

Article

Reduction of the Mass of the Proto-Quark Star during Cooling

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Abstract: The paper defines the characteristics of two proto-quark stars ($M_b = 1.22M_\odot$ and $M_b = 1.62M_\odot$) that were formed after the explosion of the supernova. It is assumed that the changes in the energy and lepton charge of the central regions of the pre-supernova star can be neglected during the implosion. The equation of state for hot quark matter is determined based on the MIT bag theory taking into account the presence of neutrinos. The possible maximum values of the central temperatures of these stars are determined. The energy of neutrinos in the studied proto-quark stars is of the order of $250 \div 300 \text{ MeV}$. The decrease in the mass of these stars during cooling is about $0.16M_\odot$ for $M_b = 1.22M_\odot$ and $0.25M_\odot$ for $M_b = 1.62M_\odot$.

Keywords: proto-quark stars; maximum temperature; mass reduction

1. Introduction

According to modern concepts the neutron and quark stars are formed during the explosion of a supernova star (SN). The first numerical simulations of the explosion of SN were carried out in the middle of the last century [1–4]. Despite of the tremendous progress of computer technology and the presence of numerous works on explosion modeling of SN, the problem has not been completely resolved. More recently, the problem of binary neutron star mergers and proto-neutron star evolution has gained attention, which like the SN explosion problem require knowledge of the equation of state of matter at finite temperature as an input in numerical simulations [5–16]. In addition, the discovery of $M \approx 2M_\odot$ pulsars during the last decade has put strong constraints on the properties of the dense matter indicating that the equation of state at high densities must be stiff [17]. Furthermore, the NICER experiment has measured the radius of PSR J0030+0451, thus constraining the equation of state of dense matter at intermediate densities [18,19].

Although it is impossible to determine the evolution of the physical system using conservation laws, in case of favorable conditions they allow to have a relation between the initial and final values of some physical parameters of the examined object. In the current work the temperature and energy reserves of a proto-quark star (PQS), which was formed during the SN explosion, have been calculated just based on the energy and lepton charge conservation laws. The decrease in its mass during cooling has been determined too. When the central dense core of a massive star loses its mechanical stability (due to neutronization or due to the dissociation of iron nuclei) it begins to shrink rapidly. During the free fall time $\tau_0 \sim (R^3/GM)^{1/2} \sim 1 \text{ sec.}$, for the radius $R \approx 5000 \text{ km}$ and the mass $M \approx 2M_\odot$ of the iron core, G is gravitational constant) the core is shrinks to nuclear densities. Atomic nuclei fall apart into separate nucleons. A proto-neutron star (PNS) is formed if the pressure of the matter stops further compression. Otherwise the compression continues until the nucleons fall apart into separate quarks. A proto-quark star is formed. In principle, the compression may continue until a black hole is formed. The mechanism of explosive ejection of the main mass of the star is not considered here.

We can only note that one of the proposed mechanisms that provides the explosive nature of the ejection of the bulk of the pre-supernova is the magnetorotational mechanism [20,21]. The current state of the problem is detailed in [22]. The purpose of our work is to define the maximum value of the temperature of the proto-quark star which has been formed after SN explosion and to calculate the decrease of its mass during full cooling.

Main assumptions:

1. The matter of the central core of the pre-supernova before implosion is opaque for neutrinos.
2. The process of compression of the central core of a pre-supernova before the formation of PQS occurs so rapidly that its total energy and lepton charge there is no enough time for significant changes.

That's why the lepton charges of the central core of a pre-supernova and a proto-quark star are almost the same. In fact, after the disintegration of atomic nuclei and then nucleons at the initial moment of the life of PQS, it completely or its central part consists of a hot superdense u, d, e^- , and e^+ plasma which is not in the β -equilibrium state.

Note that the formation of the ud phase can be accompanied by a decrease in the temperature of the medium. The equilibrium between electrons and positrons is established through a faster electromagnetic interaction. The matter had already become opaque for neutrinos before the breakdown of atomic nuclei. So the chemical equilibrium (equilibrium in β -processes) in PQS is established in a matter that is opaque for neutrinos: i.e. with full conservation of lepton charge. The ud quark composition passes into β -equilibrium state through the reactions $u + e^- \rightarrow s + \nu_e$ and $d \rightarrow u + e^- + \bar{\nu}_e$. The star can become very hot due to these reactions. This resembles the mechanism of heating of matter through non-equilibrium β processes proposed in [23]. It will be shown below that due to these processes the temperature of the proto-quark star can rise up to $7 \cdot 10^{11} K$.

