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# THE ERROR INDUCED BY USING REPRESENTATIVE PERIODS IN CAPACITY EXPANSION MODELS -SYSTEM COST, TOTAL CAPACITY MIX AND REGIONAL CAPACITY MIX, SUPPLEMENTARY MATERIAL

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## 1 Model

The test model is a network electricity optimization model. It is a so called zonal model, meaning that the electricity flows are not bound by Kirschoffs laws. The resulting optimal values for transmission connections may be interpreted as NTC values. It is the European model used in reference [1], however simplified by excluding the hydro power, and modified by introducing the temporal reduction version. In the following, the model without temporal reduction, i.e. the model with 8760 consecutive time steps, is referred to as the 'benchmark' model. The model versions with temporal reduction are referred to as 'reduced' models. With the exception of Equation 10, the model equations are the same. The benchmark version is achieved by setting the weights  $\omega_t = 1$ .

### 1.1 Sets

$I$	technologies, where $I = I_{disp} + I_{VRE} + I_{stor}$ (dispatchable-, VRE-, and storage technologies)
$R$	regions
$J_{v,r}$	classes of VRE technology $v$ in region $r$ , $v \in I_{VRE}$ , $r, r' \in R$
$T$	time steps

### 1.2 Input parameters

$\alpha_{v,j,t}$	availability factor for VRE technology $v$ , in VRE class $j$ at time $t$ , $\in [0, 1]$
$\kappa_i$	investment cost for technology $i$ , [ $\text{€}/\text{MW}$ ] or [ $\text{€}/\text{MW km}$ ] or [ $\text{€}/\text{MWh}$ ]
$\nu_i$	running cost for technology $i$ , [ $\text{€}/\text{MWh}$ ]
$\omega_t$	weight on time step $t$ , $[1, \max(T)]$
$\delta_{r,t}$	demand in region $r$ during time step $t$ , [ $\text{MWh}$ ]
$\theta_{r,r'}$	distance between regions $r, s$ , [ $\text{km}$ ]
$\rho_j$	available land for wind- and solar classes, [ $\text{MW}$ ]
$\tau$	period length, e.g. 248, for a day
$\lambda_i$	loss in transmission or storage operation, $\in [0, 1]$

, where  $i \in I$ ,  $j \in J_{v,r}$ ,  $v \in I_{VRE}$ ,  $r, r' \in R$ ,  $t \in T$

### 1.3 Variables

$n_{i,r}$	capacity of dispatchable technology $d$ in region $r$ [MW]
$n_{v,j}$	capacity of VRE technology $v$ in VRE class $j$ [MW]
$a_{r,r'}$	transmission capacity between regions $r$ and $r'$
$n_{s,r}$	capacity of storage technology $s$ in region $r$ [MWh]
$p_{d,r,t}$	electricity generated by dispatchable technology $d$ in region $r$ at time step $t$ , [MWh/hour]
$p_{v,j,t}$	electricity generated by VRE technology $i$ in region $r$ at time step $t$ , [MWh/hour]
$e_{r,r',t}$	export from region $r$ to region $r'$ at time step $t$ [MWh/hour]
$l_{s,r,t}$	storage reservoir level in region $r$ at time step $t$ , [MWh]
$o_{s,r,t}$	electricity discharged storage in region $r$ at time step $t$ , [MWh/hour]
$m_{s,r,t}$	electricity charged into storage in region $r$ at time step $t$ , [MWh/hour]

, where  $i \in I, j \in J_v, v \in I_{VRE}, s \in I_{stor}, i \in I, r, r' \in R, t \in T$  Note that the dispatch variables ( $p_{i,r,t}, e_{r,r',t}, o_{s,r,t}, m_{s,r,t}$ ) are defined as hourly values, not as the energy generated during time step  $t$ .

### 1.4 Objective function

$$\min_{i,r,t} \sum_{i \in I} \sum_{r,r' \in R} \sum_{t \in T} \kappa_i n_{i,r} + \nu_i \omega_t p_{i,r,t} + 0.5 \kappa_a \theta_{rr'} a_{rr'} \quad (1)$$

Note that the weights enter the sum for the dispatch variables.

