

Estimating galactic dark matter and redshift in Flat Space Cosmology

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Abstract: With reference to known galactic rotation speeds and previous publications on our light-speed expanding Flat Space Cosmology model, a toy model variation is presented herein for the purpose of exploring possible time-dependent relationships between galactic dark matter, visible matter, total matter, redshift, radius and angular velocity. The result of this exploration, in the form of graphs and tables, provides for remarkable correlations with current galactic observations and perhaps moves us closer to understanding the scalar nature and influence of dark matter and Lambda on the expanding universe. With reference to light speed expansion, if one is willing to re-define cosmic red shift as $[z/(1+z)]$, without considering Lambda cosmology inputs, light travel distances can be reproduced with a marginal error of +8.6% at $z=1.2$, (i.e. traditional light travel distance is 8.6% higher than our estimate), 0% at $z=11.5$ and -5.5% at $z=1200$. (i.e. traditional light travel distance is 5.5% lower than our estimate).

Keywords: Flat space cosmology; dark matter; visible matter; galactic radii; galactic angular velocity; cosmic angular velocity;

1. Introduction

Even though standard model of cosmology is standing on 5 pillars namely, big bang, inflation, super luminal expansion, dark matter and dark energy, we would like to emphasize that,

- 1) James Peebles, the famous cosmologist and 2019 Nobel laureate, strongly believes that Big bang is an inappropriate concept in understanding the universe. Readers are encouraged to visit, <https://phys.org/news/2019-11-cosmologist-lonely-big-theory.html>
- 2) Theoretically big bang, inflation, dark energy and super luminal expansion are no way connected with Planck scale which is having a major role in understanding quantum cosmology having information passing at speed of light.
- 3) Experimentally so far no one could understand big bang [1,2] inflation[3,4], dark energy, dark matter and super luminal expansion with reference to any underground or ground or satellite based experiment.
- 4) Big bang, inflation and Super luminal expansion are no way giving a clue for unifying general theory of relativity and quantum mechanics.
- 5) Even though most of the cosmological observations are being studied and understood with photons that propagate with speed of light, it is very unfortunate to say that, most of the cosmologists are strongly believing in hypothetical 'super luminal expansion' of space. Recently detected gravitational waves that are supposed to originate from massive black holes are also confirmed to be moving at speed of light. If so, superluminal expansion can be considered as a pure human intellectual concept having no experimental support.
- 6) Big bang, Inflation and Dark energy are inference based intellectual concepts having no proper physical base and probably may misguide the future generation.
- 7) Compared to Big bang, inflation, dark energy and superluminal expansion, dark matter is having some sort of physical support in terms of an unknown, unidentified and unseen elementary particle having an heuristic gravitational attractive property. In addition to that, ultimately somehow, one should suppose the existence of some kind matter for understanding the unexpected massive nature of trillions of observed galaxies.
- 8) 'Spin' is a basic property of quantum mechanics and 'rotation' is a very common experience.
- 9) It is better to understand and develop models of cosmology based on well supported physical concepts rather than extraordinary physical hypothesis that demand super-normal efforts.

2. Our five assumptions

Right from the beginning of Planck scale, cosmic boundary is moving at speed of light and growing like a ball with the following workable assumptions [5-14],[15,16,17].

Assumption-1: If $R_t \cong 2GM_t/c^2 \cong c/H_t$, $M_t \cong c^3/2GH_t$, and $tH_t \cong 1$ where M_t = cosmic total mass, R_t = cosmic radius, H_t = cosmic Hubble parameter and t = cosmic age.

Assumption-2: Cosmic angular velocity can be expressed as, $\frac{H_t}{\omega_t} \cong \Upsilon_t \cong 1 + \ln\left(\frac{H_{pl}}{H_t}\right)$ where ω_t = cosmic angular velocity and H_{pl} is the Planck scale Hubble parameter.

Assumption-3: Cosmic temperature follows the relation, $T_t \cong \frac{\hbar c^3}{8\pi G k_B \sqrt{M_t M_{pl}}}$ where $M_{pl} \cong$ Planck mass.

Assumption-4: Galactic dark matter $(M_{Gd})_t$ and visible matter $(M_{Gv})_t$ are interrelated in such way that,

$M_{Gt} \cong (M_{Gd})_t + (M_{Gv})_t$ and $\frac{(M_{Gd})_t}{(M_{Gv})_t} \cong \sqrt{\frac{(M_{Gv})_t}{M_{Xt}}}$ where $M_{Xt} \cong \frac{(M_t^3 M_{pl})^{\frac{1}{4}}}{8\pi} \cong$ Time dependent dark-visible reference mass unit.

Assumption-5: Galactic flat rotation speed can be expressed as, $\frac{V_{Gt}}{c} \cong \left(\frac{1}{2\Upsilon_t}\right)^{\frac{1}{4}} \left(\frac{M_{Gt}}{M_t}\right)^{\frac{1}{4}}$ where M_t = cosmic total mass.

3. To estimate the magnitudes of Planck scale parameters

For the Planck scale, if, total mass = $M_{pl} \cong \sqrt{\frac{\hbar c}{G}}$, based on the assumptions,

$$R_{pl} \cong \frac{2GM_{pl}}{c^2} \cong 2\sqrt{\frac{G\hbar}{c^3}} \cong 3.2325 \times 10^{-35} \text{ m} \quad (1)$$

$$H_{pl} \cong \frac{c^3}{2GM_{pl}} \cong \frac{1}{2}\sqrt{\frac{c^5}{G\hbar}} \cong 9.27445 \times 10^{42} \text{ sec}^{-1} \quad (2)$$

$$t_{pl} \cong \frac{1}{H_{pl}} \cong 2\sqrt{\frac{G\hbar}{c^5}} \cong 1.07823 \times 10^{-43} \text{ sec} \quad (3)$$

$$\Upsilon_{pl} \cong \frac{H_{pl}}{\omega_{pl}} \cong 1 \quad (4)$$

$$T_{pl} \cong \frac{\hbar c^3}{8\pi G k_B M_{pl}} \cong 5.6373 \times 10^{30} \text{ K} \quad (5)$$

$$\omega_{pl} \cong H_{pl} \cong 9.27445 \times 10^{42} \text{ rad.sec}^{-1} \quad (6)$$