2. The Equation of state of hot quark matter

The composite composition, thermodynamic characteristics, and the equation of state of a hot strange quark matter (HSQM) while it is opaque to neutrinos within the framework of the MIT quark bag model are defined in [24,25]. In these studies it was shown that the HSQM thermal energy reserves are weakly dependent on the lepton composition. The fact of taking into account the presence of muons, muon and tau neutrinos and their antiparticles in HSQM, as well as the phenomenon of neutrino oscillations, changes this dependency slightly. Therefore, in determining the integral parameters of the proto-quark star, only the electrons, electron neutrinos, and the corresponding antiparticles were taken into account, which significantly reduces the time of numerical calculation.

After the "breakdown" of the baryons and before the establishment of β -equilibrium in the quark matter the concentrations of u and d quarks n_u and n_d will be

$$n_u = n(Z + A)/A, \quad n_d = n(2A - Z)/A,$$

where A and Z are the mass and the atomic numbers of the atomic nucleus at the moment of their disintegration, n is the concentration of the baryonic charge. The concentrations of electrons and positrons are determined by the conditions of local electroneutrality and the chemical equilibrium. In the approximation of mass-less quarks, the pressure P and the energy density ε are bound by the simple relation

$$\varepsilon = 3P + 4B, \quad (1)$$

where B is the bag constant in MIT bag model. Taking into account the s quark mass, the relationship between the energy and pressure changes slightly [24,25]:

$$\varepsilon = 3P + 4B + \Delta(m_s, n_s, T), \quad (2)$$

where n_s is the concentration of the s quark. This correction is due to the presence of relatively large mass $m_s = 95$ MeV of the s quark. The calculations show that $\Delta/\varepsilon < 0.04$ [24,25]. This means that in (2) the explicit temperature dependence is weak. In opposite, the density of energy and the pressure strongly depend on the temperature at a fixed concentration of the baryon.

The mass and the radius of static isothermal superdense configurations are determined exclusively by law $P = P(\varepsilon, T)$. Therefore, in the $\Delta = 0$ approximation, all the mass-central density $\rho_c = \varepsilon_c/c^2$ curves ε_c is the density of energy at the center of the star, c is the speed of light in vacuum) of isothermal PQS coincide. Taking into account the mass of the strange quark leads only to insignificant shifts of these curves: the higher is the temperature, the greater is the shift upward of the curve $M = M(\rho_c, T_c)$ from the curve $M = M(\rho_c, T_c = 0)$. The relative shift does not exceed $6 \cdot 10^{-3}$ at the maximum of these curves at $T_c = 100$ MeV[26].

3. Methods

The integral parameters PQS (the gravitational and baryonic masses and the radius) are determined by numerical integration of the relativistic equations of hydrostatic equilibrium (3), the mass (4), the time coefficient of the Schwarzschild metric $g_{00} = e^\nu$ (5) and the baryonic charge N (6)

$$\frac{dr}{dP} = -\frac{r^2 c^2}{Gm\varepsilon} \left(1 - \frac{2Gm}{rc^2}\right) \frac{1 + \frac{4\pi r^3 P}{mc^2}}{1 + \frac{P}{\varepsilon}}, \quad (3)$$

$$\frac{dm}{dP} = 4\pi r^2 \frac{\varepsilon}{c^2} \frac{dr}{dP}, \quad (4)$$

$$\frac{d\nu}{dP} = -\frac{2}{\varepsilon + P}, \quad (5)$$

$$\frac{dN}{dP} = 4\pi r^2 \frac{n}{(1 - \frac{2Gm}{rc^2})^{1/2}} \frac{dr}{dP}, \quad (6)$$

with conditions in the center of the star $r = 0, P = P_c, m = 0, N = 0, T = T_c$. The distribution of temperature is determined by the relation

$$T(r) = T_c \sqrt{g_{00}(0)/g_{00}(r)} = T_c \exp[(\nu_c - \nu(r))/2]. \quad (7)$$

The difference $\nu_c - \nu(r)$ does not depend on the central value of ν_c , since equation (5) is linear. Therefore, in order to determine the temperature distribution it is enough to take for example, $\nu_c = 1$. Although this value does not correspond to physical reality.