### 1.5 Constraints

Generation in every hour is constrained by the capacity:

$$p_{i,r,t} \leq n_{i,r} \quad (2)$$

$$i \in I_{disp}, r \in R, t \in T$$

In the case of VRE generation, it is constrained by the capacity in each VRE class, and the availability factor in that VRE class.

$$p_{v,r,t} \leq \alpha_{v,j,t} n_{v,j} \quad (3)$$

$$v \in I_{VRE}, j \in J_v, r \in R, t \in T$$

Exported electricity is constrained by the transmission capacity:

$$e_{r,r',t} \leq \omega_t a_{r,r'} \quad (4)$$

$$\text{where } r, r' \in R, t \in T$$

In addition, wind and solar capacity are constrained by the available area in that VRE class:

$$n_{i,j} \leq \rho_j \quad (5)$$

$$i \in I_{VRE}, j \in J_r, r \in R$$

Generation must satisfy demand:

$$\omega_t \left( \sum_{v \in I_{VRE}} \sum_{j \in J_v} p_{v,j,t} + \sum_{i \in I_{disp}} p_{i,r,t} + \sum_{r' \in R} (1 - \lambda) e_{r',r,t} - e_{r,r',t} - m_{r,t} \right) \geq \omega_t \delta_{r,t} \quad (6)$$

$$\text{where } r, r' \in R, t \in T$$

which simplifies to:

$$\sum_{v \in I_{VRE}} \sum_{j \in J_v} p_{v,j,t} + \sum_{i \in I_{disp}} p_{i,r,t} + \sum_{r' \in R} (1 - \lambda) e_{r',r,t} - e_{r,r',t} - m_{r,t} \geq \delta_{r,t} \quad (7)$$

$$\text{where } r, r' \in R, t \in T$$

The storage reservoir level is constrained by the amount of storage:

$$l_{i,r,t} \leq n_{i,r} \quad (8)$$

where  $i \in I_{stor}, r \in R$

The storage level depends on the level  $i$  the previous time step, less discharge plus charging:

$$l_{i,r,t} = l_{i,r,t-1} - o_{i,r,t} + m_{i,r,t} \quad (9)$$

where  $i \in I_{stor}, r \in R$

Only for the reduced model versions: The storage level in the last time step of each period is equal to that in the first time step of each period:

$$l_{i,r,(N-1)\tau+1} = l_{i,r,N\tau} \quad (10)$$

where  $i \in I_{stor}, r \in R, N = 1, \dots, \frac{\max(T)}{\tau}$

Equation 10 is the only part where the reduced model differs from the benchmark model. In the benchmark model, there are no periods, and, hence, this constraint is not there.

## 2 Data

The study region of Europe (EU-27 + Norway and Switzerland) is defined by its wind and solar resources, as well as the limitations of the same, and a regionalization approach where each region is assigned a demand profile (based on statistics from an historical year). Europe is divided into 8 regions, see Figure 1 for the map and Table 1 for the countries included in each region. The model is applied in a greenfield exercise, i.e., there are neither existing generation capacities nor existing transmission lines between the regions. Hydro power and hydro resources are not considered, due to that it is potentially a distortion when comparing regional capacity allocation. For similar reasons, there is no offshore wind power option.

The model can invest in transmission capacity between neighboring regions, including capacities that traverse seas. There is thus the possibility to invest in transmission between UK – NOR, NOR – GER etc.

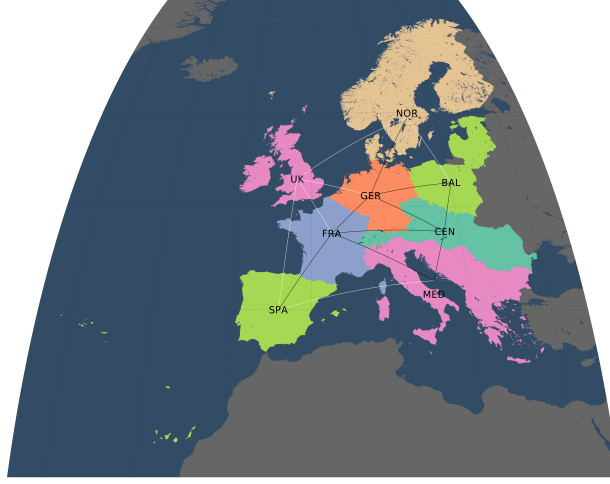
There is a procedure whereby available land for wind- and solar deployment is constrained, e.g. by removing protected areas and highly populated areas. For further details please refer to [1].

The weather data are hourly ERA-Interim data from the European Centre for Medium-Range Weather Forecasts (ECMWF), which are processed through an output function for wind and output functions for a fixed silicon solar cell. Again, details are provided in [1] and the Supplementary material therein.

The electricity demand data are generated using the machine learning approach in [1], and these regional demand time-series remain constant, so that there is no increase or decrease in demand and no alterations in the demand profile. The annual electricity demand for each region is shown in Table 1.