4. To estimate the magnitudes of current cosmic parameters

Considering, current CMBR temperature [18] as, $T_0 \cong 2.725$ K,

$$M_0 \cong \frac{1}{M_{pl}} \left(\frac{\hbar c^3}{8\pi G k_B T_0} \right)^2 \cong 9.31453 \times 10^{52} \text{ kg} \quad (7)$$

$$R_0 \cong \frac{2GM_0}{c^2} \cong 1.3834 \times 10^{26} \text{ m} \quad (8)$$

$$H_0 \cong \frac{c^3}{2GM_0} \cong 2.1671 \times 10^{-18} \text{ sec}^{-1} \cong 66.87 \text{ km.sce}^{-1} \text{Mpc}^{-1} \quad (9)$$

$$t_0 \cong \frac{1}{H_0} \cong \frac{2GM_0}{c^3} \cong 14.62 \text{ Gy} \quad (10)$$

$$\Upsilon_0 \cong 1 + \ln \left(\frac{H_{pl}}{H_0} \right) \cong 140.61 \quad (11)$$

$$\omega_0 \cong H_0 \left[1 + \ln \left(\frac{H_{pl}}{H_0} \right) \right]^{-1} \cong \frac{H_0}{\Upsilon_0} \cong 1.5412 \times 10^{-20} \text{ rad.sec}^{-1} \quad (12)$$

$$M_{X0} \cong \frac{(M_0^3 M_{pl})^{\frac{1}{4}}}{8\pi} \cong 3.623 \times 10^{38} \text{ kg} \quad (13)$$

5. Three characteristic galactic applications

Based on the assumptions, 3 characteristic properties of any galaxy can be expressed in the following way [12,13,14]. At present,

- 1) Radius of any galaxy can be expressed as [19,20],

$$R_G \cong \sqrt{\frac{GM_G}{c\omega_0}} \quad (14)$$

- 2) Angular velocity of any galaxy can be expressed as,

$$\omega_G \cong \frac{V_G^3}{GM_G} \quad (15)$$

- 3) Mean separation distance of any two neighboring galaxies can be expressed as,

$$L_{sep} \cong \left(\frac{\sqrt{V_{G1}V_{G2}}}{c} \right) \left(\frac{c}{H_0} \right) \cong \frac{\sqrt{V_{G1}V_{G2}}}{H_0} \quad (16)$$

where (V_{G1}, V_{G2}) represent the flat rotations speeds of galaxies (G_1, G_2) .

It may be noted that, according to Baryonic acoustic oscillations, current galactic separation distance is around 490 million light years [21,22]. In this context, relation (16) can be recommended for further research.

6. Galactic dark mass, visible mass, total mass, radius and angular velocity

Based on the assumptions and observed galactic flat rotations speeds taken from Spitzer Photometry and Accurate Rotation curves (SPARC) [23], in the following Table-1 we present the data for estimated galactic radii and angular velocities. Here it may be noted that, at present, for any galaxy, (ignoring the '0' subscript),

$$\begin{aligned} M_G &\cong M_{Gd} + M_{Gv} \\ &\cong M_{Gv} \sqrt{\frac{M_{Gv}}{M_{X0}}} + M_{Gv} \cong M_{Gv} \left(1 + \sqrt{\frac{M_{Gv}}{M_{X0}}} \right) \end{aligned} \quad (17)$$

$$\frac{V_G}{c} \cong \left(\frac{1}{2Y_0} \right)^{\frac{1}{4}} \left(\frac{M_G}{M_t} \right)^{\frac{1}{4}} \cong 0.2442 \left(\frac{M_G}{M_t} \right)^{\frac{1}{4}} \quad (18)$$

Replacing MOND's acceleration parameter with cosmic angular acceleration and replacing galactic visible mass with dark mass, relation (18) can be obtained. Considering a simple C++ program, relation (17) can be solved numerically. See Table -1. In most of the cases, estimated total mass of galaxy seems to be in line with galactic halo estimations made with NFW model having marginal errors (http://astroweb.cwru.edu/SPARC/WP50_M200.mrt). It can be confirmed with columns (5), (6) of Table 1 and Figure-1. Red curve is our estimation and green curve is for NFW model [24,25,26]. As total mass of galaxy is assumed to be proportional to 4th power of rotation speed, a small change in rotation speed will have large effect in galactic total mass. We have prepared figure 1 with respect to increasing galactic rotation speeds for a clear understanding. From the figure it is very clear that, for low rotation speeds, NFW estimated halo mass is on higher side and our estimated total mass is on lower side. Based on relation (14), if one is willing to consider the idea that, galactic radius is proportional to square root of its mass, it seems logical to say that, lower massive galaxies will have small radii and low rotation speeds. Here we would like appeal that, galaxies whose visible mass approaches our reference mass unit of $M_{X0} \cong 3.6 \times 10^{38}$ kg, seems to possess very little dark matter. It can be confirmed with very recent observations pertaining to NGC1052-DF2 and NGC1052-DF4 galaxies.

Recommended visible mass of NGC1052-DF2 [27,28,29] is $\approx 1 \times 10^8 M_{\text{Sun}}$ and its estimated dark mass is $\approx \left(\sqrt[3]{2 \times 10^{38}} / \sqrt[3]{3.6 \times 10^{38}} \right) \cong 1.49 \times 10^{38}$ kg. Sum of dark mass and visible mass is $\approx 0.35 \times 10^9 M_{\text{Sun}}$. Corresponding flat rotation speed is 18.5 km/sec. It needs further study with respect to NGC1052-DF2 refined data. By means of tidal mass loss [30], if dark matter shifts from satellite galaxy to its mother galaxy, based on our proposed concepts, mother galaxy's flat rotation speed must increase significantly due to increase in total matter. It is for future observational testing.

7. Understanding cosmic red shift with restricted speed of light

It may be noted that, increased redshifts and increased distances forced Edwin Hubble to propose the Hubble's law [31]. With reference to laboratory, appropriate definition of redshift (z) seems to be [32],

$$z_{\text{new}} \cong z_n \cong \frac{\lambda_O - \lambda_L}{\lambda_O} \cong 1 - \left(\frac{\lambda_L}{\lambda_O} \right) \leq 1. \quad (19)$$

$$\text{But not} \quad z_{\text{current}} \cong z_c \cong \frac{\lambda_O - \lambda_L}{\lambda_L} \cong \left(\frac{\lambda_O}{\lambda_L} \right) - 1 \quad (20)$$

Here, as usual, λ_O is the wave length of light received from observed galaxy and λ_L is the wave length of light in laboratory. Even though both relations are ad-hoc definitions, compared to relation (20), relation (19) seems to be appropriate with respect to 'light speed expansion'. Very interesting thing is that, when redshift is very small (up to $z \approx 0.01$), both relations almost all will give the same result. Important point to be noticed is that, by Hubble's time the maximum redshift noticed was 0.003.