While determining the EQS of quark matter using the MIT bag model, the u and d quarks were considered massless, for the s quark mass m_s , the bag constant B and the quark-gluon interaction constant α_c , the values $m_n = 95$ MeV, $B = 80$ MeV, $\alpha_c = 0$ are defined. The surface where $P = 0$ is considered the outer boundary of the PQS. Of course the newborn PQS will be surrounded by normal stellar matter. However, the pressure of this substance cannot significantly change the density of the quark substance on the PQS surface. Therefore the choice of the surface PQS on which $P = 0$ is justified. The radius R , the gravitational mass M , the baryonic mass M_b and the baryonic charge N_b of the PQS are determined by the ratios

$$R = r(P = 0), \quad M = m(P = 0), \quad M_b = m_n N_b, \quad N_b = N(P = 0),$$

where m_n is the mass of neutron.

In our calculations, the PQS is assumed to be isothermal, i.e. $T\sqrt{(g_{00})} = \text{const}$ in the star. If the energy losses are insignificant during the entire compression of the central core of the pre-supernova star before the formation of a proto-quark star, then the difference between the baryonic mass M_b and the gravitational mass M of the collapsing core will remain almost unchanged. At the moment of loss of stability of the central core of the pre-supernova there is: $(M_b - M)/(M_b \sim GM/(Rc^2 \approx 10^{-4} \div 10^{-3})$. The change in gravitational energy during the implosion is spent on changing the composite composition and heating the matter. Moreover, the maximum heating of the matter will be at full energy conservation. The maximum temperature PQS is determined from the condition

of equality of the masses of the central core of the pre-supernova star and the newborn PQS at the same baryon masses. If in the future the matter ejected by the SN explosion does not settle on the PQS then all the emitted energy (mainly in the form of neutrinos) will be equivalent to the mass difference between the initial and final states.

4. Results

The results of numerical calculations for the two values of the baryonic mass $M_b/M_\odot = \{1.22; 1.62\}$ and initial relative lepton charge $L = \{1/2; 26/56; 0.4\}$ (the ratio of the lepton and baryonic charges of quark matter) are given in Figure 1 and Table 1.

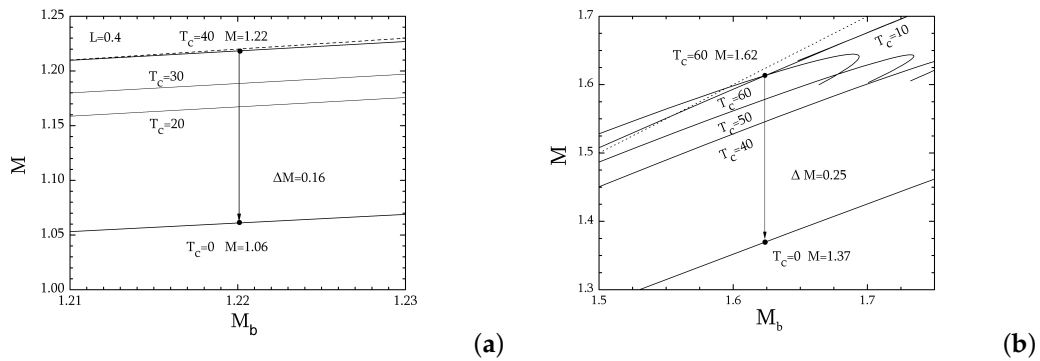


Figure 1. The dependence of the mass of the quark star M on the baryonic mass M_b for different central temperatures T_c for regions (a) $1.21 \leq M_b \leq 1.23$ and (b) $1.5 \leq M_b \leq 1.75$. The relative lepton charge L is taken as 0.4: less than 26/56 (the case of iron) in order to somehow compensate the partial escape of neutrinos before the formation PQS. This dependence for cool quark stars is also shown. The masses are given in units of the mass of the sun, and the temperatures are given in MeV. The circles mark the positions of stars with masses of 1.22 and 1.62. All curves except $T_c = 10$ MeV correspond to the uds quark stars. The dotted line corresponds to the line $M = M_b$.

