the solution for φ

$$\ddot{\varphi} + \frac{\omega}{\cos \omega(t - t_0)} \left(\frac{1}{\sin \omega(t - t_0)} + \sin \omega(t - t_0) \right) \dot{\varphi} = 0$$

$$\varphi(t) = C \int e^{-F(t)} dt + B$$

$$\begin{aligned} F(t) &= \int^t f(\zeta) d\zeta \\ &= \int^t d\zeta \frac{\omega}{\cos \omega(\zeta - t_0)} \left(\frac{1}{\sin \omega(\zeta - t_0)} + \sin \omega(\zeta - t_0) \right) \end{aligned}$$

$$= \ln |\tan(\omega t - \omega t_0)| - \ln(\cos(\omega t - \omega t_0)) + D$$

$$\varphi(t) = C \int \frac{\cos(\omega t - \omega t_0)}{|\tan(\omega t - \omega t_0)|} dt + B$$

$$\varphi(t) = -C\varpi \ln(\csc(\omega t - \omega t_0) + \cot(\omega t - \omega t_0)) \quad (6.20)$$

$$+ C\varpi \cos(\omega t - \omega t_0) + \varphi_0 \quad (6.21)$$

where $\varpi = \frac{\tan(\omega t - \omega t_0)}{\omega |\tan(\omega t - \omega t_0)|}$. For the square of the speed in Eq.(6.11), is positive, we choose only

$$v^2 = \omega \frac{|J_{\pm}| \tan^2 \omega(t - t_0)}{Q_{\pm}^4 \cos^2 \omega(t - t_0)} r^4 \quad (6.22)$$

therefore, the specific orbital energy is

$$E = \omega \frac{|J_{\pm}| \tan^2 \omega(t - t_0)}{2Q_{\pm}^4 \cos^2 \omega(t - t_0)} r^4 - \frac{M}{r}$$

References

- [1] Maldacena, J., & Susskind, L. (2013), *Cool horizons for entangled black holes*, *Fortschritte der Phys.*, 61(9), 781-811.
- [2] Andrade, T., Emparan, R., & Licht, D. (2019), Charged rotating black holes in higher dimensions, *J. High Energy Phys.* 2019(2), 76.
- [3] Cano, P. A., Chimento, S., Linares, R., Ortín, T., & Ramírez, P. F. (2020), α' corrections of Reissner-Nordström black holes, *J. High Energy Phys.*.
- [4] Liu, H., Tang, Z., Destounis, K., Wang, B., Papantonopoulos, E., & Zhang, H. (2019), *Strong cosmic censorship in higher-dimensional Reissner-Nordström-de Sitter spacetime*, *J. High Energy Phys.*.
- [5] Hawking, S. W., & Penrose, R. (1970), *The singularities of gravitational collapse and cosmology*, *Proc. Math. Phys. Eng. Sci.*.
- [6] Terashima, H. (2000), *Entanglement entropy of the black hole horizon*, *Phys. Rev. D*, 61(10), 104016.
- [7] Córdova, C., Ohmori, K., Shao, S. H., & Yan, F. (2020), *Decorated Z_2 symmetry defects and their time-reversal anomalies*, *Phys. Rev. D*, 102(4), 045019.
- [8] Zhang, J., & Zhao, Z. (2005), *New coordinates for Kerr–Newman black hole radiation*, *Phys. Lett. B*, 618(1-4), 14-22.
- [9] Gubser, S. S. (2008), *Breaking an Abelian gauge symmetry near a black hole horizon*, *Phys. Rev. D*, 78(6), 065034.
- [10] Arkani-Hamed, N., Huang, Y. T., & O’Connell, D. (2020), *Kerr black holes as elementary particles*, *J. High Energy Phys.*.
- [11] Gubser, S. S., & Rocha, F. D. (2010), *Peculiar properties of a charged dilatonic black hole in AdS 5*, *Phys. Rev. D*, 81(4), 046001.
- [12] Das, P., Sk, R., & Ghosh, S. (2017), *Motion of charged particle in Reissner–Nordström spacetime: a Jacobi-metric approach*, *EPJC*, 77(11), 735.