Figure 1: To compare estimated total mass of galaxy with NFW model

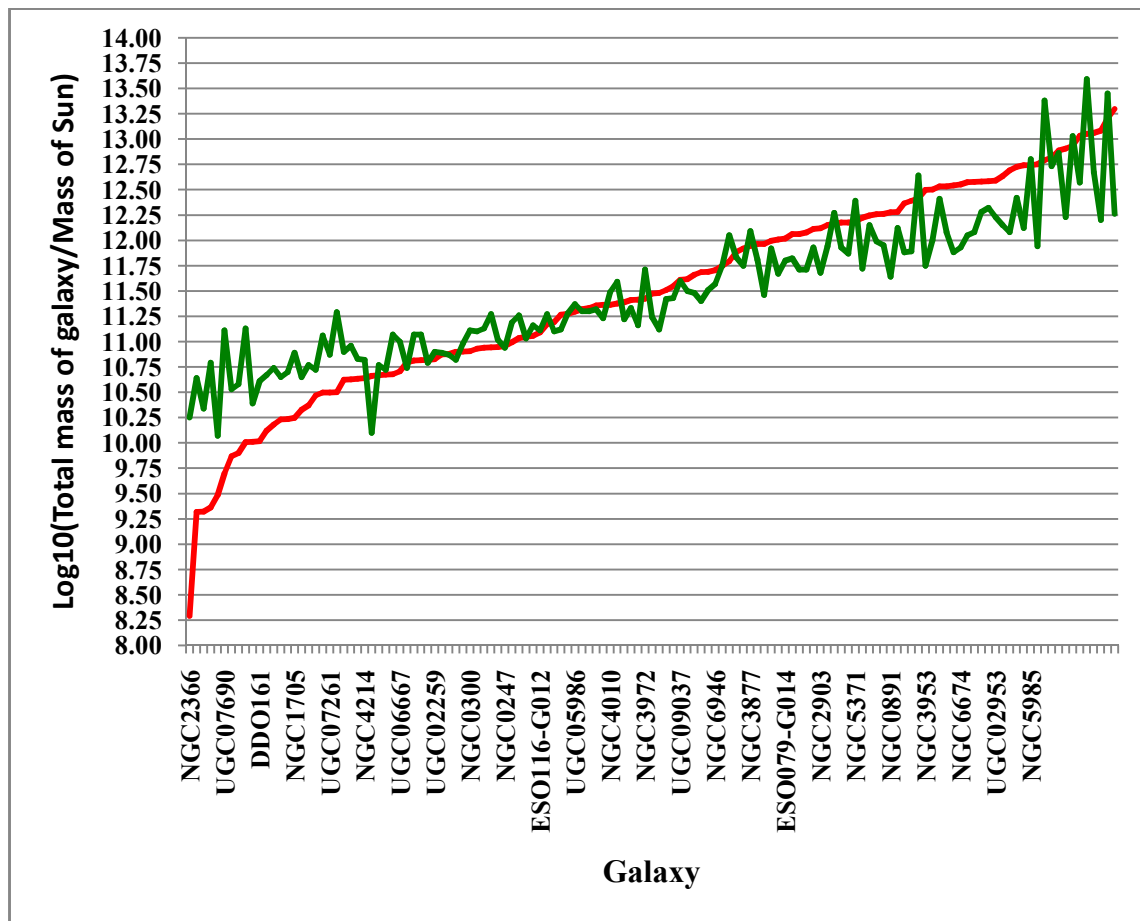


Table-1: Estimated galactic dark mass, visible mass, total mass, radius and angular velocity

Galaxy	Rotation speed (km/sec)	Error in rotation speed (km/sec)	Estimated total mass 1.989e40 (kg)	Log10 (Total mass of galaxy/ Mass of Sun)	Galactic halo mass (NFW model)	Estimated visible mass 1.989e40 (kg)	Estimated dark mass 1.989e40 (kg)	Dark mass%	Estimated radius of galaxy (kpc)	Estimated angular velocity of galaxy (rad/sec)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
D631-7	57	9.4	1.72	10.24	10.7	0.33	1.39	80.94	22.79	8.11E-17
DDO064	46.1	3.9	0.74	9.87	10.53	0.18	0.56	75.78	14.91	1.00E-16
DDO154	47	1	0.80	9.90	10.58	0.19	0.61	76.30	15.49	9.83E-17
DDO161	66.3	1.9	3.15	10.50	10.87	0.50	2.65	84.02	30.83	6.97E-17
DDO168	53.4	1.9	1.33	10.12	10.67	0.27	1.05	79.46	20.00	8.65E-17
DDO170	60	1.6	2.11	10.32	10.65	0.38	1.73	82.04	25.25	7.70E-17
ESO079-G014	175	3.5	152.90	12.18	12.39	7.28	145.62	95.24	214.80	2.64E-17
ESO116-G012	109.1	3.1	23.10	11.36	11.49	2.01	21.09	91.30	83.49	4.24E-17
ESO563-G021	314.6	11.7	1596.97	13.20	13.45	35.41	1561.55	97.78	694.19	1.47E-17
F561-1	50	2.9	1.02	10.01	11.13	0.23	0.79	77.88	17.53	9.24E-17
F563-V2	116.6	9.4	30.13	11.48	11.12	2.41	27.73	92.00	95.36	3.96E-17
F568-V1	112.3	15.8	25.93	11.41	11.16	2.17	23.76	91.61	88.45	4.11E-17
F571-8	139.7	4.3	62.09	11.79	12.05	3.95	58.15	93.64	136.88	3.31E-17
F571-V1	83.6	3.5	7.96	10.90	10.98	0.96	7.00	87.91	49.02	5.53E-17
F574-1	97.8	4.1	14.91	11.17	11.27	1.49	13.43	90.03	67.09	4.72E-17
F579-V1	112.1	13.4	25.74	11.41	11.33	2.16	23.58	91.60	88.14	4.12E-17
F583-1	85.8	3.6	8.84	10.95	11.02	1.03	7.80	88.29	51.63	5.39E-17
IC2574	66.4	2	3.17	10.50	11.29	0.51	2.66	84.05	30.92	6.96E-17