Table 1. Parameters of the proto-quark star. M_b is the baryonic mass, M is the gravitational mass, L is the initial relative lepton charge, T_c , ρ_c , and n_c are the temperature, the density and concentration of the baryonic charge in the center of the star correspondingly, $n_0 = 0.15 \text{ fm}^{-3}$ is the concentration of nucleons in the atomic nucleus, R -radius, ΔM -decrease in the mass of the proto-quark star with complete cooling, E_ν is the energy of neutrino gas, ε_ν -average neutrino energy. The table also lists the parameters of this star in the cold state.

$\frac{M_b}{M_\odot}$	L	$\frac{M}{M_\odot}$	$\frac{T_c}{10^{11}}, K(MeV)$	$\frac{n_c}{n_0}$	$\frac{\rho_c}{10^{15}}, g/cm^3$	R, km	$\frac{\Delta M}{M_\odot}$	$\frac{E_\nu}{Mc^2}$	ε_ν, MeV
1.22	0.5	1.22	3.48 (30)	3.64	1.10	9.15	0.16	5	260
1.22	26/56	1.22	3.94 (34)	3.64	1.10	9.15	0.16	4.5	257
1.22	0.4	1.22	4.64 (40)	3.65	1.10	9.15	0.16	3.7	250
1.22	-	1.06	0	3.89	0.99	8.83	-	-	-
1.62	0.5	1.62	6.15 (53)	6.06	2.12	9.29	0.25	5.2	293
1.62	26/56	1.62	6.38 (55)	5.99	2.08	9.31	0.25	4.7	288
1.62	0.4	1.62	6.69 (60)	5.95	2.06	9.31	0.25	3.9	280
1.62	-	1.37	0	4.9	1.30	9.30	-	-	-

In case of implosion of the central core of the pre-supernova star its baryonic matter can pass into the ud quark state due to the change in gravitational energy. If this energy is high enough, then all the central core matter would become the ud quark matter and the ud PQS is formed. This is how a PQS with baryonic mass of $1.62 M_\odot$ is born. Its temperature is $T_c = 10$ MeV. During the transition of the star to the uds state its temperature rises to $T_c = 60$ MeV. This heating is similar to the heat release

process proposed in Ref.[27]. While cooling down, this star loses its mass and becomes a cold quark star with a mass of $1.37 M_{\odot}$. All this is clearly seen in Figure 1b.

The proto-quark star with a mass of $1.22 M_{\odot}$ at the initial moment of its life only partially consists of ud quark matter. The change in gravitational energy turns out to be insufficient for the destruction of all the baryons of the star in ud quarks. This happens only in the central region of the star. After the transition of this region into the uds state, it will become a seed center for the transition of the entire star into the uds quark state. While cooling down, this star loses mass and becomes a cold quark star with a mass of $1.06 M_{\odot}$. All this is clearly seen in Figure 1a.

If during the implosion of the central core of a pre-supernova star the destruction of baryons into ud quarks does not occur, then a proto-neutron star will be formed. The high energy barrier between the baryonic and strange uds quark matters [28] will prevent the transformation of the proto-neutron star into a PQS. The final state of the central regions of the pre-supernova will primarily depend on its mass. This question is a topic for a separate study.

The gravitational and baryonic masses of the newborn PQS differ in the third or fourth digits, so they are just equal in the table. According to the above, for each PQS these T_c are the maximum possible values for their central temperatures. In this state the portion of the energy of the neutrino component, depending on the baryonic mass and on the lepton charge, is $4 \div 5$ percent of the total energy Mc^2 , and the average energy of the neutrino is $250 \div 290$ MeV. The mass of the PQS decreases with the leaving of neutrinos. The change in mass after complete cooling of the star is 16 and 25 percent for $M_b = 1.22 M_{\odot}$ and $M_b = 1.62 M_{\odot}$ respectively. This mainly happens due to neutrino radiation.