**Table 1:** Countries belonging to regions and annual demand for the regions in the model.

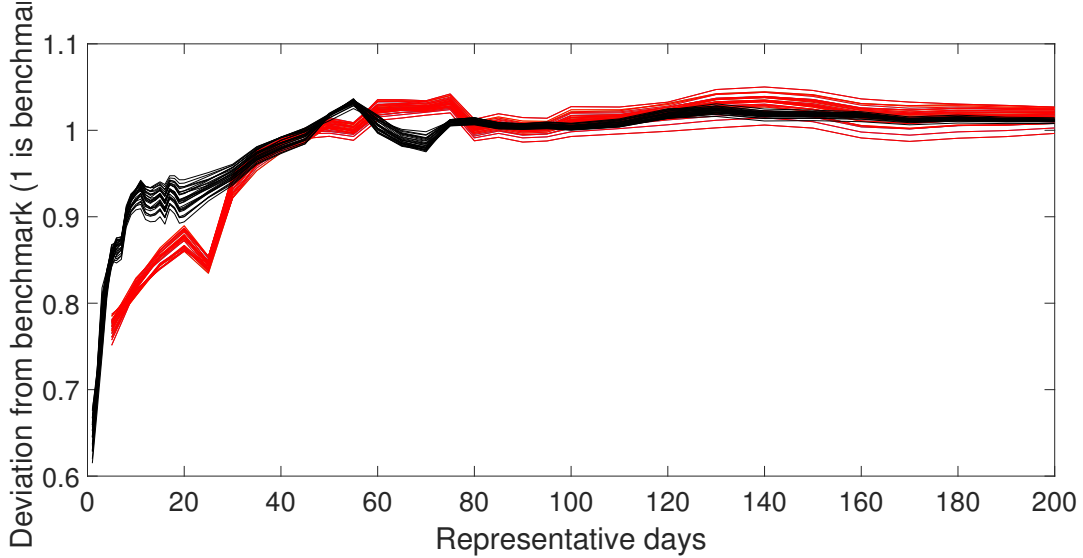
Region	Countries	Annual demand ([TWh])
NOR	Sweden, Denmark, Norway, Finland	510
FRA	France, Benelux	690
GER	Germany	1060
UK	UK, Ireland	450
MED	Italy, Greece, Bulgaria, Romania	610
BAL	Lithuania, Estonia, Latvia	240
SPA	Spain, Portugal	420
CEN	Poland, Austria, Czech Republic, Slovakia, Hungary, Switzerland	430



**Figure 1:** The 8 regions used in the case study. Transmission lines can be built between neighboring regions, including overseas connections.

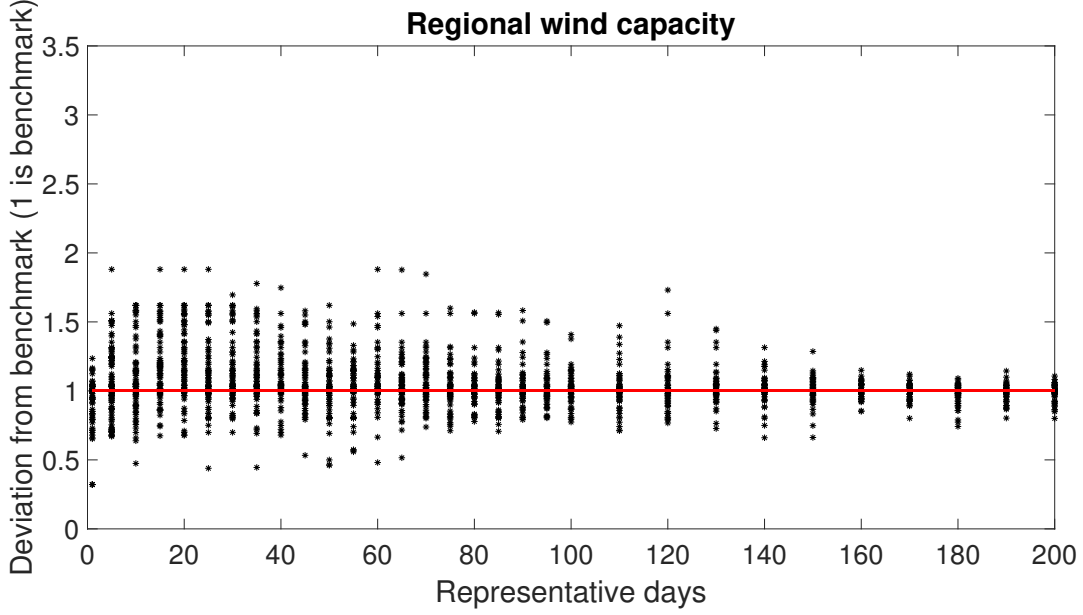
### 3 Additional results

Figure ?? shows the result of the sensitivity analysis for system cost deviation. The black lines represent the base case (same as Figure 1 in the main text), while the red lines represent the results from the sensitivity analysis, where 5-day periods were selected instead of 1-day periods. Figure ?? shows that selecting 1-day periods yields a smaller error in system cost compared to the benchmark hourly model.