IC4202	242.6	11	564.71	12.75	11.94	17.60	547.11	96.88	412.80	1.90E-17
KK98-251	33.7	1.6	0.21	9.32	10.34	0.07	0.14	66.38	7.97	1.37E-16
NGC0024	106.3	7.9	20.82	11.32	11.3	1.87	18.95	91.02	79.26	4.35E-17
NGC0055	85.6	5	8.75	10.94	11.27	1.03	7.73	88.25	51.39	5.40E-17
NGC0100	88.1	6.4	9.82	10.99	11.19	1.11	8.71	88.66	54.44	5.24E-17
NGC0247	104.9	8	19.74	11.30	11.37	1.80	17.94	90.87	77.18	4.40E-17
NGC0289	163	8	115.08	12.06	11.82	6.01	109.08	94.78	186.35	2.83E-17
NGC0300	93.3	7	12.35	11.09	11.11	1.31	11.05	89.44	61.06	4.95E-17
NGC0801	220	16.2	381.90	12.58	12.32	13.52	368.39	96.46	339.47	2.10E-17
NGC0891	216.1	5.7	355.53	12.55	11.93	12.88	342.65	96.38	327.54	2.14E-17
NGC1003	109.8	4.2	23.70	11.37	11.59	2.04	21.65	91.37	84.56	4.21E-17
NGC1090	164.4	3.7	119.09	12.08	11.71	6.15	112.94	94.84	189.57	2.81E-17
NGC1705	71.9	4.3	4.36	10.64	10.82	0.63	3.73	85.49	36.26	6.43E-17
NGC2366	50.2	3.2	1.04	10.02	10.61	0.23	0.81	77.98	17.68	9.20E-17
NGC2403	131.2	4.9	48.31	11.68	11.4	3.33	44.98	93.11	120.73	3.52E-17
NGC2683	154	8.1	91.69	11.96	11.81	5.15	86.55	94.39	166.34	3.00E-17
NGC2841	284.8	8.6	1072.56	13.03	12.57	27.10	1045.46	97.47	568.91	1.62E-17
NGC2903	184.6	5.6	189.32	12.28	11.64	8.42	180.91	95.55	239.01	2.50E-17
NGC2915	83.5	6.3	7.93	10.90	10.82	0.96	6.97	87.89	48.90	5.53E-17
NGC2976	85.4	3.3	8.67	10.94	11.13	1.02	7.65	88.22	51.15	5.41E-17
NGC2998	209.9	8.1	316.45	12.50	12	11.91	304.55	96.24	309.02	2.20E-17
NGC3109	66.2	2.6	3.13	10.50	11.06	0.50	2.63	83.99	30.74	6.98E-17
NGC3198	150.1	3.9	82.75	11.92	11.75	4.80	77.95	94.20	158.02	3.08E-17
NGC3521	213.7	15.9	340.00	12.53	12.41	12.50	327.51	96.32	320.31	2.16E-17
NGC3726	168	6.2	129.87	12.11	11.93	6.52	123.35	94.98	197.96	2.75E-17
NGC3741	50.1	2.1	1.03	10.01	10.39	0.23	0.80	77.93	17.61	9.22E-17
NGC3769	118.6	8.4	32.26	11.51	11.42	2.53	29.73	92.17	98.66	3.90E-17
NGC3877	168.4	5.1	131.11	12.12	11.68	6.56	124.55	95.00	198.90	2.74E-17
NGC3893	174	8.9	149.44	12.17	11.93	7.17	142.27	95.20	212.35	2.66E-17
NGC3917	135.9	4.1	55.61	11.75	11.75	3.66	51.95	93.41	129.54	3.40E-17
NGC3949	163	7.1	115.08	12.06	11.71	6.01	109.08	94.78	186.35	2.83E-17
NGC3953	220.8	6.1	387.49	12.59	12.23	13.65	373.84	96.48	341.95	2.09E-17
NGC3972	132.7	2.9	50.55	11.70	11.57	3.43	47.12	93.21	123.51	3.48E-17
NGC3992	241	5.2	549.96	12.74	12.12	17.29	532.67	96.86	407.38	1.92E-17
NGC4010	125.8	4.7	40.83	11.61	11.6	2.97	37.86	92.73	111.00	3.67E-17
NGC4013	172.9	7.1	145.69	12.16	12.27	7.05	138.65	95.16	209.68	2.67E-17
NGC4051	157	5.5	99.05	12.00	11.92	5.43	93.63	94.52	172.89	2.94E-17
NGC4085	131.5	4.8	48.75	11.69	11.51	3.35	45.40	93.13	121.29	3.51E-17
NGC4088	171.7	6.9	141.69	12.15	11.94	6.92	134.78	95.12	206.78	2.69E-17
NGC4100	158.2	5	102.11	12.01	11.67	5.54	96.58	94.58	175.54	2.92E-17
NGC4138	147.3	5.9	76.75	11.89	11.83	4.56	72.19	94.06	152.18	3.14E-17
NGC4157	184.7	7.2	189.73	12.28	12.12	8.43	181.30	95.56	239.27	2.50E-17
NGC4183	110.6	5.4	24.39	11.39	11.22	2.09	22.31	91.45	85.80	4.18E-17
NGC4214	80.1	5.8	6.71	10.83	10.9	0.85	5.86	87.26	45.00	5.77E-17
NGC4217	181.3	7.2	176.14	12.25	12.15	8.01	168.13	95.45	230.55	2.55E-17
NGC4559	121.2	5.1	35.18	11.55	11.43	2.68	32.50	92.38	103.03	3.81E-17
NGC5005	262.2	20.7	770.53	12.89	12.86	21.70	748.84	97.18	482.20	1.76E-17
NGC5033	194.2	3.6	231.88	12.37	11.88	9.65	222.23	95.84	264.52	2.38E-17
NGC5055	179	4.9	167.37	12.22	11.72	7.74	159.63	95.37	224.73	2.58E-17
NGC5371	209.5	3.9	314.05	12.50	11.75	11.85	302.21	96.23	307.84	2.21E-17
NGC5585	90.3	2.4	10.84	11.04	11.26	1.19	9.65	89.00	57.19	5.12E-17
NGC5907	215	2.9	348.35	12.54	11.88	12.71	335.65	96.35	324.22	2.15E-17
NGC5985	293.6	8.6	1211.39	13.08	12.2	29.41	1181.99	97.57	604.61	1.57E-17
NGC6015	154.1	7	91.93	11.96	11.46	5.16	86.78	94.39	166.56	3.00E-17
NGC6195	251.7	9.3	654.33	12.82	12.73	19.44	634.89	97.03	444.35	1.84E-17
NGC6503	116.3	2.4	29.82	11.47	11.24	2.39	27.44	91.98	94.87	3.97E-17
NGC6674	241.3	4.9	552.70	12.74	12.8	17.35	535.36	96.86	408.39	1.91E-17
NGC6946	158.9	10.9	103.93	12.02	11.8	5.61	98.33	94.61	177.10	2.91E-17
NGC7331	239	5.4	531.93	12.73	12.42	16.91	515.03	96.82	400.64	1.93E-17
NGC7814	218.9	7	374.32	12.57	12.05	13.34	360.99	96.44	336.09	2.11E-17
PGC51017	18.6	1.3	0.02	8.29	10.25	0.01	0.01	43.91	2.43	2.48E-16
UGC00128	129.3	2.8	45.57	11.66	11.48	3.20	42.37	92.98	117.26	3.57E-17