A similar thing happens with the proto-neutron star [29–31]. The energy loss of the PNS during cooling is of the same order. However, the energy of individual neutrinos is approximately 10 MeV [29–31]. In contrast to these neutrinos, the energy of neutrinos from PQS is an order higher. Therefore, due to an increase in the cross-section for the interaction of neutrinos with electrons and nucleons, the transfer of the energy of the neutrino flux from the PQS to the surrounding matter (deposition) will be more effective. This may provide the explosive nature of the ejection of the pre-supernova matter.

While cooling, the radius of PQS changes both due to a decrease in the pressure of the matter and due to a decrease in gravitational forces. While cooling the radius of PQS is changed both due to decrease of pressure of the matter and of gravitational forces. The PQS radius can either decrease or increase depending on the ratio of these changes. After complete cooling, the radius of PQS with $M_b = 1.22 M_{\odot}$ decreases by 3.5 percent, and the radius of PQS with $M_b = 1.62 M_{\odot}$ remains almost unchanged.

The central concentrations n_c of the baryon charge of the considered configurations change in different ways. After cooling, for PQS with masses $M_b = 1.22 M_{\odot}$ and $n_c = 3.64 n_0$, the concentration of the baryon charge at the center increases to a value of $n_c = 3.85 n_0$ ($n_0 = 0.15 \text{ fm}^{-3}$ is the concentration of nucleons in the atomic nucleus). For PQS with mass $M_b = 1.62 M_{\odot}$, in opposite, the central concentration of the baryon charge after cooling of the star decreases from $n_c \approx 6 n_0$ to $n_c \approx 5 n_0$, although the radii in both states are the same. At first glance, such a paradoxical result can be easily explained. On one hand, as the PQS with masses $M_b = 1.22 M_{\odot}$ cools down, the decrease in the gravitational and pressure forces of the stellar matter leads to the fact that the radii of PQS and the cold quark star remain almost unchanged. On the other hand, the concentration of the baryon charge HSQM in the surface layers of the star, where the pressure is near zero, should increase with cooling to compensate the decrease in thermal corrections to the pressure. While in PQS with $M_b = 1.22 M_{\odot}$ this is achieved by decreasing the radius, in PQS with $M_b = 1.62 M_{\odot}$ this is provided by the outflow of quarks from the inner regions to the outer layers.

5. Conclusions and Critical Remarks

For sure, during the implosion the central regions of the pre-supernova star lose both energy and lepton charge. Therefore, the numerical values of the temperature and energy of neutrinos PQS in

Table 1. are the maximum possible values of these parameters. However, the implosion occurs so quickly that these losses can be neglected in the first approximation.

Of course, the NJL model of quark matter describes the reality better than the simple MIT quark bag model. However, the simplicity of the MIT bag model allows to determine in a easy way the thermodynamic characteristics of the HSQM.

If the lepton charge and energy are conserved during the implosion of the central regions of a pre-supernova star, then a very hot proto-quark star enriched with leptons can be formed. The energy reserves of such a star can reach up to $0.25 Mc^2$, which will lead to the same decrease in its mass when it cools. Basically, this energy is carried away by neutrinos. The neutrino flux from PQS with energies of $250 \div 300$ MeV, possibly, can provide the transfer of the energy accumulated in the PQS to the surrounding non-degenerate matter. This is very important for explaining the nature of the explosive ejection of pre-supernova matter.

Since the values of the maximum masses M_{max} of hot and cold quark stars practically do not differ [26], therefore, the greatest value of the mass of the cooled PQS will be $\approx 0.75 M_{max}$. Only accretion can increase this value. If the cold quark stars are formed only after a supernova explosion, then their amount with masses of $\sim 0.75 \div 1 M_{max}$ will be significantly less in comparison with $M < 0.75 M_{max}$.

The energy losses of proto-neutron stars during the cooling are the same. Most probably the situation with proto-neutron stars is the same as for proto-quark stars. This can partially explain the relatively small number of pulsars with $M > 1.5 M_{\odot}$ if the value $M_{max} \approx 2M_{\odot}$ [32] is taken as maximum for the mass of cold neutron stars. Of course, this can be confirmed only by detailed calculations.

How much our results and conclusions correspond to reality will be confirmed only after solving the exact time problem. We hope that our work will stimulate such work.

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