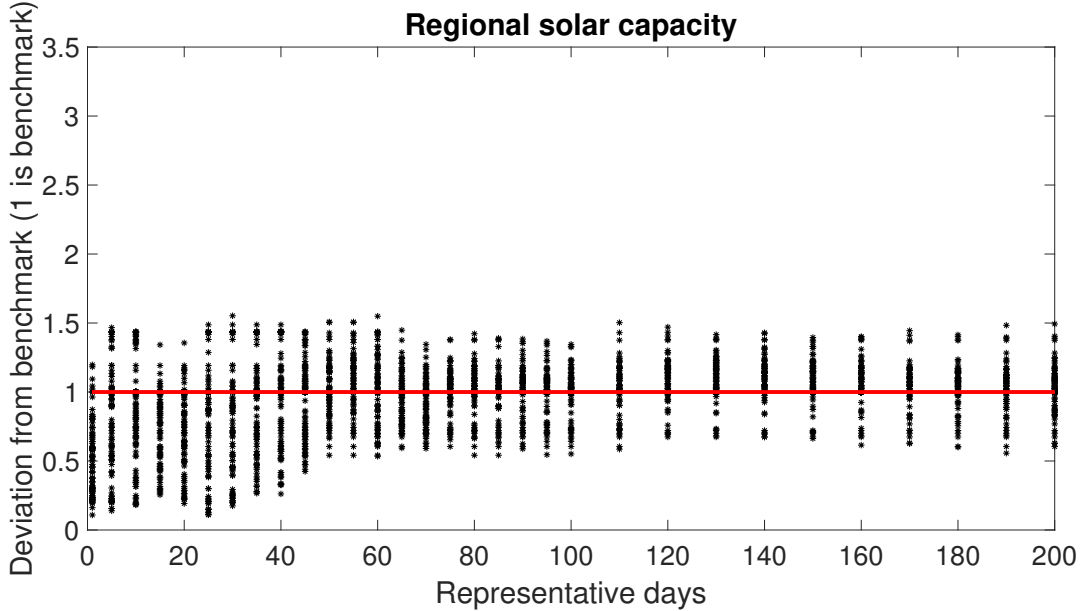


**Figure 2:** Deviation from the benchmark value for system cost, using 1-200 representative days for the base case (1-day periods, black lines) versus the sensitivity case (5-day periods, red lines). Each line represents one technology investment cost combination, see the Method section. All scenarios are highly renewable with a VRE penetration of 90%.

Figures ?? and ?? are analogous to Figure 5a and 5b in the main text, but for the threshold of regions that were omitted. Instead of showing all regions with a generation of at least 10% of demand (as Figure 5a and 5b in the main text), we here show regions with at least 40% of demand coming from wind (Figure ??) and solar Figure ??, respectively.



**Figure 3:** The deviation from benchmark for regional wind power capacity for VRE penetration of 90%. The figure shows deviation from the benchmark model results, where 1 means that there is no deviation. Each dot represents one data point, where a data point is the deviation for one region in one of the 27 cost scenarios. Data points for regions where the generation wind contributes less than 40% of the total demand are omitted. For each model version (1, 2, ... 200 days), there is a maximum of 216 data points (27 cost combinations times eight regions).



**Figure 4:** The deviation from benchmark for regional solar power capacity for VRE penetration of 90%. The figure shows deviation from the benchmark model results, where 1 means that there is no deviation. Each dot represents one data point, where a data point is the deviation for one region in one of the 27 cost scenarios. Data points for regions where the generation of solar contributes less than 40% of the total demand are omitted. For each model version (1, 2, ... 200 days), there is a maximum of 216 data points (27 cost combinations times eight regions).

## References

- [1] N. Mattsson, V. Verendel, F. Hedenus, and L. Reichenberg, “An autopilot for energy models—automatic generation of renewable supply curves, hourly capacity factors and hourly synthetic electricity demand for arbitrary world regions,” *arXiv preprint arXiv:2003.01233*, 2020.