UGC00731	73.3	2.3	4.71	10.67	10.72	0.67	4.04	85.82	37.69	6.30E-17
UGC01230	103.7	6.1	18.85	11.28	11.28	1.75	17.11	90.73	75.43	4.46E-17
UGC01281	55.2	3.5	1.51	10.18	10.74	0.30	1.22	80.22	21.37	8.37E-17
UGC02259	86.2	2.9	9.00	10.95	10.94	1.05	7.95	88.35	52.12	5.36E-17
UGC02487	332	3.5	1980.67	13.30	12.26	40.92	1939.77	97.93	773.10	1.39E-17
UGC02885	289.5	12	1145.13	13.06	12.67	28.32	1116.82	97.53	587.84	1.60E-17
UGC02916	182.7	6.9	181.64	12.26	11.95	8.18	173.47	95.49	234.12	2.53E-17
UGC02953	264.9	6	802.77	12.90	12.23	22.30	780.47	97.22	492.18	1.74E-17
UGC03205	219.6	8.6	379.13	12.58	12.28	13.46	365.69	96.45	338.24	2.10E-17
UGC03546	196.9	7.4	245.04	12.39	11.89	10.02	235.03	95.91	271.93	2.35E-17
UGC03580	126.2	3.2	41.35	11.62	11.5	2.99	38.36	92.76	111.71	3.66E-17
UGC04278	91.4	4.8	11.38	11.06	11.16	1.23	10.15	89.16	58.59	5.06E-17
UGC04305	34.5	2.7	0.23	9.36	10.79	0.08	0.16	67.14	8.35	1.34E-16
UGC04325	90.9	2.7	11.13	11.05	11.03	1.21	9.92	89.09	57.95	5.08E-17
UGC04499	72.8	2.4	4.58	10.66	10.1	0.65	3.93	85.71	37.17	6.35E-17
UGC05005	98.9	7.2	15.60	11.19	11.1	1.53	14.07	90.17	68.60	4.67E-17
UGC05253	213.71	7	340.06	12.53	12.08	12.50	327.57	96.32	320.34	2.16E-17
UGC05716	73.1	1.2	4.66	10.67	10.77	0.66	4.00	85.78	37.48	6.32E-17
UGC05721	79.7	6.6	6.58	10.82	10.79	0.84	5.74	87.19	44.55	5.80E-17
UGC05986	113	4.1	26.58	11.42	11.71	2.21	24.37	91.68	89.56	4.09E-17
UGC06399	85	3.8	8.51	10.93	11.1	1.01	7.50	88.15	50.68	5.44E-17
UGC06446	82.2	4.3	7.44	10.87	10.89	0.92	6.53	87.66	47.39	5.62E-17
UGC06614	199.8	16	259.80	12.41	12.64	10.42	249.39	95.99	280.00	2.31E-17
UGC06628	41.8	6.4	0.50	9.70	11.11	0.13	0.36	73.08	12.26	1.11E-16
UGC06667	83.8	3.1	8.04	10.91	11.11	0.97	7.07	87.94	49.25	5.51E-17
UGC06786	219.4	7.8	377.75	12.58	12.08	13.42	364.33	96.45	337.62	2.11E-17
UGC06787	248.1	4.8	617.69	12.79	13.38	18.70	599.00	96.97	431.73	1.86E-17
UGC06818	71.2	4	4.19	10.62	10.9	0.62	3.57	85.32	35.56	6.49E-17
UGC06917	108.7	3.5	22.76	11.36	11.32	1.99	20.77	91.26	82.87	4.25E-17
UGC06923	79.6	2.5	6.55	10.82	11.07	0.84	5.71	87.16	44.44	5.80E-17
UGC06930	107.2	5.1	21.53	11.33	11.3	1.91	19.62	91.11	80.60	4.31E-17
UGC06973	174.2	6.2	150.13	12.18	11.87	7.19	142.94	95.21	212.84	2.65E-17
UGC06983	109	5.8	23.01	11.36	11.23	2.00	21.01	91.29	83.33	4.24E-17
UGC07125	65.2	2.1	2.95	10.47	10.72	0.48	2.47	83.70	29.82	7.09E-17
UGC07151	73.5	2.8	4.76	10.68	11.07	0.67	4.09	85.87	37.89	6.29E-17
UGC07261	74.7	3.4	5.08	10.71	11	0.70	4.37	86.14	39.14	6.19E-17
UGC07399	103	3.3	18.35	11.26	11.12	1.71	16.64	90.66	74.41	4.49E-17
UGC07524	79.5	3.6	6.51	10.81	11.07	0.84	5.68	87.15	44.33	5.81E-17
UGC07603	61.6	2.8	2.35	10.37	10.77	0.41	1.94	82.58	26.61	7.50E-17
UGC07690	57.4	3.2	1.77	10.25	10.89	0.33	1.44	81.09	23.11	8.05E-17
UGC08286	82.4	2.3	7.52	10.88	10.87	0.93	6.59	87.69	47.62	5.61E-17
UGC08490	78.6	3.8	6.22	10.79	10.74	0.81	5.41	86.97	43.33	5.88E-17
UGC08550	56.9	1.9	1.71	10.23	10.65	0.33	1.38	80.90	22.71	8.12E-17
UGC08699	182.4	6.9	180.45	12.26	11.99	8.15	172.31	95.49	233.35	2.53E-17
UGC09037	152.3	9.6	87.71	11.94	12.09	5.00	82.72	94.31	162.69	3.03E-17
UGC09133	226.8	4.2	431.35	12.63	12.15	14.68	416.68	96.60	360.78	2.04E-17
UGC09992	33.6	3.3	0.21	9.32	10.64	0.07	0.14	66.25	7.92	1.38E-16
UGC10310	71.4	3.9	4.24	10.63	10.96	0.62	3.62	85.37	35.76	6.47E-17
UGC11455	269.4	7.4	858.72	12.93	13.03	23.34	835.39	97.28	509.04	1.72E-17
UGC11914	288.1	10.5	1123.14	13.05	13.59	27.96	1095.19	97.51	582.17	1.60E-17
UGC12506	234	16.8	488.79	12.69	12.08	15.97	472.83	96.73	384.05	1.97E-17
UGC12632	71.7	2.8	4.31	10.63	10.83	0.63	3.68	85.44	36.06	6.44E-17
UGCA444	37	4.8	0.31	9.49	10.07	0.09	0.21	69.39	9.60	1.25E-16

With reference to relation (20), relation (19) can be expressed as,

$$z_n \cong \frac{z_c}{1+z_c} \leq 1 \quad (21)$$

Based on this new definition, farthest galaxies distance can be estimated very easily. For example, see the following Table-2. We sincerely appeal that, on cosmological scales, 2.5% is not yet all a ‘serious’ error. We would like to emphasize the point that, conceptually, we are no way deviating from the basic idea of expanding

universe and receding galaxies. Only thing is that, we are confining to ‘light speed expansion’ and ‘light speed receding’. With further study, there is a scope for understanding the universe in a unified approach. Since most of the cosmological observations are being studied with photons that move at speed of light, rather than ‘working on controversial cosmic ‘acceleration’ and ‘flatness’ phenomena [33,34], it is better to work on understanding the root causes of ‘speed of light’.

Table-2: To estimate and fit the distances of farthest galaxies with re-defined redshift

Galaxy	Redshift	Standard Light travel distance (Gly)	Estimated Light travel distance (Gly)	%Error
GN-z11	11.09	13.39	13.41	-0.15
MACS1149-JD1	9.11	13.26	13.17	0.65
EGSY8p7	8.68	13.23	13.11	0.91
A2744 YD4	8.38	13.2	13.06	1.05
EGS-zs8-1	7.73	13.13	12.95	1.41
z7 GSD 3811	7.66	13.11	12.93	1.36
z8 GND 5296	7.51	13.1	12.9	1.51
SXDF-NB1006-2	7.215	13.17	12.84	2.5
GN-108036	7.213	13.07	12.84	2.5
BDF-3299	7.109	13.05	12.84	2.5
A1703 zD6	7.014	13.04	12.84	2.5
BDF-521	7.008	13.04	12.84	2.5
G2-1408	6.972	13.03	12.84	2.5
IOK-1	6.964	13.03	12.84	2.5

Richard Powell has written an online C program (<http://www.atlasoftheuniverse.com/cosmodis.c>) (version 1.1) for estimating the light travel distance. For a summary of the used formulae, one can refer: “David W Hogg, Distance Measures in Cosmology, (2000), astro-ph/9905116”. Using that program and considering a redshift of $z_c = (0.1 \text{ to } 200)$, we have prepared Figure-2. Green curve indicates the light travel distance in Lambda cosmology prepared with Omega matter = 0.32, Omega lambda = 0.68, Omega radiation = 0.0 and $H_0 = 66.87$ km/sec/Mpc. Red curve indicates our estimated light travel distance $[z_c/(1+z_c)](c/H_0) \cong [z_n](c/H_0)$ where $H_0 = 66.87$ km/sec/Mpc. As traditional redshift is increasing, error in estimated light travel distance is increasing to +8.59% at $z_c \cong 1.20$ and from there onwards, error is reaching to 0% at $z_c \cong 11.5$ to 11.55. Proceeding further, error is reaching to -5.14% at $z_c \cong 200.0$. Here, positive error’ means, traditional light travel distance is higher than our estimate and ‘negative error’ means, traditional light travel distance is lower than our estimate. This can be also be confirmed with other online cosmic redshift-distance calculators written by Aaron Robotham and Joseph Dunne (<https://cosmocalc.icrar.org/>). See the appendix for C/C++ source file.

Based on the two curves presented in Figure-2, it is certainly possible to say that, if ‘light travel distance’ is a characteristic index in Lambda cosmology, the same index, can also be understood with our re-defined red shift relation (19) and ‘light speed expansion’.

With reference to relation (21), relation (20) can be expressed as,

$$\text{Traditional redshift, } z_c \cong \frac{z_n}{1-z_n} \quad (22)$$

Relation (22) indicates that, for increasing light travel distances, z_c seems to have higher values compared to re-defined redshift z_n . See Figure 3. Red curve indicates our re-defined redshift and green curve indicates traditional redshift. To some extent, based on relations (21) and (22), it is possible to guess that, currently believed cosmic acceleration can be considered as an alternative interpretation associated with the alternative relation (20). It needs further study with respect to galactic redshifts and star rotation curves etc. Among relations (19) and (20), the correct definition can be decided with future observations.

Figure 2: Comparison of standard and estimated light travel distance

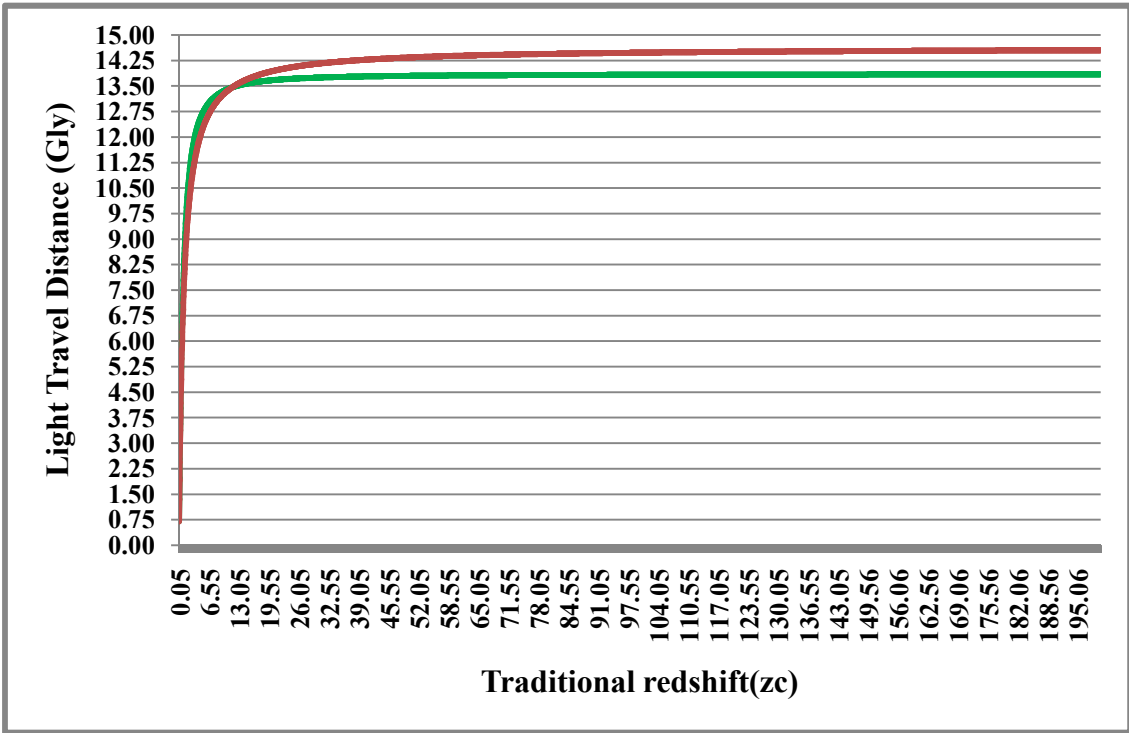
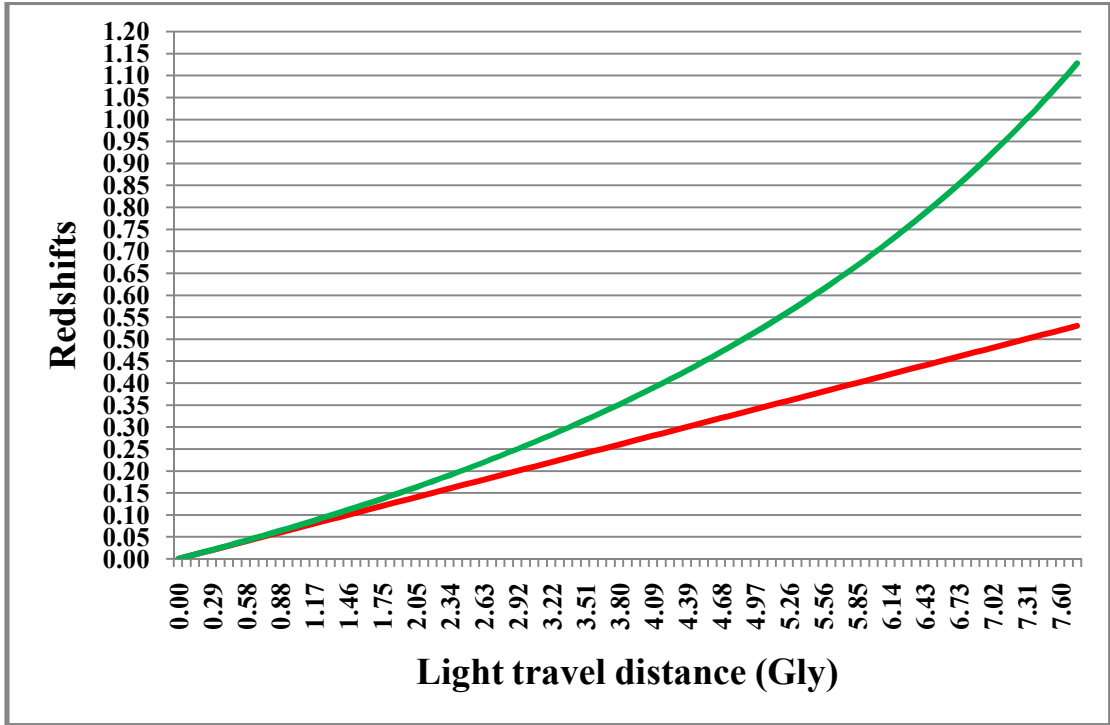


Figure 3: Comparison of standard and re-defined redshifts



8. Understanding and modifying Lambda cosmology

With reference to traditional cosmology, understanding ‘Lambda term’ or ‘cosmological constant’ is a very difficult task. We would like to emphasize that, quantum cosmology point of either Lambda term or cosmological constant can be understood very easily. By considering Planck scale as the origin and growing

Planck ball as a sequel of cosmic evolution, it is very simple to understand the cosmic physics. In this context, we appeal that,

- 1) Even though highly intuitive and impressive, there is no clarity and proper physical support for 'big bang' and 'inflation' concepts.
- 2) Lambda term can be expressed as a scalar quantity having the form, $\Lambda_t \equiv \frac{3}{R_t^2} \equiv \frac{3H_t^2}{c^2}$.
- 3) Similarly, cosmological constant can be expressed as a scalar quantity having the form, $\frac{\Lambda_t c^4}{8\pi G}$.
- 4) In our model, total mass energy density and critical energy density, both are identical.
- 5) At any stage of cosmic evolution, $\frac{3c^2 H_t^2}{8\pi G} - \frac{\Lambda_t c^4}{8\pi G} \equiv 0$. Clearly speaking, if one is willing to consider $\left(\frac{\Lambda_t c^4}{8\pi G}\right)$ as a characteristic expression for 'dark energy density', then it can be inferred that, at any stage of cosmic evolution, difference of dark energy density and mass-energy density is always 'zero' and universe is always expanding at speed of light.
- 6) Frankly speaking, if universe is really expanding with 'speed of light', unphysical Lambda term and its inherited dark energy term, both can be relinquished forever. It needs further investigation.
- 7) In Lambda cosmology, matter creation is associated with big bang. In our model, matter is being created continuously with an expression of the form, $M_t \equiv \frac{c^3}{2GH_t} \equiv \frac{c^3 t}{2G}$.
- 8) Cosmic Temperature-Time relation can be expressed as, $T_t \equiv \frac{0.18511 \times 10^{10}}{\sqrt{t}}$ K. This relation is almost similar to the main stream relation derived on big bang concepts. Only difference is that, for the same expected temperatures, our model cosmic physical processes are taking place early compared to big bang model. This idea helps in understanding the early formation of galaxies at the cosmic dawn.
- 9) Most important thing is that the characteristic cosmic expansion rate can be accurately estimated by knowing the CMBR temperature and there is no need to take the help of galactic redshifts and galactic distances.
- 10) Estimated galactic dark matter is more than 90 percent of the total mass of 'massive' galaxies. In case of least massive galaxies also, dark matter is roughly 50% of the total mass of galaxy.
- 11) Advanced galactic red shift data is raising doubts on well believed cosmic acceleration [35,36] and supporting constant rate of expansion [37]. Considering the Tolman test for surface brightness and based on the analysis of the UV SB of luminous disk galaxies from HUDF and GALEX datasets, reaching from the local universe to $z \sim 5$, recently it has been shown that the surface brightness remains constant as expected in a static universe [38,39].
- 12) One thing it is very clear that, red shift data analysis associated with references [35,36,37,38,39] is generating lot of confusion and controversy in assessing the correct rate of cosmic expansion. In this context, based on the data presented in Table 2, our proposed definition of Redshift i.e. relations (20) and (22) can be recommended further research.
- 13) Quantum cosmology point of view, Lambda cosmology needs a review at fundamental level. Our model is based on 'time reversed' black holes and seems to be well connected with General theory of relativity as well as Quantum mechanics [5,6,9].
- 14) By counting the actual number of galaxies and considering the average mass of galaxy and with minor adjustments, our model can be refined in workable way compared to Lambda model of cosmology.
- 15) In 2011, researchers found evidence that galaxies tend to rotate in a preferred direction. They uncovered an excess of left-handed, or counter-clockwise rotating, spirals in the part of the sky toward the north pole of the Milky Way. This study suggests that the shape of the Big Bang might be more complicated than previously thought, and that the early universe spun on an axis [40]. Recently, research presented at the 236th meeting of the American Astronomical Society suggests that the whole universe may be spinning. If further studies bear this out, the finding will challenge some of the fundamental assumptions of modern cosmology [41]. In this context, considering the field experts [42,43,44] advocated value of 'current cosmic angular velocity', our model can be recommended for observational search.

9. Conclusion

The result of this exploration provides for remarkable correlations with current galactic observations and perhaps moves us closer to understanding the scalar nature and influence of dark matter and Lambda on the expanding universe. Our light travel distance versus redshift graphs employ a reasonable and consistent mathematical re-definition of cosmological and galactic redshift. Our ongoing efforts to better understand the observed universe have moved us away from an unquestioning acceptance of cosmology in its standard model form. Problematic issues concerning the questionable physical nature of an infinite singularity, a 'Big Bang' characterized by cosmic inflation and its unproven 'inflaton' field, cosmic acceleration in opposition to steady light-speed expansion, the cosmological constant problem and various cosmic coincidence problems have pointed us in a different direction. The reader is encouraged to further explore Flat Space Cosmology and its model variants, one of which has been presented herein.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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Appendix on C/C++ source file for estimating and comparing cosmic redshift-distance relations

```
#include <stdio.h>
#include <math.h>
#include <conio.h>
#include <iostream.h>
#define c 299792.458
int main()
{
float z=0.0;
do{
z=z+0.05;
float H0 = 66.87; // Hubble constant (km/s/Mpc) - adjust according to taste
float OM = 0.32; // Omega(matter) - adjust according to taste
float OL = 0.68; // Omega(lambda) - adjust according to taste
float OR = 0.0; // 0.42/(H0*H0); // Omega(radiation) - this is the usual textbook value
long i; long n=10000; // Number of steps in the integral
float OK = 1-OM-OR-OL; // Omega(k) defined as 1-OM-OR-OL
float HD = 3.2616*c/H0/1000; // Hubble distance (billions of light years). See section 2 of Hogg
float a, adot; // Redshift "z", Scale Factor "a", and its derivative "adot"
float DC, DCC=0, DT, DTT=0, DA, DL, DM;
float age, size; // The age and size of the universe
for(i=n; i>=1; i--) { // This loop is the numerical integration
a = (i-0.5)/n; // Steadily decrease the scale factor
// Comoving formula (See section 4 of Hogg, but I've added a radiation term too):
adot = a*sqrt(OM*pow(1/a,3)+OK*pow(1/a,2)+OL+OR*pow(1/a,4)); // Note that "a" is equivalent to 1/(1+z)
DCC = DCC + 1/(a*adot)/n; // Running total of the comoving distance
DTT = DTT + 1/adot/n; // Running total of the light travel time (see section 10 of Hogg)
if (a>=1/(1+z)) { // Collect DC and DT until the correct scale is reached
DC = DCC; // Comoving distance DC
DT = DTT; // Light travel time DT
} }
// Transverse comoving distance DM from section 5 of Hogg:
if (OK>0.0001) DM=(1/sqrt(OK))*sinh(sqrt(OK)*DC);
else if (OK<-0.0001) DM=(1/sqrt(fabs(OK)))*sin(sqrt(fabs(OK))*DC);
else DM=DC;
age = HD*DTT; // Age of the universe (billions of years)
size = HD*DCC; // Comoving radius of the observable universe
DC = HD*DC; // Comoving distance
DA = HD*DM/(1+z); // Angular diameter distance (section 6 of Hogg)
DL = HD*DM*(1+z); // Luminosity distance (section 7 of Hogg)
DT = HD*DT; // Light travel distance
float xx;
xx=(z/(z+1))*HD;
cout<<z<<" "<<DT<<" "<<(z/(z+1))<<" "<<xx<<" "<<((DT-xx)/DT)*100<<endl;
} while (z<=200.0);
}
```